No. 239
September 1981

PREDICTION OF SHIP ROLL DAMPING–STATE OF THE ART

Professor Yoji Himeno

This research was carried out in part under the Naval Sea Systems Command General Hydrodynamics Research Program, Subproject SR 009 01 01, administered by the Naval Ship Research and Development Center Contract No. N00014-79-C-0244

It was also supported by the Japan Shipbuilding Industry Foundation

THE DEPARTMENT OF NAVAL ARCHITECTURE AND MARINE ENGINEERING

THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Prediction of Ship Roll Damping -- A State of the Art		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER 239
7. AUTHOR(s) Yoji Himeno		8. CONTRACT OR GRANT NUMBER(s) N00014-79-C-0244
9. PERFORMING ORGANIZATION NAME AND ADDRESS Nav Arch & Mar Engr, The Univ. of Michigan 2600 Draper Ann Arbor, MI 48109		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61153N R02301 SR 023 01 01
11. CONTROLLING OFFICE NAME AND ADDRESS David W. Taylor Naval Ship R&D Center (1505) Bethesda, MD 20084		12. REPORT DATE September 1981
		13. NUMBER OF PAGES 86
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Office of Naval Research 800 N. Quincy St. Arlington, VA 22217		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Sponsored by the Naval Sea Systems Command General Hydromechanics Research (GHR) Program administered by the David W. Taylor Naval Ship R&D Center, Code 1505, Bethesda, MD 20084.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

SHIP MOTIONS
ROLL DAMPING
SEPARATION

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Various methods for predicting the roll damping of a ship at forward speed is discussed. In particular, a simple method and a component analysis are described. The component analysis assumes that the damping is composed of friction damping, eddy damping, lift damping, wave damping, normal-force damping of bilge, keel, hull pressure damping due to bilge keels, and wave damping of bilge keels. Formulas for these components are derived from theoretical and experimental considerations. A listing of a computer program used to compute roll damping is included as an Appendix.

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Prediction of Ship Roll Damping --

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CONTENTS

FOREWORD

The theory for predicting the motions of a ship in a seaway is one of the triumphs of research in ship hydrodynamics. Given a rather small amount of information about a ship and the seaway, one can predict heave and pitch motions to a remarkable degree of accuracy without recourse to model tests or empirical data. Lateral-plane motions, sway and yaw, can also be predicted with reasonable accuracy.

However, when one tries to predict roll motion, one realizes what good luck we have had in analyzing heave, pitch, sway, and yaw: These are not sensitive to the effects of fluid viscosity. Roll motion is extremely sensitive to viscosity effects, especially to viscosity - induced flow separations. In addition, roll motion is strongly influenced by the presence of bilge keels, which are difficult to analyze even by the classical methods of hydrodynamics of an ideal fluid.

During the 1978-79 academic year, our Department was fortunate in having as a visiting scholar Professor Y. Himeno of the University of Osaka Prefecture. He is well-known in Japan for his research on viscous-fluid problems of ship hydrodynamics, and the Department of which he is a member is distinguished for its research on ship roll damping.

Therefore I was especially pleased when he agreed to my request that he prepare a report describing the state of the art in predicting roll damping. As in many areas of naval architecture, Japan is in the forefront in developing practical procedures for predicting ship roll damping. Professor Himeno has been closely associated with these developments.

As he makes clear in this report there are many aspects of this problem that have not yet been adequately analyzed. However, in the great tradition of Japanese naval architecture research, theory is used as far as possible, and the gaps are filled with empirical information. More research is needed, but a usable procedure for predicting roll damping is described.

In the appendix, a computer program is presented for predicting roll damping. This program from Osaka Prefecture University was tested at The University of Michigan by having an undergraduate compile it and use it. The information provided in the Appendix, together with the comments built into the program, were sufficient for this student to use the program.

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ACKNOWLEDGMENT

This survey has been prepared during the author's stay at The University of Michigan for a year. The author would like to express his heartfelt appreciation to Professor T. Francis Ogilvie, Professor William S. Vorus, Dr. Nabil Daoud, Dr. Armin W. Troesch and Mr. John P. Hackett for their valuable advice, discussions and encouragement.

The author also feels grateful to Professor Norio Tanaka and Dr. Yoshiko Ikeda at University of Osaka Prefecture.

NOMENCLATURE

[Note: Numbers in parentheses indicate equations where more information can be found about the quantity listed.]

A_R	Wave-amplitude ratio (See Fig. 4.6)
A_ϕ	Inertia coefficient in roll equation of motion (2.1)
a	Extinction coefficient (2.15)
B	Ship beam
B_E	Damping coefficient component: bare-hull eddy making (4.1)
B_F	Damping coefficient component: bare-hull skin friction (4.1)
B_L	Damping coefficient component: lift (due to forward speed) (4.1)
B_W	Damping coefficient component: bare-hull wave making (4.1)
B_{BK}	Damping coefficient component: total of bilge-keel pressure effects (4.2)
B_{BKH}	Damping coefficient component: hull pressure change due to bilge keels (4.1)
B_{BKN}	Damping coefficient component: bilge-keel normal pressure (4.1)
B_{BKW}	Damping coefficient component: bilge-keel wave making (4.1)
B_e	Equivalent linear damping coefficient (2.5)
B_ϕ	Roll-damping term in equation of motion (2.1)
B_j	Coefficients in expansion of B_ϕ , $j = 1,2,...$ (2.2)
b	Extinction coefficient (2.15)
b_{BK}	Width of bilge keel
b_I	Effective width of bilge keel (4.20)
C_B	Hull block coefficient
C_D	Drag coefficient (4.17) (See Fig. 4.9)
C_p	Pressure-difference coefficient (4.7)
C_p^+	C_p in front of bilge keel
C_p^-	C_p behind bilge keel
C_ϕ	Restoring-force coefficient in equation of motion (2.1)
c	Extinction coefficient (2.15)
d	Ship draft
Fn	Froude number
f	Empirical coefficient giving velocity increment at the bilge circle (4.20)
GM	Metacentric height (restoring-moment lever arm) (2.1)
g	Acceleration of gravity

H_0	$B/2d$ (4.7)
KG	Distance from keel to center of gravity of ship
k	Reduced frequency, $\omega L/U$
L	Ship length
ℓ_{BK}	Length of bilge keel
M_ϕ	Roll excitation moment (2.1)
N	"N-coefficient," $\Delta\phi/\phi_m^2$ (2.19)
N_{10}	Value of N for $\phi_A = 10°$ (2.20)
O	Origin of coordinates
OG	Distance downward from origin to center of gravity (4.5)
R	Bilge radius of hull
r	Mean distance from center of gravity to bilge keels (4.5)
r_{max}	Maximum distance from roll axis to hull surface (4.7)
S_0	Width of distribution of C_p on hull
T_n	Natural period of roll (2.4)
t	Time variable
U	Forward speed of ship
W	Weight of ship
α	$B_1/2A_\phi$ (2.4)
α_e	$B_e/2A_\phi$ (2.9)
β	B_2/A_ϕ (2.4)
γ	B_3/A_ϕ (2.4)
$\Delta\phi$	$\phi_{n-1} - \phi_n$ (2.15a)
ζ_A	Radiation wave amplitude (Fig. 4.6)
λ	Scale ratio of ship to model
ν	Kinematic viscosity of water
ξ_d	$\omega^2 d/g$ (4.15)
ρ	Density of Water
σ	Area coefficient of a cross-section of the hull
τ	$U\omega/g$ (4.15)
ϕ	Roll angle (2.1)
ϕ_A	Amplitude of roll motion (2.1)
ω	Frequency (rad/sec)
ω_n	$2\pi/T_n = \sqrt{C_\phi/A_\phi}$ (2.4)

Special Notations:

∇ Displacement volume of ship

^ Indicates nondimensional form of quantity.

0 [subscript] indicates value at zero forward speed.

' indicates 2-D value for a cross-section of hull.

TABLES AND FIGURES

1. INTRODUCTION

Roll motion is one of the most important responses of a ship in waves. The roll motion of a ship can be determined by analyzing various kinds of moments acting on the ship, virtual and actual mass moments of inertia, roll damping moment, restoring moment, wave excitation and other momemts caused by other modes of ship motion. Among them, the roll damping moment has been considered to be the most important term that should be correctly predicted. It is needed not only at the initial stage of ship design to secure the safety of a ship, but also to obtain a better understanding of ship motions in waves.

Since the age of W. Froude, a number of theoretical and experimental works has been made concerning the predictions of roll damping and roll motion of ships. The recent development of the "Strip Method" has made it possible to calculate almost all the terms in the equations of ship motions in waves with practical accuracy, except for the roll damping. The necessity of obtaining the roll damping of ships has been pointed out in the recent recommendations of the Seakeeping Committee of the International Towing Tank Conference (ITTC). Notwithstanding these efforts, it seems that a complete solution of this problem has not yet been reached.

Difficulties in predicting the roll damping of ships arise from its nonlinear characteristics (due to the effect of fluid viscosity) as well as from its strong dependence on the forward speed of ship. Moreover, the fact that these various effects have influences on the value of roll damping that are of the same order of magnitude makes the problem even more complicated in the absence of bilge keels.

After the classical works by Bryan [1] and Gawn [2], we can recognize an epochmaking period a couple of decades ago in the history of research on roll damping. Experiments on bilge keels by Martin [3], Tanaka [4], and Kato [5], theoretical works on the vortex flow near bilge keels by Sasajima [6], consideration of hull-friction damping by Kato [7], and study of the surface-tension effect by Ueno [8], all of these works appear in this period.

Furthermore, we can cite here Hishida's theoretical studies [9] on the wavemaking roll damping due to hull and bilge keels and Hanaoka's mathematical formulation [10] for the wave system created by an oscillatory motion of an immersed flat-plate wing with low aspect ratio. Also, an extensive series of free-roll tests at zero ship speed for ordinary ship hull forms was carried out by Watanabe and Inoue [11] and tests at forward speed by Yamanouchi [12]. Some of the results of these works have often been used even up to the present time or have given background to recent works. This fact cannot but remind us again of the difficulty of treating the ship roll-damping problem rigorously.

It can be said that the recent works started about a decade ago, mainly associated with the experimental check of the accuracy of the strip method. Much data on radiation forces acting on ship hull, including roll damping, have been accumulated through the forced oscillation tests carried out by Vugts [13], Fujii and Takahashi [14], Takaki and Tasai [15], and Takezawa *et al.* [16].

These experiments have clarified that there are still considerable differences between measured values of roll damping and those predicted by existing methods. In this period much effort has also been made for obtaining ship roll damping, for example, works by Bolton [17], Lofft [18], and Lugovski *et al.* [19], concerning the effect of bilge keels, Gersten's studies [20] on the viscous effects, and free-roll experiments by Takaishi *et al.* [21] and Tanaka *et al.* [22].

Moreover, what should be noted here is the extensive and systematic works in Japan that have recently been carried out through the cooperation of the Japan Shipbuilding Research Association, especially in the Committees of SR108, succeeded by SR125, 131 and 161 [23]. In the prediction method of ship roll damping considered there, damping is divided into several components, for instance, friction, eddy, lift, wave, and bilge-keel components. Then the total damping is obtained by summing up these component dampings predicted separately. This attempt appears to have had a certain success for ordinary ship hull forms.

The objective of this article is to describe the present state of the art in these recent attempts as well as other existing formulas for ship roll damping. Furthermore, for convenience in ship design, it is intended that the available expressions and formulas should be described in full detail as much as possible, so that their values can be calculated promptly once the particulars of a ship are given.

In Chapter 2 the various methods of representation for roll damping coefficients and their relationships are stated and rearranged in terms of an equivalent linear damping coefficient. Then, for prediction methods of roll damping, simple methods are introduced in Chapter 3, including the use of data from a regression analysis of model experiments. In Chapter 4, the newest treatment for component dampings is stated, and available formulas for each component are fully described there. Comparisons are made of measured and predicted total damping, which is the sum of the component dampings.

Chapter 5 concerns the prediction methods for ship roll motion. However, it is not the full present state of the art. The description is limited to the problem of how to use the formulas of nonlinear roll damping in order to obtain the solution of the roll equation of motion in regular or irregular sea. Finally an example of a FORTRAN statement of a computer program for the component dampings is given in the Appendix.

2. REPRESENTATION OF ROLL DAMPING COEFFICIENTS

Many ways of representing roll damping coefficients have been used, depending on whether roll damping is expressed as a linear or nonlinear form, which form of the non-dimensional expressions is to be used, and by what experimental method its value was measured, for instance, forced-roll test or free-roll experiment. Some of the expressions most commonly used are introduced here and the relationships among them are reviewed and rearranged in terms of a linearized damping coefficient.

2.1 Nonlinear Damping Coefficients

The equation of roll motion has recently been expressed as a three-degree-of-freedom form, including sway and yaw motions simultaneously. However, in order to limit the discussion here to the problem of nonlinear roll damping, we can write down the equation of the roll motion of a ship in the following simple single-degree-of-freedom form:

$$A_\phi \ddot{\phi} + B_\phi(\dot{\phi}) + C_\phi \phi = M_\phi(\omega t) \quad . \tag{2.1}$$

In Eq. (2.1), ϕ represents the roll angle (with the amplitude ϕ_A), A_ϕ the virtual mass moment of inertia along a longitudinal axis through the center of gravity and C_ϕ the coefficient of restoring moment, which is generally equal to W·GM (W the displacement weight of the ship and GM the metacentric height). Furthermore, M_ϕ stands for the exciting moment due to waves or external forces acting on the ship, ω the radian frequency and t the time. Finally, B_ϕ denotes the roll damping moment, which is now considered.

Although only the main terms of roll motion have been taken into account in Eq. (2.1), coupling terms being neglected, it can be said that Eq. (2.1) almost corresponds to that of three degrees of freedom when we consider the wave excitation term M_ϕ as the Froude form with a coefficient of effective wave slope. This is because the concept of the effective decrease of wave slope turns out, after Tasai's analysis [24], to correspond to the effect of the sway coupling terms.

We can express the damping moment B_ϕ as a series expansion of $\dot{\phi}$ and $|\dot{\phi}|$ in the form

$$B_\phi = B_1\dot{\phi} + B_2\dot{\phi}|\dot{\phi}| + B_3\dot{\phi}^2 + \dots , \tag{2.2}$$

which is a nonlinear representation. The coefficients B_1 , B_2 , ... in Eq. (2.2) are considered as constants during the motion concerned. In other words, these values may possibly depend on the scale and the mode of the motion, for instance, on the amplitude ϕ_A and the frequency ω when the ship is in a steady roll oscillation.

Dividing Eq.(2.1) by A_ϕ , we can obtain another expression per unit mass moment of inertia:

$$\ddot{\phi} + 2\alpha\dot{\phi} + \beta|\dot{\phi}|\dot{\phi} + \gamma\dot{\phi}^3 + \omega_n^2\phi = m_\phi(\omega t), \tag{2.3}$$

where

$$2\alpha = \frac{B_1}{A_\phi}, \quad \beta = \frac{B_2}{A_\phi}, \quad \gamma = \frac{B_3}{A_\phi},$$
$$\omega_n = \sqrt{\frac{C_\phi}{A_\phi}} = \frac{2\pi}{T_n}, \quad m_\phi = \frac{M_\phi}{A_\phi}. \tag{2.4}$$

In Eq. (2.4), the quantities ω_n and T_n represent the natural frequency and the period of roll, respectively.

A term of the form $\dot{\phi}/|\dot{\phi}|$ might be added to the right-hand side of Eq.(2.2). This term corresponds to the effect of surface tension at water level of the ship hull. Ueno [8] investigated this effect and concluded that the surface tension might cause a considerable error in the values of the damping coefficients when a small model is used in small amplitude of oscillation. However, this effect is not considered hereafter, because the surface tension depends strongly on the condition of the painted surface of the model hull as well as on that of the water surface, and because it can be neglected in the case of roll amplitude with moderate magnitude for a ship model of ordinary size.

To obtain the values of these coefficients of nonlinear damping directly through a steady-state forced-roll experiment, in which ϕ_A and ω are specified, we would probably need numerical techniques to fit the solution of the assumed equation to the measured data. Such an attempt does not seem to have been done. Instead, the usual way that has been taken is to assume some additional relations concerning, say, energy consumption, linearity of damping and its independence of ϕ_A , which will be described later.

2.2 Equivalent Linear Damping Coefficients

Since it is difficult to analyze strictly the nonlinear equation stated in the preceding section, the nonlinear damping is usually replaced by a certain kind of linearized damping as follows:

$$B_\phi(\dot{\phi}) = B_e\dot{\phi} \quad . \tag{2.5}$$

The coefficient B_e denotes the equivalent linear damping coefficient. Although the value of B_e depends in general on the amplitude and the frequency, because the damping is usually nonlinear, we assume B_e is constant during the specific motion concerned.

There are several ways to express the coefficient B_e in terms of the nonlinear damping coefficients B_1 , B_2 and so on. The most general way is to assume that the energy loss due to damping during a half cycle of roll is the same when nonlinear and linear dampings are used [24]. If the motion is simple harmonic at radian frequency ω ,

$$B_e = B_1 + \frac{8}{3\pi}\omega\phi_A B_2 + \frac{3}{4}\omega^2\phi_A^2 B_3 \quad . \tag{2.6}$$

For more general periodic motion, Eq. (2.6) can be derived by equating the first terms of the Fourier expansions of Eqs. (2.5) and (2.2) [15].

For convenience in analyzing the equations of lateral motions, the nondimensional forms of these coefficients are defined as follows:

$$\hat{B}_e = \frac{B_e}{\rho\nabla B^2}\sqrt{\frac{B}{2g}}\ , \quad \hat{B}_i = \frac{B_i}{\rho\nabla B^2}\left(\sqrt{\frac{B}{2g}}\right)^{2-i}, \quad \text{for } i=1,2,3\ , \tag{2.7}$$

$$\hat{\omega} = \omega\sqrt{\frac{B}{2g}}\ ,$$

where ρ , ∇ and B stand for fluid density, displacement volume and breadth of ship, respectively. Then Eq. (2.6) takes the following nondimensional form:

$$\hat{B}_e = \hat{B}_1 + \frac{8}{3\pi}\hat{\omega}\phi_A\hat{B}_2 + \frac{3}{4}\hat{\omega}^2\phi_A^2\hat{B}_3 \quad . \tag{2.8}$$

Corresponding to Eq.(2.3), we can define an equivalent linear damping coefficient $\alpha_e = B_e/2A_\phi$ per unit mass moment of inertia:

$$\alpha_e = \alpha + \frac{4}{3\pi}\omega\phi_A\beta + \frac{3}{8}\omega^2\phi_A^2\gamma \ . \tag{2.9}$$

Since these coefficients still have dimensional values (except for the second, β), the following dimensionless forms are often used, especially for the linear terms α_e and α .

$$\kappa_e = \frac{2\alpha_e}{\omega_n}, \quad \kappa_\alpha = \frac{2\alpha}{\omega_n} \quad . \tag{2.10}$$

In case of irregular roll motion, there is another approach to linearization of the roll damping expression. After the works of Kaplan [26] and others [27] [23], we assume that the difference of the damping moment between its linearized and nonlinear forms can be minimized in the sense of the least squares method. Neglecting the term B_3 for simplicity, we define the discrepancy δ in the form:

$$\delta = B_1\dot{\phi} + B_2|\dot{\phi}|\dot{\phi} - B_e\dot{\phi} \ . \tag{2.11}$$

Then we can minimize $E\{\delta^2\}$, the expectation value of the square of δ during the irregular roll motion, assuming that the undulation of the roll angular velocity, $\dot{\phi}$, is subject to a Gaussian process and that the coefficients B_e , B_1 and B_2 remain constant:

$$\frac{\partial}{\partial B_e}E\{\delta^2\} = -2(B_1-B_e)E\{\dot{\phi}^2\} - 2B_2E\{\dot{\phi}^2|\dot{\phi}|\} = 0 \quad . \tag{2.12}$$

After some calculations we can reach the form

$$B_e = B_1 + \frac{8}{\pi}\sigma_{\dot{\phi}}B_2 \ , \tag{2.13}$$

where the factor $\sigma_{\dot{\phi}}$ represents the variance of the angular velocity $\dot{\phi}$. It is claimed in recent works [23] that this form is useful for analyzing roll behavior in irregular seas.

Moreover, as an unusual way of linearization, we can equate the nonlinear expression to the linear one at the instant when the roll angular velocity takes its maximum value during steady oscillation:

$$B_e = B_1 + \omega\phi_A B_2 \ . \tag{2.14}$$

This form seems to correspond to a collocation method in a curve-fitting problem, whereas Eq.(2.6) corresponds to the Galerkin approach. Since there is a difference of about 15% between the second terms of the right hand sides of Eqs.(2.6) and (2.14), the latter form may not be valid for the analysis of roll motion. But it may be used as a simple way of analyzing forced-oscillation test data to obtain the values of these coefficients promptly from the time history of the measured roll moment.

However, the most common way to obtain these nonlinear damping coefficients through forced oscillation tests is, first, to find the equivalent linear coefficient B_e in Eq.(2.5) by assuming that the forced-roll-test system is subject to a linear equation, and, second, to fit Eq.(2.6) to the B_e values obtained by several test sequences with the amplitude ϕ_A varied. Then we can obtain the values of these damping coefficients, B_1, B_2 and so on, which are independent of the amplitude of roll oscillation.

It should be noted here that this condition, the independence of amplitude, was not stated when we derived Eq.(2.6). Therefore we might obtain different values of the coefficients from the original ones if the coefficients, especially B_2, should depend on the amplitude, particularly in the presence of bilge keels. We should keep these things in mind when we use a formula like Eq.(2.6).

2.3 Extinction Coefficients

A free-roll test is probably the simplest way to measure roll damping of ship or model. In a model test, sway and yaw motions are usually restrained to avoid the effect of the horizontal motions. On the other hand, heave and pitch motions are often kept free to avoid the error due to the sinkage force in the presence of forward speed, although it is of course desirable to make the vertical motions as small as possible. The roll axis is usually taken through the center of gravity of the model, the radius of gyration of the model is adjusted to the value of the actual ship considered, and the restoring moment lever GM is also measured through a static inclination test.

In a free-roll test, the model is rolled to a chosen angle and then released. The subsequent motion is measured. Denote by ϕ_n the absolute value of roll angle at the time of the n-th extreme value. The so-called extinction curve expresses the decrease of ϕ_n as a function of mean roll angle. Following Froude and Baker, we fit the extinction curve by a third-degree polynomial:

$$\Delta\phi = a\phi_m + b\phi_m^2 + c\phi_m^3 \qquad \text{(deg.)} \ , \tag{2.15}$$

where

$$\Delta\phi = \phi_{n-1} - \phi_n \ , \tag{2.15a}$$

$$\phi_m = [\phi_{n-1} + \phi_n]/2 \ . \tag{2.15b}$$

The angles are usually measured in degrees in this process.

The coefficients a , b and c are called extinction coefficients. The relation between these coefficients and the damping coefficients can be derived by integrating Eq.(2.1) without the external-force term over the time period for a half roll cycle and then equating the energy loss due to damping to the work done by restoring moment. The result can be expressed in the form

$$\Delta\phi = \frac{\pi}{2}\frac{\omega_n}{C_\phi}\phi_m\left(B_1 + \frac{8}{3\pi}\omega_n\phi_m B_2 + \frac{3}{4}\omega_n^2\phi_m^2 B_3\right) \ \text{(rad)} \ . \tag{2.16}$$

Comparing Eq.(2.16) with Eq.(2.15) term by term, we can obtain the relations

$$\left.\begin{aligned} a &= \frac{\pi}{2}\frac{\omega_n}{C_\phi}B_1 = \frac{\pi}{2}\frac{2\alpha}{\omega_n} = \frac{\pi}{2}\kappa_\alpha \ , \\ \frac{180}{\pi}b &= \frac{4}{3}\frac{\omega_n^2}{C_\phi}B_2 = \frac{4}{3}\beta \ , \\ \left(\frac{180}{\pi}\right)^2 c &= \frac{3\pi}{8}\frac{\omega_n^3}{C_\phi}B_3 = \frac{3\pi}{8}\omega_n\gamma \ . \end{aligned}\right\} \tag{2.17}$$

It should be noted here again that the condition for the validity of Eq. (2.17) is that the coefficients B_1 , B_2 , and α , β , ... should be independent of the roll amplitude. As we can see in the later chapters, the effect of bilge keels appears mainly in the term B_2 , and, further, the value of B_2 varies with roll amplitude. In such a case, Eq.(2.17) will not remain valid. Only the part of B_2 which is independent of the amplitude is related to the coefficient b. The other part of B_2 that is inversely proportional to the amplitude will apparently be transferred to the coefficient a , and the part proportional to the amplitude will appear in c. In place of a term-by-term comparison, therefore, it will probably be reasonable to define an equivalent extinction coefficient a_e and to compare it with the equivalent linear damping coefficient B_e as in the form

$$a_e = a + b\phi_m + c\phi_m^2 = \frac{\pi}{2}\frac{\omega_n}{C_\phi}B_e \quad . \qquad (2.18)$$

We are also familiar with Bertin's expression [25], which can be written in the form

$$\Delta\phi = N\phi_m^2 \qquad (\text{deg.}) \ . \qquad (2.19)$$

The coefficient N can be taken as a kind of equivalent nonlinear expression, and it has been called an "N-coefficient." As seen from Eq.(2.15),

$$N = \frac{a}{\phi_m} + b + c\phi_m \qquad (\text{deg.}) \ . \qquad (2.20)$$

The value of N depends strongly on the mean roll angle ϕ_m , so that its expression is always associated with the ϕ_m value, being denoted as N_{10} , N_{20} and so on, where N_{10} is the value of N when $\phi_m = 10°$, *etc*.

3. PREDICTION OF ROLL DAMPING: I. SIMPLE METHOD

When the principal dimensions of a ship form are given, the most reliable way to obtain the roll damping of the ship at present time seems to be to carry out a model experiment. Since the scale effect of the damping is considered to be associated mainly with the skin friction on the hull, which makes a small contribution to total damping, the data from the model tests can easily be transferred to the actual ship case by using an appropriate nondimensional form of roll damping, for instance, Eq.(2.8).

If model-test data are not available, it is necessary to estimate the roll-damping value by using certain kinds of prediction formula. There are two different ways of estimation at present time. One is to obtain an empirical, experimental formula directly through the analysis of model tests on actual ship forms. The other is to break down roll damping into several components and then estimate the value by summing up the values of those components individually predicted.

The latter is considered to be more rational, so that it has become the recent trend of approach in Japan. To begin with, however, some examples of the former approach are described in this chapter in order to know the magnitude of ship roll damping easily.

3.1 Watanabe-Inoue-Takahashi Formula

A couple of decades ago, Watanabe and Inoue [11] established a formula for predicting the roll damping of ordinary ship-hull forms at zero advance speed in normal-load condition, on the basis of both an extensive series of model tests and some theoretical considerations on the pressure distribution on the hull caused by ship roll motion. Their original formula has been modified slightly by them [28] so as to be applicable to a wider range of ship forms, including ships with large values of block coefficient.

Takahashi [29] proposed a form of forward-speed modification multiplier to be applied to the value at zero ship speed, thus expressing the advance-speed effect on roll damping. We may call this approach the Watanabe-Inoue-Takahashi formula.

This can be expressed in terms of an equivalent linear damping coefficient of the form

$$B_e = B_{e0}\,[1 + 0.8\{1-\exp(1-10Fn)\}\,\frac{\omega_n^2}{\omega^2}\,] \;, \qquad (3.1)$$

where B_{e0} stands for the value of B_e at zero ship speed. Its value can be expressed in terms of the extinction coefficients a and b , as follows:

$$\begin{aligned} B_{e0} &= 2\alpha_e A_\phi \\ &= \frac{2}{\pi}\,\omega_n A_\phi\,(a + \frac{\omega}{\omega_n}\,b\phi_A) \qquad (\phi_A\text{:deg}) . \end{aligned} \qquad (3.2)$$

Furthermore these coefficients can be related to the values of the N-coefficients N_{10} and N_{20} at the roll amplitudes $\phi_A = 10°$ and $20°$:

$$N_{10} = \frac{a}{10} + b, \qquad N_{20} = \frac{a}{20} + b \;, \qquad (3.3)$$

and the virtual mass moment of inertia A_ϕ can be calculated by the strip method or otherwise determined through the approximate relationship of the natural roll frequency to both the coefficient A_ϕ and the restoring moment of the ship.

The N-coefficients can be expressed after Watanabe and Inoue in the form

$$\begin{Bmatrix} N_{10} \\ N_{20} \end{Bmatrix} = \left[\begin{Bmatrix} n_{10} \\ n_{20} \end{Bmatrix} + \begin{Bmatrix} 1.5 \\ 1.0 \end{Bmatrix} \frac{A_b}{L^2}\,\sigma_0\right]\left[\ell^3\left(1 + \frac{d^2}{4\ell^2}\right) + \frac{f\,B^4}{64d}\right]\frac{L_d}{W \cdot GM \cdot T_n^2} \;, \qquad (3.4)$$

where L,B and d represent ship length, beam and draft. The quantity A_b denotes the area of the bilge keel at one side of the hull. The distance ℓ is defined:

$$\ell = KG - \frac{d}{2} \;, \qquad (3.5)$$

and the quantity f is a function of waterline area coefficient C_w :

$$f = 1 - \frac{4}{m+1} + \frac{6}{2m+1} - \frac{4}{3m+1} + \frac{1}{4m+1} \;, \qquad m = \frac{C_w}{1-C_w} \;. \qquad (3.6)$$

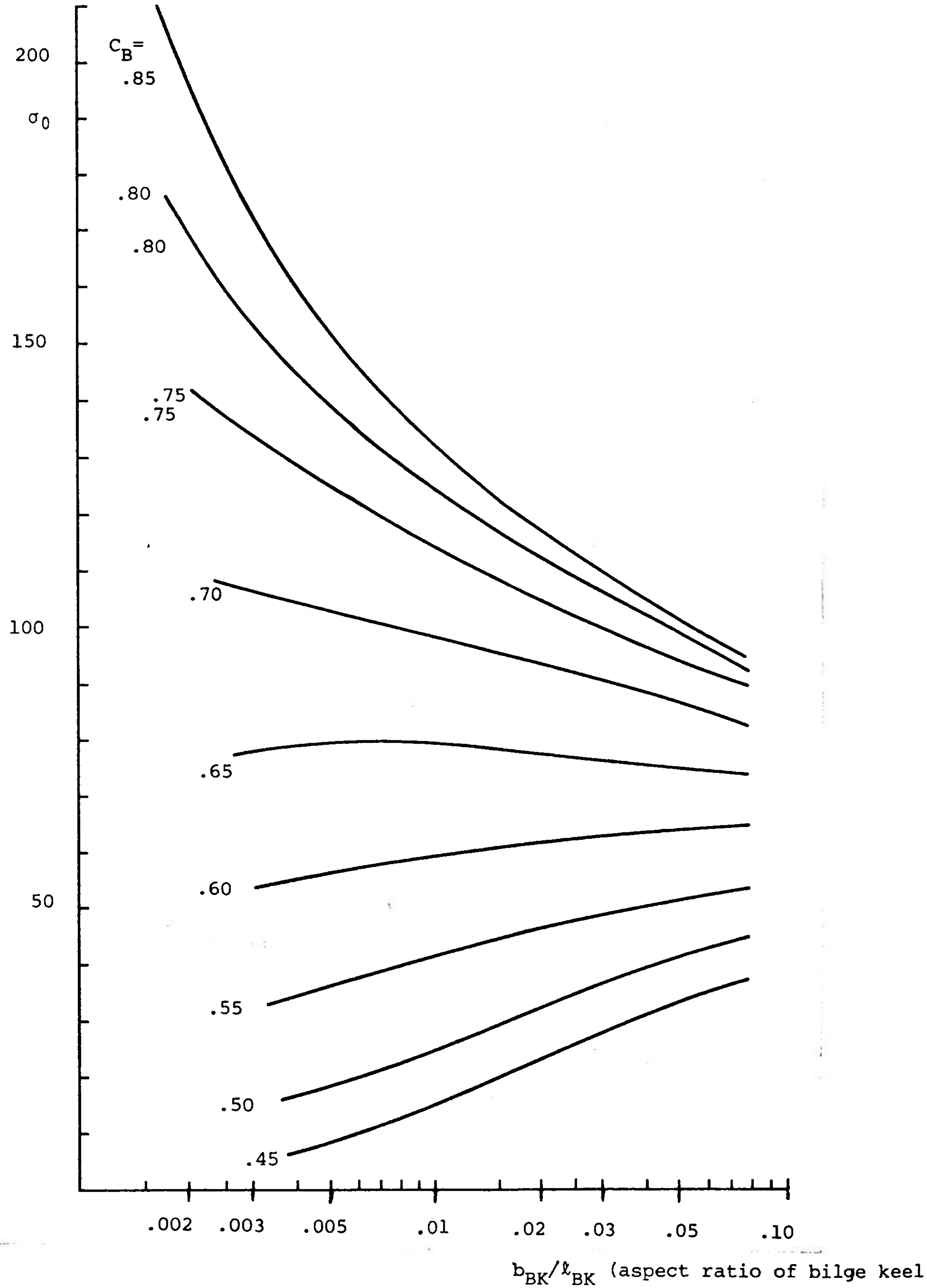

Figure 3.1. Bilge-keel efficiency in Watanabe-Inoue method

The first square bracket on the right-hand side of Eq.(3.4) corresponds to a kind of drag coefficient of nonlinear damping. The coefficient n_{10} and n_{20} stand for those of the naked hull at the roll amplitudes 10° and 20°, expressed in the form

$$\begin{Bmatrix} n_{10} \\ n_{20} \end{Bmatrix} = \begin{Bmatrix} 0.78 \\ 1.1 \end{Bmatrix} \frac{C_B d}{L} + \begin{Bmatrix} 0.03 \\ 0.02 \end{Bmatrix} , \tag{3.7}$$

where C_B represents ship block coefficient. Finally, the quantity σ_0 corresponds to a certain efficiency of the bilge keels, the value of which can be determined from Fig. 3.1, given by the authors themselves as a function of C_B and the aspect ratio b_{BK}/ℓ_{BK} of the bilge keel.

We can thus obtain the magnitude of the roll damping easily once the principal dimensions of a ship and its bilge keels are given. Since these formulas were established on the basis of a rather large amount of experimental data, it can be hoped that they will offer a reasonable estimation of roll damping in the early stage of ship design. As an example, the comparison of Takahashi's formula with some experimental data is shown in Fig. 3.2. The agreement seems to be acceptable in the ordinary range of ship speed.

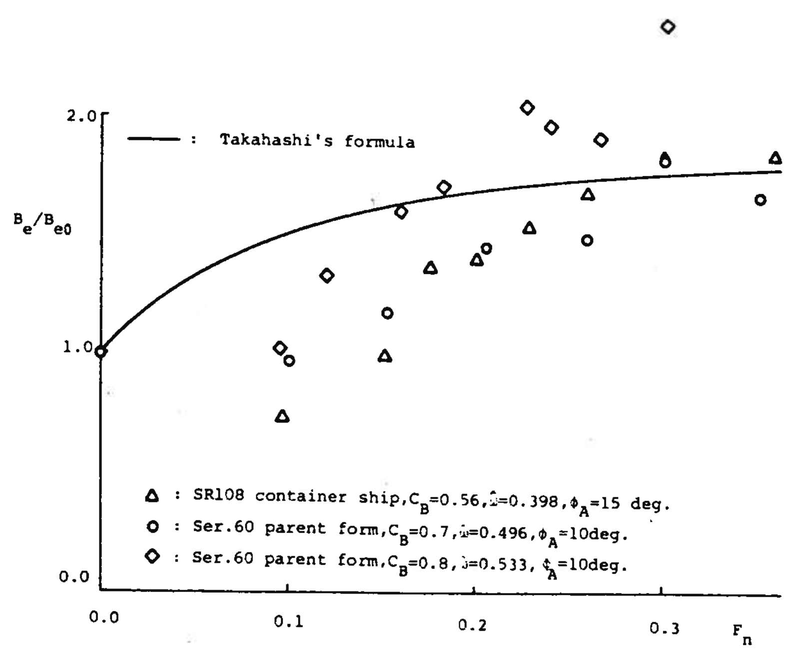

Figure 3.2. Effect of advance speed on roll damping force

The background of these formulas should also be mentioned. Eq.(3.2) can be derived from Eq.(2.17), which represents the term-by-term comparison between the coefficients B_1 , B_2 , ... and the extinction coefficients a , b and c . For its validity, however, the assumptionsare needed that the coefficients B_1 and B_2 take their values at $\omega=\omega_n$ and are constant, that is, are independent of frequency and are not affected by the roll amplitude ϕ_A . These assumptions, especially the former, might cause a misprediction for the ship rolling in a frequency range away from the ship natural frequency.

Therefore these formulas should be applied to the case of a normally loaded ship, and then only near the natural frequency, where ship rolling usually becomes important.

3.2 Tasai-Takaki's Table

As a second example of a simple method, we cite the results of Tasai and Takaki's experiment [23], which has recently been carried out for the purpose of obtaining typical values of roll damping for ordinary ship hull forms. The roll damping of four typical kinds of ship form was measured by forced-roll test at specified values of both Froude number and the roll amplitude, with frequency varied. The data obtained were fitted by regression analysis to the forms of Eq.(2.8), including two- and three-term damping coefficients.

Table 3.1 Particulars of Models

	container	cargo ship	ore carrier	tanker
length L_{pp} (m)	3.00	3.00	3.00	3.00
breadth B (m)	0.4354	0.4783	0.493	0.4719
draft d (m)	0.1628	0.1957	0.194	0.1828
displacement Δ (kg)	121.61	199.84	233.4	220.57
C_B	0.5717	0.7119	0.8243	0.8519
C_M	0.970	0.9905	0.9975	0.9946
GM (m)	0.017	0.02174	0.050	0.06077
⊗B (m)	-0.0425	-0.025	0.089	0.0993
K_L/L_{pp}	0.239	0.2172	0.2356	0.2494
K_B/B	0.382	0.3240	0.2602	0.2513
ℓ_{BK}/L_{pp}	0.25	0.25	0.25	0.37
b_{BK}/B	0.0148	0.0159	0.0142	0.00869

Table 3.1 shows the principal dimensions of the models, and Tables 3.2 and 3.3 present the nondimensional values of the nonlinear damping coefficients, taken up to the second and third terms, respectively

Table 3.2 Damping coefficients of 2nd-order approximation

F_n	COEF.	ore carrier	tanker	container	cargo ship
0	$\hat{B}_1$	0.00193	0.00161	0.00006	0
	$\hat{B}_2$	0.05667	0.05180	0.05563	0.06999
0.10	$\hat{B}_1$	0.00281	0.00272		
	$\hat{B}_2$	0.05975	0.05387		
0.15	$\hat{B}_1$		0.00276	0.00286	0.00156
	$\hat{B}_2$		0.05696	0.04702	0.05785
0.20	$\hat{B}_1$				0.00282
	$\hat{B}_2$				0.04752
0.25	$\hat{B}_1$			0.00571	0.00368
	$\hat{B}_2$			0.0321	0.04548
0.275	$\hat{B}_1$			0.00596	
	$\hat{B}_2$			0.02851	

As an example for the validity of these curve fittings, the comparison with the experiment on the $C_B = 0.71$ cargo ship model is shown in Fig. 3.3. The agreement seems to be quite reasonable, even in the region of comparatively large roll amplitude. Since the result concerns a typical ship form, it should be quite useful for the prediction of magnitude of ship roll damping at the initial stage of ship design. It can be applied to other cases of different ship form and ship speed by interpolating or extrapolating the values in the tables.

What should also be noted here is the limitation on the application of these tables. It is assumed that the damping coefficients B_1 , B_2 and B_3 in Eq.(2.8) are constant for specified ship form and Froude number. Therefore the tables do not cover the case without bilge keels, in which wave damping might prevail and the magnitude of the nonlinear terms might not be so large.

Table 3.3 Damping coefficients of 3rd-order approximation

F_n	COEF.	ore carrier	tanker	container	cargo ship
0	$\hat{B}_1$	0.00308	0.00209	0.00082	0.00061
	$\hat{B}_2$	0.03262	0.04168	0.03690	0.04908
	$\hat{B}_3$	0.12170	0.03877	0.08474	0.08994
0.10	$\hat{B}_1$	0.00359	0.00316		
	$\hat{B}_2$	0.04110	0.04453		
	$\hat{B}_3$	0.07783	0.03581		
0.15	$\hat{B}_1$		0.00344	0.00374	0.00242
	$\hat{B}_2$		0.04254	0.02531	0.03755
	$\hat{B}_3$		0.05524	0.09835	0.08755
0.20	$\hat{B}_1$				0.00332
	$\hat{B}_2$				0.03551
	$\hat{B}_3$				0.05226
0.25	$\hat{B}_1$			0.00628	0.00389
	$\hat{B}_2$			0.02125	0.04033
	$\hat{B}_3$			0.03567	0.02206
0.275	$\hat{B}_1$			0.00671	
	$\hat{B}_2$			0.01402	
	$\hat{B}_3$			0.05097	

We have seen a couple of examples of simple methods for predicting the order of magnitude of ship roll damping. Other similar methods might also be available for that purpose. To improve the accuracy of the prediction method, however, we should consider the phenomenon of roll damping from a much more physical and hydrodynamic point of view. In this sense, the concept of component damping, which will be described in the next chapter, will present more fruitful results applicable to a wider variety of cases.

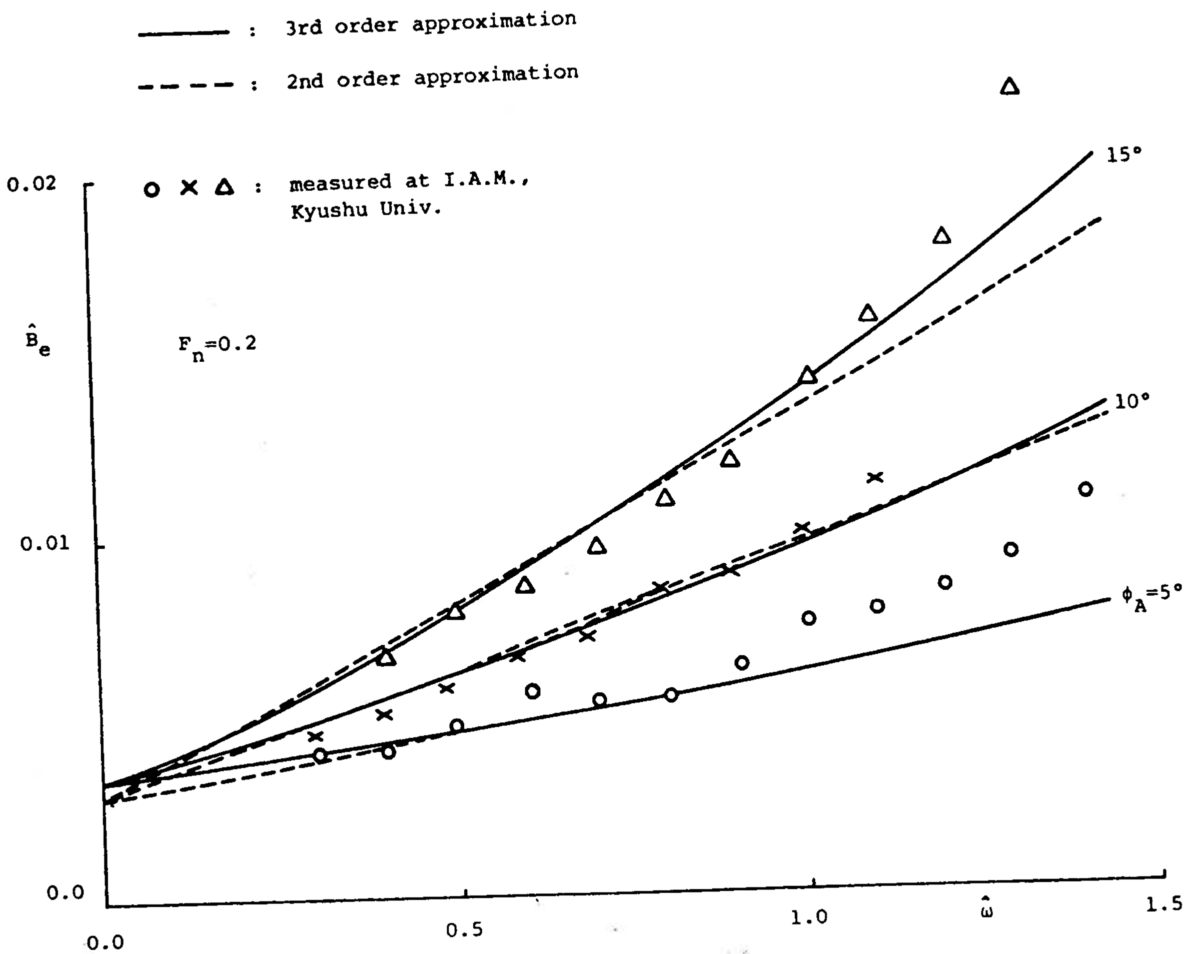

Figure 3.3. Roll damping coefficients of C_B = 0.71 ship form.

4. PREDICTION OF ROLL DAMPING: II. COMPONENT ANALYSIS

4.1 Definition of Component Damping

It has been pointed out since a couple of decades ago, as mentioned previously, that roll damping of a ship is caused by various kinds of fluid flow phenomena, for instance, skin friction of the hull, eddy shedding from the hull, free-surface waves, etc. It has also been noted that roll damping is strongly affected by the presence of bilge keels, rudder and appendages.

In recent days, the concepts of these effects have been considerably clarified and research work on the individual effects has been carried out. The objective of this chapter is to define all these effects from the recent point of view of the concept of component dampings and to describe available prediction formulas for them.

To begin with, we assume that the total roll-damping coefficient for an ordinary ship hull form can be divided into seven components, that is, friction, eddy, lift and wave damping for naked hull, and normal-force damping of bilge keels, hull-pressure damping due to bilge keels, and wave damping of bilge keels. This assumption can be expressed in terms of equivalent linear damping coefficients of the form

$$B_e = B_F + B_E + B_L + B_W + B_{BKN} + B_{BKH} + B_{BKW} \quad , \tag{4.1}$$

or we can sum up the bilge keel terms,

$$B_e = B_F + B_E + B_L + B_W + B_{BK} \quad . \tag{4.2}$$

Although these coefficients are seemingly linear, their values may vary with the roll amplitude ϕ_A and the frequency ω . For brevity the effect of appendages, except for rudder and bilge keels, is not considered here. The rudder is assumed to be included in the main hull configuration.

We can define these component dampings as follows, neglecting or including their mutual interactions:

Friction damping B_F is caused by the skin-friction stress on the hull in roll motion, so that it may possibly be influenced by the presence of waves and bilge keels.

The *eddy damping* B_E stands for the nonlinear damping ($B_2|\dot{\phi}|\dot{\phi}$) caused by the pressure variation on the naked hull, excluding the effect of waves and bilge keels. However it may apparently include the linear term B_1 , which would be transferred from the nonlinear term B_2 if a part of B_2 were inversely proportional to the amplitude ϕ_A . In the presence of ship forward speed, eddy damping represents the nonlinear part of the lifting effect of the hull itself in roll motion, whereas the linear part is defined as the lift damping B_L .

The *wave damping* B_W denotes the increment of the hull-pressure damping due to the presence of free surface waves, so that it includes the interaction between waves and eddies and between waves and lift. However, since these interactions will be very small, it will be almost linear.

Bilge keel damping B_{BK} represents the increment of pressure damping due to the presence of a pair of bilge keels. This term consists of the following three components: The *normal-force damping of bilge keels* B_{BKN} is due to the normal force on the bilge keels themselves. The second is *hull-pressure damping due to bilge keels* B_{BKH} , which corresponds to the pressure change on the hull when bilge keels are installed. Therefore this term stands for an interaction between hull and bilge keels. The rest is the *wave damping of bilge keels* B_{BKW} . Since the first two terms do not concern the free surface, this term represents the change of values of B_{BKN} and B_{BKH} due to waves. This term also includes interaction between the hull (with bilge keels) and waves.

We have presented so many component dampings that it is natural to ask which component is the most important. The answer is quite difficult because, as will be seen later, almost all the components (except B_F and B_{BKW}) are of the same order of magnitude for an ordinary ship hull form. This is the very fact that has made the prediction of ship roll damping difficult.

The subdivision of roll damping, as stated above, may not always be based on the hydrodynamic point of view but may stand rather on a practical basis for convenience in predicting the roll damping and carrying out the experiments.

However, we can still note here some remarks on these components. The terms B_L, B_W and B_{BKW} can safely be treated as nonviscous damping, while the others can be regarded as viscous damping.

Tangential stress on the hull contributes only to the term B_F. The rest of the damping is caused by the normal stress, that is, the pressure on the hull or on the bilge keels. The surface-wave effects appear mainly in the terms B_W and B_{BKW}. The other terms can be considered as free from waves, although the term B_F also includes a small wave effect in its definition.

To distinguish linear damping from nonlinear damping is quite difficult at this stage. Of course, the non-viscous dampings like B_L and B_W can be regarded as linear, but also some parts in the terms B_F and B_{BK} might be linear, due to their dependence on Reynolds number or the Keulegan-Carpenter number. The forward-speed effect is included in all terms.

These characteristics will be much better clarified after the individual prediction methods are described in the following sections.

Hereafter the subscript $_0$ represents the case of zero speed, the superscript $'$ the value in the cross-sectional plane. The roll axis passes through the center of gravity unless otherwise stated.

4.2 Friction Damping

In predicting the value of friction damping, we ignore the effect of waves and regard the ship hull form as an equivalent axisymmetric body, for which the dimensions will be defined later. Then the skin friction laws for a flat plate in steady flow are applied to the roll motion of the body, as shown by Kato [7], Takaki [30], and Schmitke [31].

We cite Kato's formula as an example. In the absence of forward speed, Kato applied Blasius' formula for laminar flow and Hughes' formula for turbulent flow to the peripheral boundary layer of a circular cylinder. Kato's formula can be expressed in terms of an equivalent linear damping coefficient as follows:

$$B_{F0} = 0.787\rho S r_s^2 \sqrt{\omega\nu}\left\{1 + 0.00814\left(\frac{r_s^2\phi_A^2\omega}{\nu}\right)^{0.386}\right\} \quad (\phi_A\text{:rad.}) , \qquad (4.3)$$

where ρ and ν represent density and kinematic viscosity of fluid. In Eq. (4.3), the first term in the brackets gives the result for the case of laminar flow, which is used for the naked model hull, while the second term gives the modification for turbulent flow by Hughes' formula, applicable to both the model hull with bilge keels and the actual ship hull. The quantities S and r_S were originally defined as the surface area and the radius of the cylinder. For the case of a ship hull, however, they represent the wetted surface area and the average radius of roll, which can be expressed approximately by the formulas

$$S = L(1.7d + C_B \cdot B) \ , \tag{4.4}$$

$$r_S = \frac{1}{\pi}\left\{(0.887 + 0.145C_B)\frac{S}{L} - 2\ OG\right\} . \tag{4.5}$$

The symbols L , d , B and C_B represent length, draft, beam and block coefficient of the ship, respectively. The vertical distance from the origin to the center of gravity, OG , is measured downward as positive.

Since in Kato's formula the friction damping coefficient does not include the amplitude ϕ_A in the case of laminar flow, the damping is apparently linear. This is because the damping is originally defined as a nonlinear form and then the nonlinearity is cancelled by the dependence of the coefficient on the roll Reynolds number.

Recently Myrhaug and Sand [32] have carried out the boundary layer calculation on a rolling circular cylinder and obtained almost the same result as Kato by solving a Stokes-type equation in case of laminar flow. Ikeda *et al.* [33] have also found the same relationship between the Stokes solution and the Blasius formula applied to the unsteady flow. Further, they have confirmed the validity of Kato's formula in practical use, through the measurements of the velocity profile in the boundary layer on two-dimensional cylinders of shiplike sections.

In the presence of forward speed, Schmitke [31] has applied the skin-friction law of turbulent flow directly to the case of ship rolling, in a manner similar to Kato's treatment. However, we here cite Tamiya's formula [34] based on the more rigorous analysis of three-dimensional boundary layers on rolling cylinder. It can be expressed in the following simple form

$$B_F = B_{F0}\ (1 + 4.1\ \frac{U}{\omega L})\ , \tag{4.6}$$

where the constant 4.1 has been determined through experiments on elongated spheroids in roll motion. The coefficient B_{F0} represents the friction damping at zero forward speed, which can be predicted by Kato's formula.

The Tamiya formula has been confirmed as giving a good prediction by Ikeda *et al.* [35], who carried out the somewhat detailed calculation of the three-dimensional turbulent boundary layer development on the axisymmetric body in its roll motion. Fig. 4.1 shows a comparison of the frictional damping as given by the prediction methods and by measurements on a spheroid model. We can observe that the Kato-Tamiya prediction method is in reasonable agreement with the measurements.

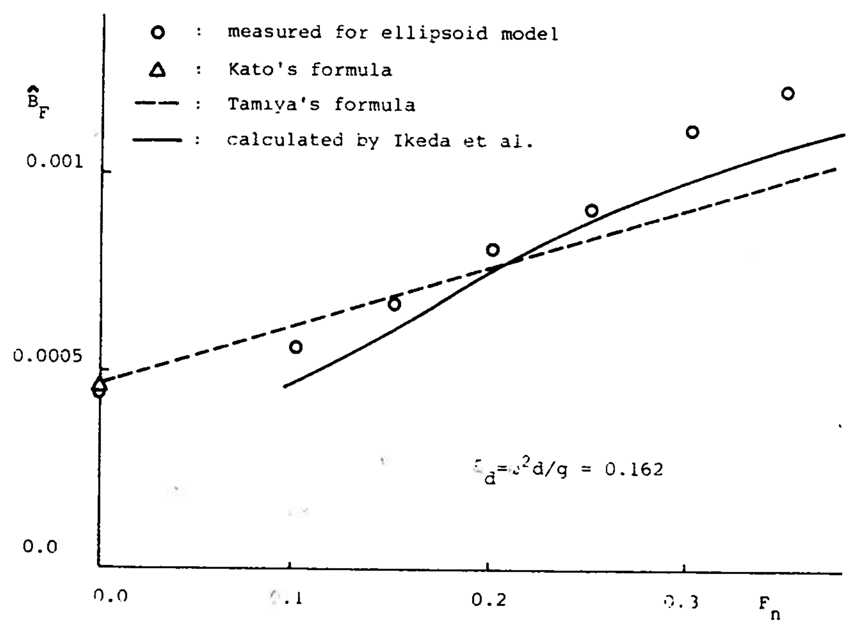

Figure 4.1. Frictional component of roll damping force

It has been claimed that this Kato-Tamiya method, Eq. (4.3) through Eq. (4.6), can safely be applied to the case of an actual ship hull form, since the ratio of friction damping to total damping is usually quite small. The exact treatment may be impossible theoretically and experimentally.

According to the above formula, the increment of frictional damping due to ship forward speed is proportional to the speed. When the frequency tends to zero at a finite speed, however, the B_F value becomes infinity, because of the dependence of the value B_{F0} on the roll Reynolds number. This point may be a problem to be studied.

It is noted that the scale effect is quite large, since, at the corresponding speed and the frequency, the non-dimensional value of the friction damping is almost inversely proportional to $\lambda^{0.75}$, where λ represents the scale ratio of the ship to the model. Therefore the value for the actual ship becomes 1/20 $\sim$ 1/30 times that for the model, so that we can safely ignore the friction damping of the actual ship.

4.3 Eddy Damping

In the absence of ship speed, this component is caused by flow separation at the bottom of the ship hull near stem and stern or at the bilge circle near the midship portion. The pressure drop in the separation region gives rise to this damping. Since early times, this has been treated in a manner similar to that for the drag problem in steady flow. Works by Bertin [25], Watanabe and Inoue [11], and Tanaka [4] have been carried out in this manner, in which the damping is assumed to have nonlinear form $B_2|\dot{\phi}|\dot{\phi}$, with the coefficient B_2 constant, depending only on the ship hull configuration.

In recent times, however, it has been found that the drag coefficient of a body in an oscillatory motion varies with the amplitude of the oscillation. The same situation may occur in the case of roll damping. Ikeda *et al.* [36] investigated this point experimentally for a number of two-dimensional cylinders with ship-like sections. In the experiments, the eddy damping was obtained by subtracting from the total measured damping (i) the wave damping, calculated or derived from the measured radiation wave height, and (ii) the calculated friction damping.

They confirmed through the analysis that the eddy damping coefficient can safely be considered to be constant in case of ship rolling. Figs. 4.2 and 4.3 show a couple of examples of the damping coefficient B'_{E0} measured for two-dimensional cylinders of shiplike sections.

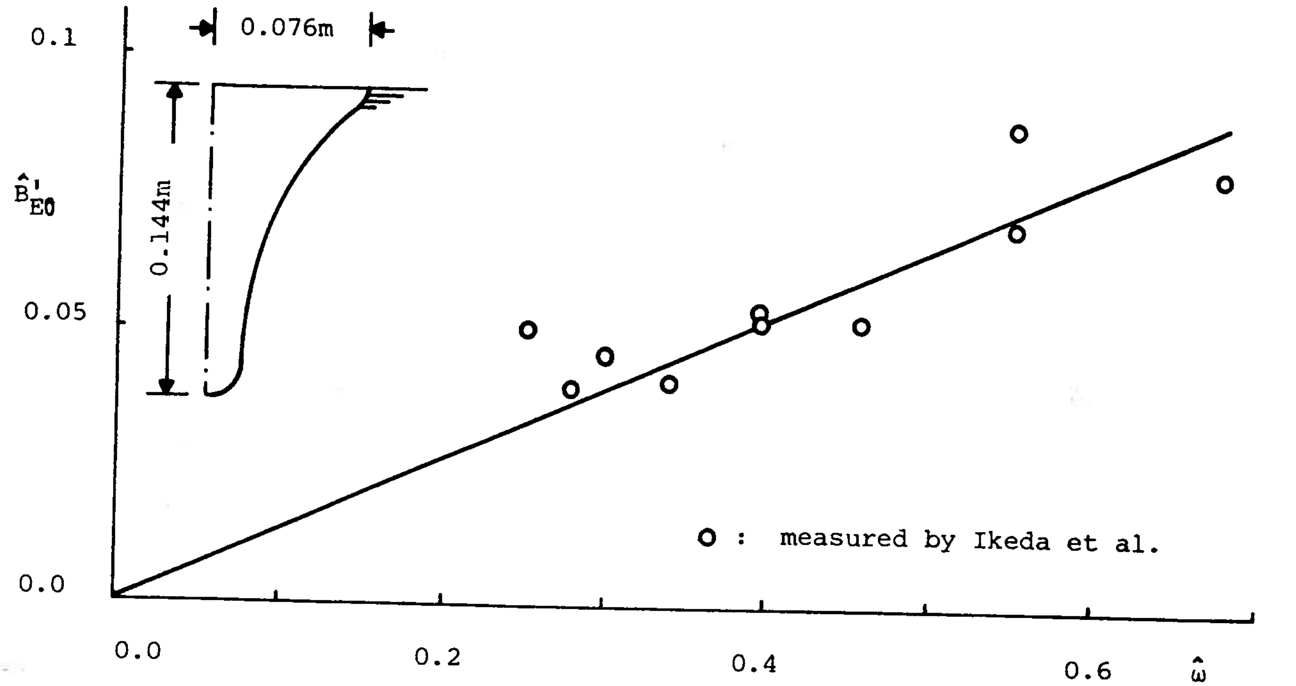

Figure 4.2. Eddy component of roll damping force for after body section with area coeffecients of 0.43.

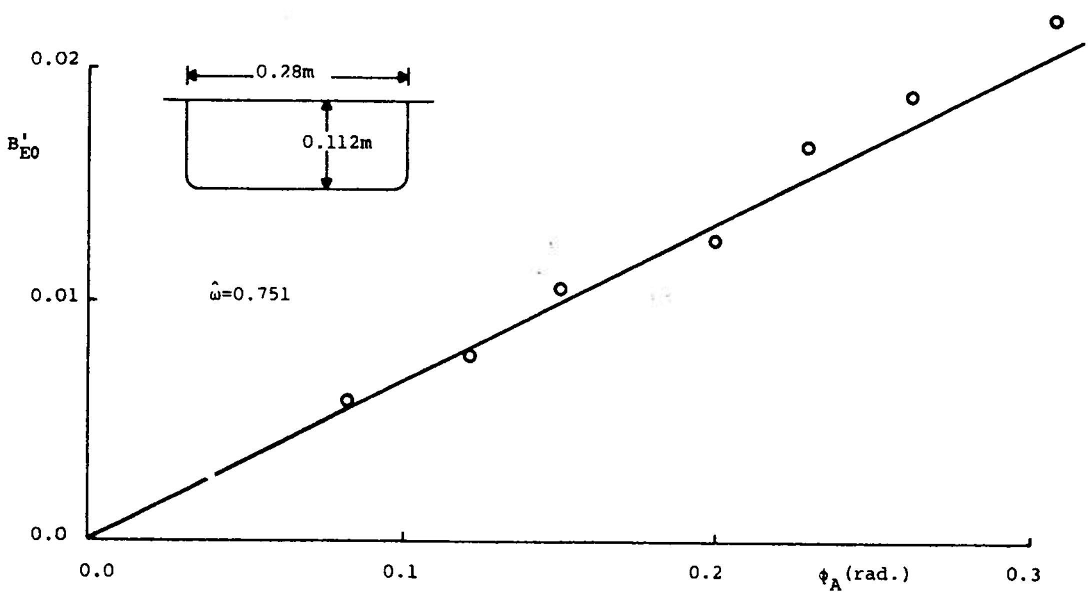

Figure 4.3. Eddy component of roll damping force for midship section, with area coefficient of 0.997.

They further proposed a new formula for the eddy damping for ordinary ship hull forms in a kind of extension of Watanabe and Inoue's approach. This can be written in terms of the 2-D cross-sectional coefficient:

$$B'_{E0} = \frac{4}{3\pi}\rho d^4 \omega \phi_A \left(\frac{r_{max}}{d}\right)^2 \cdot F\left(\frac{R}{d}, H_0, \sigma, \frac{OG}{d}\right) \cdot C_p \; , \qquad (4.7)$$

where d, r_{max}, R, H_0, σ and OG denote, respectively, draft, the maximum distance from the point G to the hull surface, bilge radius, half the beam-draft ratio, area coefficient of the section, and distance from the point O to G (downward positive). Thus the function F can be determined only by the hull shape and the pressure coefficient C_p by the maximum relative-velocity ratio on the hull; details will be stated in the Appendix. Integrating the sectional value over the ship length, we can obtain the eddy damping value for the given ship form.

In the presence of ship forward speed, on the other hand, the separated eddies flow away downstream, with the result that nonlinear damping decreases. Actually, linear lift damping prevails, as can be seen later, but we briefly consider this decrease of eddy damping.

Since the theoretical treatment is quite difficult, we can verify from the results of experiments that the amount of the decrease depends on the body shape as well as on the aspect ratio of the body. An example of this is show in Fig. 4.4, where the abscissa is the reciprocal of the reduced frequency $k = \frac{\omega L}{U}$. In the figure, eddy damping at forward speed has been derived by subtracting the lift damping (predicted separately) from the total measured damping; the wave effect has been excluded by covering the water surface with flat plates. For the case of the ship form in the figure, we can transfer the value of the extinction coefficient b , which can be obtained by free roll test, to the eddy damping coefficient B_E , because only the eddy damping among the component dampings is nonlinear in this case and the coefficient B_2 does not depend on the amplitude.

In Fig. 4.4, the values for the case of the flat plate with low aspect ratio represent the results of the measurements for a bilge-keel model by Yuasa *et al.* [48]. In this case, when U becomes large, the B_E value tends to a constant, say $B_{E\infty}$, which seems to depend on the aspect ratio of the plate.

For blunt bodies like ship forms, however, the value $B_{E\infty}$ can be regarded as zero. Ikeda *et al.* [37] have proposed an empirical formula for representing the decrease of eddy damping for arbitrary ship form:

$$B_E = B_{E0} \times \frac{0.04\omega^2 L^2}{U^2 + 0.04\omega^2 L^2} , \tag{4.8}$$

where the value B_{E0} for the zero-speed case can be predicted by Eq. (4.7).

Consequently, the eddy damping for a naked ship hull prevails only in the absence of advance speed; it decreases rapidly when the ship moves forward, so that it can be neglected in the high-speed range of Fn > 0.2. This fact has also been observed in the experiments by, say, Yamanouchi [12].

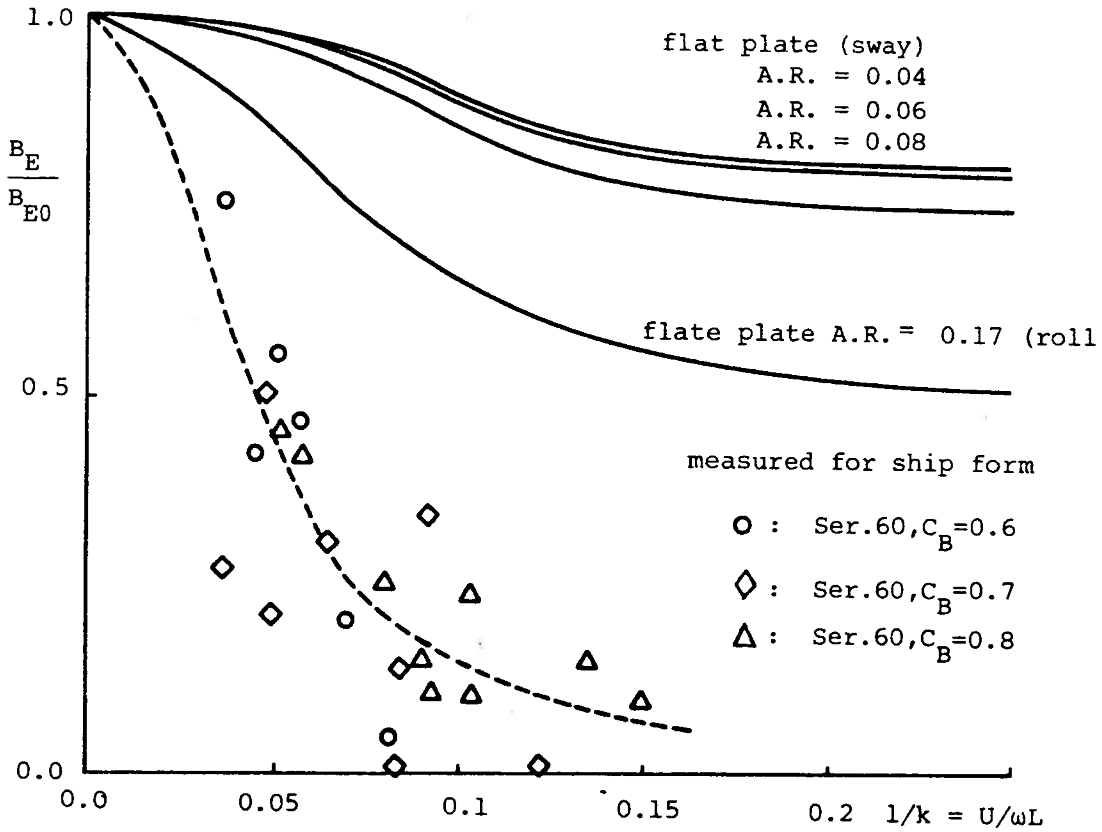

Figure 4.4. Effect of advance speed on eddy component

4.4 Lift Damping

Since a lift force acts on the ship hull moving forward with sway motion, we can imagine that a kind of lift effect occurs for ships in roll motion as well. A rigorous treatment of this is, however, still difficult for ordinary hull forms.

Yumuro *et al.* [38] derived a simple formula by applying the lateral-force formula used in the ship-maneuvering research field to the problem of roll damping. Accoring to his treatment, the damping moment M_L due to lift effect can be expressed in the form

$$M_L = \frac{1}{2}\rho L d U k_N \ell_0 \ell_R \dot{\phi} \ , \tag{4.9}$$

where

$$k_N = 2\pi \frac{d}{L} + \kappa(4.1\ B/L - 0.045) \ , \tag{4.10a}$$

$$\kappa = \begin{Bmatrix} 0 & & C_M \leq 0.92 \\ 0.1 & \text{for} & 0.92 < C_M \leq 0.97 \\ 0.3 & & 0.97 < C_M \leq 0.99 \end{Bmatrix} . \tag{4.10b}$$

In Eq. (4.9) and (4.10), k_N represents the derivative of the lift coefficient of the hull when towed obliquely. The lever ℓ_0 is defined in such a way that the quantity $\ell_0\dot{\phi}/U$ corresponds to the incidence angle of the lifting body. The other lever ℓ_R denotes the distance from the point O to the center of lift force. However, Ikeda *et al.* [37] modified the values for these levers originally assumed by Yumuro *et al.*, and they proposed another expression covering cases when the roll axis does not pass through the point O . The final prediction form can be expressed in terms of equivalent linear damping:

$$B_L = \frac{\rho}{2} U L d k_N \ell_0 \ell_R \left[1 - 1.4\frac{OG}{\ell_R} + \frac{0.7\ OG}{\ell_0 \ell_R} \right] \ , \tag{4.11}$$

where

$$\ell_0 = 0.3\,d \ , \quad \ell_R = 0.5\,d \ . \tag{4.12}$$

To obtain the lift-damping value experimentally, one must exclude wave effects by covering the free surface with flat plates or by carrying out the measurement in a low frequency range so that wave damping can be neglected.

Fig. 4.5 shows a result of measurements at low frequencies, in which friction damping has been subtracted from the data by using the prediction formula. In this figure, therefore, the experimental data represent the sum of the lift and eddy components. Since in the high speed range eddy damping can be neglected and, further, lift damping is proportional to the advance speed, we can safely regard the data at high speed as representing lift damping. The solid line in the figure shows the predicted values which agree well with the measured values in spite of the fact that the former have been derived on the basis of a very simple assumption.

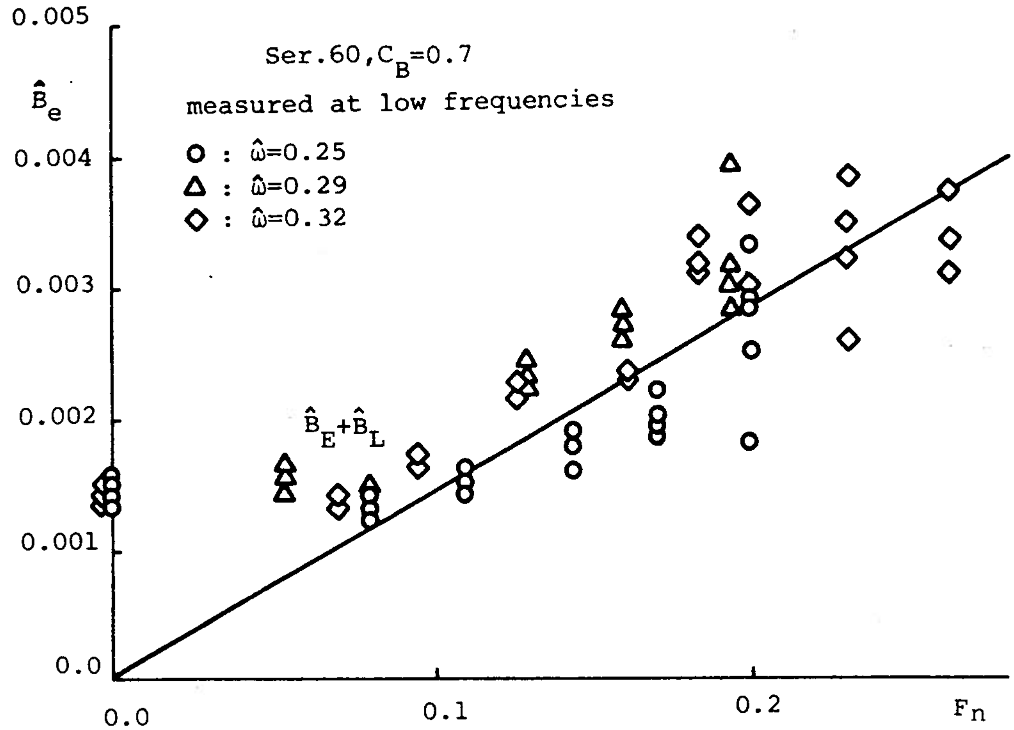

Figure 4.5. Sum of eddy and lift components of roll damping force

It can be concluded that lift damping is linear and that its coefficient is independent of ω and proportional to ship speed, so that it has an important role in the total damping at high speed. Particularly for ship forms such as a container ship or a car ferry, in which the roll natural frequency is quite low, lift damping becomes the most important component.

It must be noted finally that the prediction formula stated above may not cover all the varieties of ship forms, as pointed out by Ikeda *et al.* themselves. It fails in cases of small draft-beam ratio and of ballast condition of ships, since it is based on the assumption that the lifting effect of ship form is approximately represented by that of a flat plate with the same length and draft. It is necessary to develop a more rigorous treatment of this component.

4.5 Wave Damping

In the case of zero Froude number, the wave damping can easily be obtained by using the strip method, which has been well established recently. Otherwise we could even solve numerically the exact wave problem for a three dimensional ship hull form. In the strip method, the wave damping for a ship section is calculated from the solution of a two-dimensional wave problem, taking the form

$$B'_{W0} = \rho N_S(\ell_w - OG)^2 \quad , \tag{4.13}$$

where N_s and ℓ_w represent the sway damping coefficient and the moment lever measured from the point O due to the sway damping force.

Although the direct measurement of wave roll damping is impossible, we can use the relationship of wave damping to the radiation wave-amplitude ratio $A_R = \zeta_A/\phi_A d$, ζ_A the radiation wave amplitude, and we can compare it with the measurements by Takaki and Tasai [15]. The predicted values by strip theory agree fairly well with the measured, as shown in Fig. 4.6. Therefore it can be considered that there is little problem in predicting the wave roll damping of an ordinary ship form at zero speed by the strip method with practical accuracy.

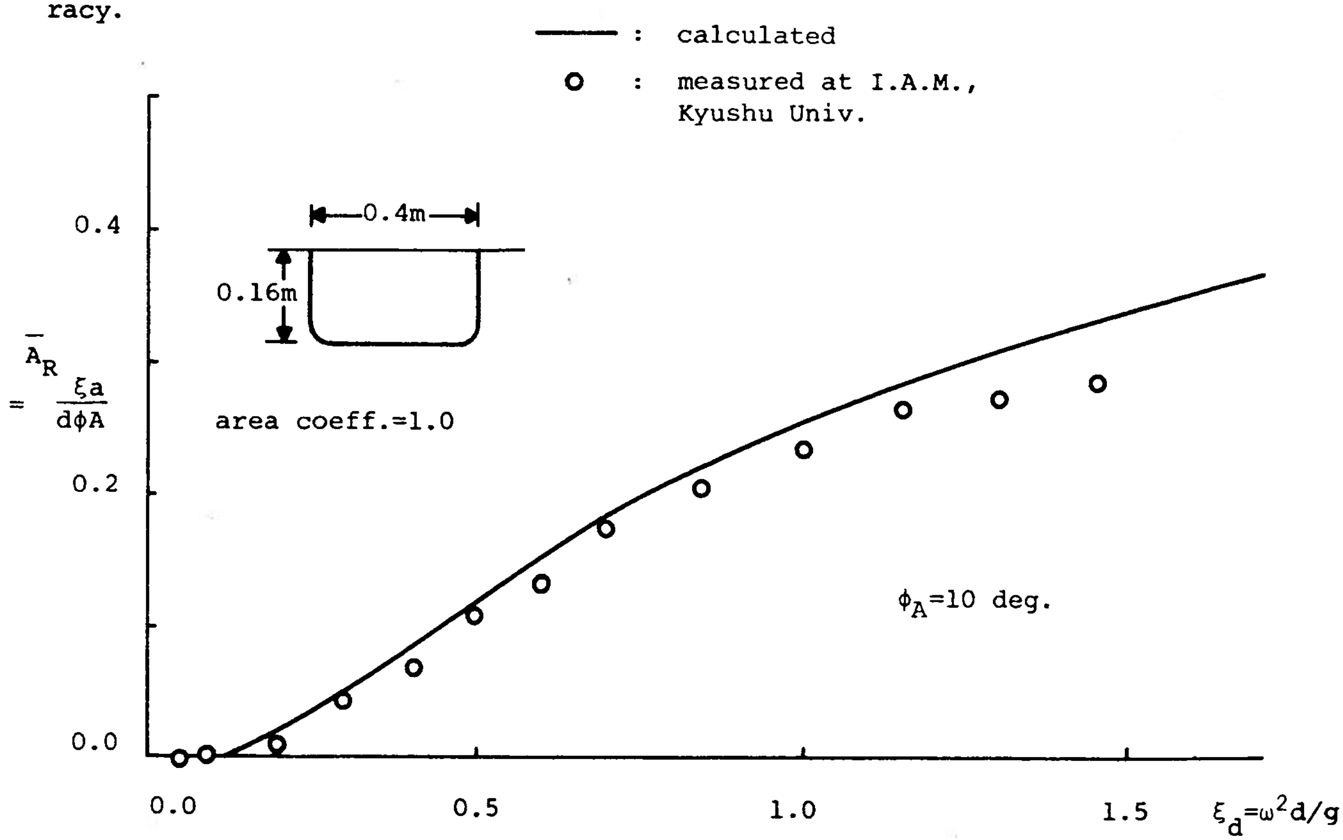

Figure 4.6. Radiation wave amplitude for Lewis form cylinder

In the presence of ship speed, on the other hand, it is quite difficult to treat the wave roll damping theoretically. Several approaches, for example, Newman and Tuck [39], Joosen [40], Maruo [41], Ogilvie and Tuck [42], etc., to improve the strip method by use of slender-body theory have recently been attempted. Troesch [43] has evaluated the solutions of Ogilvie and Tuck's theory for lateral ship motions. However, a definite improvement in the prediction of roll damping at forward speed has not been reached, since Troesch's result shows that the first higher-order correction to roll damping is zero according to the slender body theory.

A method of wave-pattern analysis to obtain the radiation potential of ship motions at forward speed has recently been proposed by Ohkusu [44]. It is hoped that this method will make it possible to measure the roll wave damping separately. Otherwise, we can approximate wave damping by subtracting all other predictable components from total damping obtained in the forced-roll test. The results show that wave damping behaves in a somewhat complicated manner, including hump/hollow undulations.

We can cite here a couple of approximate treatments for predicting the wave damping at forward speed. The first is the method in which the flow field due to roll motion is expressed by oscillating dipoles with horizontal lateral axes; then roll damping is obtained approximately from the wave-energy loss in the far field. Hishida [9] first applied this treatment to the sway motion of an axisymmetric ellipsoid to obtain the characteristics of wave roll damping at forward speed.

Ikeda *et al.* [37] calculated the energy loss in the far field due to a pair of horizontal doublets and compared the results with experiments for models of combined flat plates. Through these elementary analyses they proposed an empirical formula for roll damping of ordinary ship forms:

$$\frac{B_W}{B_{W0}} = 0.5\,[\,\{(A_2+1) + (A_2-1)\tanh(20\tau - b)\} + (2A_1 - A_2 - 1)\exp\{-150(\tau - 0.25)^2\}] \quad , \tag{4.14}$$

where

$$\left.\begin{aligned} A_1 &= 1+\xi_d^{-1.2}e^{-2\xi_d} \quad , \\ A_2 &= 0.5+\xi_d^{-1}e^{-2\xi_d} \quad , \\ \xi_d &= \omega^2 d/g \quad , \quad \tau = U\omega/g \quad . \end{aligned}\right\} \tag{4.15}$$

A comparison with experiments is shown in Fig. 4.7, in which a hump in wave damping appears near the point $\tau = 1/4$.

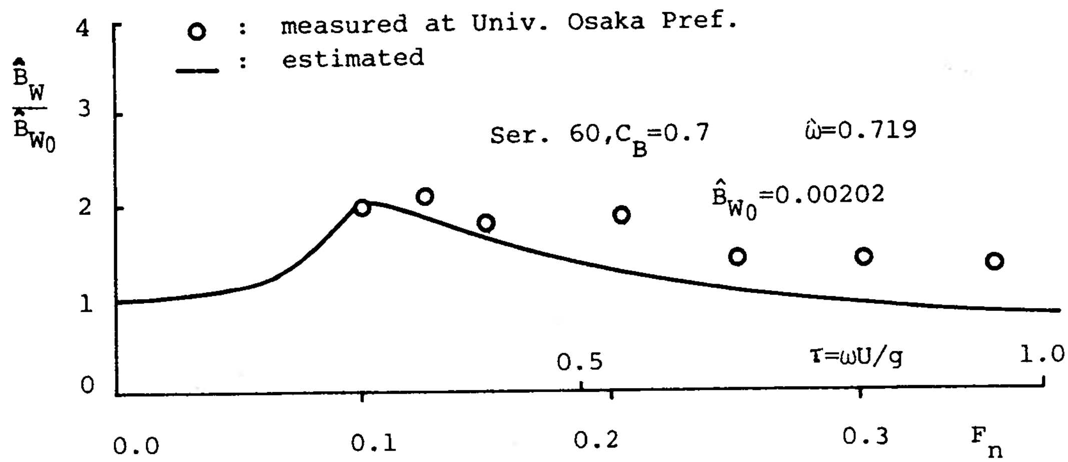

Figure 4.7. Effect of advance speed on wave component

In the second approximate method, the rolling ship hull is regarded as a lifting body and the wave-wake problem for this flow is solved. Hanaoka [10] first set the equation system for the flat plate with low aspect ratio, rolling about its longitudinal axis. Watanabe [45] has recently obtained the numerical solution of the equation, with the result that the roll damping is expressed as a sum of lift and wave damping in the form

$$B_L + B_W = F(Fn, \tau, \xi_d) \quad , \tag{4.16}$$

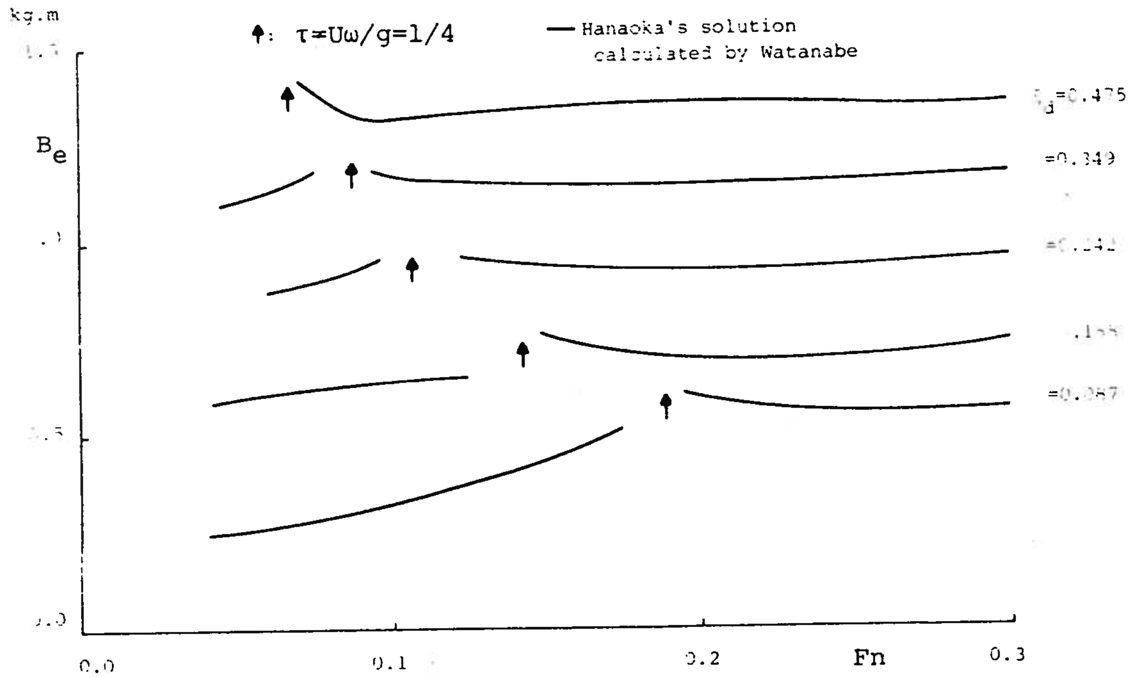

Figure 4.8. Effect of advance speed on wave and lift components

where the detailed expression is omitted here. Fig. 4.8 represents a result of Watanabe's calculation, which shows a tendency similar to that of the former method in that there is a hump at $\tau = 1/4$ and the value increases as the frequency increases. This trend also agrees completely with that of experiments.

However, it appears that there are still some difficulties to be conquered in both methods: In Hanaoka and Watanabe's method, the lift damping is based on that of a flat plate, not a ship hull, and in the method of Ikeda *et al.* there is a limitation in application to ship forms, particularly to the case of small draft-beam ratio. It is hoped to establish a more rigorous treatment.

4.6 Bilge-Keel Damping

As stated in the preceding section, bilge-keel damping is defined as an increment of damping when bilge keels are installed. It therefore includes not only the damping of the bilge keels themselves but also all the interaction effects among the bilge keels, the hull and the waves. Tanaka [4] and Kato [5] separately proposed empirical formulas for bilge-keel damping in which the effect of a variety of ship forms was partly taken into account in terms of a modification coefficient. Cox and Lloyd [46] also obtained a formula for the bilge-keel drag at zero ship speed using the Martin [3] and Ridjanovic [47] experimental data. Sasajima [6] attempted to formulate the hull pressure change due to eddy shedding from the edge of a bilge keel. Watanabe and Inoue [11], as mentioned previously, also dealt with this problem as an extension of Bryan's treatment. Hishida [9] discussed wave-damping due to bilge keels in terms of a pair of dipoles on the hull with axes tangential to the hull.

It can be concluded through these works that bilge-keel damping is not merely a quadratic nonlinear form, but that it depends on the roll amplitude and the frequency in a more complicated manner, and further that the effect of ship forward speed is not so large as we might expect.

The physical meanings of these facts have been much more clarified by the recent works of Yuasa *et al.* [48], Ikeda *et al.* [49], and Fujino *et al.* [50], which are based on recent developments in research on bluff-body drag in oscillatory motion. These works will be described in the subsequent sections.

4.7 Normal-Force Damping of Bilge Keel

To begin with, let us consider the case of zero ship speed. Much work on the drag force on a bluff body in oscillatory motion has recently been carried out, mainly in the ocean engineering field [51], [52], [53], [54]. It has also been attempted to apply the results of these works to the problem of bilge-keel drag.

Let the coefficient C_D of a body be defined in the form

$$F = C_D \cdot \frac{1}{2} \rho A |v| v \quad , \tag{4.17}$$

where F represents the drag force, A the area of the body projected onto the crossplane normal to direction of motion, and v the velocity of motion. Although C_D is assumed to be constant during the specified motion, its value is known to vary with the period parameter or Keulegan-Carpeter number, VT/D (V is the maximum speed $|v_{max}|$, T is the period and D the maximum projected breadth), contrary to the case of steady flow. In the case of periodic oscillatory motion, especially of bilge-keel motion, the parameter becomes $\pi r\phi_A/b_{BK}$ when we substitute $r\phi_A\omega$, $\frac{2\pi}{\omega}$ and $2b_{BK}$ for V , T and D respectively, where r represents the mean distance from G to the bilge keel and b_{BK} the breadth of the bilge keel. This means that the parameter can be regarded as a sort of amplitude ratio since it depends no longer on the period of the oscillation.

The drag of the bilge keel can be expressed by the following formula, which was obtained by Ikeda *et al.* [35] (including the case of an oscillating flat plate), as shown in Fig. 4.9:

$$C_D = 22.5\,\frac{b_{BK}}{\pi r\phi_A} + 2.40 \quad . \tag{4.18}$$

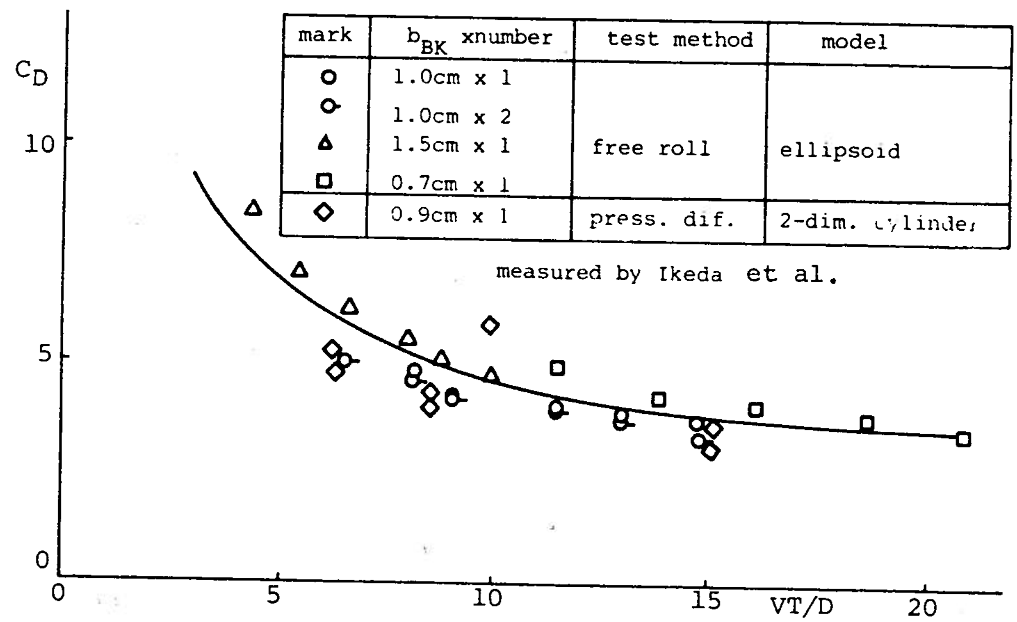

Figure 4.9. Drag coefficient of bilge keel

Its validity has also been confirmed by Takaki's experiment [54] except for large amplitude, and a dependence on the amplitude ratio has appeared also in the Cox and Lloyd formula.

However, in order to obtain the normal-force damping of the bilge keel installed on a ship hull with comparatively small bilge radius, it is necessary to consider some modifications to Eqs. (4.17) and (4.18). Ikeda *et al.* [34] assumed that the area A in Eq. (4.17) can be replaced by b_{BK} per unit length and the velocity should be multiplied by an empirical coefficient f of velocity increment at the bilge circle. Then the damping takes the form

$$B'_{BKN_0} = \frac{8}{3\pi}\rho r^2 b_{BK}^2 \omega f^2 \left\{\frac{22.5}{\pi f} + 2.40 \frac{r\phi_A}{b_{BK}}\right\} , \tag{4.19}$$

$$f = 1 + 0.3 \exp\left\{-160(1-\sigma)\right\} , \tag{4.20}$$

where σ denotes the area coefficient of the ship hull section.

Fujino *et al.* [50] have recently assumed in a somewhat different way that the quantity D in the definition of the period parameter should be replaced by $b_I = b_{BK} + \frac{Rb_{BK}}{R + b_{BK}}$, where R is the bilge-circle radius; they include the effect of the mirror image with respect to the bilge circle, instead of taking $D = 2b_{BK}$, as Ikeda *et al.* did. Moreover, Fujino *et al.* defined f as the average velocity increment at the position of the bilge keel and they calculated its value for several ship sections using the finite element method. The expression for damping, however, takes a form similar to Eq. (4.19).

Considering the contribution of the amplitude ϕ_A in Eq. (4.19), the first term in the bracket is independent of the amplitude, so that it becomes apparently a linear term. This is because the effect of period parameter on the drag coefficient occurs in the damping terms, or, in other words, the B_2 coefficient varies with amplitude so that a part of B_2 is seemingly transferred into the B_1 coefficient. Therefore Eq. (2.17), expressing the term-by-term comparison between the damping and extinction coefficients, does not hold in this case in the sense of their original definition. For the case of a naked hull, as seen previously, the effect of period parameter is fortunately small, so that we can safely take B_2 as a constant.

The values predicted by Eq. (4.19) for the case of a circular cylinder with bilge keels agrees fairly well with the measured values, as shown in Fig. 4.10.

In the presence of ship forward speed, on the other hand, it is known from the work of Yuasa *et al.* [48] that the C_D value of bilge keels decreases slightly, as can be seen in Fig. 4.4. Instead, linear damping appears, due to the lifting effect of bilge keels. Yuasa *et al.* applied the approximation of low-aspect-ratio wings to the case of bilge keels with the result that the normal-force damping can be expressed in the form

$$B_{BKN} = B_{BKN_0} + \frac{\pi}{2}\rho b_{BK}^2 r^2 U \quad . \tag{4.21}$$

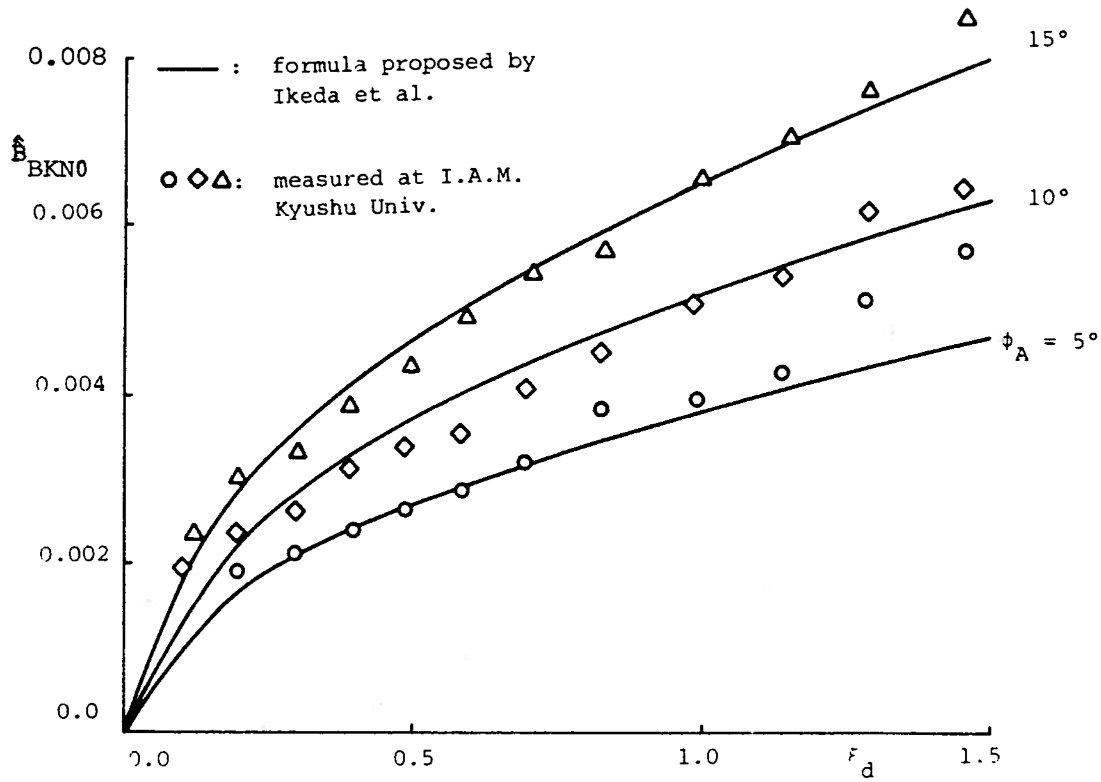

Figure 4.10. Component due to normal force on bilge keel

A similar form was also adopted by Schmitke [31]. Although the decrease of nonlinear damping is not represented in Eq. (4.21), the contribution of the linear lifting term seems to agree with the experimental results, as shown in Fig. 4.11, in which the ordinate C_{De} denotes an equivalent nonlinear drag coefficient of bilge keel.

Recently Fujino *et al.* [50] have shown an impressive way of numerically treating the drag of bilge keels at forward speed. They have assumed that the flow around a bilge keel corresponds to the steady flow around a low-aspect-ratio wing with breadth b_I as defined before and with length equal to that of the bilge keel. Taking the effect of the velocity increment at the bilge circle into the expressions of the incident angle and the incoming velocity of the wing, they have obtained the numerical solution of the bilge keel drag

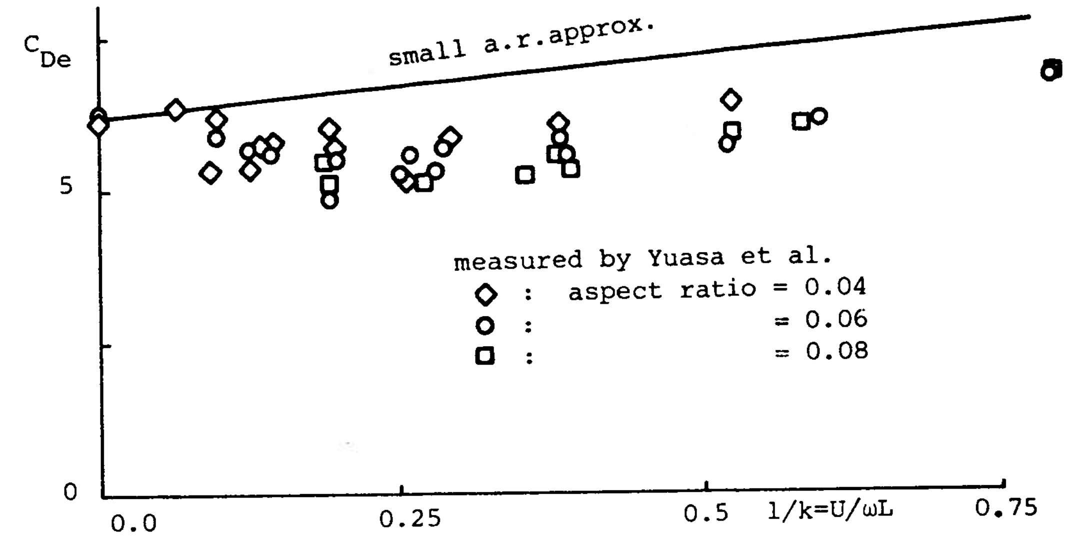

Figure 4.11. Effect of advance speed on drag coefficient of bilge keel

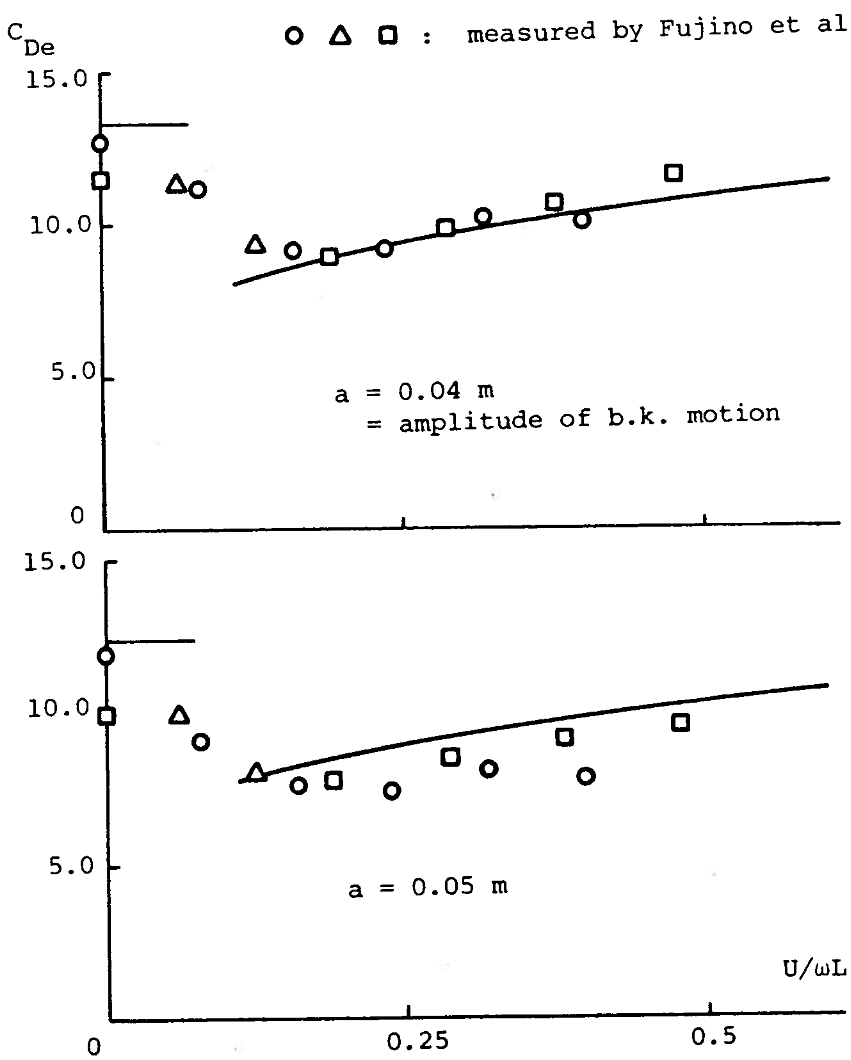

Figure 4.12. Fujino's prediction for normal force on bilge keel

on the basis of Bollay's nonlinear wing theory [56]. As we can see in Fig. 4.12, the results seem to be quite promising. In the figure, the estimated value at zero ship speed comes from their method, mentioned previously.

In conclusion, it can be noted here that the slight increase of bilge keel drag in the high-speed range is due to the linear or nonlinear lifting effect; the problem of the slight decrease of drag at low speed is still left for future study. As a practical treatment at the present time, one might assume that the change of bilge-keel drag with ship speed is negligible, since these two effects cancel each other for ordinary dimensions of bilge keels, as seen in Fig. 4.11.

4.8 Hull-Pressure Damping Due to Bilge Keels

Although this component may be a part of the eddy damping due to the interaction between bilge keels and ship hull, it is convenient to consider it as an independent component for obtaining a prediction formula. According to the works of Ikeda *et al.*, the difference in the hull pressure with and without bilge keels is regarded as an effect of the bilge keels. The hull pressure difference p can be defined as follows, by analogy with C_D for the case of normal-force damping of bilge keels:

$$p = C_p \frac{1}{2} \rho |v_\phi| v_\phi \quad . \tag{4.22}$$

Here, v_ϕ represents the instantaneous relative velocity at bilge circle, which can be assumed to be $v_\phi = fr\dot{\phi}$, with the coefficient f standing for the velocity increment ratio, as stated in the previous section. It is noted that the expression for pressure, Eq. (4.22), does not include the term $\rho\frac{\partial\Phi}{\partial t}$ (Φ is the velocity potential), since this term seems at present to make a small contribution to the damping in the absence of a free surface; further study on this point may be needed.

The distribution of the pressure-difference coefficient C_p on the ship hull, Eq. (4.22), takes the shape shown in Fig. 4.13, according to the experiments by Ikeda *et al.* [49] and Goda and Miyamoto [57]. The positive pressure C_p^+ in front of the bilge keel is not affected by the displacement of the bilge-keel motion, while the negative value C_p^- at the rear of the bilge keel and its effective extent S_0 around the hull depend strongly on the value of period parameter, as does the drag coefficient of the bilge keels.

Ikeda *et al.* made assumptions on the shape of the C_p distribution and on the values of C_p^+ and C_p^- to establish a formula for the damping due to this pressure difference, as follows:

$$\left.\begin{aligned} B_{BKH_0} &= \frac{4}{3\pi}\rho r^2 d^2 \omega \phi_A f^2 I \quad , \\ I &= \frac{1}{d^2}\int C_p \, \ell_0 \, ds \quad . \end{aligned}\right\} \qquad (4.23)$$

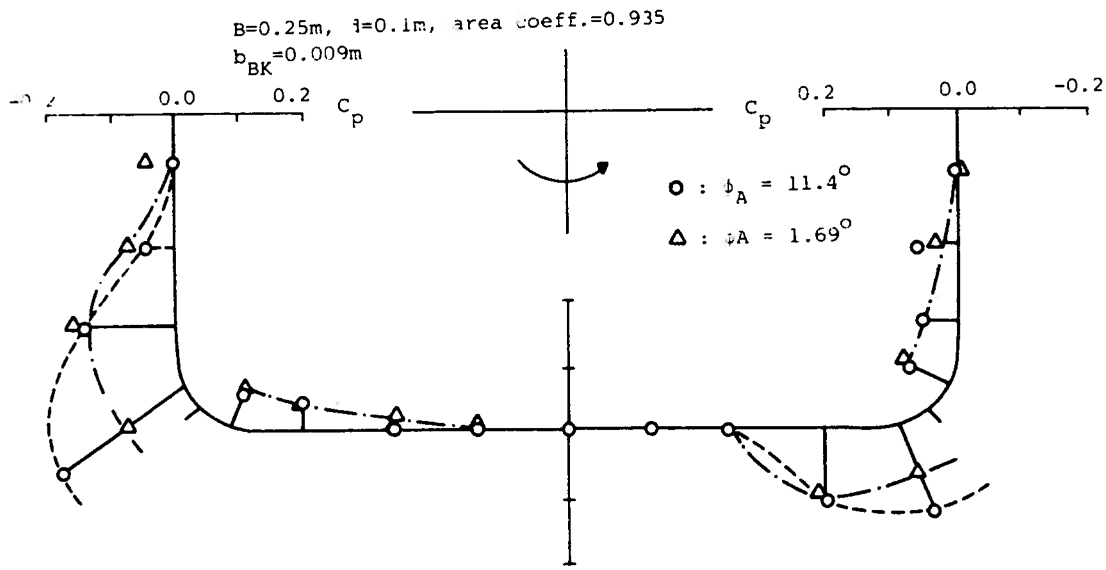

Figure 4.13. Pressure distribution on hull induced by bilge keel

The Integration of I must be done around the whole girth, with the integrand C_p multiplied by the moment lever ℓ_0 about the rotation axis. The detailed expression will be stated in the Appendix. The value of I can be determined as a function of the hull shape and the period parameter of the motion. Therefore it includes not only the apparent nonlinear term of ϕ_A, similar to that of the previous section, but also the higher-order terms of the third and fourth powers of ϕ_A, although their contribution is small.

Since it is difficult to measure this component separately, the prediction for the total bilge-keel damping without the effects of surface waves, that is, B_{BKN} plus B_{BKH} , can be compared with experiments, as shown in Fig. 4.14. The agreement between them seems to be fairly good.

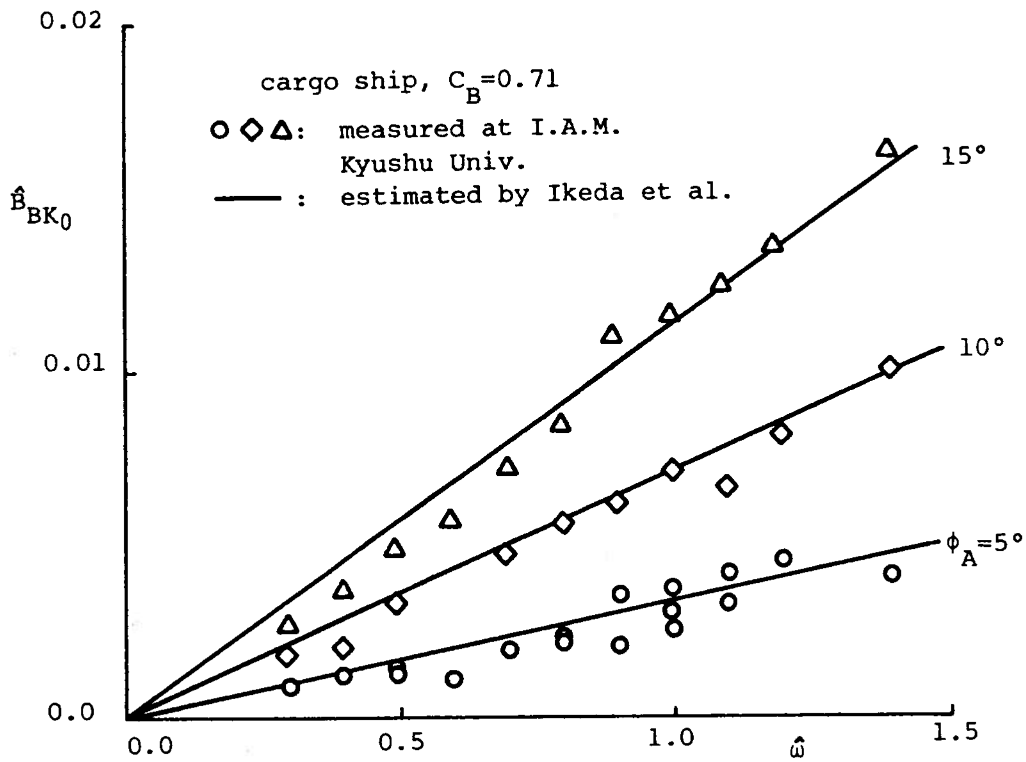

Figure 4.14. Effect of bilge keel on roll damping coefficient at zero Froude number

The research for this component in the presence of ship forward speed does not seem to have been done. However, by analogy with the case of the drag coefficient one may suppose the pressure difference between front and rear surfaces of a bilge keel decreases slightly and that the width S_0 of its distribution on the hull also decreases slightly with forward speed. Consequently hull-pressure damping decreases by a corresponding amount. This amount, however, will compensate the increment of normal-force damping due to the lifting effect, since it is known that total damping of bilge keels does not vary much. We have just to remember here that the nature of bilge-keel damping changes from being nonlinear to being more nearly linear as ship speed increases.

4.9 Wave Damping of Bilge Keel

As an example of the study of this component in early times, we cite Hishida's theoretical treatment [9]. Let the radiation waves due to the roll motion of a bare two-dimensional cylinder be described by $\zeta_H \cos(k_o y - \omega t)$ and the change when bilge keels are installed by $\zeta_{BK} \cos(k_o y - \omega t + \varepsilon)$, where k_0 represents the wave number, ω^2/g , y is the horizontal axis, t the time and ε a phase lag. Then the amplitude ζ_A of the resultant wave can be expressed in the following form:

$$\zeta_A^2 = \zeta_H^2 + \zeta_{BK}^2 + 2\zeta_H \zeta_{BK} \cos\varepsilon \quad . \tag{4.24}$$

According to Hishida's consclusion, although the value of ζ_{BK} may be as large as that of ζ_H in some cases, the magnitude of ζ_A and thus of the damping may not always increase with the addition of bilge keels, because of the difference of phase of the wave systems.

Takaki [23] has recently carried out the calculation of roll damping of ship-like sections with bilge keels using the close-fit method. From the results, as shown in Fig. 4.15, it can be said that linear theory agrees with experiments only in case of small roll amplitude. There is still a considerable discrepancy for larger amplitude, probably due to the non-linear effect of wave damping.

However, it can be noted that for bilge keels with ordinary breadth of $b_{BK} = B/60$ to $B/80$, we can safely neglect the wave effect of bilge keels, since the contribution of this component is usually quite small compared to the viscous damping caused by bilge keels.

In the presence of ship speed, no study of this component has been made. For ship forms with relatively large-sized bilge keels, like a warship, it may be necessary to take the wave effect into consideration in cases both with and without advance speed.

At the present time, however, it can be concluded that the total damping increment due to bilge keels can be predicted as the sum of two bilge-keel dampings, B_{BKN} and B_{BKH} .

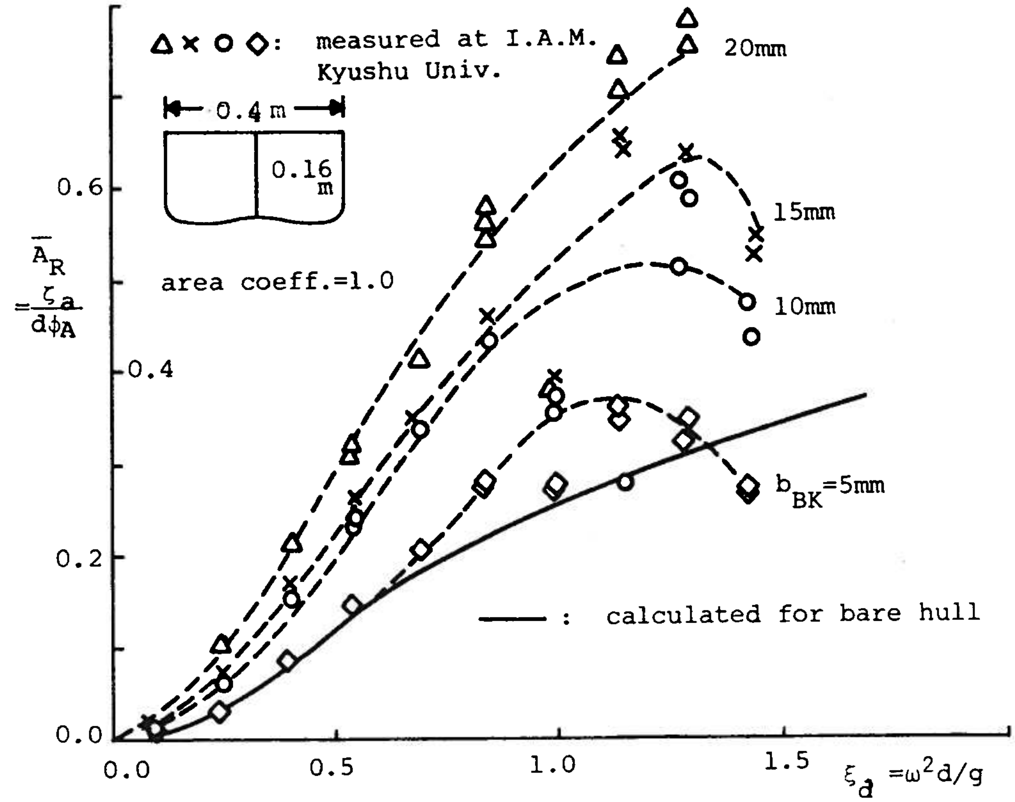

Figure 4.15. Radiation wave amplitude for cylinder with bilge keels

4.10 Prediction of Total Damping

The component dampings that were introduced in the preceding sections can be summed up to give the total damping of ship rolling. Either Eq. (4.1) or Eq. (4.2) can be used:

$$B_e = B_F + B_E + B_L + B_W + B_{BKN} + B_{BKH} + B_{BKW}\ , \tag{4.1}$$

$$B_e = B_F + B_E + B_L + B_W + B_{BK}\ . \tag{4.2}$$

In addition, we can add the contribution of appendages to these expressions, as Schmitke [31] has done. Although not every individual component has been well-established for all ship forms, it is worthwhile to note that the concept of dividing the damping into these components should enable us to predict ship roll damping more accurately than by the simple methods. At present, we can indeed choose valid prediction formulas for the component dampings; these will be identified in the next section. Here we discuss the general properties of the prediction procedure and the component dampings.

In the absence of ship speed, since only the lift term B_L becomes zero, the other terms are predicted by integrating sectional values along the length of the ship, as in the strip method (however, in Kato's formula, the friction damping for the whole ship form is still available). Therefore we can also obtain the longitudinal distribution of the roll damping. We can compare it with the experiments on a divided model, as performed in the Ship Research Institute of Japan [23].

At forward speed, however, the modification to the individual components is made for the whole ship form, not for each ship section. In this case we cannot obtain the damping distribution, which presents an important problem for future study.

Comparisons of these component dampings are shown schematically in Figure 4.16 with the abscissa Fn, in Fig. 4.17 with frequency $\hat{\omega}$, and in Fig. 4.18 with the amplitude ϕ_A. Although bilge-keel dampings are shown as nearly constant in Fig. 4.16, the linear part increases as Fn becomes large. This means that in Fig. 4.18 the values of bilge keel dampings in the limiting case $\phi_A = 0$ increase but the slopes of the corresponding curves decrease as Fn increases.

It can also be seen from Fig. 4.18 that the linear dampings consist of B_L, B_F, B_W and B_{BKW}, as well as parts of B_{BKN} and B_{BKH}. The nonlinear part with the constant B_2 coefficient includes B_E and parts of B_{BK}. The terms B_F and B_{BK} also depend on the frequency as shown in Fig. 4.17, although they appear to be linear with respect to amplitude. This can be realized from the facts that they were originally defined as being nonlinear but then the coefficients varied with Reynolds number in the case of B_F and with the period parameter in the case of B_{BK}.

The scale effect on roll damping is an important problem. No detailed experiments on actual ships have been carried out to determine the component dampings, so that we do not yet have an exact picture. However it can be considered as in the case of an oscillatory flat plate that dampings like B_E and B_{BK} that are generated by separating bubbles may not be affected by the scale difference between model and ship. The almost linear dampings like B_L and B_W are of course independent of the scale effect. Therefore only the frictional damping suffers from the scale effect. For instance,

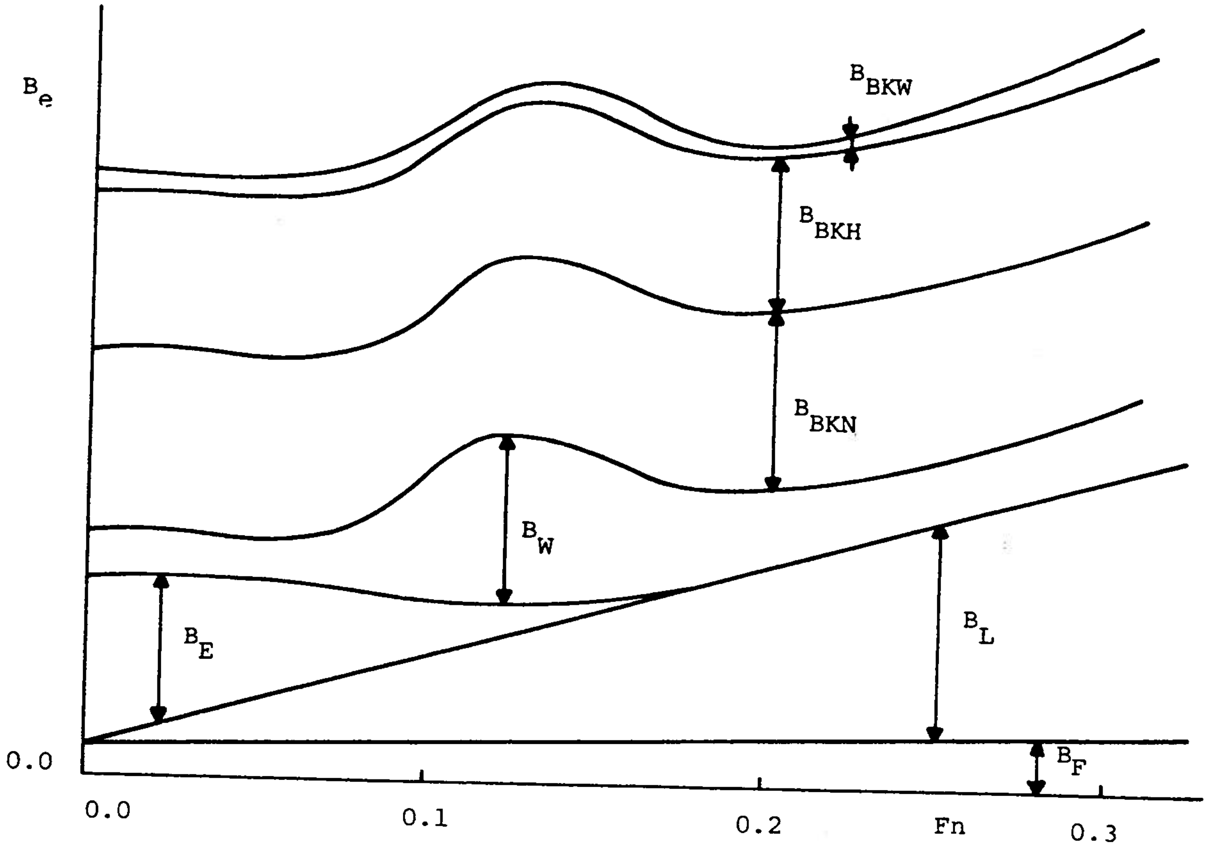

Figure 4.16. Schematic view of roll damping components with advance speed

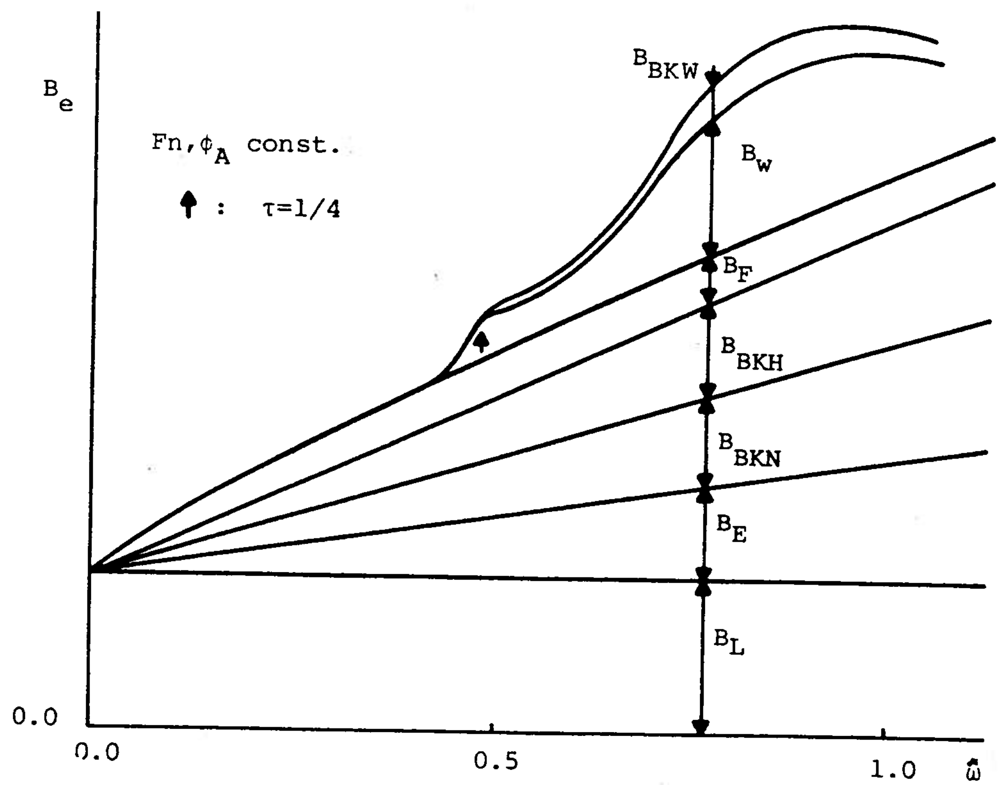

Figure 4.17. Effect of roll frequency on roll damping components

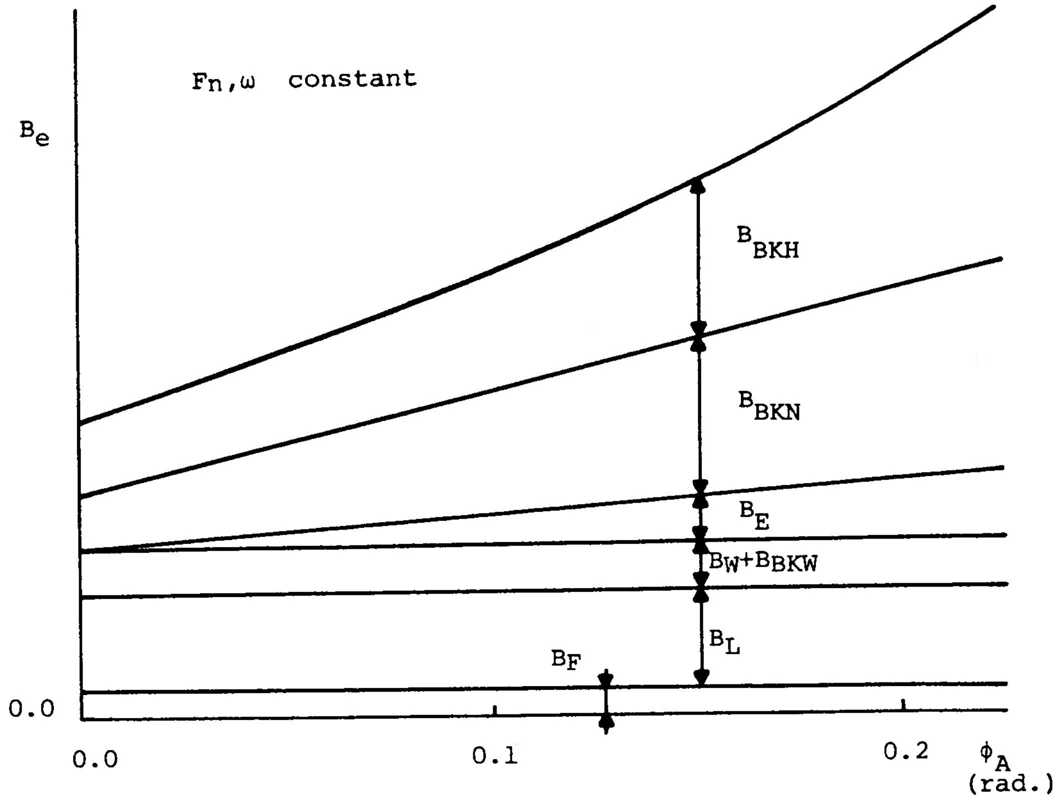

Figure 4.18. Nonlinear effect of roll damping components

when the ship roll damping is to be predicted by scaling up from the model value, the $\hat{B}_e$ value for the ship can be obtained only by replacing the B_F value by that of the actual ship under the same conditions of Fn , $\hat{\omega}$ and ϕ_A as for the model, although B_F may be neglected in the ship case.

Concerning the independence among component dampings, there still remain problems. All the interaction effects have been included in proper components by their definitions in Sec. 4.1. However, in the present prediction formulas, the interactions are usually ignored, except for the hull-pressure change due to bilge keels, which can be regarded as the largest interaction term. The other interaction effects may exist, and so they may affect the independence of components. However, we can expect their magnitudes not to be large, since almost all the component dampings have been determined and formulated through corresponding experiments that already implied the interaction effects.

4.11 Comparison with Experiment

Although it is still necessary to improve the prediction method for component dampings, we can at present adopt an adequate set of prediction formulas for component dampings for ordinary ship hull forms. The following is an example of the second (best) method which can be chosen in a practical point of view.

At first we can safely use Kato and Tamiya's formula for the frictional damping B_F. Ikeda and others' method can be taken for B_E. The lift damping B_L is given by the formula of Yumuro, modified by Ikeda and others. The wave damping can be obtained by the strip method in case of zero Froude number and by Ikeda and others' formula at advance speed. The latter is preferred over Hanaoka and Watanabe's method, which does not yet yield an adequate result in magnitude, in spite of the fact that it is based on a more rigorous assumption.

For the bilge keel damping, Fujino's treatment seems to be the most reasonable one at present time. However, since it does not include hull-pressure damping due to bilge keels, B_{BKH}, an alternative way is to adopt Ikeda and others' method for predicting B_{BKN} and B_{BKH} at zero forward speed and to assume that the bilge keel damping does not vary with forward speed. Furthermore we can neglect both the wave damping of bilge keels B_{BKW} and the effect of the other appendages on the hull.

Using the set of formulas thus chosen, we can calculate the roll damping of a ship once the hull form and the roll conditions like Fn, $\hat{\omega}$, and ϕ_A are given. A computer program for this is presented in the Appendix.

Examples of results are shown in Figs. 4.19 and 4.20 with the abscissa Fn and in Figs. 4.21 and 4.22 with the frequency as the abscissa. We can recognize from Figs. 4.19, 4.20 and 4.21 that the magnitude and even the tendency of the roll damping are predicted by this method to sufficient practical accuracy. At the same time, however, Fig. 4.22 shows that there still appears some discrepancy in the trend, probably of the wave damping at forward speed. Moreover it has been reported by Ikeda *et al.* themselves [37] that this method fails in cases of small draft-beam ratio or in ballast condition, because of the difficulty of predicting the lift damping.

In conclusion, it can be said that this method can safely be applied to the case of ordinary ship hull form with single screw and rudder if the ship is in its normally loaded condition.

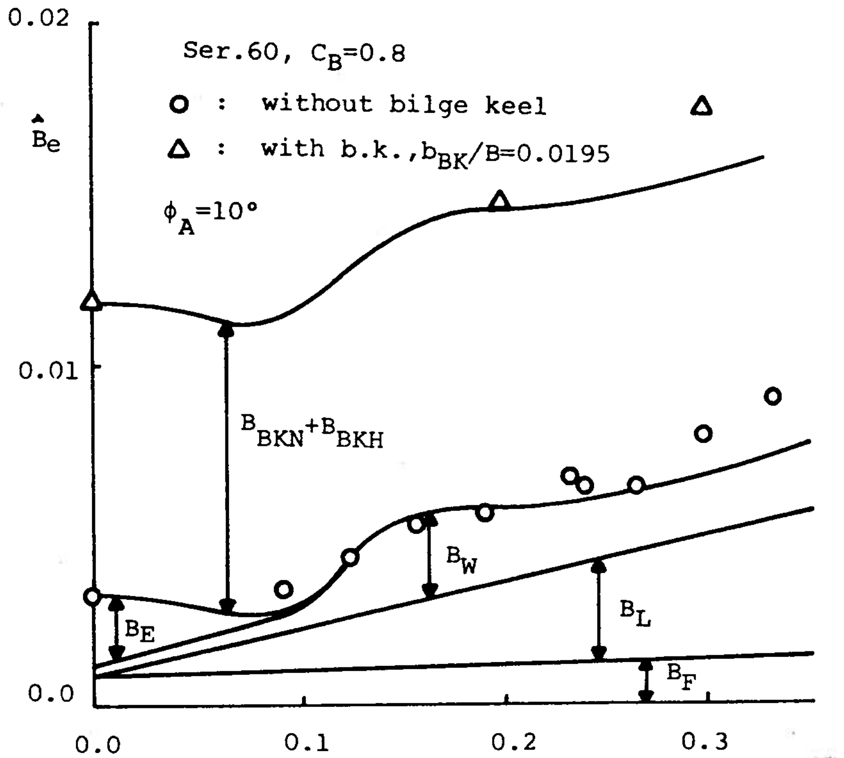

Figure 4.19. Comparison of measured and estimated roll damping coefficient as functions of advance speed

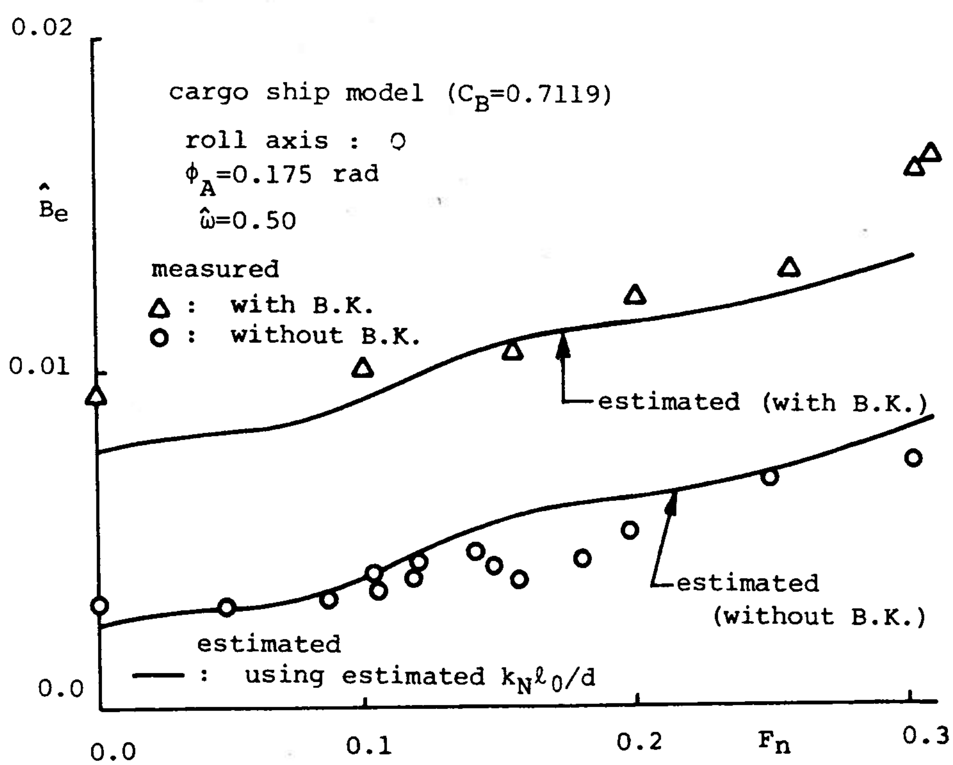

Figure 4.20. Roll damping coefficient for cargo ship model at forward speed

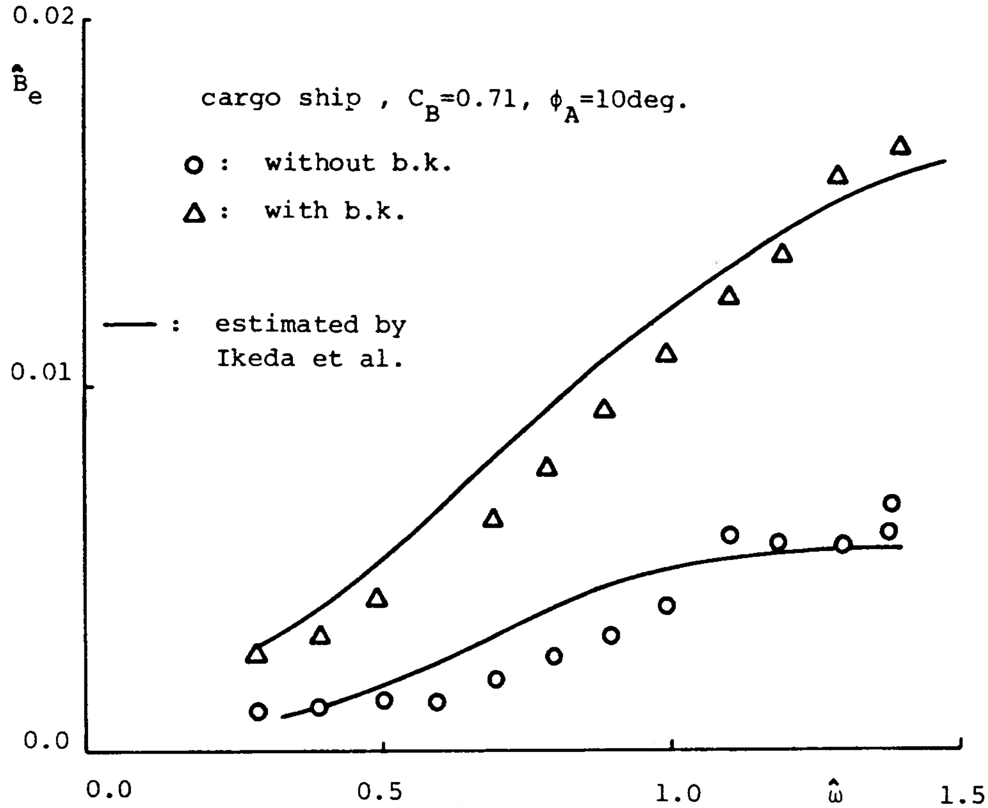

Figure 4.21 Comparison between measured and estimated roll damping coefficent at zero Froude number.

Measurements made at the Research Institute for Applied Mechanics of Kyushu University.

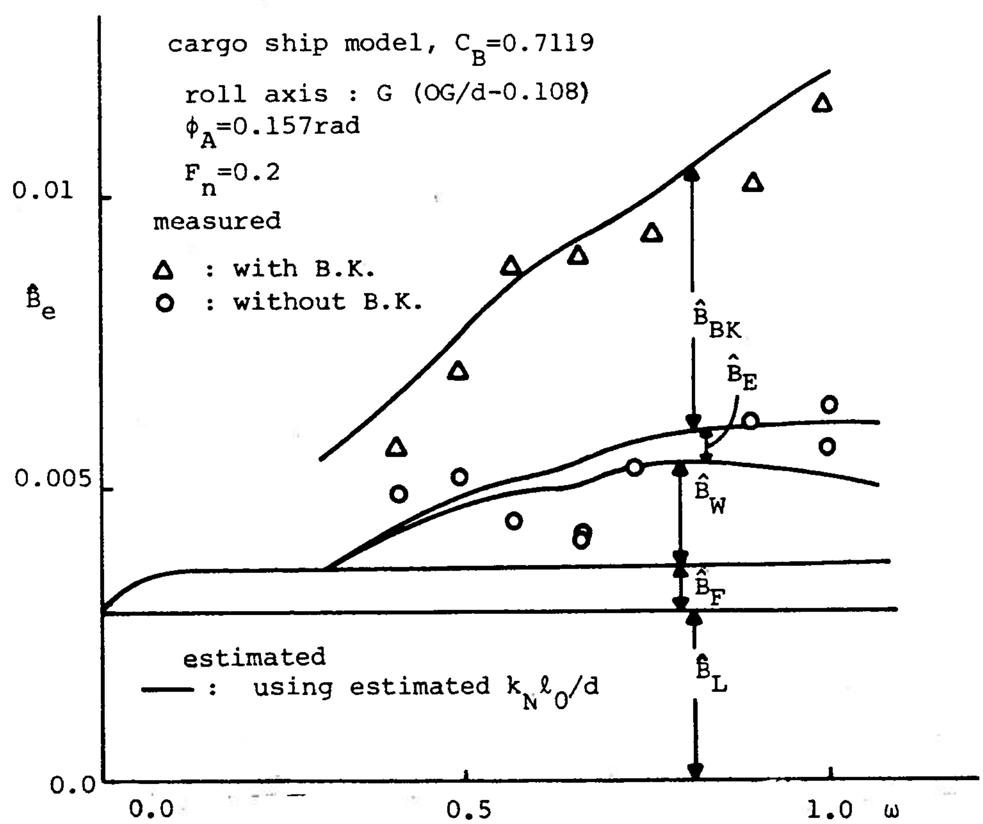

Figure 4.22 Roll damping coefficient for cargo ship model Fn = 0.2

5. TREATMENT OF NONLINEAR ROLL DAMPING IN PREDICTION OF ROLL MOTION

Since roll damping of a ship is quite nonlinear, as we have seen in the preceding chapters, special treatment is necessary in the calculation of ship roll response in regular or irregular seas. Although there will appear other similar nonlinearities in the restoring moment and the wave excitation, we concern ourselves hereafter only with the nonlinearity of roll damping in the analysis of roll motion.

To begin with, the roll response in regular seas can usually be obtained from solution of the simultaneous equations of ship sway, roll and yaw motions, for example, in the following linear form,

$$\begin{bmatrix} A_{22} & B_{22} & C_{22} \\ A_{42} & B_{42} & C_{42} \\ A_{62} & B_{62} & C_{62} \end{bmatrix} \begin{Bmatrix} \ddot{y} \\ \dot{y} \\ y \end{Bmatrix} + \begin{bmatrix} A_{24} & B_{24} & C_{24} \\ A_{44} & B_{44} & C_{44} \\ A_{64} & B_{64} & C_{64} \end{bmatrix} \begin{Bmatrix} \ddot{\phi} \\ \dot{\phi} \\ \phi \end{Bmatrix} + \begin{bmatrix} A_{26} & B_{26} & C_{26} \\ A_{46} & B_{46} & C_{46} \\ A_{66} & B_{66} & C_{66} \end{bmatrix} \begin{Bmatrix} \ddot{\psi} \\ \dot{\psi} \\ \psi \end{Bmatrix} = \begin{Bmatrix} F_2 \\ M_4 \\ M_6 \end{Bmatrix} e^{i\omega_e t}, \quad (5.1)$$

where y , ϕ and ψ denote the displacement in sway, roll and yaw, respectively, all expressed in complex form. In Eq. (5.1) the coefficients of wave exciting forces, F_2 , M_4 , M_6, are expressed by an appropriate method as functions of ship form and speed, the wave amplitude h_A , encounter angle, and encounter frequency ω_e . The inertial coefficients in the radiation forces, A_{ij} $(i,j=2,4,6)$, can also be calculated by a similar method as functions of ω_e . Then we can easily solve the equations to obtain the responses, provided that the equations are linear.

However the roll damping coefficient, B_{44} , is usually expressed in a form implying dependence on the roll amplitude, ϕ_A:

$$B_{44} = B_e(U, \omega_e , \phi_A). \quad (5.2)$$

In this case the equation is nonlinear and must be solved by some kind of iteration technique. Then the responses thus obtained depend on wave amplitude h_A in a nonlinear manner. This means that the nondimensional roll-response amplitude ϕ_A/kh_A (k: the wave number) should be stated for the h_A value concerned.

Takaki and Tasai [5] adopted the quadratic or cubic form,

$$B_e = B_1 + \frac{8}{3\pi}\omega\phi_A B_2 + \frac{3}{4}\omega^2\phi_A^2 B_3 \quad , \tag{2.6}$$

to express the roll damping coefficient B_e of Eq. (5.2) and they determined the values of B_1 , B_2 and/or B_3 by the experiment. If we use the prediction formulas stated above to obtain the constant values of B_1 , *etc.*, in Eq. (2.6), however, a regression analysis with wide variations of frequency and amplitude is needed, since the present prediction formulas are not always expressed in the simple form of Eq. (2.6) but are given as a combination of formulas for individual components in somewhat complicated manner. In order to obtain only the roll response in regular seas, such a transfer of damping expression should not be necessary. We need only the value of the equivalent linear damping at the specified values of ship speed, frequency and amplitude, as in Eq. (5.2).

We now proceed to the case of irregular seas. To predict the ship response in short-term irregular seas, the energy-spectrum analysis based on the principle of linear superposition has been applied. Let the energy spectrum of the incoming long-crested waves be denoted by $S_w(\omega)$. Then we can obtain the energy spectrum of the ship response $S(\omega)$ in the form

$$S(\omega) = |A(\omega)|^2 S_w(\omega) \quad . \tag{5.3}$$

The frequency ω of the waves can be transformed into the encounter frequecy ω_e of the ship. The response amplitude operator $A(\omega)$ represents the ship response per unit magnitude of input wave. In particular, it becomes ϕ_A/kh_A (k: the wave number) in the case of roll response.

The equation system is nonlinear when $A(\omega)$ depends on the amplitude of the wave and thus on the spectrum $S_w(\omega)$. Several attempts have been made to treat this nonlinearity. The most simple way, by Fukuda *et al.* [29], is to take the $A(\omega)$ value as that in the case of the wave height ($2h_A$) equal to the significant wave height, $H_{1/3}$. This corresponds to assuming that the roll damping B_e is constant, equal to the actual value associated with a roll amplitude ϕ_A corresponding to the prescribed wave height. The ω-dependence of the roll damping, in this case, is taken into account exactly.

Yamanouchi [58] applied the regular perturbation technique to the nonlinear terms in the equation of motion. As another approach, the concept of the equivalent linear damping, as stated in Chap. 2, has been applied to this case:

$$B_e = B_1 + \sqrt{\frac{8}{\pi}} B_2 \sigma_{\dot{\phi}} \quad . \tag{2.13}$$

This has been derived from the assumption that the difference between linear and nonlinear damping can, in the sense of least-squares method, be minimized during the roll motion, which is assumed to be a Gaussian process. This treatment also requires an iteration method of solution, in which the variance $\sigma_{\dot{\phi}}$ of the roll angular velocity is obtained in the form

$$\sigma_{\dot{\phi}}^2 = 2 \int_0^{\infty} S(\omega)\, \omega^2 d\omega \tag{5.4}$$

and should be reevaluated in each iteration stage. According to Takagi and Yamamoto's work [23], this method gives reasonable results in case of irregular seas.

Kaplan [26] proposed that the variance be replaced by the quantity

$$\sigma_{\dot{\phi}} = \omega_n \sigma_{\phi} \quad . \tag{5.5}$$

His justification is that roll motion occurs largely in a narrow band near the natural frequency ω_n .

It should be noted here again that the coefficients B_1 and B_2 are assumed to be constant. To apply this method, therefore, it is necessary to rearrange formulas to obtain a regression analysis on the equivalent linear damping in a manner similar to that stated before. For the case of a bare hull, however, this might fail, because the roll damping is less nonlinear and can hardly be expressed in a form like Eq. (2.6) with constant coefficients. At the same time, this method cannot represent the ω-dependence of the roll damping so well as the first simple approach.

Dalzell [59], Haddra [60], and Lewison [61] have proposed expressing the roll damping in a cubic form without quadratic term, $B_1\dot{\phi} + B_3\dot{\phi}^3$, for convenience in the statistical analysis of roll motion in irregular seas.

6. CONCLUSION

In this survey, recent trends and results of research on ship roll damping have briefly been stated. In Chapter 2 the representation of roll damping coefficients was summarized and some limitations on the definitions of the coefficients were discussed. The simple methods stated in Chapter 3 could give a rough estimation of the magnitude of roll damping at an early stage of ship design.

Chapter 4 was concerned with the importance of the concept of component dampings and recent works on them. Then a method of predicting the ship roll damping of ordinary hull forms was introduced and compared with experiments, the resulting agreement being reasonable from a practical point of view. Chapter 5 was devoted to comments on the treatment of the nonlinearity of roll damping in the calculation of ship roll motion.

In conclusion, it can be said that it is possible at present to estimate ship roll damping for ordinary hull forms with reasonable accuracy. However, there still remain problems in case of ballast condition of a ship and for unconventional ship forms. From the hydrodynamics point of view, the linear or nonlinear lifting force on a body in oscillatory motion and wave damping at forward speed are still important problems.

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Note: The following abbreviations are used below:

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T West-Japan SNA: Transactions of the West-Japan Society of Naval Architects

J Kansai SNA: Journal of the Kansai Society of Naval Architects, Japan

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APPENDIX: An Example of Computer Program on Ship Roll Damping

This appendix provides the detailed expressions that have been omitted in the previous chapters. Then a computer program of the proposed method stated in Section 4.11 will be shown as an example.

The friction damping B_F according to the Kato-Tamiya method can be written in the form

$$B_{F_0} = B_{F_0} \ (1 + 4.1\,U/\omega L) \ , \tag{A1}$$

$$B_{F_0} = \frac{4}{3\pi}\rho\, S\, r_S^3 \phi_A \omega C_f \ , \tag{A2}$$

$$C_f = 1.328 \left[\frac{3.22 r_S^2 \phi_A^2 \omega}{2\pi\nu}\right]^{-1/2} , \tag{A3}$$

and S and r_S are defined in Eqs. (4.4) and (4.5).

For the eddy damping at zero ship speed, the Ikeda *et al.* formula can be expressed in the following form,

$$B_{E_0} = \frac{4\rho d^4 \omega \phi_A}{3\pi} C_R$$

$$= \frac{4\rho d^4 \omega \phi_A}{3\pi} \left\{\left[1 - f_1 \frac{R}{d}\right]\left[1 - \frac{OG}{d}\right] + f_2\left[H_0 - f_1 \frac{R}{d}\right]^2\right\} C_p \left[\frac{r_{max}}{d}\right]^2 , \tag{A4}$$

where

$$C_p = 0.5 \left[0.87 \exp\,(-\gamma) - 4 \exp\,(-0.187\gamma) + 3\right] , \tag{A5}$$

$$f_1 = 0.5\left[1 + \tanh\{20(\sigma - 0.7)\}\right] ,$$

$$f_2 = 0.5 \left[1 - \cos\pi\,\sigma\right] - 1.5 \left[1 - \exp\{-5(1-\sigma)\}\right] \sin^2\pi\,\sigma , \tag{A6}$$

and the velocity-increment ratio γ can be expressed:

$$r = \frac{\sqrt{\pi} f_3}{2d\left(1 - \frac{OG}{d}\right)\sqrt{H_0 \sigma^1}}\left(r_{max} + \frac{2M}{H}\sqrt{A^2 + B^2}\right) , \tag{A7}$$

with

$$
\left.\begin{aligned}
&M=\frac{B}{2(1+a_1+a_3)}\ , \quad H_0'=\frac{H_0}{1-OG/d}\ , \qquad \sigma'=\frac{\sigma-OG/d}{1-OG/d}\\
&H = 1 + a_1^2 + 9a_3^2 + 2a_1(1 - 3a_3)\cos 2\psi - 6a_3\cos 4\psi,\\
&A = -2a_3\cos 5\psi + a_1(1-a_3)\cos 3\psi + \{(6-3a_1)a_3^2 + (a_1^2-3a_1)a_3 + a_1^2\}\cos\psi,\\
&B = -2a_3\sin 5\psi + a_1(1-a_3)\sin 3\psi + \{(6+3a_1)a_3^2 + (3a_1+a_1^2)a_2 + a_1^2\}\sin\psi,\\
&r_{max} = M\left[\{(1+a_1)\sin\psi - a_3\sin 3\psi\}^2 + \{(1-a_1)\cos\psi + a_3\cos 3\psi\}^2\right]^{1/2}
\end{aligned}\right] \quad \text{(A8)}
$$

$$
\left.\psi=\begin{cases} 0 = \psi_1 & (\text{when } r_{max}(\psi_1) \geq r_{max}(\psi_2)),\\ \frac{1}{2}\cos^{-1}\dfrac{a_1(1+a_3)}{4a_3} = \psi_2 & (\text{when } r_{max}(\psi_1) < r_{max}(\psi_2)),\end{cases}\right] \quad \text{(A9)}
$$

$$
f_3 = 1 + 4\exp\{-1.65\times 10^5(1-\sigma)^2\}\ . \qquad \text{(A10)}
$$

The forward-speed effect is stated in Eq. (4.8).

The lift damping is given in Eqs. (4.11) and (4.12).

The wave damping at zero speed should be predicted by the strip method; the computer program presented here does not include this term. The modification of wave damping to account for forward speed is stated in Eqs. (4.14) and (4.15).

For the bilge-keel damping B_{BK} , only the terms B_{BKN} and B_{BKH} are used here; we neglect the wave effect B_{BKW}. The assumption is made that B_{BK} is constant with ship speed. The expression of B_{BKN} is given in Eqs. (4.19) and (4.20). B_{BKH} can be expressed:

$$
B_{BKH_0} = \frac{4}{3\pi}\rho\, r^2 d^4 \omega\, \phi_A \cdot I \cdot f^2\ , \qquad \text{(A11)}
$$

where

$$I = \int C_p \ell_0 ds = -A\ C_p^- + B\ C_p^+ \quad ; \tag{A12}$$

the coefficient f is given in Eq. (4.20), and

$$\left.\begin{aligned} C_p^+ &= 1.2 \quad , \\ C_p^- &= C_p^+ - C_D = -22.5\ \frac{b_{BK}}{\pi r f \phi_A} - 1.2 \ . \end{aligned}\right\} \tag{A13}$$

Further,

$$\left.\begin{aligned}
A &= (m_3 + m_4) m_8 - m_7^2 \quad , \\
B &= \frac{m_4^3}{3(H_0 - 0.215 m_1)} + \frac{(1-m_1)^2 (2m_3 - m_2)}{6(1 - 0.215 m_1)} + m_1 (m_3 m_5 + m_4 m_6) \quad , \\
m_1 &= R/d,\ m_2 = OG/d,\ m_3 = 1 - m_1 - m_2,\ m_4 = H_0 - m_1 \quad , \\
m_5 &= \frac{0.414 H_0 + 0.0651 m_1^2 - (0.382 H_0 + 0.0106) m_1}{(H_0 - 0.215 m_1)(1 - 0.215 m_1)} \quad , \\
m_6 &= \frac{0.414 H_0 + 0.0651 m_1^2 - (0.382 + 0.0106 H_0) m_1}{(H_0 - 0.215 m_1)(1 - 0.215 m_1)} \quad , \\
m_1 &= \begin{cases} S_0/d - 0.25\pi m_1 & (S_0 > 0.25\pi R) \ , \\ 0 & (S_0 \le 0.25\pi R) \ , \end{cases} \\
m_8 &= \begin{cases} m_7 + 0.414 m_1 & (S_0 > 0.25\pi R) \ , \\ m_7 + \sqrt{2}\ \{1 - \cos(S_0/R)\}\, m_1 & (S_0 \le 0.25\pi R) \ . \end{cases}
\end{aligned}\right\} \tag{A14}$$

The bilge-circle radius R and the distance r from the roll axis to the bilge keel are given in the form

$$R = \left\{\begin{array}{ll} 2d\sqrt{\dfrac{H_0(\sigma - 1)}{\pi - 4}} & (R<d,\ R<B/2) \ , \\ d & (H_0 \ge 1,\ R/d>1) \ , \\ B/2 & (H_0 \le 1,\ R/d>H_0) \ , \end{array}\right\} \tag{A15}$$

$$r = d\left[\left\{H_0 - \left(1 - \frac{\sqrt{2}}{2}\right)\frac{R}{d}\right\} + \left\{1 - \frac{OG}{d} - \left(1 - \frac{\sqrt{2}}{2}\right)\frac{R}{d}\right\}^2\right]^{1/2} \qquad \text{(A16)}$$

A computer program can be written in the following form:

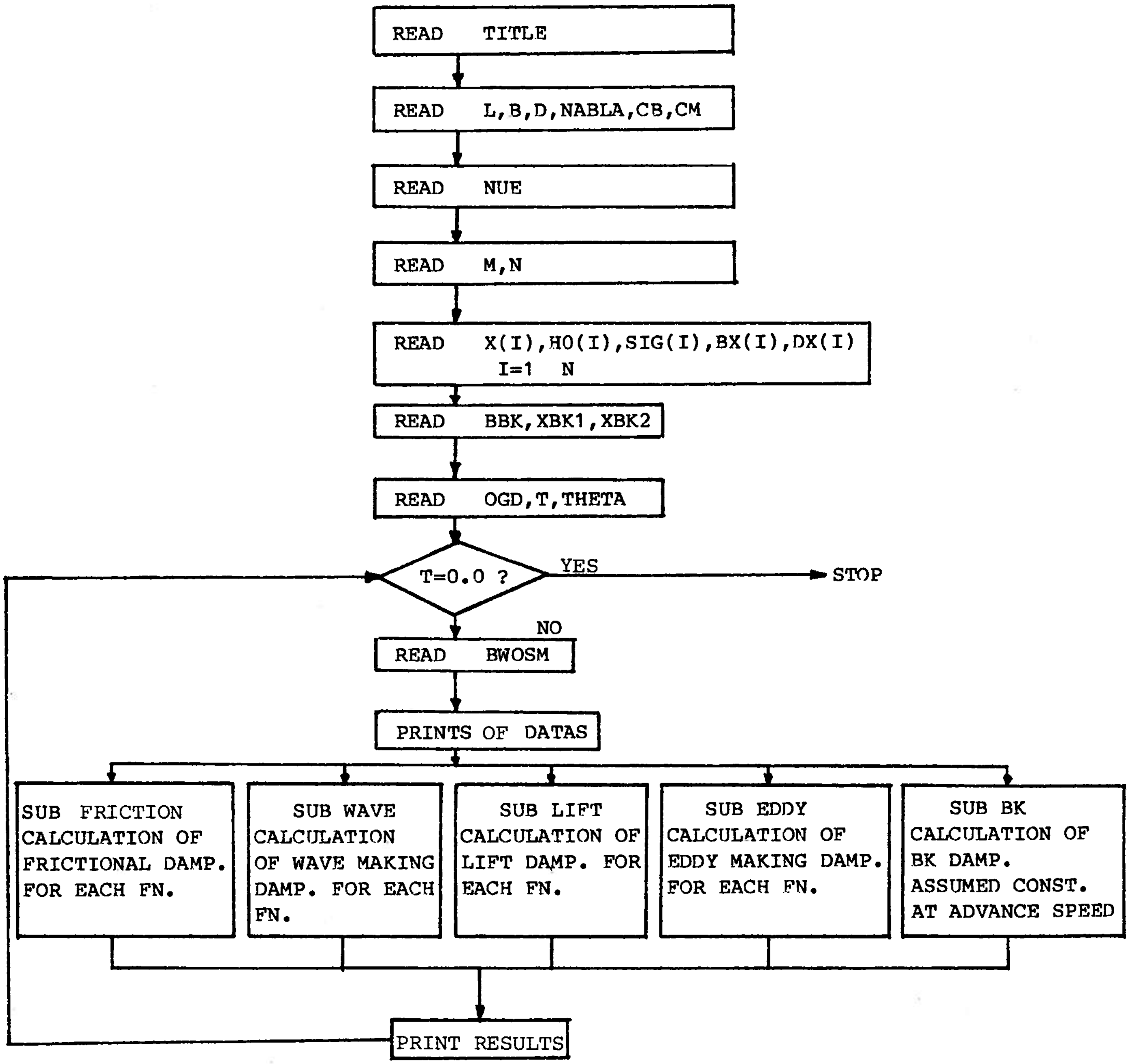
(1) INPUT

1) Title (70 words) — In put type 70A1
2) L,B,D,NABLA,CB,CM — 6F10.0
   - L ; length b ; breadth D ; draft (m)
   - NABLA ; displacement volume (m^3)
   - CB ; block coeff. CM ; midship coeff.
3) NUE — F10.0
   - NUE ; kinematic viscosity (m^3/s)
   - Empirical formula
     - $\nu=0.0178/(1+0.0336t+0.000221t^2)$ cm^2/s
     - t ; temperature (°C)
4) M,N — 2I 5
   - M ; the number of calculation points of Fn (interval: Fn=0.02)
   - N ; the number of input data of sections (= the number of data cards of 5)
5) X(I),HØ(I),SIG(I),BX(I),DX(I) — 5F10.0
   - X(I) ; the number of S.S (0.0 at A.P.&10.0 at F.P.)
   - HØ(I) ; half breadth/draft (=BX(I)/DX(I)
   - SIG(I) ; area coefficient (=cross section area/BX(I)DX(I)
   - BX(I) ; breadth (m)
   - DX(I) ; draft (m)
6) BBK,XBK1,SBK2 — 3F10.0
   - BBK ; breadth of bilge keel (m)
   - XBK1 ; SS number of aft end of bilge keel
   - XBK2 ; SS number of fore end of bilge keel
7) OGD,T,THETA — 3F10.0
   - OGD ; =OG/d
   - T ; roll period (sec)
   - THETA ; roll amplitude (rad)
8) BWOSM — F10.0
   - BWOSM ; $\hat{B}_W$ at Fn=0.0 calculated by potential theory

(2) OUTPUT

1) prints of input data
2) distribution of the coefficient C_R of the eddy making component.
   ($C_R=M_{RE}/(0.5\rho d^4\ \dot{\phi}|\dot{\phi}|$, M_{RE} ; roll moment due to the eddy making component)
3) distributions of $\Delta\hat{B}_{BK}$ and $\Delta\hat{B}_N/\Delta\hat{B}_{BK}$.
   - $\Delta\hat{B}_{BK}$; B_{BK} for unit length
   - $\Delta\hat{B}_N$; $\hat{B}_N$ for unit length
4) each component $\hat{B}_F,\hat{B}_W,\hat{B}_E,\hat{B}_L,\hat{B}_{BK}$ — for each Fn
   rates of each component — for each Fn
   total roll damping coefficient $\hat{B}_{44}$ — for each Fn

Flow chart of the program



The program is written in FORTRAN IV, as follows.

```
C        *****ESTIMATION OF ROLL DAMPING*****
C               CODED BY Y.IKEDEA.
      REAL L,NUE,NABLA
      DIMENSION   X(25),HO(25),SIG(25),BX(25),DX(25),B44HAT(100),
     *                 BWHAT(100),BEHAT(100),BLHAT(100)
     *            ,FN(100),TITLE(70),BFHAT(100)
         READ (5,100) (TITLE(I),I=1,70)
  100      FORMAT (70A1)
C      ***PRINCIPAL DIMENSIONS*** L=LENGTH,B=BREADTH,D=DRAFT,NABLA=VOLUM
C                          *** CB=BLOCK COEFF.,CM=MIDSHIP COEFF..
         READ (5,101) L,B,D,NABLA,CB,CM
  101      FORMAT (6F10.0)
         READ (5,103) NUE
  103      FORMAT (F10.0)
C     *** NUMBERS OF FN AND DATAS ***
         READ (5,104) M,N
  104      FORMAT (2I5)
C     *** PARAMETERS OF EACH SECTION *** HO=B/2D,SIG=S/BD,BX=BREADTH
C                                               DX=DRAFT
         DO 1   I=1,N
    1    READ (5,105) X(I),HO(I),SIG(I),BX(I),DX(I)
  105      FORMAT (5F10.0)
C      *** BILGE KEELS DATA ***  BBK=BREADTH OF B.K.,XBX1=X OF B.K. END
C                                XBK2=X OF B.K. END(FOR)
         READ (5,106) BBK,XBK1,XBK2
  106      FORMAT (3F10.0)
    5    CONTINUE
C      *** CONDITION ***  OGD=OG/D,T=PERIOD,THETA=AMP. OF ROLL
         READ (5,102) OGD,T,THETA
  102      FORMAT (3F10.0)
         IF (T.LT.0.000001)  STOP
C      *** WAVE MAKING COMPONENT AT FN=0  ***
         READ (5,103) BWOSM
         WRITE (6,200)  (TITLE(I),I=1,70)
  200    FORMAT  (1H1,///,7X,5H*****,2X,70A1,2X,5H*****)
         WRITE (6,300) L,B,D,NABLA,CB,CM
  300    FORMAT  (1H ,//,5X,8H**DATA**,2X,2HL=,F8.5,2X,2HB=,F8.5,2X,2HD=
     *             ,F8.5,2X,6HNABLA=,F8.5,2X,3HCB=,F8.5,2X,3HCM=,F8.5)
         WRITE (6,301) OGD,T,THETA,NUE,BWOSM
  301    FORMAT  (1H ,5X,5HOG/D=,F8.3,2X,2HT=,F8.3,2X,6HTHETA=,F8.3,3HRAD
     *             ,2X,4HNUE=,F12.8,2X,6HBWOSM=,F15.10)
         WRITE (6,302)  BBK,XBK1,XBK2
  302    FORMAT (1H ,5X,4HBBK=,F8.5,2X,5HXBK1=,F8.3,2X,5HXBK2=,F8.3)
         WRITE  (6,304)
  304    FORMAT  (1H ,//,4X,25H**DATAS OF EACH SECTION**)
         DO 15  I=1,N
   15    WRITE  (6,303) X(I),HO(I),SIG(I),BX(I),DX(I)
  303    FORMAT(1H ,4X,3HSS=,F8.3,3X,3HHO=,F8.5,3X,6HSIGMA=,F8.5,3X,2HB=
     *             ,F8.5,3X,2HD=,F8.5)
C      *********************************************************
        OMEGA=6.28318/T
         DO 2   I=1,M
    2    FN(I)=0.0+0.02*FLOAT(I-1)
         CALL FRICT   (L,B,D,CB,NABLA,OGD,NUE,OMEGA,FN,BFHAT,M)
         CALL WAVE    (L,D,OMEGA,BWOSM,FN,BWHAT,M)
         CALL LIFT    (L,B,D,CM,NABLA,OGD,FN,BLHAT,M)
         CALL EDDY    (X,HO,SIG,BX,DX,B,D,NABLA,OGD,OMEGA,THETA,FN,
     *                 BEHAT,M,N,L)
```

```
      IF  (BBK.LT.0.0000000001) GO TO 20
      CALL BK      (X,HO,SIG,BX,DX,B,D,NABLA,OGD,OMEGA,THETA,BBK,
     *             XBK1,XBK2,BBKHAT,M,N,L)
   20 IF (BBK.LT.0.0000000001) BBKHAT=0.0
      DO 3  I=1,M
    3 B44HAT(I)=BFHAT(I)+BWHAT(I)+BLHAT(I)+BEHAT(I)+BBKHAT
C    *******************************************************
      WRITE (6,201)
  201   FORMAT (1H ,///,4X,2HFN,6X,5HBFHAT,5X,5HBWHAT,5X,5HBEHAT,5X,
     *          5HBLHAT,5X,6HBBKHAT,4X,2H**,3X,6HBF/B44,4X,6HBW/B44,
     *          4X,6HBE/B44,4X,6HBL/B44,2X,7HBBK/B44,4X,2H**,2X,
     *          6HB44HAT,2X,2H**)
      DO 4  I=1,M
       BFF=BFHAT(I)/B44HAT(I)
       BWW=BWHAT(I)/B44HAT(I)
       BEE=BEHAT(I)/B44HAT(I)
       BLL=BLHAT(I)/B44HAT(I)
       BBKK=BBKHAT  /B44HAT(I)
    4 WRITE(6,202)FN(I),BFHAT(I),BWHAT(I),BEHAT(I),BLHAT(I),BBKHAT,
     *           BFF,BWW,BEE,BLL,BBKK,B44HAT(I)
  202   FORMAT (1H ,2X,F6.3,2X,F8.5,2X,F8.5,2X,F8.5,2X,F8.5,2X,F8.5
     *          ,3X,2H**,2X,F8.5,2X,F8.5,2X,F8.5,2X,F8.5,2X,F8.5,2X,
     *         2H**,1X,F8.5,1X,2H**)
      GO TO 5
      END
```

```
C***************************************************************************
      SUBROUTINE LIFT (L,B,D,CM,NABLA,OGD,FN,BLHAT,M)
C        LIFT COMPONENT
C        REF.  Y.IKEDA ET AL (JZK.NO.143)
       REAL  L,NABLA,KAPA,KN,LO,LR
       DIMENSION  FN(100),BLHAT(100)
       IF (CM.LE.0.92)  KAPA=0.0
       IF  (CM.LE.0.97.AND.CM.GT.0.92)  KAPA=0.1
       IF  (CM.GT.0.97)  KAPA=0.3
       KN=6.28319*D/L+KAPA*(4.1*B/L-0.045)
       OG=OGD*D
       LO=0.3*D
       LR=0.5*D
       DO 1  I=1,M
    1  BLHAT(I)=L*D*KN*LO*LR*FN(I)*0.5/(NABLA*B**2)*SQRT(0.5*L*B)*(1.0-
     *         1.4*OG/LR+0.7*OG**2/(LO*LR))
       RETURN
       END
```

```
C***********************************************************************
      SUBROUTINE  WAVE  (L,D,OMEGA,BWOSM,FN,BWHAT,M)
C         WAVE MAKING COMPONENT
C         REF  Y.IKEDA ET AL (JZK.NO.243)
      REAL  L,LOMEGA
      DIMENSION  FN(100),BWHAT(100)
      GUZAID=OMEGA**2*D/9.80665
      A1=1.0+GUZAID**(-1.2)*EXP(-2.0*GUZAID)
      A2=0.5+GUZAID**(-1.0)*EXP(-2.0*GUZAID)
      DO 1  I=1,M
      LOMEGA=OMEGA*FN(I)*SQRT(L/9.80665)
    1 BWHAT(I)=BWOSM*0.5*(((A2+1.0)+(A2-1.0)*TANH(20.0*(LOMEGA-0.3)))+
     *          (2.0*A1-A2-1.0)*EXP(-150.0*(LOMEGA-0.25)**2))
      RETURN
      END
```

```
C*****************************************************************************
      SUBROUTINE FRICT   (L,B,D,CB,NABLA,OGD,NUE,OMEGA,FN,BFHAT,M)
C          FRICTIONAL COMPONENT
C          REF. H.KATO (JZK.NO.102) AND S,TAMIYA ET AL (JZK.NO.132)
       REAL NABLA,NUE,L
       DIMENSION  FN(100),BFHAT(100)
       SF=L*(1.7*D+CB*B)
       RF=((0.887+0.145*CB)*(1.7*D+CB*B)-2.0*OGD*D)/3.145
       DO 1    I=1,M
     1 BFHAT(I)=0.78*SF*RF**2*SQRT(OMEGA*NUE*B/19.6133)/(NABLA*B**2)
     *           *(1.0+4.1*FN(I)/OMEGA*SQRT(9.80665/L))
       RETURN
       END
```

```
C*****************************************************************************
      SUBROUTINE EDDY   (X,HO,SIG,BX,DX,B,D,NABLA,OGD,OMEGA,THETA,FN,
     *                   BEHAT,M,N,L)
C           EDDY MAKING COMPONENT
C          REF.   Y,IKEDA ET AL   (JZK.NO.142,JZK.NO.143)
       REAL NABLA,LTHETA,L
       DIMENSION X(25),HO(25),SIG(25),BX(25),DX(25),FN(100),BEHAT(100),
     *            RMAX1(2),V(2),CR(25),CR1(25),X1(30)
       DO 1  J=1,N
       AHO=HO(J)/(1.0-OGD)
       SIGMA=(SIG(J)-OGD)/(1.0-OGD)
       E=(AHO-1.0)/(AHO+1.0)
       E2=E**2
       A=4.0*SIGMA*(1.0-E2)/3.1415+E2
       O=-A/(A+3.0)
       O2=SQRT(O**2-(A-1.0)/(A+3.0))
       A3=O+O2
       A1=E*(1.0+A3)
       AM=BX(J)/(1.0+A1+A3)*0.5
       AA1=A1*(1.0+A3)/A3*0.25
       IF(AA1.GT.1.0)  AA1=1.0
       IF(AA1.LT.-1.0)  AA1=-1.0
       DO 2  I=1,2
       LTHETA=0.5*ARCOS(AA1)
       IF (I.EQ.1)  LTHETA=0.0
       AH=1.0+A1**2+9.0*A3**2+2.0*A1*(1.0-3.0*A3)*COS(2.0*LTHETA)-6.0*A3
     *      *COS(4.0*LTHETA)
       AA=-2.0*A3*COS(5.0*LTHETA)+A1*(1.0-A3)*COS(3.0*LTHETA)+((6.0-3.0*
     *   A1)*A3**2+(A1**2-3.0*A1)*A3+A1**2)*COS(LTHETA)
       BB=-2.0*A3*SIN(5.0*LTHETA)+A1*(1.0-A3)*SIN(3.0*LTHETA)+((6.0+3.0*
     *   A1)*A3**2+(3.0*A1+A1**2)*A3+A1**2)*SIN(LTHETA)
       V(I)=2.0*AM*SQRT(AA**2+BB**2)/AH
     2 RMAX1(I)=AM*SQRT(((1.0+A1)*SIN(LTHETA)-A3*SIN(3.0*LTHETA))**2
     *          +((1.0-A1)*COS(LTHETA)+A3*COS(3.0*LTHETA))**2)
       RMAX=RMAX1(1)
       VMAX=V(1)
       IF   (RMAX1(1).LE.RMAX1(2)) GO TO 8
       GO TO 9
     8 RMAX=RMAX1(2)
       VMAX=V(2)
     9 CONTINUE
       RMEAN=2.0*DX(J)*(1.0-OGD)*SQRT(AHO*SIGMA/3.1415)
       P1=VMAX/RMEAN
       P2=RMAX/RMEAN
       PP3=P1+P2
       IF (SIGMA.LT.0.99) GOTO 20
       GAMMA=(1.0+4.0*EXP(-165000.0*(1.0-SIGMA)**2))*PP3
       GOTO 21
    20 GAMMA=PP3
    21 CP=0.5*(0.87*EXP(-GAMMA)-4.0*EXP(-0.187*GAMMA)+3.0)
       F1=0.5*(1.0+TANH(20.0*(SIG(J)-0.7)))
       F2=0.5*(1.0-COS(3.1415*SIG(J)))-1.5*(1.0-EXP(-5.0*(1.0-SIG(J))))*
     *      SIN(3.1415*SIG(J))**2
       R=2.0*DX(J)*SQRT(HO(J)*(SIG(J)-1.0)/(-0.8584))
       RD=R/DX(J)
       IF   (HO(J).LE.1.0.AND.RD.GE.AHO)  R=0.5*BX(J)
       IF   (HO(J).GT.1.0.AND.RD.GE.1.0)  R=DX(J)
       RD=R/DX(J)
```

```
      1  CR1(J)=RMAX**2/DX(J)**2*CP*((1.0-F1*RD)*(1.0-OGD-F1*RD)+F2*
      *        (HO(J)-F1*RD)**2)
         WRITE (6,452)
    452  FORMAT (1H ,//,4X,4H****,31HLONGITUDINAL DISTRIBUTION OF CR,
      *          4H****)
         DO 10  J=1,N
     10  WRITE  (6,453) X(J),CR1(J)
    453  FORMAT (1H ,4X,3HSS=,F8.5,4X,3HCR=,F8.5)
         DO 3  K=1,21
         X1(K)=0.0+0.5*FLOAT(K-1)
         MAX=N
      3  CALL HOKAN1 (X,CR1, 25,MAX,X1(K),CR(K),DAM,1,0)
         CR(1)=1.5*(1.0-OGD)
         CR(21)=1.5*(1.0-OGD)
         SAM=0.0
         DO 4  K=1,10
         K2=2*K
         K1=K2-1
         K3=K2+1
         SAM1=CR(K1)+4.0*CR(K2)+CR(K3)
      4  SAM=SAM+SAM1
           CRT=SAM/60.0
         BEHAT(1)=4.0*L*D**4/3.0/3.1415*OMEGA*SQRT(B/19.6)/NABLA/B**2*CRT
      *          *THETA
         DO  5   I=2,M
         AK=OMEGA/FN(I)*SQRT(L/9.8)
      5  BEHAT(I)=BEHAT(1)*(0.04*AK)**2/((0.04*AK)**2+1.0)
         RETURN
         END
```

```
C*******************************************************************************
       SUBROUTINE BK  (X,HO,SIG,BX,DX,B,D,NABLA,OGD,OMEGA,THETA,BBK,
     *                  XBK1,XBK2,BKHAT,M,N,L)
C              DAMPING DUE TO BILGE KEELS.
C              REF. Y.IKEDA ET AL  (KZK.NO.161, KZK.NO.165)
        REAL NABLA, M1,M2,M3,M4,M5,M6,M7,M8,L
        DIMENSION  X(25),HO(25),SIG(25),BX(25),DX(25),BBKHAT(25),XBK(25)
     *          ,RATIO(25)
        XBK(1)=XBK1
        XBK(11)=XBK2
        DO 1  I=2,10
      1 XBK(I)=XBK(I-1)+(XBK2-XBK1)*0.1
        MAX=N
        DO 2  I=1,11
         CALL HOKAN1  (X,HO, 25,MAX,XBK(I),HO1,DAM,1,0)
         CALL HOKAN1  (X,SIG, 25,MAX,XBK(I),SIG1,DAM,1,0)
         CALL HOKAN1  (X,DX, 25,MAX,XBK(I),DX1,DAM,1,0)
         CALL HOKAN1  (X,BX, 25,MAX,XBK(I),BX1,DAM,1,0)
        R=2.0*DX1*SQRT(HO1*(SIG1-1.0)/(-0.8585))
        RD=R/DX1
        IF (HO1.LE.1.0.AND.RD.GE.HO1)  R=0.5*BX1
        IF  (HO1.GT.1.0.AND.RD.GE.1.0)  R=DX1
        RD=R/DX1
        F=1.0+0.3*EXP(-160.0*(1.0-SIG1))
        RBK=DX1*SQRT((HO1-0.2929*RD)**2+(1.0-OGD-0.2929*RD)**2)
        M1=RD
        M2=OGD
        M3=1.0-M1-M2
        M4=HO1-M1
        M5=(0.414*HO1+0.0651*M1**2-(0.382*HO1+0.0106)*M1)/((HO1-0.215
     *      *M1)*(1.0-0.215*M1))
        M6=(0.414*HO1+0.0651*M1**2-(0.382+0.0106*HO1)*M1)/((HO1-0.215
     *      *M1)*(1.0-0.215*M1))
        SO=0.3*(3.1415*F*RBK*THETA)+1.95*BBK
        M7=SO/DX1-0.25*3.1415*M1
        R1=0.25*3.1415*R
        IF (SO.LT.R1)  M7=0.0
        M8=M7+0.414*M1
        IF (SO.LT.R1)  M8=M7+1.414*(1.0-COS(SO/R))*M1
        A=(M3+M4)*M8-M7**2
        BB=M4**3/3./(HO1-0.215*M1)+(1.0-M1)**2*(2.0*M3-M2)/6.0/(1.0-0.215
     *      *M1)+M1*(M3*M5+M4*M6)
        CPPLAS=1.2
        CPMINS=-22.5*BBK/(3.1415*RBK*F*THETA)-1.2
        CD=CPPLAS-CPMINS
C       *** BBKHAT FOR UNIT LENGTH ***
        RATIO(I)=RBK*BBK*CD/(RBK*BBK*CD+0.5*DX1**2*(-A*CPMINS+BB*CPPLAS))
      2 BBKHAT(I)=8.0*RBK**2*OMEGA*SQRT(B/19.6)*THETA*F**2/(3.0*3.1415
     *          *NABLA*B**2)*(RBK*BBK*CD+0.5*DX1**2*(-A*CPMINS+BB*CPPLAS
     *          ))
        WRITE (6,100)
    100 FORMAT (1H ,//,4H****,35HLONGITUDINAL DISTRIBUTION OF BBKHAT,
     *         4H****)
        DO 7  I=1,11
      7 WRITE  (6,101) XBK(I),BBKHAT(I),RATIO(I)
    101 FORMAT  (1H .4X,3HSS=,F8.5,3X,7HBBKHAT=,F13.8,3X,22HNORMAL FORCE/
     *TOTAL BK=,F13.8)
C       ****  BBKHAT FOR THREE DIMENSIONAL SHIP FORM  ****
```

```
      SAM =0.0
      DO 3 I=1,5
      I2=2*I
      SAM1=BBKHAT(I2-1)+4.0*BBKHAT(I2)+BBKHAT(I2+1)
    3 SAM=SAM+SAM1
      BKHAT=SAM*(XBK2-XBK1)*0.1/3.0*L*0.1
      RETURN
      END
```

```
C*************************************************************************
      *******************************************************************
      SUBROUTINE HOKAN1 (X1,Y1,MAX,N,X,Y,YX,M1,M2)
C     LAGRANGE 3 POINTS INTERPOLATION
      DIMENSION X1(MAX),Y1(MAX),WX(3),WY(3)
      N1=N-1
        DO 10  I=2,N1
        IF (X.LE.X1(I)) GO TO 1
   10   CONTINUE
    1 I1=I-1
        IF (X.GT.X1(N1)) I1=N-2
      I2=I1+2
        DO 20 I=I1,I2
      II=I+1-I1
      WX(II)=X1(I)
   20 WY(II)=Y1(I)
        IF(M1.NE.1) GO TO 2
        CALL LAG3(WX,WY,X,Y)
    2   CONTINUE
        IF(M2.NE.1)  RETURN
      YX=0.0
      RETURN
      END
```

```
C*******************************************************************************
      SUBROUTINE LAG3  (WX,WY,X,Y)
      DIMENSION WX(3),WY(3)
      Y=0.0
        DO 11  I=1,3
      W=1.0
      Z=1.0
        DO 12  J=1,3
        IF(J.EQ.I) GO TO 12
      W=W*(X-WX(J))
      Z=Z*(WX(I)-WX(J))
   12   CONTINUE
      Y=Y+WY(I)*W/Z
   11   CONTINUE
      RETURN
      END
```

TABLE OF CONTENTS
(SEQUENTIAL)

TABLE OF CONTENTS
(ALPHABETICAL)


```
EXECUTION TERMINATED    14:17:20   T=0.479   RC=0   $0.43

$SIG
```