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STATIC ANALYSIS OF MARINE RISERS

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ABSTRACT

The static riser design problem is formulated and solved. A two dimensional, small slope, small deflection, linear model for bending of vertical circular tubular beams under tension, internal mud-static and external hydrostatic pressure is used to analyze the behavior of risers under external time invariant hydrodynamic loads. At the present state of the art, the prediction of hydrodynamic loads exerted on a circular cylinder moving in a viscous fluid, is possible only in certain cases that have been studied experimentally. To overcome this difficulty and study the design problem per se, a parametric approach has been adopted. The load profile along the riser is described by a general n^{th} degree polynomial expression of the water depth and a series of loading parameters.

The resulting static riser boundary value problem has been solved in terms of the Airy functions of the first and second kind. In addition to the exact solution, an approximate solution is derived. The approximate solution, called cable approximation to the riser static problem, violates two of the four riser boundary conditions. Two different methods have been developed to correct the cable approximation and make it satisfy all four boundary conditions. Each solution, the exact and the two corrected approximations, is applicable for different ranges of the design variables. The corrected cable approximations provide a satisfactory solution to the riser static boundary value problem and can be easily implemented numerically for values of the design variables for which the exact solution is numerically unstable [1].

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NOMENCLATURE

C_D	Drag coefficient
D	Average diameter of riser
D_B	Outer diameter of buoyancy modules
D_i, D_o	Inner and outer riser diameters
E	Young's Modulus
f_x	External hydrodynamic load per unit length in the x direction
I	Riser cross-sectional area moment of inertia
K(0),K(1)	Riser curvature approximation at lower and upper ends
L	Riser's length
M	Bending moment
M_0, M_1	Bending moment at the lower and upper riser ends
p	Dimensionless vertical coordinate along the riser
P_e	Effective tension
Q	Shear force
S_y	Riser's material yield strength
T(0)	Actual tension at the riser's lower end
TTR	Tension at the top of the riser
U	Riser's lateral displacement
V	Fluid velocity
W_e	Effective weight of riser per unit length
W_m	Weight of drilling mud per unit length
W_R	Weight of riser per unit length
z	Vertical coordinate along the riser

Greek Symbols

$\alpha_1, \alpha_2, \ldots \alpha_n$ Hydrodynamic load parameters

β Dimensionless effective riser weight

$\gamma_B, \gamma_M, \gamma_W$ Specific weight of buoyancy modules material, mud and water respectively

γ_R Specific weight of riser material

Δ Static offset of drilling vessel

τ Dimensionless effective weight at lower end of riser

INTRODUCTION AND OUTLINE

Continuous demand for new energy resources and rising of oil prices in recent years have made the recovery of oil from the sea bed economically attractive even in deep waters. The marine riser is the structure connecting the offshore platform with the well at the sea bed during drilling or production operations. The first riser was installed in 1949 [2] at a 20 ft depth. Nowadays risers of 4000 ft are in operation and 7000 ft risers are considered for installation and expected to be in place in some cases for at least 20 years. However, the technical understanding of riser behavior is still inadequate and the design problem has not been satisfactorily solved as yet. This results in riser failures causing higher oil recovery costs and environmental and safety hazards.

The solution of the riser design problem can be achieved in the following steps.

(1) Proper formulation of the static and dynamic riser behavior.

(2) Derivation of a model for the prediction of the external hydrodynamic forces exerted on the riser.

(3) Solution of the mathematical problem resulting from concatenation of the above two models.

Upon completion of step (3) the riser design problem per se can be studied in steps (4) through (7).

(4) Derivation of the design constraints e.g. stress, buckling, and positivity constraints.

(5) Formulation of the optimization problem.

(6) Identification of the design feasibility domain.

(7) Derivation of the optimum design.

The marine riser design problem has not as yet been satisfactorily solved for the following reasons.

(1) No adequate mathematical model for description of the static and dynamic behavior of risers has been numerically implemented.

(2) No general theoretical or phenomenological [3] model exists for prediction of the hydrodynamic loads exerted on circular cylinders moving in a viscous fluid.

(3) Some essential design constraints have not been formulated properly i.e. Euler bucking of columns in tension due to internal static pressure [4].

(4) Experimental studies of scaled riser models cannot simulate the actual phenomenon [5].

(5) On site measurements are technically impractical if not infeasible.

In spite of the above difficulties, numerous investigators have studied the riser problem [Refs. 6 to 24]. The typical approach used in the literature consists of:

(1) Implementation of a two-dimensional, small slope, small deflection linearized Euler-Bernoulli type of beam model.

(2) Simplification of the hydrodynamic load model and,

(3) Solution of the resulting riser problem.

The only effort to formulate the design problem and solve the optimization problem has been made by the author of this report in [25,26].

This work is the first of a series of reports of the Department of Naval Architecture and Marine Engineering of the University of Michigan which are written in an effort to properly formulate and solve the riser design problem. These reports are used as references in the graduate Ocean Engineering course.

Our method of solution and design approach is based on the following four tools which we have recently developed.

(1) A three dimensional, large deflection nonlinear model which describes the dynamic behavior of risers and takes into account its extensional oscillations [27].

(2) A mathematical model for the column buckling of vertical tubular beams in tension due to internal pressure [4]. This model was used to prove that for a given riser design there exists a riser length

such that global buckling may occur due to internal pressure even if the actual stress is tensile at all points in the riser.

(3) A generalized model describing the external hydrodynamic loads in terms of unknown parameters. As was stated earlier, no satisfactory method for prediction of the hydrodynamic loads exerted on risers exists. However, ranges for the above parameters can be estimated. This method does not solve the problem but shifts it from the level of analysis to the level of design. The method which we use in studying the design problem is specifically suitable for design problems involving parameters which vary over wide ranges [25,26].

(4) A design optimization method based on monotonicity analysis [28,29] which eliminates some of the fundamental drawbacks of the currently used numerical optimization methods in structural problems. Numerical methods are based on Finite Element Analysis and seek improvement of existing designs. Their disadvantages are:

  (a) They are purely numerical methods and do not allow for analytic solution or parametric optimization.
  (b) They do not necessarily converge to the global optimum.
  (c) They do not define the feasibility domain.
  (d) They cannot be used to derive design rules, that is, analytic relations between design variables and parameters for the optimum design.

The method we use based on monotonicity analysis [28,29] eliminates all the above drawbacks of the currently used design methods. The potential of this method and its applicability to the riser design problem has been demonstrated in recent publications [25,26]. This design method is especially fruitful when the objective and the constraints are expressed in closed form. For this reason in this series of reports on riser design we will seek analytical instead of numerical solution. In addition, wherever possible, we will also develop approximate analytical solutions which we will implement only if they compare well with the exact solution.

In this report, which is the first in the riser design series, the static riser boundary value problem is formulated and solved.

The marine riser, the various components of the offshore drilling or production facility which have some influence on the design and performance of risers and the environmental conditions, are described in section I. In section II the mathematical problem is formulated. That is the riser model and the hydrodynamic load model are developed and the boundary conditions are defined. In section III the exact solution of the boundary value problem is derived in terms of the Airy functions of first and second kind. In the last section of this report the cable approximation solution is developed. This approximation is corrected near the ends since it violates two boundary conditions. This approximation is very satisfactory and has been implemented in the static riser design optimization problem [1].

I. THE MARINE RISER DESIGN PROBLEM

I.1 Description of the Riser System

The configuration of an offshore drilling system varies depending on the type of the supporting structure, the site of operation and the environmental conditions. However, the principle of the riser design is independent of the overall drilling system configuration. The major components which make up the system are the following six: (see Figure 1)

a. The Marine Riser. It is a long tubular beam connecting the supporting structure with the well head at the seabed and is composed of rigid steel pipes with an average length of 40 feet and outer diameter between 16 and 42 inches. These pipes, made out of forged weldable steel, are connected by the riser connectors which are designed to minimize installation time and to provide joints able to withstand high tension loads. In areas of high bending stresses - near the end systems - flexible joints are used instead. The riser diameter determines the magnitude of the external hydrodynamic force and, along with the thickness of the pipes, it defines the weight of the riser per unit length and the area of cross-section of the pipes.

Usually, depending on the depth of the water and the size of the riser, additional buoyancy is needed to diminish the required tension at the top of the riser. This buoyancy is provided by floating modules mounted on the riser pipes at the expense of a substantial increase of the hydrodynamic forces [5].

b. The Kill and Choke Lines. They are high pressure pipes needed to control sudden increases of the well pressure. They run along the riser and are mounted directly on the connectors through which they exert concentrated moments and forces on the riser. In recent design the kill and choke lines are mounted inside the riser [2].

c. The Drilling Mud. It circulates between the riser and the drill string and inside the latter. The mud exerts on the riser static pressure force, Coriolis and centrifugal forces due to the riser's local rotation, and vertical and torsional frictional forces due to its viscosity. Of these forces, only the first is significant.

Similar forces are exerted by the mud on the drill string. However, the presence of the mud inside and outside the drill pipe significantly reduces the

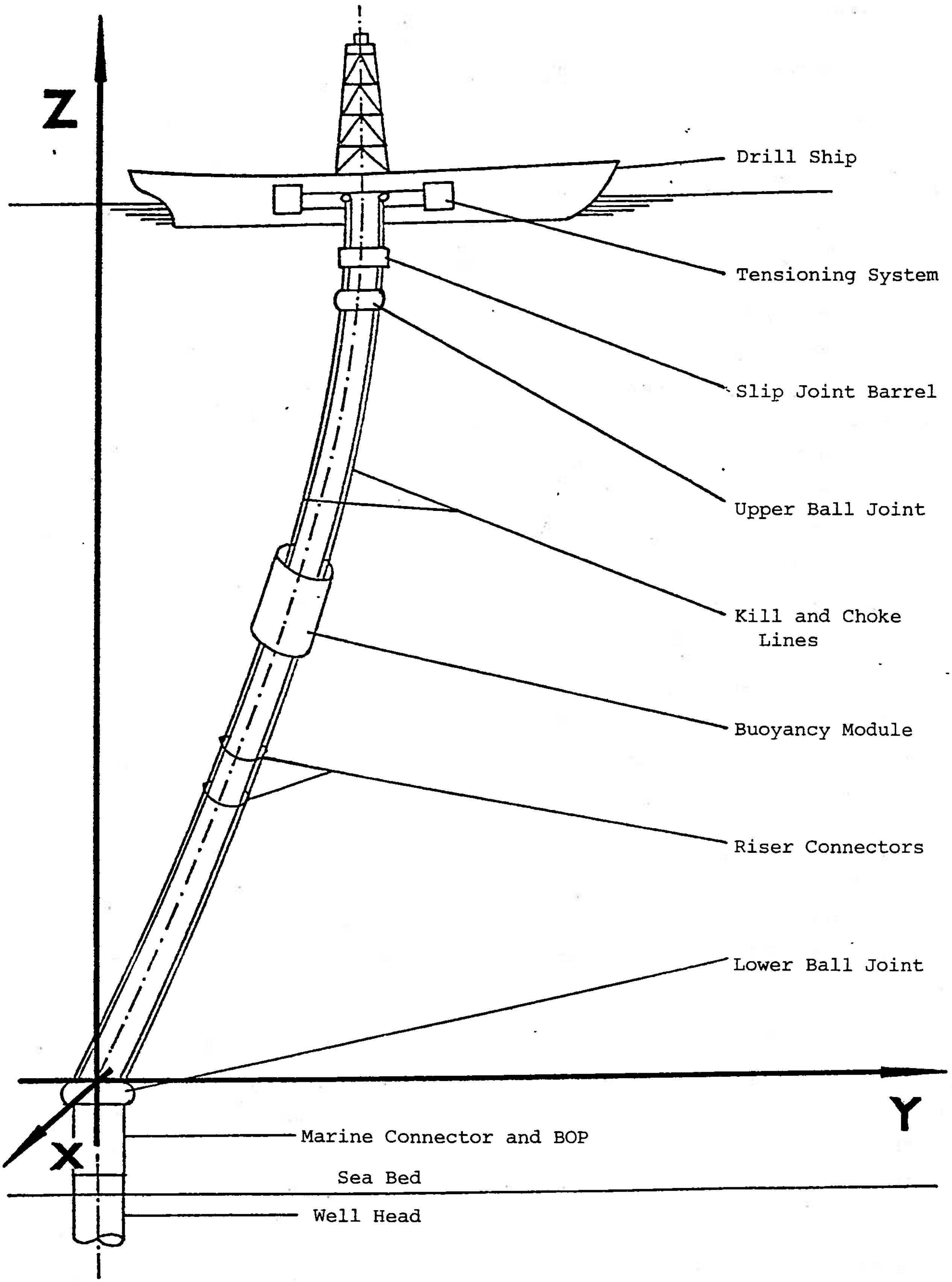

Figure 1. Drilling System

effect of the mud static pressure.

The choice of the drilling mud with the optimal physical properties is a very difficult engineering problem. The mud should be dense enough to:

1) protect the riser from the drill string, and
2) prevent a blowout in case of a sudden increase of the pressure of the well.

On the other hand, if the mud is too dense,

1) it will not lubricate and cool the bit properly,
2) it will not carry away the cuttings at an adequate rate,
3) it may be lost through semipermeable formations in the well, and
4) it will significantly diminish the effective tension of the riser (see Section II).

The drilling mud may be water based or oil based and its density usually varies between 9 and 15 lb/gal, with an average of 12 lb/gal and an absolute maximum of 18 lb/gal.

d. The Drill String. It is the instrument that the riser protects from the environmental forces and guides to the wellhead. Its outer diameter is 4 to 6 inches. It may come in contact and wear out the riser, unless the drill collars are properly spaced.

e. The Upper End System. It consists of a supporting structure, a tensioning system, a slip joint, a ball joint, and a variable size buoy.

The motion of the supporting structure - e.g. a drill ship, a semisubmersible or a submersible - has two components: (1) a fast small amplitude periodic one due to the surface waves, and (2) a slow large amplitude nonperiodic one due to the wind, the ocean currents, and the wave second order drift forces. This motion is controlled by the ship controller's action or the mooring system.

The tensioning system, which is housed on the supporting structure, holds the upper end of the riser through the moonpool and provides part of the necessary tension to keep the riser tight and prevent buckling.

The slip joint compensates for the vessel's heave motion and diminishes the effect of the variation of the tension at the top of the riser due to imperfections of the tensioning system.

The upper ball joint alleviates excessive bending moments due to the rolling and pitching of the supporting structure.

The upper buoy is a variable buoyancy tank, found a few hundred feet below the free surface. It provides additional tension at the top of the riser but is subject to wave forces which may be significant. Consequently, it is designed to be of variable size, and depending on the environmental conditions, its volume is adjusted to provide maximum tension within the structural limits.

f. The Lower End System. It consists of a ball joint, a Blow Out Preventer (BOP), and a marine connector.

The ball joint is a stress alleviation device at the riser's lower end which is an area of high bending stresses.

The blowout preventer, placed on top of the well head, is used primarily to control large pressure experienced during drilling. The BOP is connected to the riser's lower end through the marine connector and to the well through the well-head connector.

I.2 Description of the Environment

The sources of environmental excitations, which may influence the riser directly or indirectly by exerting forces on other components of the offshore drilling system are the following:

a. Ocean Current. Its speed is usually not greater than 2 knots and in most cases its maximum value does not exceed 3 knots. Timewise, the current is slowly varying, while depthwise it may change considerably and even reverse its direction. It exerts significant hydrodynamic forces on the riser and the supporting structure.

b. Surface Waves. They induce oscillatory forces on the riser. The wave spectrum is in general a function of the wave frequency, ω_o, and its direction of propagation, θ_o [30]. In the presence of a current, the properties of the waves and the sea spectrum change [31]. Under certain conditions, the computation of the properties of the modified waves and spectrum is possible [32] and should be implemented in the riser analysis [22].

c. Strong Winds and Gusts. They offset the drill ship from its original place and the riser from its vertical position.

Other sources of excitation, not so often encountered, are the following:

a. Internal Waves. They propagate on the surface between different density layers [33]. Internal wave currents may be as strong as the ocean surface currents [34].

b. Microseismic Waves. They are surface standing waves formed by the vibrations of the bottom of a tank or of the ocean. They generate a second order pressure term, which does not decay exponentially with depth, but is constant in the whole domain [35,36].

c. Tides. They can influence the riser in two ways. First, by varying the water level, and second, by generating currents of considerable speed. The latter is significant for shallow water drilling operations.

d. Volcanic Waves or Tsunamis [37]. Their existence is rather improbable. However, when such waves occur, the probability of surviving of the offshore drilling system is low [15]. It is necessary to disconnect the riser from the wellhead and move the supporting structure and the riser out of the wave system.

I.3 Preliminary Riser Design

In the preliminary riser design, the following basic variables, parameters and constants are involved in general.

a. Variables:

1) The outer diameter and the thickness of the riser pipes.
2) The tension at the top of the riser provided by the tensioning system.
3) The total buoyancy of the buoyancy tanks and their distribution along the riser.
4) The density of the drilling mud.
5) The offset of the supporting structure.
6) The properties of the upper and lower end systems which define the boundary conditions.

b. Parameters:

1) The water depth and the riser length.
2) The upper and lower limits of the drilling mud density [25,26].
3) The properties of the riser material and in particular, Young's Modulus, density and yield stress.
4) The ocean current profile description parameters.
5) The ocean surface wave properties.
6) The internal wave characteristics.
7) The Response Amplitude Operators (RAO) of the supporting structure in all six degrees of freedom.

c. Constants:

1) The intensity of the gravity field g.
2) The water density and kinematic viscosity.

The task of the designer is to compute the values of the design variables for some particular values of the parameters. To achieve this task, the designer should solve several problems related to the riser performance. The most important of these are the following:

a. Model the static and dynamic behavior of the riser taking into account all the significant forces [14,19].

b. Predict the hydrodynamic forces exerted on the riser under various flow conditions [5].

c. Use the derived mathematical models in (a) and (b) to predict the riser's static and dynamic response [21].

d. Calculate the resultant stresses in the material due to the static and dynamic response of the riser [15].

e. Determine the natural frequencies of the riser and find their dependence on the various design parameters [20].

f. Study the possibility of local buckling.

Each of the above problems has been identified and studied early in the research on marine risers [6]. Many computer programs which calculate the stresses in the riser exist [19]. However, none of these problems is fully understood.

Finally the riser design must be optimized for minimum weight, cost or minimum maximum expected stress.

In this work the static riser problem is formulated and solved.

II. FORMULATION OF THE STATIC RISER PROBLEM

The mathematical model for the riser's static response derived in this section is a two-dimensional, linearized, small slope small deflection model. It is a special case of the general three dimensional nonlinear model derived in reference [27]. However, for the sake of completeness of this report the riser static linear model will be derived from first principles in this section. The derivation of the riser model is based on the following assumptions:

a. Shear forces are small. Consequently, the riser is modelled as a Bernoulli-Euler type beam and not as a Timoshenko beam [38].

b. The riser material is isotropic, homogeneous and linear elastic.

c. The presence of the drill string is neglected [19].

d. Thermal stresses are negligible.

e. Coriolis, centrifugal and frictional forces due to the motion of the drilling mud are small.

II.1 Static Equilibrium Equations

In addition to the above assumptions, the following ones are made in order to derive a linearized model.

a. Deflections of the riser are small.

b. Slopes of the riser are small. The consequence of assumptions a. and b. is that squares or products of deflections and/or slopes can be neglected.

c. The torsional moment and the extension of the riser due to the tension are small and can be omitted from the calculations.

d. External forces in the tangential direction are negligible in comparison to those in the normal direction (see Figure 2).

e. Viscous damping forces, which are known to be proportional to the square of the relative velocity of the riser with respect to the water, can be linearized.

As a result of assumptions b., c. and d., the equations of motion of the riser in the (x,z) and (y,z) planes are decoupled, the damping forces are lin-

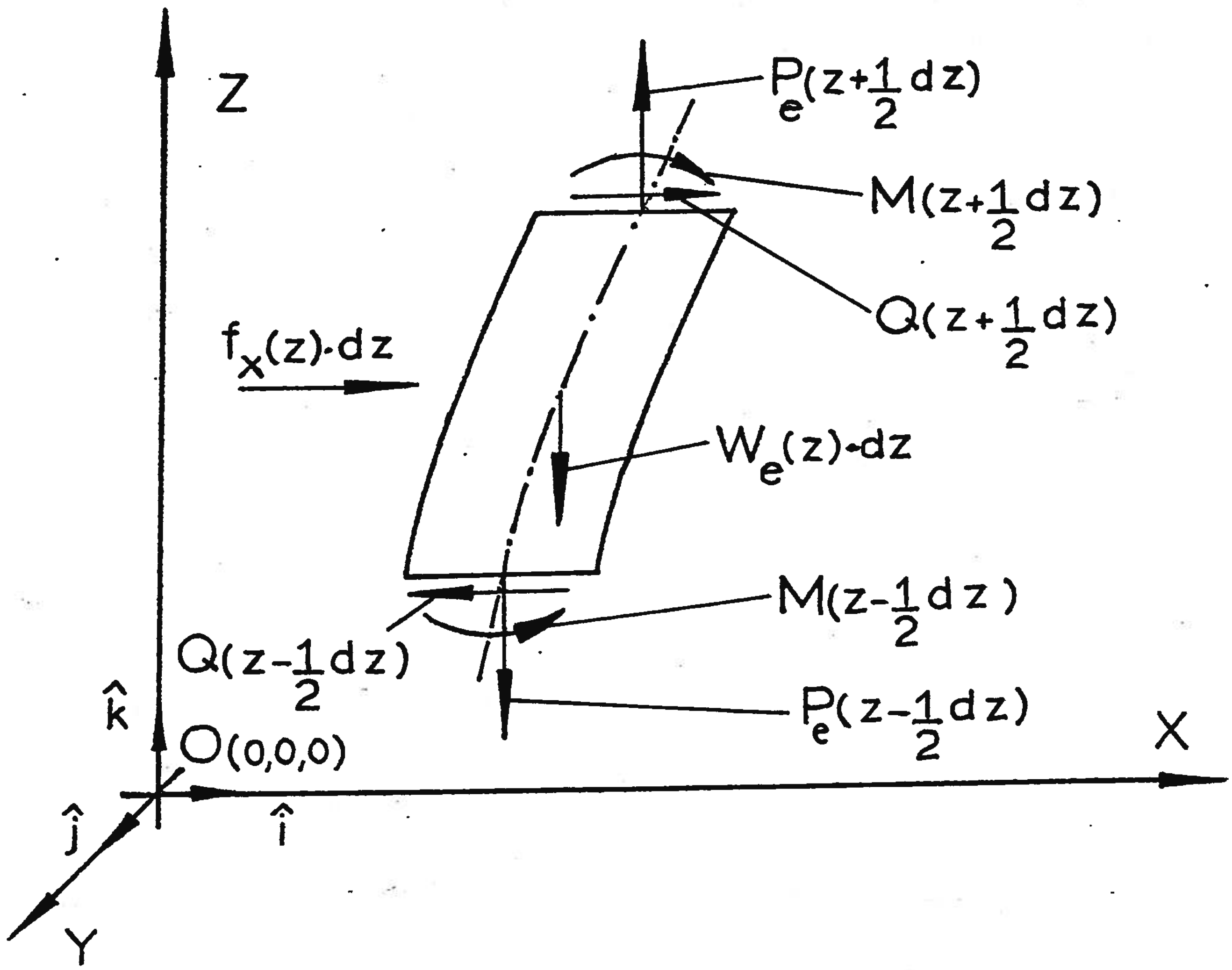

Figure 2. Free Body Diagram for a Differential Element dz

earized and the lift forces due to vortex shedding are treated as external excitation.

Some additional assumptions, related to the discontinuities of the properties of the riser its effective weight and tension are discussed later in this section.

Based on the above assumptions we can derive the linearized differential equations of equilibrium of risers. (Figure 2).

Equilibrium of moments

$$\frac{dM}{dz} - P_e \frac{dU}{dz} + Q = 0 \qquad \text{(II-1)}$$

Equilibrium of forces in the x direction:

$$\frac{dQ}{dz} = f_x \qquad \text{(II-2)}$$

Equilibrium of forces in the z direction:

$$\frac{dP_e}{dz} = W_e \qquad \text{(II-3)}$$

where:

$$W_e \equiv W_R + W_m + W_B - B \qquad \text{(II-4)}$$

$$P_e(z) \equiv T(z) + \gamma_w \frac{\pi D_o^2}{4} (h_w - z) - \gamma_m \frac{\pi D_i^2}{4} (h_m - z) \qquad \text{(II-5)}$$

U is the deflection of the riser in the x direction,

$T(z)$ is the actual tension in the riser,

$M(z)$ is the bending moment in the Y direction,

$Q(z)$ is the shear force in the X direction,

$f_x(z)$ is the hydrodynamic force per unit length exerted on the riser.

In addition,

$$W_R = \gamma_R \frac{\pi}{4} (D_o^2 - D_i^2) \qquad \text{(II-6)}$$

is the weight of the riser per unit length,

$$W_m = \gamma_m \frac{\pi}{4} D_i^2 \qquad \text{(II-7)}$$

is the weight of the drilling mud per unit length

$$W_B = \gamma_B \frac{\pi}{4} (D_B^2 - D_o^2) \qquad \text{(II-8)}$$

is the weight of the buoyancy modules per unit length and

$$B = \gamma_w \frac{\pi}{4} D_B^2 \qquad \text{(II-9)}$$

The linearized constitutive relation of bending is:

$$M(z) = EI \frac{d^2U}{dz^2} \qquad \text{(II-10)}$$

where:

$$I = \frac{\pi}{64} (D_o^2 - D_i^2) \qquad \text{(II-11)}$$

Finally the boundary conditions are:

$$U(0) = 0 \qquad \text{(II-12)}$$

$$U(L) = \Delta \qquad \text{(II-13)}$$

$$\frac{d^2U}{dz^2}(0) = 0 \qquad \text{(II-14)}$$

$$\frac{d^2U}{dz^2}(L) = 0 \qquad \text{(II-15)}$$

Controversial issues in the riser design are the roles of the effective tension $P_e(z)$ and the effective weight per unit length $W_e(z)$. In equation (II-1) it is $P_e(z)$ and not $T(z)$, that is the effective and not the actual tension, that affects the equilibrium of moments. In effect, equations (II-1) and

(II-5) show that the external hydrostatic force has a stiffening effect on the riser similar to that of the actual tension while the internal mud-static pressure has an opposite effect. In addition, in the equation of equilibrium of forces (II-3) in the z direction the effective and not the actual weight of the riser per unit length appears. These statements are proved in reference [27] for the general three dimensional case and the proof is not repeated here. Suffice to say that equations (II-1) and (II-3) can be proved by integrating the static pressure forces from the water and the drilling mud on the external and internal riser surfaces respectively.

At this point we may also explain the effect of the buoyancy modules. The outer diameter of the modules, D_B, appears in W_e since they reduce the riser weight in water. However, D_B does not appear in equation (II-5) since the modules are not rigid and do not contribute to the effective stress $P_e(z)$.

Further, from equation (II-5) we have

$$P_e(z) \equiv T(z) + \rho_w g A_o(z)\,[h_w - z] - \rho_m g A_i(z)\,[h_m - z] \qquad \text{(II-16)}$$

where

$$A_o(z) = \pi D_o^2/4 \qquad \text{(II-17)}$$

and

$$A_i(z) = \pi D_i^2/4 \qquad \text{(II-18)}$$

In addition equations (II-3) and (II-4)

$$\frac{dP_e(z)}{dz} = W_e(z) = W_R(z) + W_B(z) + W_m(z) - B(z) \qquad \text{(II-19)}$$

Eliminating $P_e(z)$ from equations (II-16) and (II-19) we get:

$$\frac{dT(z)}{dz} = W_R(z) + W_B(z) - \rho_w \frac{\pi(D_B^2 - D_o^2)}{4} - \qquad \text{(II-20)}$$
$$- \rho_w g \frac{d}{dz}\left(A_o(z)\right)[h_w - z] + \rho_m g \frac{d}{dz}\left(A_i(z)\right)[h_m - z]$$

Integrating equation (II-19) we get

$$P_e(z) = \int_0^z W_e(\xi)d\xi + P_e(0) \tag{II-21}$$

Similarly from equation (II-20)

$$T(z) = \int_0^z \left[W_R(\xi) + W_B(\xi) - \rho_w g\frac{\pi(D_B^2-D_o^2)}{4}\right]d\xi - \int_0^z \rho_w g\frac{d}{d\xi}\left(A_o(\xi)\right) \cdot [h_w-\xi]d\xi$$

$$+ \int_0^z \frac{dW_m(\xi)}{d\xi} [h_m-\xi]d\xi + T(0) \tag{II-22}$$

We can make the following remarks on equation (II-22)

1. The tension at the top of the riser $(z=L)$ is equal to the weight of the riser in vacuum plus the integrals of the effects of variations or discontinuities of the internal and external riser surfaces.

2. The required tension at the top of the riser is reduced only by the buoyancy of the modules and not by that of the riser.

3. The actual tension $T(z)$ in the riser can be reduced only by short buoyancy modules. Long continuous modules mounted all along the riser cannot serve this purpose.

4. Only a small part of the total mud weight is carried by the riser. In addition the third term on the right-hand side of equation (II-22) indicates that discontinuities of the internal riser surface should be avoided.

II.2 External Loads

It was explained in the introduction of this report that our knowledge of the hydrodynamic forces exerted on circular cylinders moving in a viscous fluid is limited [5]. In order to derive a general solution for the static riser problem we have adopted the following method.

The external hydrodynamic force, $f_x(z)$, is proportional to the square of the relative fluid velocity. Along the riser (depthwise) we can approximate the force profile by a n^{th} degree polynomial* as in equation (II-23).

$$f_x(z) = \frac{1}{2}\rho_w C_D V^2 D_B \left[\alpha_0 + \alpha_1 \frac{z}{L} + \alpha_2\left(\frac{z}{L}\right)^2 + \ldots + \alpha_n\left(\frac{z}{L}\right)^n\right] \qquad \text{(II-23)}$$

where V is a characteristic velocity and C_D is the drag coefficient [5].

II.3 The Boundary Value Problem

If we assume that the riser properties are constant, that is independent of z, then we can combine equations (II-1), (II-2), (II-3) and (II-9) to get the fourth order differential equation with variable coefficients which describes the riser's response to external static loads:

$$\frac{d^2}{dz^2}\left[EI\frac{d^2U}{dz^2}\right] - \frac{d}{dz}\left[\left(W_e z + P_e(0)\right)\frac{dU(z)}{dz}\right] = f_x(z) \qquad \text{(II-24)}$$

In dimensionless form equation (II-24) becomes:

$$\frac{d^4U}{dp^4} - \frac{d}{dp}\left[(\beta p + \tau)\frac{dU}{dp}\right] = \frac{L^4}{EI} f_x(p) = \frac{L^4 D_B}{EI} C_h\, h(p) \qquad \text{(II-25)}$$

where:

$$\beta = \frac{W_e L^3}{EI} \qquad \text{(II-26)}$$

$$\tau = \frac{P_e(0)L^2}{EI} \qquad \text{(II-27)}$$

$$C_h = \frac{1}{2}\rho_w C_D V^2 \qquad \text{(II-28)}$$

$$p = \frac{z}{L} \qquad \text{(II-29)}$$

and $h(p) = \alpha_0 + \alpha_1 p + \alpha_2 p^2 + \ldots + \alpha_n p^n$ (II-30)

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*Other classes of functions may be used to approximate the profile of the external static load; e.g. exponential functions [18]. The method of solution presented in the following section is applicable for any function $f_x(z)$.

Equation (II-25) is subject to the following boundary conditions which are imposed by the fixed lower end of the riser, the static excursion of the drill ship, Δ , and the upper and lower ball joints

$$U(0) = 0 \tag{II-31}$$

$$U(1) = \Delta \tag{II-32}$$

$$\frac{d^2U}{dp^2}(0) = 0 \tag{II-33}$$

$$\frac{d^2U}{dp^2}(1) = 0 \tag{II-34}$$

In conclusion, equation (II-25) subject to the boundary conditions (II-31) to (II-34) models the riser response to the static load defined by equation (II-23).

III. EXACT SOLUTION OF THE BOUNDARY VALUE PROBLEM

Equation (II-25) subject to the boundary conditions (II-31) to (II-34) can be solved in terms of well known functions. The method of solution presented in this section is applicable for any forcing function h(p) . Equation (II-25) can be written as

$$\frac{d^4U}{dp^4} - \frac{d}{dp}\left[(\beta p + \tau)\frac{dU}{dp}\right] = \delta_0 + \delta_1 p + \delta_2 p^2 + \ldots + \delta_n p^n \qquad \text{(III-1)}$$

where $$\delta_i = \frac{L^4 D_B}{EI} C_h \alpha_i \qquad \text{(III-2)}$$

Integrating (III-1) once we get:

$$\frac{d^3U}{dp^3} - (\beta p + \tau)\frac{dU}{dp} = \sigma + \sum_{j=1}^{n+1} \delta_{j-1} p^j / j \qquad \text{(III-3)}$$

where σ is a constant of integration.

Equation (III-3) is equivalent to

$$\frac{d^2y}{dx^2} - xy = \sigma\beta^{-2/3} + \sum_{j=1}^{n+1} \delta_{j-1}\beta^{-2/3}(\beta^{-1/3}x - \beta^{-1}\tau)^j / j \qquad \text{(III-4)}$$

where

$$x = \beta^{1/3}p + \beta^{-2/3}\tau \qquad \text{(III-5)}$$

and

$$\frac{dU(p)}{dp} = y(p) \qquad \text{(III-6)}$$

Rewriting the right hand side of equation (III-4) in terms of powers of x gives:

$$\frac{d^2y}{dx^2} - xy = \sigma\beta^{-2/3} + \sum_{i=0}^{n+1} h_i x^i \qquad \text{(III-7)}$$

The solution to equation (III-7) is given by (III-8) below where the first two terms constitute the homogeneous and the rest the particular solution:

$$y(x) = c_1 Ai(x) + c_2 Bi(x) + \pi(\sigma\beta^{-2/3} + h_0)\{-Ai(x)\int^x Bi(\xi)d\xi$$

$$+ Bi(x)\int^x Ai(\xi)d\xi\} + h_1\pi\ \{-Ai(x)\int^x \xi Bi(\xi)d\xi$$

$$+ Bi(x)\int^x \xi Ai(\xi)d\xi\} + \dots + h_{n+1}\ \pi\{-Ai(x)\ \int^x \xi^{n+1}Bi(\xi) +$$

$$+ Bi(x)\int^x \xi^{n+1}Ai(\xi)d\xi\} \qquad \text{(III-8)}$$

Here $Ai(x)$ and $Bi(x)$ are the Airy functions, and c_1 and c_2 are constants.

From (III-6) and (III-8) we get:

$$U(p) = \beta^{-1/3}\int^x y(\xi)d\xi\ \Big|_{x = \beta^{1/3}p + \beta^{-2/3}\tau} \qquad \text{(III-9)}$$

and

$$\frac{d^2U(p)}{dp^2} = \beta^{1/3}\ \frac{dy(x)}{dx}\ \Big|_{x = \beta^{1/3}p + \beta^{-2/3}\tau} \qquad \text{(III-10)}$$

where

$$\frac{dy(x)}{dx} = c_1 Z_1''(x) + c_2 Z_2''(x) + \pi(\sigma\beta^{-2/3} + h_0)\ Z_3''(x) + \ h_1 Z_4''(x) + \dots$$

$$+ \pi h_{n+1} Z_{n+4}''(x) \qquad \text{(III-11)}$$

$$\int^x y(\xi)d\xi = c_3 + c_1 Z_1(x) + c_2 Z_2(x) + \pi(\sigma\beta^{-2/3} + h_0)Z_3(x) + \pi h_1 Z_4(x)$$

$$+ \dots + \pi h_{n+1} Z_{n+1}(x) \qquad \text{(III-12)}$$

$$Z_1(x) = \int^x Ai(\xi)d\xi \qquad \text{(III-13a)}$$

$$Z_2(x) = \int^x Bi(\xi)d\xi \qquad \text{(III-13b)}$$

$$Z_{n+1}(x) = -\int^{x} Ai(\xi) \int^{\xi} \eta^{n+4} Bi(\eta) d\eta d\xi + \int^{x} Bi(\xi) \int^{\xi} \eta^{n+4} Ai(\eta) d\eta d\xi \quad \text{(III-14)}$$

Finally the four constants of integration σ, c_1, c_2, and c_3 can be determined by solving the following system of linear equations:

$$[A]\ [C] = [R] \quad \text{(III-15)}$$

where

$$[A] = \begin{bmatrix} 1 & Z_1(x0) & Z_2(x0) & \pi\beta^{-2/3} Z_3(x_0) \\ 1 & Z_1(x_1) & Z_2(x_1) & \pi\beta^{-2/3} Z_3(x_1) \\ 0 & Z_1''(x_0) & Z_2''(x_0) & \pi\beta^{-2/3} Z_3''(x_0) \\ 0 & Z_1''(x_1) & Z_2''(x_1) & \pi\beta^{-2/3} Z_3''(x_1) \end{bmatrix} \quad \text{(III-16)}$$

$$x_0 = \beta^{-2/3}\tau \quad \text{(III-17)}$$

$$x_1 = \beta^{1/3} + \beta^{-2/3}\tau \quad \text{(III-18)}$$

$$[C]^T = [c_3\ c_1\ c_2\ \sigma] \quad \text{(III-19)}$$

and

$$[R]^T = -\pi \sum_{i=0}^{n+1} h_i \cdot \left[Z_{i+3}(x_0)\ Z_{i+3}(x_1)\ Z_{i+3}''(x_0)\ Z_{i+3}''(x_1) \right] \quad \text{(III-20)}$$

The solution presented in this section has been implemented numerically for broad ranges of values of the design variables β and τ [1]. For high values of x the asymptotic expressions of $Ai(x)$ and $Bi(x)$ were used [39]. Negative values of x were also considered in order to evaluate the global static buckling loads of risers.

IV. APPROXIMATE SOLUTION OF THE BOUNDARY VALUE PROBLEM

In the previous section, the exact solution to the static riser boundary value problem - defined by the differential equation (III-1) subject to the boundary conditions (II-31) to (II-34) - was derived and can be used to evaluate the bending stresses in the riser needed in the formulation of the design problem.

The exact solution is a function of the Airy functions of first and second kind and their integrals and integrals of their moments. However, the exact solution is very complicated and it is very hard for the designer to realize the significance of the design variables, design parameters and loading parameters and the effect of their variation without a considerable amount of numerical computations. For this reason a simpler approximate solution to the static riser problem is developed and presented in this section. The accuracy of this method, studied in [1], and its simplicity makes it very attractive to the designer particularly in the formulation of the optimization problem [25,26].

IV.1 Cable Approximation

The first term in the fourth order linear differential equation with variable coefficients which describes the static riser behavior, (III-1), is due to the riser bending rigidity, EI. The bending moment that the riser can sustain $EId^2U(z)/dz^2$ is significant only in places along the riser where the riser curvature is significant. This is in general true near the upper and lower ends of the riser. Consequently, the first term in equation (III-1) is comparable in magnitude to the second term, which is due to the effective tension in the riser, only near the boundaries i.e. near $z=0$ and $z=L$. This observation made many investigators [14] conclude that the first term in equation (III-1) can be omitted in comparison to the second one, thus reducing the governing differential equation to a second order linear one with variable coefficients.

$$-\frac{d}{dz}\left[\left(W_e z + P_e(0)\right)\frac{dU}{dz}\right] = f_x(z) \qquad \text{(IV-1)}$$

Equation (IV-1) can now satisfy only two of the four required boundary conditions defined by equations (II-31) to (II-34). Namely,

$$U(0) = 0 \qquad \text{(IV-2)}$$

and

$$U(L) = \Delta \qquad \text{(IV-3)}$$

The boundary value problem defined by the differential equation (IV-1) and the boundary conditions (IV-2) and (IV-3) describes the static behavior of a cable. Consequently, the solution to the cable B.V.P. is called Cable Approximation to the riser B.V.P.

We can rewrite the cable B.V.P. in the following dimensionless form:

$$-\frac{d}{dp}\left[(\beta p + \tau)\frac{dU}{dp}\right] = \frac{L^4}{EI} f_x(pL) \qquad \text{(IV-4)}$$

where β , τ and p are defined by equations (II-26), (II-27) and (II-29) respectively and $f_x(pL)$ is the external time independent load. For the general case studied in section III where the load is described by a n^{th} degree polynomial,

$$f_x(z) = \frac{1}{2}\rho_w C_D V^2 D_B \left[\alpha_0 + \alpha_1 \frac{z}{L} + \alpha_2\left(\frac{z}{L}\right)^2 + \ldots + \alpha_n\left(\frac{z}{L}\right)^n\right] \qquad \text{(IV-5)}$$

the cable B.V.P. becomes:

$$-\frac{d}{dp}\left[(\beta p + \tau)\frac{dU}{dp}\right] = ah(p) \qquad \text{(IV-6)}$$

subject to the boundary conditions

$$U(0) = 0 \qquad \text{(IV-7)}$$

$$U(L) = \Delta \qquad \text{(IV-8)}$$

where:

$$a = \frac{L^4 D_B}{EI} C_h \qquad \text{(IV-9)}$$

$$C_h = \frac{1}{2}\rho_w C_D V^2 \qquad \text{(IV-10)}$$

and

$$h(p) = \alpha_0 + \alpha_1 p + \alpha_2 p^2 + \alpha_3 p^2 + \ldots + \alpha_n p^n \qquad \text{(IV-11)}$$

The solution to the cable B.V.P. is:

$$U(p) = -a \int_0^p \frac{H(\xi)d\xi}{\beta\xi+\tau} + C_1 \frac{1}{\beta} \ln \left(\frac{\beta}{\tau} p + 1\right) \qquad \text{(IV-12)}$$

where:

$$C_1 = \frac{\Delta + a \int_0^1 \frac{H(\xi)d\xi}{\beta\xi+\tau}}{\frac{1}{\beta} \ln \left(\frac{\beta}{\tau} + 1\right)} \qquad \text{(IV-13)}$$

$$H(p) = \int_0^p h(\xi)d\xi \qquad \text{(IV-14)}$$

and ξ is a dummy variable.

IV.2 Range of Validity of the Cable Approximation

Equation (IV-12) is not a solution to (IV-6) for negative values of τ. Obviously the denominator in the first term and the argument of the natural logarithm in the second term of the right hand side of equation (IV-12) may become zero or negative for $\tau<0$ and some value of ξ or p between 0 and 1. When $\tau<0$ the effective tension at the lower end of the riser $P_e(0)$ will be less or equal to zero. A riser may not buckle for negative values of $P_e(0)$ [4] due to its bending rigidity. A cable though, will collapse. Consequently for $\tau<0$ the cable approximation is invalid. It should be emphasized at this point that the cable collapses when the effective tension and not the actual one becomes negative. That is a cable heavier than water may not collapse because it is actually supported by the external hydrostatic pressure.

Even for values of $\tau>0$ the cable approximation is not correct near the riser ends, that is for $p\cong 0$ and $p\cong 1$, because it violates the two boundary conditions (II-33) and (II-34). Two types of corrections are proposed and modelled in the following sections. In both corrections concentrated bending moments are applied at the riser ends in order to make the riser curvature satisfy equations (II-33) and (II-34). From the cable approximation and the riser bending rigidity we can evaluate the required end corrections. However, the riser response to these corrections can be evaluated in two ways. The first method formulated in section IV.3 is simple mathematically but is valid only for high and moderate values of the tensile stress. The second method developed in section IV.4 is more complicated but is acceptable even for relatively low positive values of τ . For low positive or negative values of τ neither of the proposed corrections is acceptable. In this case the exact solution developed in section III must be used. Obviously it is difficult and expensive to implement the exact solution numerically. In reference [1] the exact solution, the cable approximation and the two corrected cable approximations are compared. In reference [1] we show explicitly the limits of applicability of each one of the four solutions and we develop design curves which can readily be used to derive the preliminary riser design particulars.

IV.3 End Corrections for High and Moderate Tensile Stresses

The exact boundary value problem is defined by equations (IV-15) to (IV-19)

$$\frac{d^4U}{dp^4} - \frac{d}{dp}\left[(\beta p + \tau)\frac{dU}{dp}\right] = ah(p) \tag{IV-15}$$

$$U(0) = 0 \tag{IV-16}$$

$$U(1) = \Delta \tag{IV-17}$$

$$\frac{d^2U(0)}{dp^2} = 0 \tag{IV-18}$$

$$\frac{d^2U(1)}{dp^2} = 0 \tag{IV-19}$$

The cable approximation is the solution to the following boundary value problem

$$-\frac{d}{dp}\left[(\beta p + \tau)\frac{dU}{dp}\right] = ah(p) \tag{IV-20}$$

$$U(0) = 0 \tag{IV-21}$$

$$U(1) = \Delta \tag{IV-22}$$

where $h(p)$ and a are defined by equations (IV-6) and (IV-9) respectively.

Let $K(0)$ and $K(1)$ be the values of the riser curvature evaluated by the cable approximation. Then the bending moments at the riser ends are

$$M_0 = \frac{EI}{L^2} K(0) \tag{IV-22}$$

and

$$M_1 = \frac{EI}{L^2} K(1) \tag{IV-23}$$

and not zero as the proper boundary conditions (IV-18) and (IV-19) state. To correct this discrepancy we will impose concentrated moments $-M_0$ and $-M_1$ at the lower and upper riser ends respectively, calculate the riser response and superpose it to the cable approximation. This superposition is possible because the system is linear. The boundary value problem that we will have to solve in order to find the riser response near the upper end is:

$$\frac{d^4U}{dp^4} - \frac{d}{dp}\left[(\beta p + \tau)\frac{dU}{dp}\right] = 0 \tag{IV-24}$$

$$\frac{d^2U}{dp^2}(1) = \frac{-M_1L^2}{EI} \tag{IV-25}$$

$$\frac{d^2U}{dp^2}(0) = 0 \tag{IV-26}$$

$$U(1) = 0 \tag{IV-27}$$

$$U(0) = 0 \tag{IV-28}$$

The above problem can be solved exactly in terms of Airy functions of the first and second kind. However, as explained in reference [1] the numerical implementation of the exact solution is not stable for high values of $(\beta^{1/3} + \beta^{-2/3}\tau)$ and/or $(\beta^{-2/3}\tau)$. For high values of the tensile force we can assume that near the riser ends the quantity $\beta p+\tau$ varies slowly. Consequently, for the upper end correction we can set

$$\beta p + \tau \cong \beta + \tau = A\ . \tag{IV-29}$$

Then the boundary value problem (IV-24) to (IV-28) reduces to the following one:

$$\frac{d^4U}{dp^4} - A\,\frac{d^2U}{dp^2} = 0 \qquad \text{(IV-30)}$$

$$\frac{d^2U(1)}{dp^2} = \frac{-M_1L^2}{EI} \qquad \text{(IV-31)}$$

$$\frac{d^2U(0)}{dp^2} = 0 \qquad \text{(IV-32)}$$

$$U(1) = 0 \qquad \text{(IV-33)}$$

$$U(0) = 0 \qquad \text{(IV-34)}$$

The solution to the boundary value problem defined by equations (IV-30)-(IV-34) is:

$$U(p) = -\frac{K(1)}{\beta+\tau}\,\frac{\sinh\sqrt{\beta+\tau}\;p}{\sinh\sqrt{\beta+\tau}} + \frac{K(1)}{\beta+\tau}\,p \qquad \text{(IV-35)}$$

and the curvature is

$$\frac{d^2U(p)}{dp^2} = -K(1)\,\frac{\sinh\sqrt{\beta+\tau}\;p}{\sinh\sqrt{\beta+\tau}} \qquad \text{(IV-36)}$$

Similarly the problem that we must solve in order to correct the cable approximation near the lower riser end is:

$$\frac{d^4U}{dp^4} - \frac{d}{dp}\left[(\beta p + \tau)\,\frac{dU}{dp}\right] = 0 \qquad \text{(IV-37)}$$

$$\frac{d^2U(1)}{dp^2} = 0 \qquad \text{(IV-38)}$$

$$\frac{d^2U(0)}{dp^2} = \frac{-M_oL^2}{EI} \qquad \text{(IV-39)}$$

$$U(0) = 0 \qquad \text{(IV-40)}$$

$$U(1) = 0 \qquad \text{(IV-41)}$$

For high values of the tensile force we can assume that near $p=0$ the quantity $\beta p+\tau$ varies slowly and can be set approximataely equal to τ

$$\beta p + \tau \cong \tau = B \qquad \text{(IV-42)}$$

Then the boundary value problem (IV-37) to (IV-41) reduces to the following one:

$$\frac{d^4U}{dp^4} - B\,\frac{d^2U}{dp^4} = 0 \tag{IV-43}$$

$$\frac{d^2U(0)}{dp^2} = \frac{-M_0L^2}{EI} \tag{IV-44}$$

$$\frac{d^2U(1)}{dp^2} = 0 \tag{IV-45}$$

$$U(0) = 0 \tag{IV-46}$$

$$U(1) = 1 \tag{IV-47}$$

The solution to the reduced boundary value problem is:

$$U(p) = -\frac{K(0)}{\tau}\,\frac{\sinh\sqrt{\tau}(1-p)}{\sinh\sqrt{\tau}} + \frac{K(0)}{\tau}(1-p) \tag{IV-48}$$

and the curvature is:

$$\frac{d^2U(p)}{dp^2} = -K(0)\,\frac{\sinh\sqrt{\tau}(1-p)}{\sinh\sqrt{\tau}} \tag{IV-49}$$

In order to derive the corrected cable approximation we have to superpose the cable solution (IV-12) and the end corrections (IV-35) and (IV-48). Thus the corrected cable approximation for high values of the tensile force is:

$$U(p) = \left[-a\int_0^p \frac{H(\xi)d\xi}{\beta\xi+\tau} + C_1\,\frac{1}{\beta}\,\ln\left(\frac{\beta}{\tau}\,p+1\right)\right] + \\ + \left[-\frac{K(1)}{\beta+\tau}\,\frac{\sinh\sqrt{\beta+\tau}\;p}{\sinh\sqrt{\beta+\tau}} + \frac{K(1)}{\beta+\tau}\,p\right] + \\ + \left[-\frac{K(0)}{\tau}\,\frac{\sinh\sqrt{\tau}(1-p)}{\sinh\sqrt{\tau}} + \frac{K(0)}{\tau}(1-p)\right] \tag{IV-50}$$

where C_1 is defined by equation (IV-13),

$$K(0) = -\frac{a\alpha_0}{\tau} - \frac{\beta}{\tau^2}\,C_1 \tag{IV-51}$$

and

$$K(1) = -\frac{ah(1)}{(\beta+\tau)} - \frac{\beta}{(\beta+\tau)^2}\left(C_1 - aH(1)\right) \qquad \text{(IV-52)}$$

Similarly, using equations (IV-36) and (IV-49) we get the corrected cable approximation to the riser curvature:

$$\frac{d^2U(p)}{dp^2} = \left[\frac{-ah(p)}{(\beta+\tau)^2} - \frac{\beta}{(\beta+\tau)^2}\left(C_1 - aH(1)\right)\right] + \left[-K(1)\frac{\sinh\sqrt{\beta+\tau}\,p}{\sinh\sqrt{\beta+\tau}}\right] + \left[-K(0)\frac{\sinh\sqrt{\tau}\,(1-p)}{\sinh\sqrt{\tau}}\right] \qquad \text{(IV-53)}$$

IV.4 End Corrections for Low Positive Tensile Stresses

As discussed in the previous section the numerical implementation of the exact solution is not stable for high values of

$$x_1 = \beta^{1/3} + \beta^{-2/3}\tau \qquad \text{(IV-54)}$$

and/or

$$x_0 = \beta^{-2/3}\tau \quad [1] \; . \qquad \text{(IV-55)}$$

For such values of β and τ we have to use the cable approximation and correct it. For high values of τ the correction formulae developed in section IV.3 are satisfactory. However, x_0 and/or x_1 may have high values for high values of β and low positive τ values. In this case we must solve the boundary value problems defined by equations (IV-24) to (IV-28) and (IV-43) to (IV-47) exactly using the Airy functions. The procedure for solving these two boundary value problems is similar to the one developed in section III and the numerical implementation is stable for even higher values of x_0 and/or x_1 [1].

Upper End Correction

The solution to the problem defined by equations (IV-24) to (IV-28) is:

$$U^u(p) = \beta^{-1/3}\{c_3^u + c_1^u Z_1(x) + c_2^u Z_2(x) + \pi\sigma^u\beta^{-2/3}Z_3(x)\}\Big|_{x = \beta^{1/3}p + \beta^{-2/3}\tau} \qquad \text{(IV-56)}$$

and the curvature is:

$$\frac{d^2U^u(p)}{dp^2} = \beta^{1/3}\{c_1^u Z_1''(x) + c_2^u Z_2''(x) + \pi\sigma^u\beta^{-2/3} Z_3''(x)\}\Big|_{x = \beta^{1/3}p + \beta^{-2/3}\tau} \tag{IV-57}$$

where Z_1 , Z_2 , Z_3 are defined by equations (III-13) and (III-14) and c_1^u , c_2^u , c_3^u and σ^u are the solutions to the following system of linear equations:

$$[A]\begin{bmatrix} c_3^u \\ c_1^u \\ c_2^u \\ \pi\sigma^u\beta^{-2/3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -K(1)\beta^{-1/3} \end{bmatrix} \tag{IV-58}$$

where

$$A = \begin{bmatrix} 1 & Z_1(x_0) & Z_2(x_0) & Z_3(x_0) \\ 1 & Z_1(x_1) & Z_2(x_1) & Z_3(x_1) \\ 0 & Z_1''(x_0) & Z_2''(x_0) & Z_3''(x_0) \\ 0 & Z_1''(x_1) & Z_2''(x_1) & Z_3''(x_1) \end{bmatrix} \tag{IV-59}$$

<u>Lower End Correction</u>

The exact solution to the boundary value problem defined by equation (IV-43) to (IV-47) is:

$$U^L(p) = \beta^{-1/3}\{c_3^L + c_1^L Z_1(x) + c_2^L Z_2(x) + \pi\sigma^L\beta^{-2/3} Z_3(x)\}\Big|_{x = \beta^{1/3}p + \beta^{-2/3}\tau} \tag{IV-60}$$

and the curvature is:

$$\frac{d^2U^L(p)}{dp^2} = \beta^{1/3}\{c_1^L Z_1''(x) + c_2^L Z_2''(x) + \pi\sigma^L\beta^{-2/3} Z_3''(x)\}\Big|_{x = \beta^{1/3}p + \beta^{-2/3}\tau} \tag{IV-61}$$

where c_1^L , c_2^L , c_3^L and σ^L are the solutions to the following system of linear equations:

$$[A]\begin{bmatrix} c_3^L \\ c_1^L \\ c_2^L \\ \pi\sigma^L\beta^{-2/3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -K(1)\beta^{-1/3} \\ 0 \end{bmatrix} \qquad \text{(IV-62)}$$

In order to derive the corrected cable approximation we have to superpose the cable solution (IV-12) and the end corrections (IV-56) and (IV-60). Thus the corrected cable approximation for low positive values of the tensile force is:

$$U(p) = \left[-a\int_0^p \frac{H(\xi)d\xi}{\beta\xi+\tau} + C_1 \frac{1}{\beta} \ln\left(\frac{\beta}{\tau} p+1\right) \right] + \beta^{-1/3}\left[c_3^u + c_1^u Z_1(x) + c_2^u Z_2(x) + \right.$$

$$\left. + \pi\sigma^u\beta^{-2/3} Z_3(x) \right]_{x = \beta^{1/3}p + \beta^{-2/3}\tau} \qquad \text{(IV-63)}$$

$$+ \beta^{-1/3}\left[c_3^L + c_1^L Z_1(x) + c_2^L Z_2(x) + \pi\sigma^L\beta^{-2/3} Z_3(x) \right]_{x = \beta^{1/3}p + \beta^{-2/3}\tau}$$

Similarly, using equations (IV-57) and (IV-61) we get the corrected cable approximation to the riser curvature:

$$\frac{d^2U(p)}{dp^2} = \left[\frac{-ah(p)}{(\beta+\tau)} - \frac{\beta}{(\beta+\tau)^2} \left(C_1 - aH(p)\right) \right] +$$

$$+ \beta^{1/3}\left[c_1^u Z_1''(x) + c_2^u Z_2''(x) + \pi\sigma^u\beta^{-2/3} Z_3''(x) \right]_{x = \beta^{1/3}p + \beta^{-2/3}\tau} +$$

$$+ \beta^{1/3}\left[c_1^L Z_1''(x) + c_2^L Z_2''(x) + \pi\sigma^L\beta^{-2/3} Z_3''(x) \right]_{x = \beta^{1/3}p + \beta^{-2/3}\tau}$$

(IV-64)

CONCLUSIONS AND FURTHER WORK

In this report the static marine riser problem has been modelled and solved. The exact solution developed in section III can be implemented numerically over limited ranges of the values of the dimensionless variables β and τ due to numerical instability of the Airy functions of the second kind for high values of its arguments. To solve the riser problem for high values of β and τ the cable approximation was developed in section IV. Two different types of corrections were used in order to achieve satisfactory approximations. One makes the cable approximation applicable for low positive values of the tensile force and the other for moderate and high values of the tensile force.

All three solutions have been implemented numerically in reference [1] and the ranges of validity of each one has been found. These solutions are used in [1] to do a parametric static riser design analysis and generate design curves.

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