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Low-speed motion of a ship leads to a singular perturbation problem as speed approaches zero, since waves occur only in a boundary layer of vanishing thickness at the free surface. The complete solution consists of the singular expansion superposed on a regular expansion. The latter (the "naive expansion") by itself satisfies all conditions in the lower half-space below the undisturbed free surface, but it does not represent the boundary layer. For the case of a two-dimensional surface-piercing body, it is shown that the

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20. Abstract (continued)

regular perturbation series fails to satisfy the body boundary condition in a small wetted region just above the level of the undisturbed free surface. This fact leads to a nonhomogeneous body boundary condition that must be satisfied by the singular expansion. Without such a condition, the singular part of the solution (which represents the real wave motion) would satisfy purely homogeneous conditions and thus would be indeterminate.

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WATER WAVES GENERATED BY A SLOWLY MOVING TWO-DIMENSIONAL BODY

Part I

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CONTENTS

PRINCIPAL NOMENCLATURE

[Note: Equation numbers are given below where it may help to identify the point of first introduction of a symbol.]

$C(\theta)$	Curvature of body surface
C	$C(0)$
$\tilde{F}(z)$	Complex potential corresponding to $\tilde{\Phi}(x,y)$ (24)
$f_j(z)$	Complex potential corresponding to $\phi_j(x,y)$ (15)
g	Gravitational constant
$g_j(z)$	Complex potential, a part of $f_j(z)$ (16)
$\tilde{g}(z)$	Complex potential, a part of $\tilde{F}(z)$ (25)
$H(x)$	Free-surface shape from exact solution (5)
$\overline{H}(x)$	Free-surface shape from naive expansion (9)
$\tilde{H}(x)$	Free-surface shape from wavelike solution $\tilde{\Phi}(x,y)$ (21) (31)
$h_j(z)$	Complex potential, a part of $f_j(z)$ (16)
$\tilde{h}(z)$	Complex potential, a part of $\tilde{F}(z)$ (25)
K	$d^3R/d\theta^3$, evaluated at $\theta = 0$ (37)
$k(z)$	$[f_0'(z)]^{-2}$ (27)
L	Typical body dimension
$p_n(x)$	Function defined on $y = 0$ (13)
$R(\theta)$	Function describing body shape (1)
$R_0(\theta)$	Function describing shape of double body (34)
r	Radius in cylindrical coordinates
U	Forward speed of body (or speed of stream past fixed body)
x,y	Cartesian coordinates (nondimensional)
z	$x + iy$
$\gamma_n(x,y)$	$Re\{g_n(z)\}$ (46)
ε	F^2/gL, the small parameter of the problem
$\eta_n(x)$	Coefficient of ε^n in the naive expansion of $H(x)$ (9)
$\Theta(z)$	Phase function, $-(1/\varepsilon)\int_{z_1}^{z} k(u)\,du$ (28)
θ	Angle coordinate in cylindrical coordinates

$\Phi(x,y)$	Velocity potential for the exact problem
$\overline{\Phi}(x,y)$	Velocity potential for the naive expansion (8)
$\tilde{\Phi}(x,y)$	Velocity potential for wavelike motion (20) (30)
$\phi_n(x,y)$	Coefficient of ε^n in naive expansion of $\overline{\Phi}(x,y)$ (8)
$\psi_n(x,y)$	$Re\{h_n(x,y)\}$ (47)

I. INTRODUCTION

The problem of ship-wave generation at very low speeds has fundamental importance, although it is not of much interest to naval architects. At very low speeds, there is little wave generation at all, and so low-speed wave resistance need not be considered in the design of a ship. However, the usual methods of predicting wave resistance at finite speed are not valid as speed approaches zero, that is, the mathematical solutions on which these methods are based are not uniformly valid at low speed. So there is a question how low the speed must be for the standard methods to break down.

There is another practical reason for investigating the low-speed problem: Inui and Kajitani (1977) found that the propagation of ship-generated waves can be predicted more accurately if the waves are considered to propagate on the nonuniform stream around a double body (the actual part of the body below the undisturbed free surface plus its mirror image) rather than on a uniform stream. A low-speed analysis leads naturally to such an approach. It may be that "low speed" to a mathematician may not be very low at all to a naval architect.

Over a period of decades, numerous authors have discussed the low-speed limit of solutions obtained by standard perturbation techniques. Such solutions are based on the assumption that all perturbation velocities are asymptotically small in magnitude compared to the forward speed U of the body (or, if the reference frame is fixed to the body, compared to the speed U of the streaming flow past the body). The implications of this assumption become unclear if then one lets $U \to 0$.

Salvesen (1969) showed for a particular case that this is a singular limit. Initially, he considered the forward speed as fixed. Then he expanded the problem statement in terms of a small-disturbance parameter, worked out the explicit solution to three terms, and finally examined the behavior of the expansion as $U \to 0$. At moderate speed, the second and third terms provided small corrections to the linear solution. Then, as U decreased, the second-order term became larger and larger until it dominated the first-order term. At still lower speed, the third-order term began to dominate the second-order term. It appeared that the problem was becoming more and more nonlinear as speed decreased more and more. However, from a physical point of view, it is clear that the fluid disturbance becomes smaller and smaller as speed decreases. This apparent paradox is a result of nonuniformity of the expansion as $U \to 0$.

Ogilvie (1968) proposed a method of solution that was expected to remove the nonuniformity. He reasoned as follows: At very low speed, the free surface is hardly disturbed at all, and so a first approximation should be equivalent to the double-body flow. In such a flow, if the fluid is ideal, the magnitude of the perturbation velocity is everywhere proportional to U . So one should not assume *a priori* that the perturbation velocities vanish much more rapidly than U in the limit. Rather, one should take the double-body flow as the first approximation. Then perhaps it is reasonable to assume that free-surface effects are small perturbations of this nonuniform base flow.

This was a new assumption, which Ogilvie implemented on a highly pragmatic basis: He argued that there was little chance of solving the wave problem unless it were a linear problem, and so he tried to determine under what conditions this would in fact be the case, and he obtained a partial solution for the problem of a fully submerged two-dimensional hydrofoil (Salvesen's problem).

Effectively, Ogilvie divided the flow field into two parts, the two being characterized by vastly different length scales. In the double-body flow, the fluid motion can be described on a scale comparable to body dimensions. The wave motion, however, is described appropriately on a scale comparable to wavelength. A suggestion of what this scale ought to be can be obtained from classical (finite-speed) wave theory: Waves on a stream of speed U typically have wavenumber $\kappa = g/U^2$ and the corresponding wavelength is $\lambda = 2\pi/\kappa = 2\pi U^2/g$, where g is the gravitational constant. It seems appropriate then to introduce a small parameter $\varepsilon = F^2 = U^2/gL$, where L is a typical body dimension and F is, of course, a Froude number. In the wave part of the solution, differentiation of a field variable has an order-of-magnitude effect equivalent to multiplication by the wavenumer, which is $O(\varepsilon^{-1})$. This fact was used systematically to order terms in the several conditions that constitute the statement of the wave problem.

This division of the problem into two distinct parts is based on a plausible argument, but several fundamental questions remain unanswered;

(1) One can develop a perturbation expansion of the solution without introducing the concept of the rapidly varying wave field. The expansion thus developed has been called the "naive expansion." It appears on first sight to satisfy the formal conditions of the problem. However, it cannot possibly represent a wave motion (with short waves), and so it is incomplete or possibly quite incorrect. How can one show formally that it is incomplete if it satisfies the partial differential equation and all of the boundary condition? How does one determine the relationship between it and a truly wavelike solution?

(2) Dagan (1972) follows much the same reasoning as Ogilvie, but he argues (persuasively) that the base flow should correspond to the two-term naive expansion.

(3) Keller (1979) developed a ray theory based on the same fundamental concept. Like Dagan, he used the two-term naive expansion for the base flow. However, he argued that Ogilvie's linear free-surface condition on the wave solution is defective: It contains terms on one side that are rapidly varying, but the terms on the other side of the equation are not rapidly varying, and so, as Keller pointed out, they cannot be equal (unless both sides equal zero, which is not an acceptable resolution of the difficulty). So Keller set up the wave problem in such a way that the free-surface condition would be homogeneous in the rapidly varying field quantities. This then leads to difficulties in predicting how waves are generated. The only mechanism (from a mathematical point of view) must exist at the body surface. However, a ray theory cannot be used to predict what happens near a body in the presence of the free surface, since the governing partial differential equation is elliptic, and the pseudo-hyperbolic character of the ray-theory solution is not valid there. The ray theory can be used only to predict how water waves will propagate once they have been generated.

(4) It is not clear whether Ogilvie's approach resolved the apparent paradox in Salvesen's perturbation expansion.

In this paper, we shall provide some answers to these questions, although the actual solution is presented is a subsequent paper, Chen and Ogilvie (1982). In particular, we shall show the following:

(1) The naive expansion in fact does not satisfy all of the boundary conditions of the problem.

(2) In showing that the boundary conditions are not completely satisfied by the naive expansion, we obtain a nonhomogeneous boundary condition that will have to be satisfied by the wavelike part of the solution.

(3) Asymptotically, as $\varepsilon \to 0$, Ogilvie's solution does not represent a wavelike motion. We now believe that, in a strict asymptotic sense, the wave resistance of a submerged body is small of exponential order in this limit. We have not been able to obtain a relationship between the expansion for $\varepsilon \to 0$ and Salvesen's expansion.

(4) The double-body flow is not analytic at the intersection of the body and the undisturbed free surface. The singularity is very weak, but subsequent terms in the naive expansion exhibit singularities there that are less weak.

Following Keller (1979), we conclude that the free-surface condition on the wave part of the solution must indeed be homogeneous. Wave generation is accomplished through the body boundary condition. Baba and Takekuma (1975) and Maruo and Fukazawa (1979) followed Ogilvie's approach, in which the waves arise from a nonhomogeneous free-surface condition; their results appear to be questionable.

Two more points should be noted: (i) Very short waves are strongly affected by surface tension, which we neglect. But we are interested ultimately in the problem of ship-generated waves, and even very "short" waves in that case should be long enough not to be subject to serious alterations by surface tension. (ii) In the submerged-body problem, our conclusions may not be valid if the submergence is very small (comparable to wavelength).

II. PROBLEM FORMULATION

We consider a slow streaming flow past a two-dimensional body. Let the body surface be given by an equation of the form

$$r = R(\theta) \,, \tag{1}$$

where $r = (x^2+y^2)^{1/2}$ and θ is the polar angle measured counterclockwise from the x axis. The stream has speed U in the positive x direction. The undisturbed free surface coincides with the x axis.

All lengths are nondimensionalized by a length L, which may be taken as any characteristic body dimension. We seek an asymptotic solution expressed in terms of the parameter

$$\varepsilon = F^2 = U^2/gL \,, \tag{2}$$

where F is the Froude number and g is theacceleration of gravity. The velocity potential is taken as $LU\Phi(x,y)$, and so Φ is also nondimensional. The shape of the free surface will be given as $y \doteq \varepsilon H(x)$.

The potential satisfies the Laplace equation,

[L] $$\Phi_{xx} + \Phi_{yy} = 0 \quad \text{in the fluid domain,} \tag{3}$$

the body boundary condition,

[B] $$\frac{\partial \Phi}{\partial n} = 0 \quad \text{on the body,} \tag{4}$$

the dynamic free-surface condition,

[H] $$H(x) = \frac{1}{2}\left\{1 - \Phi_x^2 - \Phi_y^2\right\}\Big|_{y=\varepsilon H(x)} \,, \tag{5}$$

the free-surface condition combining [H] with the kinematic condition:

[F] $$\Phi_y + \varepsilon\left\{\Phi_x^2\Phi_{xx} + 2\Phi_x\Phi_y\Phi_{xy} + \Phi_y^2\Phi_{yy}\right\} = 0 \quad \text{on} \quad y = \varepsilon H(x) \,, \tag{6}$$

and a radiation condition,

[R] $$|\Phi - x| \to 0 \quad \text{as} \quad x \to -\infty \quad \text{and/or} \quad y \to -\infty \,. \tag{7}$$

The *naive expansion* exhibits one kind of approach to solving such a problem. Assume that $\Phi(x,y)$ and $H(x)$ can be expanded in series in ε :

$$\Phi(x,y) = \overline{\Phi}(x,y;\varepsilon) \sim \sum_{n=0}^{N} \varepsilon^n \phi_n(x,y) ; \tag{8}$$

$$H(x) = \overline{H}(x;\varepsilon) \sim \sum_{n=0}^{N} \varepsilon^n \eta_n(x) . \tag{9}$$

When these expansions are simply substituted into the problem statement, one obtains the following set of problems:

[L] $$\phi_{n_{xx}} + \phi_{n_{yy}} = 0 \quad \text{outside of the body,} \quad y < 0 ; \tag{10}$$

[B] $$\frac{\partial \phi_n}{\partial n} = 0 \quad \text{on the body;} \tag{11}$$

[H] $$\eta_0(x) = \frac{1}{2}\{1 - \phi_{0_x}^2\} , \tag{12a}$$

$$\eta_1(x) = - \phi_{0_x}\phi_{1_x} , \tag{12b}$$

$$\eta_2(x) = - \phi_{0_x}\phi_{2_x} - \frac{1}{2}\{\phi_{1_x}^2 + \phi_{1_y}^2\} - \frac{1}{2}\{1 - \phi_{0_x}^2\}[\phi_{0_x}\phi_{1_{xy}} + \phi_{1_y}\phi_{0_{yy}}]$$
$$- \frac{1}{8}\{1 - \phi_{0_x}^2\}^2\{\phi_{0_x}\phi_{0_{xyy}} + \phi_{0_{yy}}^2\} , \tag{12c}$$

all to be satisfied on $y = 0$;

[F] $$\phi_{0_y} = 0 , \tag{13a}$$

$$\phi_{1_y} = \frac{1}{2}\phi_{0_{xx}}\{1 - 3\phi_{0_x}^2\} = \frac{\partial}{\partial x}(\eta_0\phi_{0_x}) \equiv p_1'(x) , \tag{13b}$$

$$\phi_{2_y} = \frac{1}{2}\phi_{1_{xx}}\{1 - 3\phi_{0_x}^2\} - 3\phi_{0_x}\phi_{1_x}\phi_{0_{xx}}$$
$$= \frac{\partial}{\partial x}(\eta_0\phi_{1_x} + \eta_1\phi_{0_x}) \equiv p_2'(x) , \tag{13c}$$

also to be satisfied on $y = 0$;

[R] $$\left.\begin{aligned} |\phi_0 - x| &\to 0 , \\ |\phi_n| &\to 0 , \; n \geq 1 , \end{aligned}\right\} \quad \begin{aligned} &x \to -\infty \\ &\text{and/or} \\ &y \to -\infty . \end{aligned} \tag{14a, 14b}$$

Of course, (12) and (13) can be continued for larger values of n .

The first term in the expansion, $\phi_0(x,y)$, is the solution of the rigid-wall problem, in which the free surface is replaced by a wall. We sometimes refer to this problem also as the "double-body" problem. Notwithstanding the interpretation in terms of a hypothetical rigid wall at $y = 0$, there is a corresponding free-surface disturbance, given by (12a). However, it is not a wavelike disturbance. In fact, no matter how far this method of solution is pursued, there will never be any real waves.

Clearly, this is not the correct solution, since we expect to find waves behind the body. Nevertheless, the expansion (8) will have an important role later, and so we note that, in principle, it can be solved completely. The ϕ_0 problem can be solved by standard numerical methods. To proceed further, let

$$\phi_j(x,y) = Re\{f_j(z)\} , \tag{15}$$

where $f_j(z)$ is a function of the complex variable $z = x + iy$. Also, set

$$f_j(z) = g_j(z) + h_j(z) , \quad j > 0 , \tag{16}$$

where

$$h_j(z) = \frac{1}{\pi} \int_F \frac{ds\ p_j(s)}{s - z} , \tag{17}$$

and F is the part of the x axis outside of the body. Then we have

$$h_j'(z) = \frac{1}{\pi} \int_F \frac{ds\ p_j'(s)}{s - z} \underset{y \uparrow 0}{\rightarrow} - ip_j'(x) + \frac{1}{\pi} ⨍_F \frac{ds\ p_j'(s)}{s - x} , \tag{18}$$

the last integral being a principal-value integral. When these results are substituted into (13b) and (13c), we find that

$$Im\{g_j'(x-i0)\} = 0 . \tag{19}$$

Thus $g_j(z)$ satisfies conditions just like $f_0(z)$, the body boundary condition being written so that the combination $g_j + h_j$ satisfies (11). The numerical method used to solve for $f_0(z)$ can be used for all of the $g_j(z)$.

An alternative method of obtaining $h_j(z)$ will be presented later.

The expansions (8) and (9) are not complete. It will be necessary to augment them with terms that represent real waves and to obtain appropriate conditions to be satisfied. An attempt to do this was presented by Ogilvie (1968), and several investigators have followed his approach. Unfortunately, this does not lead to the desired results. It is worthwhile to consider why, which we do in the next section.

III. THE ANALYSIS OF OGILVIE (1968)

Ogilvie assumed that the solution consisted of (i) the rigid-wall solution and (ii) a wave term:

$$\Phi(x,y) = \phi_0(x,y) + \varepsilon^2\tilde{\Phi}(x,y;\varepsilon) , \tag{20}$$

$$H(x) = \eta_0(x) + \varepsilon\tilde{H}(x;\varepsilon) . \tag{21}$$

The new terms were assumed to have the property that

$$\tilde{\Phi}_x , \ \tilde{\Phi}_y = O(\tilde{\Phi}/\varepsilon) ; \ \tilde{H}' = O(\tilde{H}/\varepsilon) . \tag{22}$$

This assumption corresponds to the expectation that the waves will be very short, with wavelength that is $O(\varepsilon)$ (and thus wavenumber that is $O(\varepsilon^{-1})$).

When (20) and (21) are substituted into the free-surface conditions, the free-surface condition is found to be:

$$\varepsilon^2\tilde{\Phi}_y + \varepsilon^3\phi_{0x}^2\tilde{\Phi}_{xx} = \varepsilon\frac{\partial}{\partial x}(\eta_0\phi_{0x}) = \varepsilon p_1'(x) . \tag{23}$$

In this condition, ϕ_0 is to be evaluated on $y = 0$ and $\tilde{\Phi}$ on $y = \varepsilon\eta_0(x)$. In principle, it is wrong to evaluate $\tilde{\Phi}$ on $y = 0$, but Ogilvie (and others following him) have devised arguments indicating that perhaps this can be done anyway.

For the case of a submerged body (the only case considered by Ogilvie), one can construct a partial solution, as shown by Ogilvie. Let

$$\tilde{\Phi}(x,y;\varepsilon) = Re\{\tilde{F}(z)\} , \tag{24}$$

$$\tilde{F}(z) = \tilde{g}(z) + \tilde{h}(z) , \tag{25}$$

with $\tilde{h}(z)$ given by:

$$\tilde{h}'(z) = -\frac{1}{\pi i\varepsilon}\int_{-\infty}^{\infty} ds \ p_1'(s) \int_{-\infty}^{z}\frac{d\zeta \ k(\zeta)}{s-\zeta} e^{i[\Theta(z)-\Theta(\zeta)]} , \tag{26}$$

and

$$k(z) = [f_0'(z)]^{-2} , \tag{27}$$

$$\Theta(z) = -\frac{1}{\varepsilon}\int_{z_1}^{z} k(u) \ du . \tag{28}$$

The value of z_1 is arbitrary, since it vanishes when the difference, $[\Theta(z) - \Theta(\zeta)]$ is introduced. When $\tilde{h}'(z)$ is used in (23), it is found that it satisfies the nonhomogeneous condition, leaving only a homogeneous free-surface condition to be satisfied by $\tilde{g}(z)$. Of course, $\tilde{h}(z)$ does not satisfy the body boundary condition in general, and $\tilde{g}(z)$ must be determined so that the complete solution does satisfy that condition.

The partial solution $\tilde{h}(z)$ was constructed in much the same way that one finds solutions of problems with pressure distributions imposed on the free surface (see Wehausen and Laitone (1960)). So one might assume that it represents a true wave motion that decays rapidly with depth. If that were true, the term $\tilde{g}(z)$ would not be really necessary, since $\tilde{h}(z)$ represents extremely short waves (if it represents waves at all). In such a case, we would have found the solution.

In fact, if one were to compute the solution represented by $\tilde{h}(z)$, it would undoubtedly exhibit a wavelike behavior. But this is not true in an asymptotic sense, as we now show. First, we note that $\tilde{h}'(z)$ appears superficially to be $O(\varepsilon^{-1})$ (see (26)). Now integrate by parts, first with respect to ζ and then with respect to s . The result is the following:

$$\tilde{h}'(z) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{ds\; p_1'(s)}{s-z} - \frac{e^{i\Theta(z)}}{\pi}\int_{-\infty}^{z} d\zeta\; e^{-i\Theta(\zeta)}\int_{-\infty}^{\infty}\frac{ds\; p_1''(s)}{s-\zeta} \quad .$$

The first term is clearly $O(1)$ and it does not represent a wave motion. The second term appears to be of the same order of magnitude, but another integration by parts (with respect to both ζ and s) leads to the result:

$$\tilde{h}'(z) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{ds\; p_1'(s)}{s-z} + \frac{i\varepsilon}{\pi k(z)}\int_{-\infty}^{\infty}\frac{ds\; p_1''(s)}{s-z} - \frac{i\varepsilon e^{i\Theta(z)}}{\pi}\int_{-\infty}^{z} d\zeta\; e^{-i\Theta(\zeta)} \cdot\int_{-\infty}^{\infty}\frac{ds}{s-\zeta}\left(\frac{p_1'''(s)}{k(\zeta)} - \frac{p_1''(s)k'(\zeta)}{k^2(\zeta)}\right) \quad . \tag{29}$$

So the second term, also not wavelike, is $O(\varepsilon)$. The process can clearly be continued indefinitely, and we see that we obtain a series in increasing powers of ε , none of the terms representing a wave motion.

The solution in (26) corresponds closely to the naive expansion. The first term in (29) is identical to $h_1'(z)$ in (18). The following terms in (29) could be obtained from a naive expansion of the linear problem stated in (23).

Thus Ogilvie's (1968) approach does not solve the problem of determining the wave motion at very low speed. The results of evaluating (26) numerically are not known, and they just might give reasonable predictions in the actual physical situation, although they would be inconsistent from a perturbation-theory point of view. It also appears that reasonable results might be obtained if the body were surface-piercing, for then the integrations by parts would not yield the trivial result found in (29). But that is not really the direction in which to look for a valid approach.

A fundamental error in the above analysis appears to have been made in the assumption in (20) and (21). One should write:

$$\Phi(x,y) = \bar{\Phi}(x,y;\varepsilon) + \varepsilon^2\tilde{\Phi}(x,y;\varepsilon) , \tag{30}$$

$$H(x) = \bar{H}(x;\varepsilon) + \varepsilon\tilde{H}(x;\varepsilon) . \tag{31}$$

That is, the wave term must be added to the complete naive expansion (or to at least two terms in it). When (30) and (31) are substituted into the free-surface conditions, one obtains:

$$\varepsilon^2\tilde{\Phi}_y + \varepsilon^3\bar{\Phi}_x^2\tilde{\Phi}_{xx} = 0 . \tag{32}$$

That is, the free-surface condition is homogeneous. The right-hand side of (23) has been absorbed into the problem for ϕ_1 in the naive expansion (see (13b)). Keller (1979) argued this for different reasons. It may also be noted that Dagan (1972) insisted that the coefficient in (32) (or (23)) should be $\bar{\Phi}_x^2$ and not $\phi_{0_x}^2$, again for different reasons. We agree with Keller and with Dagan.

The implication of (32) is that there is no "equivalent pressure distribution" on the free surface causing the creation of waves. In the case of the submerged body, there appears to be no mechanism that can lead to the creation of waves — a conclusion that is compatible with the result expressed in (29). For the case of a surface-piercing body, we may expect to find a nonhomogeneous condition in the neighborhood of the intersection of the body and the free surface. This we shall show explicitly in the next section.

IV. A NONHOMOGENEOUS CONDITION FOR THE WAVE POTENTIAL

As already noted, it appears that the "naive" expansion, (8) and (9), satisfies the boundary conditions of the problem, leaving a homogeneous problem for the wave potential. This is an unsatisfactory situation, since it is not clear then how the waves ever come into existence. We need a nonhomogeneous boundary condition to impose on the wave potential.

A closer examination of the naive expansion will show that it does not completely satisfy the body boundary condition in the case of a surface-piercing body. It is the purpose of this section to show this and to derive the nonhomogeneity in the wave problem.

The clue to accomplishing this task can be found in (13a): If we find a function ϕ_0 that satisfies the Laplace equation in the lower half-space outside of the body, that function can be continued analytically into the upper half-space because of its property in (13a). We suppose that this function satisfies the body boundary condition in $y < 0$. There is no guarantee that this function also satisfies the body boundary condition in $y > 0$. And it must be noted that part of the wetted surface of the body does in fact lie in the upper half-space, since, from (5), $H(x) = 1/2$ (and thus $y = \varepsilon/2$) at a stagnation point. Similar conclusions can be drawn from (13b), (13c), etc., although the situation is not so obvious.

A. THE ϕ_0 PROBLEM

From (13a), we can write:

$$\phi_0(x,y) = \phi_0(x,-y) . \tag{33}$$

We assume that ϕ_0 satisfies the Laplace equation in the lower half-space (outside of the body) and that it satisfies the body boundary condition there, (11). Our purpose now is to compute $\partial\phi_0/\partial n$ on that part of the body for which $y > 0$.

We assume for now that the function defining the body, $R(\theta)$ (see (1)), has at least three continuous derivatives in a range $|\theta| \leq \varepsilon/2$. The most interesting case is that $d^3R/d\theta^3 \neq 0$ at $\theta = 0$. We assume that the body surface is vertical at $\theta = 0$, which implies that $dR/d\theta = 0$ at $\theta = 0$.

As already noted, ϕ_0 represents the streaming flow around the double body. In $-\pi \leq \theta \leq 0$, (1) describes both the actual body and the double body. We introduce the following description of the double body, applicable in both $\theta < 0$ and $\theta > 0$:

$$r = R_0(\theta) = \begin{cases} R(\theta) & -\pi < \theta < 0 , \\ R(-\theta) , & 0 < \theta < \pi . \end{cases} \tag{34}$$

In a neighborhood of $\theta = 0$, we can expand $R(\theta)$ and $R_0(\theta)$:

$$R(\theta) = R(0) + \frac{1}{2!}\theta^2 R''(0) + \frac{1}{3!}\theta^3 K + \ldots , \tag{35}$$

$$R_0(\theta) = R(0) + \frac{1}{2!}\theta^2 R''(0) - \frac{1}{3!}|\theta|^3 K + \ldots , \tag{36}$$

where

$$K = \left.\frac{d^3R}{d\theta^3}\right|_{\theta=0} . \tag{37}$$

We now compute $\partial\phi_0/\partial n$ on $r = R(\theta)$. This quantity vanishes for $\theta \leq 0$, but not otherwise. For the moment, it is convenient to write $\phi_0 = \phi_0(r,\theta)$. In polar coordinates, we can write the normal derivative:*

$$\frac{\partial\phi_0}{\partial n} = \frac{-R(\theta)\phi_{0_r} + [R'(\theta)/R(\theta)]\phi_{0\theta}}{\{[R(\theta)]^2 + [R'(\theta)]^2\}^{1/2}} . \tag{38}$$

We now evaluate $\phi_{0_r}(R(\theta),\theta)$ in terms of values on $(R_0(\theta),\theta)$. For example,

$$\phi_{0_r}(R(\theta),\theta) = \phi_{0_r}(R_0(\theta),\theta) + [R(\theta) - R_0(\theta)]\phi_{0_{rr}}(R_0(\theta),\theta) + \ldots . \tag{39}$$

From (35) and (36), we have

$$R(\theta) - R_0(\theta) = \frac{1}{3}K\theta^3 + \ldots , \quad \theta > 0 . \tag{40}$$

Carrying out details in this way, we find that

$$\left.\frac{\partial\phi_0}{\partial n}\right|_{r=R(\theta)} = \frac{K}{[R_0(\theta)]^2}\left(\theta^2\phi_{0_\theta}(R_0(\theta),\theta) + \frac{1}{3}\theta^3\phi_{0\theta\theta}\right) + O(\theta^4) . \tag{41}$$

We note also that

*We take the unit normal outward from the fluid.

$$R_0(\theta) = R_0(0) + O(\theta^2) \quad ,$$

$$\phi_{0_\theta}(R_0(\theta),\theta) = \phi_{0_\theta}(R_0(0),0) + \theta\{R_0'(0)\phi_{0_{r\theta}} + \phi_{0_{\theta\theta}}\} + O(\theta^3)$$

$$= \theta\phi_{0_{\theta\theta}}(R_0(0),0) + O(\theta^3) \quad ,$$

the last result following from the facts that $\phi_{0_\theta}(R_0(0),0) = 0$ and $R_0'(0) = 0$. Thus (41) simplifies:

$$\left.\frac{\partial\phi_0}{\partial n}\right|_{r=R(\theta)} = \frac{4K}{3[R_0(0)]^2}\,\theta^3\phi_{0_{\theta\theta}}(R_0(0),0) + O(\theta^4) = O(\theta^3) \quad . \tag{42}$$

This quantity will have to be canceled by a contribution from the wave potential.

If it happens that $K = 0$, the leading-order term in (42) will be $O(\varepsilon^5)$, provided that $d^5R/d\theta^5 \neq 0$ at $\theta = 0$.

In a formal sense, one can claim that ϕ_0 does indeed satisfy the body boundary condition, even for $y > 0$, since ϕ_0 is simply the first term in an asymptotic solution, and the body boundary condition requires only that $\partial\phi_0/\partial n = o(1)$ on $r = R(\theta)$. This is a legitimate point of view, of course. If we follow it, we would then expect the quantity in (42) to show up in the statement of a higher-order problem, and we might suppose that we could then cancel it with an appropriate contribution from the higher-order solution. However, it does not work out this way. At each stage of the solution of this problem, we shall find another quantity more or less like that in (42), always in violation of the body boundary condition for $0 < y < \varepsilon/2$.

Another special case is possibly of comparable interest: Let the shape in $y < 0$ be described as above, but assume that the body is continued above $y = 0$ by vertical walls. Then we can write for the body geometry:

$$r = R(\theta) = \begin{cases} R_0(\theta) & , \quad \theta < 0 \,, \\ R_0(0)\sec\theta & , \quad \theta > 0 \,. \end{cases} \tag{43}$$

In this case, the body curvature is not generally continuous at $\theta = 0$. From the general formula for curvature,

$$C(\theta) = \frac{[R(\theta)]^2 - R(\theta)R''(\theta) - 2[R'(\theta)]^2}{\{[R(\theta)]^2 + [R''(\theta)]^2\}^{3/2}} \,,$$

we have for $\theta \uparrow 0$:

$$C(-0) = \frac{R(0) - R''(-0)}{R^2(0)} , \tag{44}$$

whereas for $\theta \downarrow 0$ we have:

$$C(+0) = 0 .$$

If these happen to be equal, the preceding analysis covers the case. So let us now assume that the limiting curvature from below is not zero. The analysis is still similar to the preceding, and we find that, for $\theta > 0$,

$$\left.\frac{\partial \phi_0}{\partial n}\right|_{r=R(\theta)} = C(-0)\,\theta^2 \left\{\frac{1}{2}[R(0)]^2 \phi_{0_{rr}}(R_0(0),0) - \phi_{0_{\theta\theta}}\right\} + O(\theta^3) . \tag{45}$$

So the error in satisfying the body boundary condition above $y = 0$ is $O(\varepsilon^2)$ in this case.

In deriving some of the above results, we implied that $\phi_0(r,\theta)$ can be expanded in Taylor series. This is not really true at the stagnation points, where in fact this potential is not analytic unless the function $R_0(\theta)$ is also analytic. However, the singularity is extremely weak, and the expansions used above are valid as far as they have been carried. This point is discussed further in the Appendix.

B. THE ϕ_1 PROBLEM

In the ϕ_1 problem, there is an error in satisfying the body boundary condition just like the error described above in the ϕ_0 problem. Since we must multiply ϕ_1 by ε in the expansion of $\overline{\Phi}(x,y;\varepsilon)$, this error is $O(\varepsilon^4)$ (for the case of continuous curvature at $\theta = 0$).

There is another error in the ϕ_1 problem. Because of the nonhomogeneous free-surface condition on ϕ_1 , (13b), we find an error of lower order of magnitude than above when we try to satisfy the body boundary condition above $y = 0$. This error is not negligible even if $d^3R/d\theta^3 = 0$ at $\theta = 0$, and so we assume that this derivative does vanish there. However, we assume that the curvature at $\theta = 0$ does not vanish, that is, $C(0) \neq 0$.

We again write $\phi_1(x,y)$ in terms of a function of a complex variable, as in (15) and (16):

$$\phi_1(x,y) = Re\{f_1(z)\} = Re\{g_1(z)\} + Re\{h_1(z)\} .$$

It is also convenient to introduce the notation:

$$\gamma_1(x,y) = Re\{g_1(z)\} , \quad (46)$$

$$\psi_1(x,y) = Re\{h_1(z)\} . \quad (47)$$

Instead of introducing an expression like (17) for $h_1(z)$, we now set

$$h_1(z) = -\frac{i}{2} f_0'(z)\{1-[f_0'(z)]^2\} . \quad (48)$$

Then we observe that

$$h_1'(z) = \psi_{1x} - i\psi_{1y} = -\frac{i}{2} f_0''(z)\{1-3[f_0'(z)]^2\} . \quad (49)$$

Breaking the last expression into real and imaginary parts and using properties of $f_0(z)$ already established, we find that

$$Re\{h_1'(x)\} = \psi_{1x}(x,0) = 0 ; \quad (50a)$$

$$-Im\{h_1'(x)\} = \psi_{1y}(x,0) = \frac{1}{2}\phi_{0xx}\{1-3\phi_{0x}^2\}\Big|_{y=0} = \phi_{1y}(x,0) . \quad (50b)$$

The last result shows why we introduced the expression in (48): It takes care of the nonhomogeneous part of the free-surface condition, just as the expression in (17) did. The new form of $h_1(z)$ also has the useful property given in (50a). (If the form in (17) has this property, it is certainly not obvious.) The other part of the complex potential satisfies a homogeneous free-surface condition:

$$-Im\{g_1'(x)\} = \gamma_{1y}(x,0) = 0 . \quad (51)$$

The function $\gamma_1(x,y)$ must be determined so that (51) is satisfied and so that the sum $\gamma_1(x,y)+\psi_1(x,y)$ satisfies the body boundary condition on $\phi_1(x,y)$.

The contrast between (50a) and (51) is crucial to what follows: $\psi_1(x,y)$ is an odd function of y (except possibly for an additive constant), whereas $\gamma_1(x,y)$ is even with respect to y . It is this fact that leads to the major error in the satisfying of the body boundary condition, as we now demonstrate. We should determine $\gamma_1(x,y)$ so that it satisfies (51) and also so that

$$\frac{\partial \gamma_1}{\partial n} = - \frac{\partial \psi_1}{\partial n} \quad \text{for} \quad -\pi < \theta < 0 \quad . \tag{52}$$

Just as with ϕ_0 , we cannot prescribe γ_1 further, since the determination of γ_1 in $y < 0$ completely determines γ_1 in $y > 0$. From (50a) and (51), it then follows directly that

$$\frac{\partial \gamma_1}{\partial n} = + \frac{\partial \psi_1}{\partial n} \quad \text{for} \quad 0 < \theta < \pi \quad . \tag{53}$$

So γ_1 does not cancel the effect of ψ_1 in this region. In fact, it precisely doubles it. Thus we find that

$$\frac{\partial \phi_1}{\partial n} = 2 \frac{\partial \gamma_1}{\partial n} = 2 \frac{\partial \psi_1}{\partial n} \quad \text{for} \quad 0 < \theta \quad . \tag{54}$$

Since $\psi_1(x,y)$ has been completely specified already, we can conveniently use it for the calculation of $\partial\phi_1/\partial n$ on the body above the line $y = 0$. This normal velocity component on the body surface will ultimately have to be canceled by the wave-motion solution.

The expression for the normal velocity component on $r = R(\theta)$, given previously in the form in (38), can be rewritten:

$$\left.\frac{\partial \psi_1}{\partial n}\right|_{r=R(\theta)} = - \mathcal{Re}\left\{e^{i\theta} h_1'(z) \frac{R(\theta) - iR'(\theta)}{\sqrt{R^2 + R'^2}}\right\} \quad . \tag{55}$$

We need to evaluate the right-hand side just for small θ (or y). A few subsidiary results are useful:

$$h_1'(z) \simeq - \frac{i}{2} \{\phi_{0_{xx}}(x_0,0) + iy\phi_{0_{xxx}}(x_0,0)\} \quad , \quad \text{with} \quad x_0 = R(0) \; ; \tag{56}$$

$$e^{i\theta} \simeq 1 + i\theta \simeq 1 + \frac{iy}{x_0} \; ; \tag{57}$$

$$R(\theta) - iR'(\theta) \simeq x_0 - i\theta R''(0) \simeq x_0 - \frac{iyR''(0)}{x_0} \; ; \tag{58}$$

$$[R^2 + R'^2]^{-1/2} \simeq \frac{1}{x_0} \quad . \tag{59}$$

Substituting all of these back into (55), we find that

$$\left.\frac{\partial \psi_1}{\partial n}\right|_{r=R(\theta)} \simeq - \frac{1}{2} y \{\phi_{0_{xxx}}(x_0,0) + C(0)\phi_{0_{xx}}(x_0,0)\} \quad , \tag{60}$$

One further simplification can be made. From the boundary condition on ϕ_0 (see the Appendix), we have

$$\phi_{0_{xxx}}(x_0,0) = -3C(0)\phi_{0_{xx}}(x_0,0) \ . \tag{61}$$

Using this in (60) and then finally returning to (54), we have our desired result:

$$\left.\frac{\partial\phi_1}{\partial n}\right|_{r=R(\theta)} \simeq 2\,y\,C(0)\ \phi_{0_{xx}}(x_0,0) \quad \text{for} \quad 0 < \theta \ . \tag{62}$$

Since the upper limit of the wetted body surface is at $y = \varepsilon/2$, we see that the quantity in (62) is $O(\varepsilon)$. However, it must also be multiplied by ε in the asymptotic expansion of the full solution, and so the error in satisfying the body boundary condition is $O(\varepsilon^2)$.

The potential $\phi_1(x,y)$, like $\phi_0(x,y)$, is not analytic at $(x_0,0)$. It is interesting to compare their behavior in a neighborhood of this point. If we set $\zeta = (x-x_0) + iy$, we find from the Appendix that $f_0(z)$ has a term that is proportional to $\zeta^4 \log \zeta$. On the other hand, $f_1(z)$ will contain whatever singularities are contained in both $g_1(z)$ and $h_1(z)$. Using the result of the Appendix in (48), we find that the leading singular term in $h_1(z)$ is proportional to $\zeta^3 \log \zeta$, which is still a very weak singularity. However, $g_1(z)$ must have a term of the form $\zeta^2 \log \zeta$. To show this, we note that (60) is valid for y positive or negative. This same expression gives $\partial\gamma_1/\partial n$ for $\theta > 0$ but it gives the negative of $\partial\gamma_1/\partial n$ for $\theta < 0$. Therefore $\partial\gamma_1/\partial n$ must be proportional to $|y|$. Such a property requires that $g_1(z)$ have the form stated.

V. CONCLUSIONS

There are two principal conclusions:

(1) The solution of the complete problem should be broken into two terms, as in (30) and (31), with the first representing the so-called naive expansion. This procedure leads to a homogeneous free-surface condition on the wave part of the solution.

(2) The naive expansion fails to satisfy the body boundary condition on the small segment between $y = 0$, the level of the undisturbed free surface, and the actual location of the stagnation point, which is located at $y = \varepsilon/2$. The normal component of velocity on the body, resulting from this fact, is given approximately by (62). The wave part of the solution has to cancel this velocity component.

The wave solution is derived by Chen and Ogilvie (1982). It may be noted that the form of their solution differs in one respect from that supposed in (62): The power of ε in the second term of the solution is initially taken as unknown, a quantity to be determined during the solution process. In fact, the wave term turns out to be higher order than stated in (62). Its order of magnitude is completely specified eventually by the requirement that there be a stagnation point on the body at $y = \varepsilon/2$.

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APPENDIX. SINGULARITIES IN THE NAIVE EXPANSION

In the first term of the naive expansion, $\phi_0(x,y)$, there is a stagnation point at $x = x_0 = R(0)$. This is to be expected, of course, in view of the interpretation of ϕ_0 in terms of the double-body (rigid-wall) flow. It is indicated explicitly by the combination of the conditions (11) and (13a). A stagnation-point flow is so elementary and so well-behaved that one can easily overlook the fact that it may well be singular (in a mathematical sense). We show this explicitly here.

We rewrite the body boundary condition on ϕ_0 , (13a), in a form similar to (55):

$$\left.\frac{\partial\phi_0}{\partial n}\right|_{r=R_0(\theta)} = -\,Re\left(e^{i\theta}\, f_0'(z)\,\frac{R_0(\theta) - iR_0'(\theta)}{\sqrt{R_0^2 + R_0'^2}}\right) = 0 \,. \tag{A1}$$

Since we are considering the flow around a double body, we have taken the equation describing the body as in (34). Our purpose is to show that, in order to satisfy this condition, $f_0(z)$ cannot be analytic at $(x_0,0)$. Furthermore, we wish to determine the nature of the nonanalytic behavior there.

Our approach is to expand the quantities in (A1) with respect to the singular point. The expansion of $R_0(\theta)$ has already been given in (36), from which the expansion of $R_0'(\theta)$ can be found trivially. Of course, there is no difficulty in expanding the factor $\exp(i\theta)$ for small θ . It is then convenient to change from θ to y by using the relationship

$$\begin{aligned} y &= R_0(\theta)\,\sin\theta \\ &= x_0\theta + \left(\frac{R_0''}{2} - \frac{x_0}{6}\right)\theta^3 + \dots \,. \end{aligned} \tag{A2}$$

(When not otherwise noted, derivatives of $R_0(\theta)$ are to be evaluated at 0 .) Putting all of this together, we obtain

$$e^{i\theta}\,\frac{R_0(\theta) - iR_0'(\theta)}{\sqrt{R_0^2 + R_0'^2}} = 1 + iCy + \frac{1}{2}\,y^2\left(\frac{2R_0''}{x_0^3} - \frac{R_0''^2}{x_0^4} - \frac{1}{x_0^2} \pm \frac{iK}{x_0^3}\right) + \dots \,. \tag{A3}$$

We note that corresponding to (A2) we can also write:

$$x - x_0 = R_0(\theta)\cos\theta - x_0$$
$$= -\frac{1}{2}\,\theta^2 x_0^2 C \mp \frac{1}{6}K\theta^3 + \ldots \; . \qquad \text{(A4)}$$

In these expressions, C is the body curvature at $\theta = 0$:

$$C = C(0) = \frac{x_0 - R_0''(0)}{x_0^2} \; . \qquad \text{(A5)}$$

(*Cf*. (44).) We assume that curvature is continuous for the double body.

We can satisfy the boundary condition if we set, with $\zeta = z - x_0$,

$$f_0(z) = A_2\zeta^2 + A_3\zeta^3 + A_4\zeta^4 + \ldots + (\log\zeta)[B_4\zeta^4 + \ldots] \; . \qquad \text{(A6)}$$

We write the derivative of this expression,

$$f_0'(z) = 2A_2\zeta + 3A_3\zeta^2 + (4A_4 + B_4)\zeta^3 + \ldots + 4B_4\zeta^3\log\zeta + \ldots \; . \qquad \text{(A7)}$$

We evaluate this on the body, using (A2) and (A4), and substitute into the body boundary condition. We find that the coefficient of each power of y equals zero if

$$A_2 = \frac{1}{2}\,\phi_{0xx}(x_0,0) \; ; \quad A_3 = -\,C\,A_2 \; ; \quad B_4 = \frac{2K}{3\pi x_0^3}\,A_2 \; . \qquad \text{(A8)}$$

That is,

$$f_0(z) = \frac{1}{2}\,\phi_{0xx}(x_0,0)\left\{\zeta^2 - C\zeta^3 + A_4\zeta^3 + \ldots + \frac{2K}{3\pi x_0^3}\,\zeta^4\log\zeta + \ldots\right\} . \qquad \text{(A9)}$$

The fourth derivative of $f_0(z)$ is undefined (infinite) at $(x_0,0)$. Thus we have found the leading singular term in the double-body-flow potential.

From (A9), we find immediately that

$$f_0''(x_0 + i0) = \phi_{0xx}(x_0,0)$$

(a trivial result), and also that

$$f_0'''(x_0 + i0) = -\,3\,C(0)\,\phi_{0xx}(x_0,0) \; .$$

Since the last expression also equals $\phi_{0xxx} + i\phi_{0xxy}$, and $\phi_{0xxy}(x,0) = 0$ from (13a), we have the result needed in (61).

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