General Equilibrium Theory of the Term Structure of Interest Rates

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Business Administration)
in The University of Michigan
2012

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For my parents, Tina and Wei-Li, and my brother Stone. For my wife, Arlene, and
our little peanut, Yuan Yuan. This work is only made possible by your
unconditional love and support over the years.

ACKNOWLEDGEMENTS

The author is indebted to his committee members Robert Barsky, Robert Dittmar, Reuven Lehavy, Haitao Li, and Francisco Palomino for their guidance and insight. The chapters of my dissertation have benefited from discussions and correspondences with Kenneth Ahern, Alessandro Barbarino, Sugato Bhattacharyya, John Cochrane, Jonathan Cohn, Eric Leeper, Erica Li, Deborah Lucas, M.P. Narayanan, Paolo Pasquariello, Amiyatosh Purnanandam, Uday Rajan, Clemens Sialm, and Tyler Shumway. The author would also like to express his gratitude to his colleagues for endless hours of discussion and entertainment, Stefanos Delikouras, Gi Hyun Kim, Di Li, Maciej Szefler, Daniel Weagley, and Chen Xue. Finally, thanks to all seminar participants at the Michigan Finance Brownbag series, the 2011 FMA Doctoral Student Consortium, the 2011 FMA main program, and the 8th Annual Corporate Finance Conference Doctoral Poster Session at Washington University. All the usual disclaimers apply.

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ABSTRACT

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This dissertation consists of three essays examining the interactions between macroe-conomy and the term structure of interest rates. In the first two theoretical essays, I investigate the effects of fiscal policy and monetary policy on the nominal and real yield curves. In the first essay on fiscal policy, bond supply and the government spending shock drive bond risk premia in a production economy where a fraction of the households are constrained to consume their entire after-tax labor income. Ricardian equivalence breaks down in the model, and fiscal policy becomes relevant to the real economy. This friction increases the consumption risk of the marginal pricers, and it results in large term premia on nominal bonds. In the second essay, co-authored with Erica Li and Francisco Palomino, the influence of monetary policy on the real term premia and the inflation risk premia is studied in the New-Keynesian framework with nominal wage and price rigidities. Under rigidities, both the monetary policy shock and the permanent productivity shock generate positive covariances between the marginal utility to consume and real long-term yields, leading to positive real term premia. However, their implications on inflation risk premia are the opposite: policy shocks generate negative inflation risk premia while permanent productivity shocks

generate positive inflation risk premia. The final essay, which is empirical, tests the expectations hypothesis of the term structure by examining bond return predictability using fiscal policy variables. Built on the theory provided in the first essay, I regress excess bond returns on government spending shocks and revenue shocks as well as the conditional volatility on the spending shocks. I find evidence against the expectations hypothesis in that fiscal policy shocks have explanatory power on future bond returns. The result is robust after controlling for monetary policy as well as financial variables such as the *Cochrane and Piazzesi* (2005) "return-forecasting" factor.

CHAPTER I

Does Fiscal Policy Matter for Treasury Bond Risk Premia?

1.1 Abstract

Fiscal policy affects Treasury bond risk premia. I examine the impact of government spending and financing on bond risk premia via a dynamic stochastic general equilibrium (DSGE) model. Bond supply becomes relevant to bond risk premia through limited market participation by the non-Ricardian households in the model. I derive two key insights. First, government spending shocks generate positive covariances of marginal utility to consume with inflation and nominal yields, making both nominal bonds and long-term bonds, respectively, poor hedges against consumption risk. Therefore, investors demand positive risk premia for holding inflation and interest rate risk. Second, the presence of non-Ricardian households helps resolve the bond premium puzzle: the calibrated model generates 10-year term premia matching the levels of term premia observed in the data on average.

1.2 Introduction

As of November 2010, the United States government has roughly nine trillion dollars of outstanding debt held by the public, representing approximately 66% of

U.S. Gross Domestic Product (GDP). The heavy debt burden generates great debate among academics, politicians, and the public on when the current path of spending will become unsustainable as debt obligations and interest payments accumulate over time. A central issue in this debate is the influence fiscal policy has on bond risk premia, which make up the cost of borrowing for the government. There is anecdotal evidence from the recent financial crisis that government spending and tax policy affected interest rates. In March 2010, after rounds of stimulus spending, interest rates jumped due to weak demand in Treasury bond auctions as investors called for austerity[1]. Furthermore, the Treasury bond market reacted unfavorably to the Bush tax-cut extension announcement by the Obama administration, and long-term rates rose sharply in early December 2010[2]. In this case, bond investors perceived that the tax-cut extension would negatively impact tax revenue in the short term, making it more difficult for the government to pay down the current debt level.

In this paper, I propose a dynamic stochastic general equilibrium (DSGE) model to examine the effects of fiscal policy on the term structure of interest rates and associated risk premia. I focus on two specific aspects of fiscal policy: shocks to government spending[3] and the systematic response of the fiscal authority to these shocks via the fiscal policy rule. Theoretically, the relationship between fiscal policy and the nominal term structure remains an open question. Given the path of government spending, Ricardian equivalence states that the financing decision of debt versus taxes is irrelevant to aggregate consumption because infinitely-lived agents with perfect foresight will adjust their savings accordingly to undo any fiscal actions taken by the government. In traditional general equilibrium models, where households are Ricardian, fiscal policy is neutral with respect to inflation dynamics and the nominal term structure. As a result, bond supply does not affect nominal bond prices. On the

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[1]WSJ article, "Treasury Crushed after Auction," March 24th, 2010.

[2]Bloomberg article, "Treasuries Fall on Tax-Cut Extension...," Dec 7th, 2010.

[3]I use the term government spending loosely in this paper. What I have in mind specifically is government purchases sans transfers. Thanks to Bob Barsky for pointing this out.

other hand, the fiscal theory of the price level equates the government's outstanding debt obligations deflated by the price level to its expected real surpluses using the fiscal valuation equation in partial equilibrium[4]. In this framework, fiscal policy and bond supply affect nominal prices, but the model is silent on consumption growth so it cannot help determining interest rates.

To fill this theoretical gap, I extend the fiscal theory of the price level into general equilibrium[5] by endogenizing bond supply and taxes, and I break Ricardian equivalence in the model by introducing a class of agents with limited market participation. The heterogeneity of the consumers follows *Mankiw* (2000), such that they consist of optimizing households who have the ability to save by investing in nominal Treasury bonds, as well as rule-of-thumb households who are constrained to consume their entire after-tax labor income. The households are called savers and spenders, respectively, and the presence of the spenders leads to fiscal policy non-neutrality such that debt becomes a predetermined state variable in the households' decision-making process. Each period, both types of households decide the quantity of goods to consume and the hours of labor to supply to firms in order to maximize their lifetime utilities over consumption and labor.

In the benchmark model, outlined in section 4, the production sector is populated with a continuum of monopolistic competitive firms producing intermediate goods to supply to a single final good-producing firm, which aggregates the intermediate goods into the unique consumption good for the households. The intermediate-good firms adjust prices according to the *Calvo* (1983) process, under which only a fraction of the firms are allowed to maximize present value of their expected profits by choosing the optimal price each period. This is the standard New Keynesian setup that leads

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[4]See *Cochrane* (2001) and *Cochrane* (2005) for a detailed exposition on fiscal theory of the price level. The present value condition is $\frac{\text{Nominal Debt}}{\text{Price Level}} = $ PV of Real Surpluses, and it can be derived from the government's intertemporal budget constraint.

[5]I accomplish this in the spirit of the cashless model of *Cochrane* (2005) such that money demand is zero. This eliminates the cash-in-advance constraint in the model. *Cochrane* (2005) has shown that the government's fiscal valuation equation alone can determine the price level in the economy.

to monetary policy non-neutrality in the real economy. This setup will allow me to compare impulse responses of various endogenous variables following government spending and monetary policy shocks. In the simplified version of the model presented in section 3, I make monetary policy neutral by assuming a representative firm which can adjust its price to the optimal level every period to focus on the mechanism underlying fiscal policy and bond risk premia.

To close the model, the government is made up of a monetary authority and a fiscal authority. The monetary authority sets the nominal short-term interest rate using a simple Taylor rule with contemporaneous feedbacks from inflation and the output gap plus a monetary policy shock which represents any unexpected deviations of the nominal short rate. The balance sheet of the monetary authority is irrelevant in the model, so quantitative easing is not driving the results. The fiscal authority chooses the amount of current period lump-sum taxes to collect as a function of debt maturing in the same period and the current realization of the spending shock. The coefficients on maturing debt and spending shocks represent the policy rule parameters that determine the fraction of government obligations to be financed through taxes instead of through new debt issuance. Implementing this simple rule, a tight fiscal policy is described by high policy rule parameters, and the government finances itself more through taxes resulting in less debt outstanding.

How does uncertainty in government spending affect bond risk premia? I answer this question by analyzing the simplified model in which the government spending shock is the only exogenous process driving the economy. For nominal bonds, investors require compensation for inflation risk that erodes nominal payoffs. Positive government spending shocks lead to lower consumption while debt supply and taxes increase. At the same time, positive spending shocks decrease the present value of expected future surpluses. Lower total surpluses generate inflation because the amount of nominal debt maturing in the current period is predetermined and known; thus,

the current price level has to adjust upwards for the equality to hold in the government's budget constraint. Under this scenario, inflation is high exactly when savers wish to consume more but high inflation makes payoffs on nominal bonds low in real terms, and the positive covariance between marginal utility of consumption and inflation generates positive inflation risk premia. Moreover, when the fiscal authority tightens fiscal policy by increasing the policy rule parameters, inflation risk premia on nominal bonds decrease. Tighter fiscal policy means the government uses higher taxes to finance spending as opposed to issuing more nominal debt; less debt means less inflation, ceteris paribus. Since the threat of inflation is mitigated by a more stringent fiscal policy, savers demand lower risk premia in return for holding nominal government bonds, and inflation risk premia decrease.

The term premium is positive in the model because investors are compensated for holding long-term Treasury debt over short-term debt. The mechanism underlying positive term premia starts with positive government spending shocks that result in higher taxes and greater bond supply as the fiscal authority issues more bonds to finance higher spending. Given their budget constraint, higher taxes and investing more in Treasury bonds mean lower consumption and higher marginal utility to consume for the savers. At the same time, nominal interest rates rise and bond prices fall as the government induces savers to save more in anticipation of higher taxes in the future. This means that, ex-ante, long-term bonds are a poor hedge against consumption risk because their payoffs are low relative to short-term bonds precisely when the savers would like to consume more. The positive covariance between the marginal utility of consumption and nominal yields allows long-term bonds to command positive term premia. In addition, tighter fiscal policy reduces term premia on long-term bonds similar to inflation risk premia. Increasing policy parameters means less debt outstanding. Less debt translates into lower interest rates, and the covariance between the marginal utility of consumption and nominal yields decreases

resulting in smaller term premia.

The simplified model is convenient to disentangle the economic intuition behind how government spending shocks and bond supply drive bond risk premia. However, it is insufficient to generate model implied moments that are useful in terms of matching the data. I build the benchmark model in order to verify that shocks to fiscal policy under the non-Ricardian regime contribute to the model's ability to produce large term premia close to that is observed empirically. The benchmark model extends the simple model by the inclusion of nominal price rigidities and monopolistic competition in the production sector. I further augment the households' preferences from CRRA utilities to Epstein-Zin recursive utilities and add stochastic volatility to government spending shocks. The benchmark model is able to solve the "bond premium puzzle" by matching the unconditional mean and standard deviation of term premium on 10-year bonds while simultaneously matching the macroeconomic moments. In a similar setup, *Rudebusch and Swanson* (2010) document that the use of Epstein-Zin preferences in the presence of long-run risk can generate large term premium in DSGE models. However, their results come at a cost of a large coefficient of relative risk aversion of over 100. In this paper, the benchmark model matches the unconditional macro and finance moments with a risk aversion parameter of 32 because the savers are unable to perfectly smooth their consumption, due to the non-Ricardian regime, and demand higher risk premia for holding financial assets. The benchmark model is also helpful in deciphering the relative magnitude of responses of bond risk premia following monetary policy and fiscal policy shocks. Impulse responses of term premia on long-term bonds following one-standard deviation monetary policy and government spending shocks show that spending shocks are indeed important in the presence of monetary policy coordination.

The joint modeling of the yield curve and macroeconomic variables has received much attention since *Ang and Piazzesi* (2003), where the authors connect latent term

structure factors to inflation and the output gap. More recently, many term structure studies incorporate monetary policy elements in their models using the fact that the nominal short rate is the monetary policy instrument. However, these models are generally silent on the effects of fiscal policy on the term structure despite evidence suggesting that it has nontrivial effects on interest rates. The primary contribution of this paper is establishing the link between fiscal policy and risk premia on nominal bonds, namely the term premium and the inflation risk premium. The model shows loose fiscal policy and high government spending cause investors to demand higher returns in exchange for holding Treasury securities.

The rest of this paper is organized as follows. Section 2 discusses related literature. Section 3 presents the model where government spending shocks are the only exogenous shocks driving the economy. Section 4 summarizes the results from the extended model. Impulse responses following various exogenous shocks are examined. Section 5 concludes.

1.3 Related Literature

This paper is most closely related to the literature on term structure and bond risk premia in equilibrium. *Campbell* (1986) specifies an endowment economy in which utility maximizing agents trade bonds of different maturities. When the exogenous consumption growth process is negatively autocorrelated, term premia on long-term bonds are positive, generating upward sloping yield curves because they are bad hedges against consumption risk compared to short-term bonds. The intuition is straightforward; high current consumption growth means low expected future consumption growth and low prices for long-term bonds. On the other hand, high current consumption growth means marginal utility of consumption is high now. Therefore, long-term bonds always have low payoffs in the states of the world where investors want to consume more. More recently, *Piazzesi and Schneider* (2006), using

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Epstein and Zin (1989) preferences, show that inflation is the driver that generates a positive term premium on nominal long-term bonds. Negative covariance between consumption growth and inflation translates into high inflation when consumption growth is low and marginal utility to consume is high. *Wachter* (2006) generates upward sloping nominal and real yield curves employing habit formation. In her model, bonds are bad hedges for consumption as agents wish to preserve the previous level of consumption as current consumption declines.

The models in all of the above papers are endowment economies. *Rudebusch and Swanson* (2008) and *Rudebusch and Swanson* (2010) examine bond risk premia in general equilibrium where utility-maximizing agents supply labor to profit-maximizing firms to produce consumption goods. The best-fit model in the latter paper is successful in matching the basic empirical properties of the term structure, this comes at a cost of employing an extremely high parameter of relative risk aversion. *Hsu and Palomino* (2012) examine risk premia on real bonds in a DSGE setting, integrating price stickiness into the model so monetary policy is non-neutral on the real economy. Calibrated to TIPS data, they find that productivity growth shocks and monetary policy shocks generate negative term premis on real bonds. Finally, *Palomino* (2010) studies optimal monetary policy and bond risk premia in general equilibrium with New-Keyesian techniques. He shows that the welfare-maximizing monetary policy affects inflation risk premia depending on the credibility of the monetary authority in the economy as well as the representative agent's preference.

More broadly, this paper also contributes to the literature on the effect of macroeconomic variables on the term structure of interest rates. *Ang and Piazzesi* (2003), in a reduced-form no-arbitrage framework, show that when the short rate follows a simple Taylor rule, inflation and output gap can explain a significant portion of the movements in the short end and the middle part of the yield curve. *Bekaert, Cho, and Moreno* (2006) solve a forward-looking New-Keynesian model nested within the

no-arbitrage term structure framework and conclude that the unobserved inflation target is closely related to the level factor whereas monetary policy shocks affect the slope and curvature factors.

A growing body of literature documents the impact of fiscal policy and bond supply on interest rates. Employing a no-arbitrage model with government budget deficit as one of the factors, *Dai and Philippon* (2006) find that the 10-year rate increases by 40–50 basis points as a result of a 1% increase to debt to GDP ratio. Importantly, for the purposes of this paper, the authors are able to show that fiscal shocks affect the long end of the yield curve by changing future expected short rates as well as inflation risk premia. In contrast, *Engen and Hubbard* (2004) conduct a regression-based analysis of the impact of U.S. government debt and deficits on interest rates and find that a 1% increase in the debt to GDP ratio results in a 3 basis point increase in the real 10-year interest rate. In a similar study on the quantitative effect of government debt and deficits have on long term rates, *Laubach* (2009) confirm the findings of *Engen and Hubbard* (2004) employing forward rates and projected debts and deficits. Most recently, *Krishnamurthy and Vissing-Jorgensen* (2010) examine the safety premium in Treasury bond yields. They conclude that changes in supply of government debt have significant effects on yield spreads of various fixed income assets including long-term bonds.

1.4   Simplified Model

Here I present a general equilibrium model with heterogenous agents and a representative firm with no price rigidities in the economy. The monetary authority sets the short term nominal rate following a simple Taylor rule, while the fiscal authority has the ability to issue one-period as well as longer-maturity bonds to satisfy the government's budget constraint. The only shock driving the endogenous variables in the model is the government spending shock, $\epsilon_{g,t+1}$

Households

There are two types of households in the economy; following *Mankiw* (2000), I will call them savers and spenders[6] according to their budget constraints. The savers have the ability to save current income in order to smooth future consumption by purchasing government bonds. In contrast, the spenders are required to consume their entire after tax income. The representative agent of the savers maximizes lifetime utility by solving the following:

$$\max E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{{C_{t+j}^o}^{1-\gamma}}{1-\gamma} - \frac{{N_{t+j}^o}^{1+\omega}}{1+\omega} \right) \right] \tag{1.1}$$

subject to the budget constraint:

$$P_t C_t^o + \sum_{j=1}^{\infty} Q_t^{(j)} \left[B_t(t+j) - B_{t-1}(t+j) \right] = W_t P_t N_t^o + B_{t-1}(t) - P_t T_t^o + P_t \Psi_t,$$

where the o superscript denotes the optimizing household. β is the time discount factor, γ is the coefficient of relative risk aversion, and ω is the inverse of the Frisch elasticity of labor supply. C_t and N_t are consumption and labor, respectively. P_t is the price level in the economy. $B_t(t+1)$ is the amount of nominal bonds outstanding at the end of period t and due in period $t+1$. W_t refers to real labor income, which is the same across households in the economy. T_t is real taxes and transfers to the government, and Ψ_t is dividend income coming from the firms.

The budget constraint states that the agent has periodic income from labor and dividends, as well as bonds maturing at time t, and long-term bonds repurchased by the government at time t before they are due. The agent then decides how much to consume after taxes and how much to pay for newly issued bonds at time t at price $Q_t^{(j)}$.

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[6]For the rest of the paper, I will interchangeably refer to savers and spenders as optimizing households and rule-of-thumb households, respectively.

Writing out the Lagrangian, denoted by L, the first order conditions are:

$$\frac{\partial L}{\partial C_t^o} : \frac{1}{C_t^{o\gamma}} - \lambda_t P_t = 0 \Rightarrow \lambda_t = \frac{1}{C_t^{o\gamma} P_t} \tag{1.2}$$

$$\frac{\partial L}{\partial N_t^o} : -N_t^{o\omega} + \lambda_t P_t W_t = 0 \Rightarrow W_t = C_t^{o\gamma} N_t^{o\omega} \tag{1.3}$$

$$\frac{\partial L}{\partial B_t(t+1)} : -\lambda_t Q_t^{(1)} + E_t[\beta \lambda_{t+1}] = 0 \Rightarrow Q_t^{(1)} = E_t\left[\beta \left(\frac{C_t^o}{C_{t+1}^o}\right)^\gamma \frac{P_t}{P_{t+1}}\right] \tag{1.4}$$

$$\frac{\partial L}{\partial B_t(t+2)} : -\lambda_t Q_t^{(2)} + E_t[\beta \lambda_{t+1} Q_{t+1}^{(1)}] = 0$$

$$\Rightarrow Q_t^{(2)} = E_t\left[\beta \left(\frac{C_t^o}{C_{t+1}^o}\right)^\gamma \frac{P_t}{P_{t+1}} Q_{t+1}^{(1)}\right], \tag{1.5}$$

where λ_t is the Lagrangian multiplier for the budget constraint.

The representative agent of the spenders maximizes the same lifetime utility:

$$\max E_t\left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^r{}^{1-\gamma}}{1-\gamma} - \frac{N_{t+j}^r{}^{1+\omega}}{1+\omega}\right)\right],$$

while following a different budget constraint such that

$$P_t C_t^r = W_t P_t N_t^r - P_t T_t^r,$$

where the r superscript now refers to the rule-of-thumb household. The parameters β, γ, and ω are the same across both types of households. The level of consumption each period simplifies from the budget constraint to be:

$$C_t^r = W_t N_t^r - T_t^r, \tag{1.6}$$

and the first order condition relating wage and labor supply is:

$$W_t = C_t^{r\gamma} N_t^{r\omega}. \tag{1.7}$$

Finally, the aggregate consumption, labor supply and tax in the economy can be expressed by the following weighted average of the corresponding variables of each type of households:

$$C_t = \mu C_t^r + (1-\mu)C_t^o \tag{1.8}$$

$$N_t = \mu N_t^r + (1-\mu)N_t^o \tag{1.9}$$

$$T_t = \mu T_t^r + (1-\mu)T_t^o, \tag{1.10}$$

where μ denotes the fraction of rule-of-thumb consumers in the economy.

Firms

A representative firm in the economy produces the consumption good and sells to the households using labor as the only input in its production function. The firm's objective is to maximize profit for its shareholders, thus it faces the following optimization problem:

$$\max_{N_t} \quad P_t Y_t - W_t P_t N_t \tag{1.11}$$

$$\text{s.t.} \quad Y_t = N_t^\alpha, \tag{1.12}$$

where Y_t is output, and α is the labor share of output. The resulting first order condition is $W_t = \alpha N_t^{\alpha-1}$.

Monetary Policy

The monetary authority sets the short term interest rate according to a simple Taylor rule:

$$i_t = \imath + \rho_\pi \pi_t, \tag{1.13}$$

where π_t is inflation, and it is defined as $\pi_t = log(P_t) - log(P_{t-1})$, or change in log price level.

Government

In the presence of long-term bonds, the government's flow budget constraint balances resources with uses:

$$P_t T_t + Q_t^{(1)} B_t(t+1) + \cdots + Q_t^{(\infty)} B_t(t+\infty)$$
$$= B_{t-1}(t) + Q_t^{(1)} B_{t-1}(t+1) + \cdots + Q_t^{(\infty)} B_{t-1}(t+\infty) + P_t G_t,$$

where G_t is consumption by the government or government spending. G_t is not productive in the model economy. To explain the intuition on the meaning of this equation, I rearrange the terms above to get

$$B_{t-1}(t) - \sum_{j=1}^{\infty} Q_t^{(j)} [B_t(t+j) - B_{t-1}(t+j)] = P_t(T_t - G_t). \tag{1.14}$$

I then rewrite the budget constraint in its present value form as

$$\frac{B_{t-1}(t)}{P_t} + E_t \left[\sum_{j=1}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^\gamma \frac{B_{t-1}(t+j)}{P_{t+j}} \right] = E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^\gamma S_{t+j} \right], \tag{1.15}$$

where S denotes the primary surplus,

$$S_t = T_t - G_t. \tag{1.16}$$

The present value condition tells us that, in any given period, the government's fiscal liability has to be endorsed by the present value of expected real surpluses from now to infinity. Derivation of equation (1.15) can be found in the appendix[7].

Following *Cochrane* (2001), I make the assumption of geometrically declining debt structure to further simplify the government budget constraint:

$$B_{t-1}(t+j) = \varphi^j B_{t+j-1}(t+j). \tag{1.17}$$

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[7]This is a variant of the original present value equation derived by *Cochrane* (2001) in which consumption growth is exogenously fixed.

Furthermore, it can be shown that the fraction of debt issued at time t maturing at time $t + j$ is

$$\frac{B_t(t+j) - B_{t-1}(t+j)}{B_{t+j-1}(t+j)} = \varphi^{j-1}(1-\varphi). \tag{1.18}$$

Substituting (1.17) into the present value version of the government budget constraint in (1.15) and (1.18) into the intertemporal budget constraint, multiplying the latter by $\frac{\varphi}{1-\varphi}$ and adding, I arrive at the following:

$$\left(1 + \frac{\varphi}{1-\varphi}\right)\frac{B_{t-1}(t)}{P_t} = E_t\left[\sum_{j=0}^{\infty}\beta^j\left(\frac{C_t^o}{C_{t+j}^o}\right)^{\gamma}S_{t+j}\right] + \frac{\varphi}{1-\varphi}S_t, \tag{1.19}$$

which is only expressed in terms of the amount of debt outstanding at the end of period $t - 1$ that is due at time t. By applying the geometrically declining maturity structure, the present value budget constraint is free of long-term bonds.

Fiscal Policy

The fiscal authority decides the lump-sum tax by the linear fiscal policy rule,

$$\tau_t = \rho_b d_{t-1}(t) + \rho_g g_t, \tag{1.20}$$

which is a simple version of the generalized fiscal rule outlined in *Woodford* (2003), where $d_{t-1}(t)$ is the amount of real debt, defined as the nominal amount deflated by the price level outstanding at the end of period $t-1$ and due in period t. The variable g_t is log real government spending such that $g_t = log(G_t)$. g_t is exogenously specified to follow an $AR(1)$ process with mean θ_g:

$$g_{t+1} = (1 - \phi_g)\theta_g + \phi_g g_t + \sigma_g \epsilon_{g,t+1}, \tag{1.21}$$

where $\epsilon_{g,t+1}$ is a $\mathcal{N}(0,1)$ random shock.

Market Clearing

In this economy, total output has to equal total consumption plus total government spending:

$$Y_t = C_t + G_t. \tag{1.22}$$

Log-Linearization

The system presented above is highly non-linear. In order to solve for the endogenous variables in the model in closed form, I log-linearize the system. For ease of exposition, all lower-case variables are log quantities of their capitalized counterparts unless specified otherwise, and capitalized variables without the time subscript denote their steady state values.

The optimizing consumers' first order conditions are:

$$w_t = \gamma c_t^o + \omega n_t^o, \tag{1.23}$$

$$-i_t = log\beta + logE_t \left[e^{-\gamma \Delta c_{t+1}^o - (\pi_{t+1} + \pi^*)} \right], \tag{1.24}$$

where w_t is log real wage, c_t^o is log consumption, n_t^o is log labor supply, and $\pi^* = log(\Pi^*)$ is log inflation target. The rule-of-thumb consumers log consumption level can be found by linearizing (1.6) around the steady state:

$$C_t^r = W_t N_t^r - T_t^r$$

$$C(1 + c_t^r) = WN(1 + w_t + n_t^r) - T(1 + \tau_t^r)$$

$$c_t^r = \frac{WN}{C}(w_t + n_t^r) - \frac{T}{C}\tau_t^r, \tag{1.25}$$

where, following *Gali, Valles, and Lopez-Salido* (2007), I explicitly assume $C^r = C^o = C$ and $N^r = N^o = N$ in steady state. τ_t is log taxes. Furthermore, the linearized labor supply optimality condition for the spenders is:

$$w_t = \gamma c_t^r + \omega n_t^r. \tag{1.26}$$

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The log-linearized aggregation equations are the following:

$$c_t = \mu c_t^r + (1 - \mu) c_t^o \tag{1.27}$$

$$n_t = \mu n_t^r + (1 - \mu) n_t^o. \tag{1.28}$$

Finally, by combining (1.23), (1.26) and (1.27), I have the same wage demand equation as in the case of a representative agent:

$$w_t = \gamma c_t + \omega n_t. \tag{1.29}$$

The firm's production function and first order condition are log-linearzized as

$$y_t = \alpha n_t, \tag{1.30}$$

$$w_t = log\alpha + (\alpha - 1) n_t, \tag{1.31}$$

respectively, where y_t is log output.

To log-linearize the government's budget constraint, I start with the righthand-side of the present value equation:

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^\gamma S_{t+j} \right] = E_t \underbrace{\left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^\gamma \frac{S_{t+j}}{S_t} \right]}_{H_t} S_t$$

$$= \left\{ 1 + \beta H e^{logE_t \left[e^{-\gamma \Delta c_{t+1}^o + s_{t+1} - s_t + h_{t+1}} \right]} \right\} S_t.$$

By definition, H_t is the present value of the expected real primary surplus ratio between periods t and $t+j$ discounted by the real stochastic discount factor. Denoting $\Upsilon_t = logE_t \left[e^{-\gamma \Delta c_{t+1}^o + s_{t+1} - s_t + h_{t+1}} \right]$, this means

$$H_t = H e^{h_t} = 1 + \beta H e^{\Upsilon_t}.$$

Solving for h_t in the equation above, and applying a first order Taylor series expansion to the righthand-side, I have:

$$
\begin{aligned}
h_t &= log(1 + \beta H e^{\Upsilon_t}) - log(H) & (1.32) \\
&= \overline{\eta_h} + \eta_h log E_t \left[e^{-\gamma \Delta c^o_{t+1} + s_{t+1} - s_t + h_{t+1}} \right], & (1.33)
\end{aligned}
$$

where H is the deterministic steady state of H_t. Please see the appendix for detailed derivations of h_t as well as the functional forms of the coefficients $\overline{\eta_h}$ and η_h.

Given the definition of h_t, I log-linearize the government's present value budget constraint by rearranging terms and taking logs on both sides of equation (1.19) to get

$$
log \left(1 + \frac{\varphi}{1 - \varphi} \right) + log \frac{D}{S} + d_{t-1}(t) - (\pi_t + \pi^*) - s_t = log \left[H e^{h_t} + \frac{\varphi}{1 - \varphi} \right],
$$

where $D_{t-1}(t)$ is the real government debt outstanding at time $t - 1$ and due at time t. D and S are values of real debt and real primary surplus in the steady state, respectively. π_t is inflation, and π^* denotes the inflation target of the central bank. As a reminder, φ is the geometric maturity parameter. Applying a first order Taylor expansion on the righthand-side of the above equation, the fully log-linearized fiscal equation is

$$
log \left(1 + \frac{\varphi}{1 - \varphi} \right) + log \frac{D}{S} + d_{t-1}(t) - (\pi_t + \pi^*) - s_t = \overline{\eta_\varphi} + \eta_\varphi h_t.
$$

All derivations and functional forms can be found in the appendix.

The log-linearized primary surplus and market clearing conditions are, respec-

tively:

$$s_t = \frac{T}{S}\tau_t - \frac{G}{S}g_t \tag{1.34}$$

$$y_t = \frac{C}{Y}c_t + \frac{G}{Y}g_t, \tag{1.35}$$

where the capitalized variables without time subscripts are steady state values.

1.4.1 The Impact of Fiscal Policy and Government Spending Shocks

I solve the simple model analytically employing the method of undetermined coefficients. The result is a collection of policy functions for the endogenous variables in the model that are affine in the state variables. The state variables are the spending shock, which is exogenous, and the level of one-period debt maturing at time t, which is predetermined. For convenience, in the remainder of the paper, I will refer to $d_{t-1}(t)$ as maturing debt and $d_t(t+1)$ as outstanding debt. Notice the former is a state variable, and the latter is endogenously determined. Details of the solution technique are provided in the appendix.

The Real Economy

When $\mu = 0$, there are no spenders present, and the model economy reduces to a traditional RBC model with a government sector. In this case, the fiscal policy rule is irrelevant and bond supply does not matter to inflation, consumption, output, labor supply, or real wage. The coefficient loadings on maturing debt, $d_{t-1}(t)$, are zero for these endogenous variables. Furthermore, the nominal term structure is flat as bond risk premia are negligible under this scenario.

When the spenders are included in the model, aggregate consumption is determined endogenously by four equations: the wage demand equation, the firm's production function, the firm's first order condition, and the market clearing condition, equations (1.29), (1.30), (1.31) and (1.35), respectively. Similar to traditional RBC

models, aggregate consumption is not affected by the amount of debt maturing, and its loading on $d_{t-1}(t)$ is zero. If households are populated only by the optimizing consumers, then consumption of the optimizers, which determines the pricing kernel, equals to aggregate consumption leading to bond supply neutrality, and fiscal policy has no effect on the term structure and bond risk premia. However, the presence of rule-of-thumb consumers in the economy disengages the one-to-one relationship between consumption of the optimizing households and aggregate consumption. In addition, since the consumption of the spenders is directly impacted by bond supply and fiscal policy through taxes, consumption dynamics of the savers also vary with bond supply and fiscal policy. In other words, the inclusion of rule-of-thumb households is necessary to induce fiscal policy non-neutrality on the pricing kernel.

Figure A.1 shows the comparative statics of the coefficient loadings of consumption by the optimizing households, inflation, and real outstanding debt as the parameter μ changes. Higher value of μ means a greater proportion of the agents in the economy are rule-of-thumb consumers who cannot save their income to smooth consumption. In the absence of rule-of-thumb consumers, or when μ is zero, c_d^o is also zero as fiscal policy is neutral. As μ increases, the loading of savers' consumption on debt increases while its loading on spending shocks becomes more negative. As expected, a positive spending shock lowers the consumption of optimizers so the coefficient c_g^o is negative. On the other hand, the positive correlation between consumption and debt maturing can be explained by the fact that the optimizing households are Ricardian. More debt due today means less taxes in the future, and the savers actually consume more today as opposed to saving for the future.

[**Figure A.1 Here**]

Furthermore, consumption by the optimizers affect the inflation dynamics through the savers' Euler equation. As the economy becomes more populated with spenders,

the response of inflation to the spending shock becomes more positive, and the response to maturing debt becomes more negative. Similarly, a greater share of spenders leads short term outstanding debt to react more strongly to the government spending shock, but the auto-regressive coefficient, d_d, actually decreases. This can be explained by the fact that higher μ means a smaller fraction of the households demand holding short term debt.

Figure A.2 plots the comparative statics of the coefficients of present value of surplus ratios (h_t), inflation, and outstanding debt on g_t and $d_{t-1}(t)$ as the persistence parameter of the government spending shock, ϕ_g, varies. As the spending shock becomes more persistent, or as ϕ_g increases, d_g decreases as the fiscal authority issues less short term real debt at time t to keep the debt repayment at time $t+1$ low. However, this also results in persistence of the short-term debt such that d_d increases in ϕ_g. At the same time, higher persistence of the spending shock means higher future taxes and the present value of surplus ratio increases holding current surplus constant. This results in lower inflation through the government budget constraint given the amount of maturing debt is fixed.

[**Figure A.2 Here**]

The comparative statics of the coefficients of present value of consumption, outstanding debt, inflation, and present value of surplus ratios in terms of the fiscal variable ρ_b are shown in figure A.3. The parameter ρ_b is the proportion of maturing real debt to be financed by taxes in a given period. As ρ_b increases, consumption of the spenders decreases due to higher taxes and induces the savers to consume more. Higher taxes today means higher surplus today and lowers the present value of surplus ratio so h_g and h_d are decreasing in ρ_b. Again, through the government budget constraint, lower h_g leads to higher π_g given that $d_{t-1}(t)$ is fixed at time t. Last, higher value of ρ_b means more fiscal discipline and the autoregressive coefficient on

debt, d_d decreases while d_g increases because more outstanding debt is a result of higher government spending as opposed to debt rollover.

[**Figure A.3 Here**]

The comparative statics of the endogenous variables to the second fiscal parameter, ρ_g, the fiscal authority's responses to the spending shock, is plotted in figure A.4. Since ρ_g is also the coefficient of taxation on the spending shock, it has a one-to-one relationship with τ_g. Furthermore, given the spending shock at time t, higher tax directly increases the current surplus so s_g is also increasing in ρ_g. At the same time, higher tax reduces the consumption by the rule-of-thumb households through their budget constraint. The optimizing households, on the other hand, are Ricardian. They observe higher current tax and infer a stronger fiscal stance by the fiscal authority of not issuing more debt to finance current spending. They consume more and save less now. Thus, the demand of short term debt and total debt go down resulting in lower d_g and h_g as ρ_g increases. Finally, the coefficient of inflation on g_t decreases with ρ_g to maintain equality of the government budget constraint holding maturing debt constant.

[**Figure A.4 Here**]

Figure A.5 shows the comparative statics of the impact multipliers of outstanding real debt, inflation, and present value of future surplus ratios as functions of the Taylor rule parameter, ρ_π. As the central bank tightens monetary policy by raising ρ_π, bond prices decrease and the demand for risk-free bonds rise as consumption and tax stay unchanged for the optimizing households. As a result, higher ρ_π raises d_g and d_d. Furthermore, given the increase in savings at time t will lead to higher taxes at time $t+1$, this means greater short term debt translates into smaller surplus next period. Thus, h_d is declining in ρ_π. While h_d is decreasing, h_g is increasing

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because total debt outstanding is an increasing function of ρ_π given greater demand for short term bonds. Last, from the government's budget constraint (A.10), holding s_t constant and conditional on the amount of real debt maturing, $d_{t-1}(t)$, greater h_t has to be offset by lower inflation. Thus, π_g and π_d are moving in opposite direction of π_g and π_d, respectively, as ρ_π increases.

[**Figure A.5 Here**]

The Term Structure of Interest Rates

Figures A.7 and A.9 illustrate bond risk premia in the term structure. To facilitate better understanding of the impact of the macroeconomy on bond risk premia, I utilize analytical solutions to demonstrate the mechanism underlying the connection between households and the bond market.

The decomposition of nominal bond yields consists of real yields, expected inflation, and inflation risk premium. In closed form, I have:

$$i_t^{(n)} = r_t^{(n)} + \frac{1}{n}\left\{ E_t\left[\pi_{t,t+n}\right] + cov_t(m_{t,t+n}, \pi_{t,t+n}) - \frac{1}{2}var_t(\pi_{t,t+n}) \right\},$$

where the conditional covariance of the marginal rate of consumption substitution between times t and $t+n$ with inflation during the same period gives us the compensation for inflation risk for holding n-period to maturity nominal bonds. To understand inflation risk premium in the current model, I study this covariance term using matrices employing the following notation:

$$\pi_s = \begin{bmatrix} \pi_d \\ \pi_g \end{bmatrix}, \ \Gamma = \begin{bmatrix} \Gamma_d \\ \Gamma_g \end{bmatrix}, \ \lambda = \begin{bmatrix} 0 \\ \lambda_g \end{bmatrix}, \ \Phi = \begin{bmatrix} d_d & d_g \\ 0 & \phi_g \end{bmatrix}, \text{ and } \Sigma = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_g \end{bmatrix}^2.$$

Under this specification, the one-period real pricing kernel and inflation can be written as:

$$m_{t,t+1} = -\Gamma_0 - \Gamma^T S_t - \lambda^T \Sigma^{\frac{1}{2}} \epsilon_{t+1},$$

and

$$\pi_{t,t+1} = \pi_{t+1} = \pi + \pi_s^T S_{t+1},$$

respectively, where $S_t = [d_{t-1}(t) \ g_t]^T$. Under conditions such that $\Phi = \Phi^T$, the covariance term has an analytical solution which can be found in *Hsu and Palomino* (2012). In the current model, however, this is not feasible since the state variables are correlated. To calculate the inflation risk premium on the long end of the yield curve for a n-period bond, I recursively solve for the covariance term starting from the one-period case such that:

$$
\begin{aligned}
cov_t(m_{t,t+j+1}, \pi_{t,t+j+1}) &= cov_t(m_{t,t+j}, \pi_{t,t+j}) - \lambda^T \Sigma \pi_s - \\
&\quad \Gamma^T (\mathbf{I}_2 - \Phi^T)^{-1} (\mathbf{I}_2 - \Phi^{Tj}) \Sigma \pi_s - \\
&\quad \lambda^T \Sigma \Phi^T (\mathbf{I}_2 - \Phi^T)^{-1} (\mathbf{I}_2 - \Phi^{Tj}) \pi_s - \\
&\quad \Gamma^T (\mathbf{I}_2 - \Phi^T)^{-1} (\mathbf{I}_2 - \Phi^{Tj}) \Sigma \Phi^T (\mathbf{I}_2 - \Phi^T)^{-1} (\mathbf{I}_2 - \Phi^{Tj}) \pi_s,
\end{aligned}
$$

where $j = 1, ..., n-1$ and $cov_t(m_{t,t+1}, \pi_{t,t+1}) = -\lambda^T \Sigma \pi_s = -\gamma c_g^o \sigma_g^2 \pi_g$.

From the closed-form solution, the market price of risk on spending shocks, λ_g, and the inflation response to the same shocks drive the inflation risk premium. Since c_g^o is negative and π_g is positive in the baseline calibration, a positive spending shock increases the marginal utility of consumption as well as inflation. Therefore, inflation is high precisely at a time when marginal utility to consume is high, meaning nominal bonds are not a very good hedge against inflation risk, relative to real bonds in this economy. As a result, nominal bond prices are low and inflation risk premium is high. The mechanism through which positive inflation risk premium is generated in the simple model is outlined in figure A.6.

[Figure A.6 Here]

The comparative statics of inflation risk premium on a 1-period bond is displayed

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in figure A.7. Parameters ϕ_g, ρ_π and φ affect inflation risk premium only through the inflation channel but not the savers' consumption channel, while parameters μ, ρ_b and ρ_g affect inflation risk premium through both. The increasing persistence parameter and the Taylor rule parameter lower inflation resulting in a lower inflation risk premium. Increasing the maturity structure parameter, φ, on the other hand, generates inflation risk premium. When the proportion of rule-of-thumb households increase with μ, consumption of the optimizers decreases leading to a higher inflation risk premium. Lastly, increasing the fiscal policy parameters controls inflation, and the inflation risk premium decreases.

[**Figure A.7 Here**]

While the inflation risk premium is the compensation the investors require from the government for holding nominal bonds, term premium is the compensation for risk the investors require in return for holding long term bonds over short term bonds. Typically, I can extract components of the average spread between bonds of different maturities by the following:

$$
E\left[i_t^{(n)} - i_t\right] = \frac{1}{n}\left\{\sum_{k=1}^{n-1}(n-k)E\left[cov_t\left(m_{t,t+1}^{\$}, i_{t+1}^{(n-k)}\right)\right] - \frac{1}{2}\sum_{k=1}^{n-1}(n-k)^2 E\left[var_t\left(i_{t+1}^{(n-k)}\right)\right]\right\},
$$

where the covariance between the 1-period nominal pricing kernel and nominal interest rate is the term premium. Applying properties of the geometric series, I can compute the term premium in closed form:

$$
\sum_{k=1}^{n-1}(n-k)E\left[cov_t\left(m_{t,t+1}^{\$}, i_{t+1}^{(n-k)}\right)\right] = -(n-1)\lambda^{\$T}\Sigma\left(\mathbf{I}_2 - \Phi^T\right)^{-1}
$$

$$
\left[\mathbf{I}_2 - \frac{1}{n-1}\Phi^T\left(\mathbf{I}_2 - \Phi^T\right)^{-1}\left(\mathbf{I}_2 - \Phi^{T(n-1)}\right)\right]\Gamma^{\$},
$$

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where

$$\Gamma^\$ = \begin{bmatrix} \Gamma_d^\$ \\ \Gamma_g^\$ \end{bmatrix} \text{ and } \lambda^\$ = \begin{bmatrix} 0 \\ \lambda_g^\$ \end{bmatrix}.$$

For $n = 2$,

$$
\begin{aligned}
E\left[cov_t\left(m_{t,t+1}^\$, i_{t+1}\right)\right] &= -\lambda^{\$T}\Sigma\Gamma^\$ \\
&= -\lambda_g^\$\sigma_g\Gamma_g^\$ \\
&= -(\gamma c_g^o + \pi_g)\sigma_g^2\left[(\gamma c_d^o + \pi_d)d_g - \gamma c_g^o(1 - \phi_g) + \pi_g\phi_g\right].
\end{aligned}
$$

Long-term bonds are worse at hedging consumption risk when compared to short-term bonds if the marginal rate of consumption substitution between times t and $t+1$ is high while long-term bonds are expected to have cheaper prices at time $t+1$. Therefore, the investor requires a risk premium at time t for holding long-term debt, generating positive term premium. Since $\lambda_g^\$$ is positive, a positive government spending shock raises the marginal rate of consumption substitution while simultaneously increasing the yield on long term bonds, thus lowering the prices. The investors understand that long term bonds have lower payoffs exactly when the marginal utility of consumption is high and require a higher risk premium for holding those bonds. In other words, government spending shocks increase the covariance between the marginal rate of consumption substitution and long term yields resulting in higher term premia. Figure A.8 diagrams the channels through which a positive term premium is realized.

[**Figure A.8 Here**]

Figure A.9 plots comparative statics of term premium for a 2-period bond. When μ is increasing, consumption of the optimizing households decreases. Long term rates also increase making them cheaper, generating a positive covariance with the pricing kernel, and resulting in positive term premium. When government spending

shocks are persistent, more long-term bonds are outstanding leading to a higher term premium. For ρ_b and ρ_g, as the fiscal authority tightens fiscal policy by raising those parameters, less outstanding long-term debt is issued following a positive spending shock thus lowering term premium. As the monetary authority raises ρ_π, inflation decreases, which in turn drives the nominal pricing kernel higher while decreasing nominal rates. Long term bonds become a better hedge for consumption risk and term premia declines. Finally, higher value of φ leads to greater outstanding long-term bonds in the economy making their yields even higher and driving up term premium.

[**Figure A.9 Here**]

1.5   The Benchmark Model

The benchmark model presented in this section is built upon the traditional general equilibrium framework with four additional layers of complexity. The first feature is the saver-spender dichotomy introduced in the simplified model in the previous section. Rule-of-thumb consumers in the economy are included in the model to remove the one-to-one mapping between consumption of the optimizing consumers, which drives the dynamics of the pricing kernel, and aggregate consumption. In the absence of these rule-of-thumb consumers, fiscal policy is neutral and does not influence the real economy and bond risk premia.

The second feature of the full model is the introduction of monopolistic producers and price rigidities in the setting of a New-Keynesian economy. The New-Keynesian framework is the workhorse of modern monetary economics. The microfoundations underlying the New-Keynesian framework generates frictions in the firm's first order condition relating its real marginal cost to its marginal product of labor. The resulting output gap between actual output and output under flexible prices summarizes

aggregate economic activity. The output gap is forward-looking and is driven by the expectation of the future output gap as well as the real short-term interest rate[8]. The monetary authority implements the Taylor rule and sets the nominal short rate as a function of inflation and output gap. The nominal short rate in turn affects the real short rate thus making monetary policy non-neutral with respect to output gap and the real economy.

Disengaging monetary policy neutrality by augmenting the saver-spender model with the New-Keynesian framework, I assess the implications of fiscal policy on bond risk premia in the presence of an effective monetary authority. The Taylor rule used in the full model is:

$$i_t = \imath + \rho_\pi \pi_t + \rho_x x_t + u_t,$$

where x_t is the output gap, and u_t is the monetary policy shock following an autoregressive process. Impulse responses of the term premium following a positive government spending shock are contrasted with those following a positive monetary policy shock to provide the relative strength of the two policies.

A robust finding in the empirical term structure literature is the predictability of excess bond returns in the observed data, which implies time-varying risk premia for nominal bonds. One of the objectives in building the full model is to endogenously generate time variation in bond risk premium missing from the analytical solutions due to log-linearization. To that end, I solve the extended model numerically using Dynare++, which applies perturbation methods[9] to the non-linear system of equations to compute the policy functions for the endogenous variables. Furthermore, under first and second[10] order approximations, risk premium is zero and constant,

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[8]For a detailed exposition on the New-Keynesian framework, see *Clarida, Gali, and Gertler* (1999) and *Gali* (2008) where the IS equation and the New-Keynesian Phillips Curve are derived.

[9]*Aruoba, Fernández-Villaverde, and Rubio-Ramírez* (2006) report that perturbation methods are highly accurate and provide speed improvements unmatched by value function iteration or projection methods. See *Judd* (1998) for the mathematical theory behind the perturbation methods with applications to continuous- and discrete-time economic models.

[10]The first order approximation can be viewed as the certainty equivalent such that the policy

respectively. It is only under third or higher order approximations the model produces time-variation such that the coefficients in the functions governing risk premia are non-zero. However, given the calibrated parameter values, the consequent unconditional variances on inflation risk premium and term premium are too small in the model. I modify the economic agents' preferences from CRRA utilities to *Epstein and Zin* (1989) utilities and make government spending shocks heteroskedastic to make bond risk premia more volatile.

Indeed, I introduce two additional features to the extended model to amplify the magnitude of time-variation in risk premium: Epstein-Zin preferences for the households and stochastic volatility on government spending shocks. Instead of maximizing the present value of expected utility as in equation (1.1), the representative agent of the savers has recursive utility of the form:

$$V(C_t^o, N_t^o) = \left\{ (1 - \beta)U(C_t^o, N_t^o)^{1-\psi} + \beta E_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1-\psi}{1-\gamma}} \right\}^{\frac{1}{1-\psi}},$$

where

$$U(C_t^o, N_t^o) = \left[\frac{C_t^{o1-\psi}}{1-\psi} - \frac{N_t^{o1+\omega}}{1+\omega} \right]^{\frac{1}{1-\psi}}.$$

Under this notation, ψ is the inverse of elasticity of intertemporal substitution (EIS) governing the agent's preference for early or late resolution of uncertainty while γ is the coefficient of risk aversion. As before, superscript o denotes the consumption and labor supply of the optimizing households. It should be noted that the representative agent of the spenders also has recursive preferences, and the two households share the same parameter values. To insert stochastic volatility into government spending shocks, I augment equation (1.21) by replacing σ_g with $\sigma_{g,t}$, which follows a square

functions are independent of the variance-covariance matrix of the underlying shocks. *Schmitt-Grohé* (2005) shows that first order approximations of the expected values of endogenous variables are equal to their non-stochastic steady-state values. Moreover, the second order approximation would result in cross product terms involving volatility to be zero by construction.

root process such that:

$$\sigma^2_{g,t+1} = (1 - \phi^g_\sigma)\theta^g_\sigma + \phi^g_\sigma \sigma^2_{g,t} + \sigma^g_\sigma \epsilon_{\sigma,t+1}.$$

This is analogous to the volatility process used by *Bansal and Yaron* (2004) to generate time-varying economic uncertainty in the long run risk model. Detailed derivation of the households' first order conditions with Epstein-Zin preferences can be found in the appendix.

Outside of yielding time-varying risk premium, I choose to solve the extended model with third order perturbation methods for the following reasons. *Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao* (2009) compare different solution techniques for computing the equilibrium dynamics of a stochastic growth model with recursive utilities and document that third order perturbation is the best approach balancing accuracy and speed. Also, *Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe* (2009) conclude that stochastic volatility shocks only enter into the decision rules with coefficients different from zero in a third order approximation. To study the role of stochastic volatility on expected excess bond returns, I need to employ the third order approximation. Finally, *Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez* (2010) demonstrate that one needs at least a second order approximation for risk aversion to affect the decision rules in the case of Epstein-Zin preferences. Under first order approximation, there is no difference between the decision rules coming from CRRA or recursive utilities with the same value of elasticity of intertemporal substitution.

1.5.1 Calibration

Table A.1 presents parameter values for the calibrated benchmark model. Most parameters are standard with values within the range found in the literature. The

discount factor and coefficient of risk aversion are set to 0.995 and 32, respectively. The baseline value for elasticity of wages with respect to labor supply, ω, is set to 0.2 following *Gali, Valles, and Lopez-Salido* (2007). For the Epstein-Zin preferences, the inverse of the elasticity of intertemporal substitution (EIS), ψ is set to 0.633.

[**Table A.1 Here**]

In the New-Keynesian economy, the Calvo parameter is set to 0.66, which means roughly two-third of the firms each period are not allowed to optimally adjust their prices while the rest have the ability to choose the profit-maximizing price. In other words, parameter α dictates the degree of price rigidity present in the economy. Furthermore, the firms are monopolistic and can charge a markup on the price. The price markup parameter is set to -0.9.

In the baseline setting, the weight of rule-of-thumb households, μ, is 0.5. This is within the estimated range of percentage of rule-of-thumb households in the U.S. economy[11]. For the policy response of inflation in the monetary rule, I set the Taylor rule coefficient to 1.25, which satisfies the Taylor principle of greater than one-to-one response of the nominal short rate to inflation, and is common to the empirical literature. The response of the monetary policy rule on the output gap is 0.3 while the constant in the monetary policy rule is set to 0.005. Moreover, the autoregressive parameter on the monetary policy shock is 0.5 with a standard deviation of 0.003. Last, the inflation target for the central bank is 22 basis points per quarter.

The fiscal policy parameters on maturing debt and government spending are 0.33 and 0.1, respectively. Those are taken from the simple fiscal rule used by *Gali, Valles, and Lopez-Salido* (2007), which the authors obtain from VAR-based estimates employing empirical data. Furthermore, the autoregressive coefficient on the spending shock, ϕ_g, is set to 0.9 to match the half-life of the responses of government spending[12].

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[11]See *Mankiw* (2000).
[12]See *Gali, Valles, and Lopez-Salido* (2007).

The persistence parameter on the productivity shock, on the other hand, is set at 0.98 with a standard deviation of 0.005. The debt maturity structure parameter, φ, is set to 0.58. This means for the amount of nominal debt maturing in a given period, 42% is issued one period ago while 58% is issued from previous periods, and the amount issued declines geometrically going back in time. This is in line with the values from *Cochrane* (2001) where the author suggests φ to be less than 1.

The outcome of the calibration is shown in table A.2. I match the model-generated unconditional mean and standard deviation of each of the following endogenous variables to average quarterly data from $1971 : IV$ to $2010 : III$: the consumption-to-GDP ratio, inflation, the 1- and 2-year nominal yields.

[**Table A.2 Here**]

1.5.2 Analysis

The Bond Premium Puzzle

It is known that the term premium generated by the traditional DSGE models used in macroeconomics is too small and not volatile enough when compared to empirical assessment of term premium inferred from data. *Rudebusch and Swanson* (2010) (RS) refers to this as an example of the "bond premium puzzle." In a New-Keynesian DSGE model similar to the one presented here, RS solve the bond premium puzzle by augmenting the standard DSGE model with Epstein-Zin preferences and the inclusion of long-run risks in productivity and inflation. Although their model is able to closely match both the macroeconomic and financial moments simultaneously, RS achieve the best fit of their model using coefficients of relative risk aversion of over 200. While it is not uncommon to see high estimations of the risk aversion parameter in other general equilibrium term structure models with Epstein-Zin preferences, it is clear the risks faced by the economic agents in these models fail to capture the quantity of risk with which U.S. households have to contend.

As pointed out by RS, this could signal failure of the representative-agent DSGE framework because consumption volatility is not homogenous across the population drawn from U.S. data[13]. Implementing the extended model outlined above, the model generates high levels of term premium with large volatilities on long term bonds to match empirical observations. Also using Epstein-Zin preferences, the optimizing households in the model economy only has a coefficient of risk aversion of 32, a much more reasonable value than required in the RS model.

[**Table A.3 Here**]

Table A.3 compares the unconditional expectations and standard deviations of the term premium on 10-year bonds generated by the benchmark model in this paper to the best fit model in *Rudebusch and Swanson* (2010), as well as U.S. data.

The Effect of Monetary Policy

Figure A.10 plots the impulse responses of consumption by savers, inflation, 10-year bond yields, and the term premium on 10-year bonds as a result of one standard deviation shocks to government spending, monetary policy, and productivity. Comparing the left column and the middle column, government spending shocks and monetary policy shocks respectively, it is clear that the monetary policy shock affects the macroeconomic variables such as savers' consumption and inflation more, while the magnitude of responses by the financial variables is greater following the government spending shock. Furthermore, the government spending shock has non-trivial consequences on the term premium in the presence of an active monetary policy. In other words, the monetary authority cannot negate completely the increase in inflation or term premium as a result of higher government spending.

The directions of the responses are in line with expectations given the simple model in the previous section and the existing literature on monetary policy. A

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[13]*Malloy, Moskowitz, and Vissing-Jorgensen* (2009) show that the consumption volatility of the stockholders is higher than that of the non-stockholders.

positive government spending shock lowers consumption through higher taxes and generates inflation as a result of the government taking on more debt. Long term bond prices decrease exactly when the marginal utility to consume is high thus elevating the term premium on long-term bonds. A positive monetary policy shock also lowers consumption due to a higher nominal short-term interest rate via the Taylor rule. The higher interest rate suppresses inflation as it negatively affects economic activity. Long-term rates on 10-year nominal bonds increase, making the term premium positive in absolute terms. However, because the risk-free rate is higher following a positive monetary policy shock, the term premium decreases in relative terms.

[**Figure** A.**10 Here**]

What Is Generates the Term Premia

In order to understand how the model is generating the large unconditional term premium, I systematically test each of the model features by switching them off and checking the model responses. The most striking difference between the extended model in this paper and the long-run risk model of RS is the inclusion here of the non-Ricardian households. As expected, when all consumers in the economy are savers, the model reverts to a Ricardian regime, and the term premium drops significantly. Indeed, from the impulse response functions of the various structural shocks, the price on long-term bonds decreases less following an one standard deviation shock to government spending in a model without spenders than in a model with spenders. In other words, the presence of the non-Ricardian households amplifies the effects of government spending shocks on long-term bond prices thus generating large term premia.

The role of stochastic volatility is critical in allowing the extended model to replicate the large unconditional standard deviation of term premium observed in the data. When the stochastic process governing the government spending shocks is

set to be conditionally homoskedastic, the average standard deviation on term premium drops and becomes negligible in this setting. This shows that, although the higher order perturbation methods endogenously generate time variation in risk premia, stochastic volatility is needed for the extended model presented here to generate appreciable heteroskedasticity in risk premia to match the data, which is consistent with previous literature on modeling stochastic volatility in production economies. *Fernández-Villaverde and Rubio-Ramírez* (2007) examine the "Great Moderation" in the U.S. economy by estimating dynamic general equilibrium models with built-in heteroskedastic shocks. They find that time-varying volatility contributes to the reduction of observed standard deviation of output growth in the data but has little effects on the level of growth.

Similar to the RS finding with regard to the effect of technology shocks on the term premium, the impulse responses show technology shocks are the most important shocks driving the dynamics of term premium in the extended model. Figure A.10 shows that the responses of savers' consumption, inflation, 10-year bond yields, and 10-year bond term premia are orders of magnitude larger following a technology shock compared to those after a government spending shock or a monetary policy shock.

1.6   Conclusion

Investors require compensation in exchange for holding nominal bonds due to consumption risk and inflation risk, and shocks to government spending directly impact bond risk premia by simultaneously affecting the households' marginal utility of consumption and the price on nominal bonds. In this paper, I develop a dynamic stochastic general equilibrium model to study the transmission mechanism underlying the effects of fiscal policy on bond risk premia. Positive government spending shocks increase marginal utility of consumption while decreasing bond prices; the positive covariance between the stochastic discount factor and yields make bonds a

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poor hedge against consumption risk, resulting in a positive term premium. Also, positive government spending shocks increase marginal utility of consumption while putting upward pressure on inflation, which erodes returns on nominal bonds in real terms. The positive covariance between the stochastic discount factor and inflation make nominal bonds risky due to inflation, leading to a positive inflation risk premium. In addition, the model shows that both the term premium and inflation risk premium decrease when the government tightens fiscal policy by financing spending with higher taxes as opposed to issuing more debt.

I numerically solve the extended model with saver-spender households, real price rigidities, Epstein-Zin preferences, and stochastic volatility in government spending shocks. When calibrated to macroeconomic and financial moments, the model is capable of generating unconditional term premium that matches the data both in mean and in volatility. In comparison to the long-run risk model of *Rudebusch and Swanson* (2010), the inclusion of saver-spender households helps to generate a large unconditional term premium using a reasonable level of relative risk aversion.

Government spending shocks in the model contribute directly to time variation in term premium. This time variation implies that government spending shocks will predict bond risk premia and, by extension, bond returns. Using excess holding period returns on long-term Treasury bonds to proxy for bond risk premia, I empirically verify the predictability of government spending shocks on future bond returns. As expected, government spending shocks constructed from macroeconomic data are statistically significant across different bond maturities in the predictive regressions regardless of specification. Furthermore, the coefficient loadings on government spending shocks are positive as anticipated by theory: positive government spending shocks increase the marginal utility to consume while generating inflation thus making nominal bonds risky, leading to higher expected returns.

CHAPTER II

What Do Nominal Rigidities and Monetary Policy Tell Us about the Real Yield Curve?

2.1 Abstract

We provide a theoretical analysis of the implications of monetary policy on the term premia in real bonds and the inflation risk premia in nominal bonds. Monetary policy has real effects in an economy characterized by recursive preferences, nominal wage and price rigidities, and a nominal interest-rate policy rule. Positive monetary policy shocks increase the marginal utility to consume while making long-term real bonds cheap, leading to positive real term premia. However, inflation is low due to the positive policy shocks, and the inflation risk premia in nominal bonds are negative. Productivity growth shocks in conjunction with wage rigidity generate a positive covariance between consumption growth and the price of real long-term bonds, resulting in an upward sloping real yield curve and positive real term premia. At the same time, inflation is high following the negative productivity growth shocks when consumption growth is low, and the inflation risk premia in nominal bonds are positive. Overall, the effects of the productivity growth shocks dominate those of the policy shocks, and the real term premia and the inflation risk premia are positive, on average. Finally, more responsive monetary policies reduce the government's cost of

borrowing when issuing nominal debt, and reduce the diversification benefits of real bonds.

2.2   Introduction

Default-free real interest rates of short and long maturities are of fundamental importance in macroeconomics, finance, and policymaking. These rates can help us understand the willingness to substitute consumption over time in the economy, represent a benchmark to compare financial asset expected returns, and may play an important role in the transmission mechanism of monetary policy, among others. This importance radically contrasts with our understanding of the economic drivers of these rates. While the empirical literature has been limited by data availability, the theoretical literature suffers from the lack of a satisfactory model connecting asset prices to economic dynamics. In this paper, we explore an equilibrium model characterized by nominal price and wage rigidities to (i) understand how these rigidities affect several properties of real bond yields and, (ii) gain insights into how monetary policy can affect these properties through nominal rigidities.

Treasury inflation protected securities (TIPS) have been issued by the United States government since 1997. We use TIPS as a proxy for real bond yields and compute different descriptive statistics of TIPS yields to make comparisons with U.S. nominal government bond yields.[1] Our empirical analysis for the 1999-2008 period shows that long-term TIPS yields have been higher than short-term TIPS yields on average, the spreads between long- and short-term TIPS yields are smaller than comparable spreads for nominal bond yields, the TIPS yields are as volatile as nominal bond yields, and are highly correlated for comparable maturities. In addition,

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[1]Strictly speaking, TIPS are not exactly comparable to real bond yields given their particular inflation indexation procedure and their embedded optionality. The comparison of TIPS and nominal government bond yields can be affected by the difference in liquidity in the two bond markets. These differences are not captured in our theoretical model.

a principal component analysis shows that nominal and TIPS yields are mainly driven by two common factors. We link these properties of bond yields to nominal rigidities and monetary policy in our theoretical model.

Wage and price rigidities and monetary policy play an important role in our model capturing the empirical properties above. First, wage rigidities generate positive average spreads between long and short-term real yields, which are lower than comparable spreads for nominal yields. It results from positive term and inflation risk premia for permanent productivity shocks. Second, the volatility of real bond yields is increased by monetary policy shocks in the presence of rigidities, but not by shocks to an inflation target. Third, wage rigidities increase the correlations between real and nominal bonds. Fourth, changes in the response of monetary policy to economic conditions affect the magnitudes of these findings.

Our model is based on *Li and Palomino* (2012) and contains four important ingredients. First, a representative household with *Epstein and Zin* (1989) recursive preferences over consumption and labor. As shown by *Tallarini* (2000), it allows us to keep a low level of elasticity of intertemporal substitution to match macroeconomic dynamics and a high degree of risk aversion to match high compensations for risk in financial assets. Second, *Calvo* (1983) rigidities on nominal wages and prices. The representative household has market power to set its wages from supplying labor, but at each point of time faces the probability of not being able to adjust these wages optimally. Firms have market power to set their product prices, but at each point of time face the probability of not being able to adjust these prices optimally. Third, a *Taylor* (1993) interest rate rule describes monetary policy. As a result of nominal rigidities, monetary policy has effects on real economic activity and then real bond yields. Fourth, the economy is affected by three types of shocks: permanent and productivity shocks, policy shocks, and inflation target shocks. Equilibrium in the economy implies that real and nominal yields are driven by these shocks and

depend on preference, production, and policy parameters. While the first ingredient is common in asset pricing models, the last three ingredients are standard in New Keynesian models for the analysis of monetary policy. Our model calibration matches important first and second moments of macroeconomic variables and the high Sharpe ratio observed in the stock market. The implied average nominal yield curve is upward sloping, but the spread between long- and short-term yields and yield volatility are lower than in the data.

The average spread between long and short-term real bond yields captures real term premia and provides a measure of the difference in borrowing costs for the government of issuing long- vs. short-term bonds. The average spread between nominal and real bonds adjusted by inflation captures inflation risk premia and provides a measure of the difference in borrowing cost for the government of issuing nominal vs. real bonds. The model ability to capture positive real term and inflation risk premia crucially depends on wage rigidities and permanent productivity shocks. Both premia are negative in the absence of this rigidity.

Wage rigidities induce a procyclical but mean reverting labor demand and countercyclical inflation with respect to permanent productivity shocks. Bad news for productivity growth is bad news for consumption growth and labor demand, and generates inflation. Labor demand decreases since wages (marginal costs) are higher. However, expected future labor demand increases since wages will adjust downwards and generates positive expected consumption growth. Simultaneously, since wages are higher, producers increase their product prices to restore their markup, generating inflation. The positive effect on expected consumption growth increases interest rates and decreases real bond returns. Therefore, real term premia are positive since real bond returns are low and marginal utility is high at the same time. The increase in prices reduces the real return of nominal bonds. Therefore, inflation risk premia are positive since real returns on nominal bonds are low and marginal utility is high

at the same time.

The unconditional volatilities of nominal v.s. real yields and long-term v.s. short-term yields are examined as an application of the model. In general, the shock to the inflation target has significant impact on the volatilities of nominal long-term bonds. This is due to the fact that the inflation target shocks generate volatility in inflation, making nominal yields more volatile, and are highly persistent, making long-term bonds more volatile. Monetary policy, on the other hand, has significant impact on the volatility ratios of nominal yields over real yields. When the policy rule is very responsive to inflation, the volatility of inflation decreases resulting in more stable nominal yields. The effects of stronger response to the output gap by the policy rule are the opposite: volatility ratios of nominal yields over real yields increase with the reaction coefficient.

There are possible diversification benefits of investing in real bonds for a constrained investor's portfolio. As a first step to study the diversification benefits of real bonds, we calculate the unconditional correlations between real returns on real bonds and real returns, excess of inflation, on nominal bonds. We find that these return correlations are the lowest in a fully flexible economy without permanent productivity shocks. This means the diversification benefits of TIPS are the greatest in the absence of wage and price rigidities. When there are permanent productivity shocks, the return correlations are the lowest when only price is sticky, and this is especially true for long-term bonds. Under the benchmark specification, strong monetary policy response to inflation keeps inflation risk premium low, and returns between real and nominal bonds become more correlated. Persistence in monetary policies also has the same effect on returns of long-term bonds: inflation risk premium is low, and nominal bonds behave more like real bonds resulting in more correlated returns.

The paper is organized as follows. Section 2.3 presents some descriptive statistics

for nominal government bonds and treasury inflation protected securities. The principal component analysis in this section shows that two factors capture most of the variability of nominal and TIPS yields. Section 2.4 describes the economic model and its equilibrium conditions. Section 2.5 presents the analysis and section 2.6 concludes. The appendix contains all proofs.

2.3   Some Descriptive Statistics

We use United States monthly data for real and nominal bond yields from 1999 to 2008. The term structure series was obtained from monthly data on bond yields for yearly maturities from 1 to 20 years. The nominal and real yields are obtained following the procedure in *Gurkaynak, Sack, and Wright* (2006) and *Gurkaynak, Sack, and Wright* (2008), respectively. These data are published on the Federal Reserve website. The short-term nominal interest rate is the 3-month T-bill from the Fama risk-free rates database. TIPS with maturities between 2 and 4 years are only available since 2004. Table B.1 reports the average yields and the standard deviation of yields for TIPS and nominal bonds. We report statistics computed for the sample $1999-2008$ and $2004-2008$, given concerns about liquidity in the TIPS market in the early period. The table shows upward sloping average curves and similar volatilities for TIPS and comparable nominal bonds. The nominal yield curve is steeper than the TIPS curve, suggesting positive inflation risk premia in nominal bonds.

[**Table** B**.1 Here**]

Table B.2 shows the variability of nominal bond and TIPS yields captured by their three principal components when these components are computed for TIPS and nominal bonds separately and jointly. Two principal components in the joint analysis can capture most of the variability of TIPS and nominal bond yields. The

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first principal component explains a significantly larger fraction of yield variability than the second component.

[**Table** B.**2 Here**]

Figures B.1 and B.2 present the loadings of TIPS and nominal bond yields for different maturities for the 1999-2008 and 2004-2008 periods, respectively. The two figures show that the loadings on the first principal component for TIPS and nominal bonds are very similar in magnitude across all maturities. However, the magnitude increases slightly with maturity for the 1999-2008 period, and decreases considerably with maturity for the 2004-2008 period. The difference can be driven by the fact that the 2004-2008 period includes TIPS with 2, 3, and 4 years to maturity. The second principal component has bond yield loadings that are significantly higher for nominal bonds than for TIPS for comparable maturities. The loadings are decreasing with maturity for the 1999-2008 period and increasing with maturity for the 2004-2008 period. Tables B.3 and B.4 complete the analysis showing regressions of TIPS and nominal bond yields on the first three principal components. The adjusted R^2's show that most of the variability of both TIPS and nominal bonds is captured by these components. The explanatory power for 20-year TIPS and 30 year nominal bond yields is lower, suggesting that the long-end of the two curves has an additional important driving factor.

[**Tables** B.**3 and** B.**4 Here**]

2.4   Economic Model

We model a production economy where households derive utility from the consumption of a basket of goods and disutility from supplying labor to the production sector. The labor market is characterized by nominal wage rigidities. The production sector is characterized by monopolistic competition and nominal price rigidities.

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Wage and price rigidities generate real effects of monetary policy. When nominal prices and/or wages are not adjusted optimally, price and wage inflation generates distortions that affect production decisions. Since inflation is determined by monetary policy, different policies have different implications for real activity, and then real and nominal interest rates for all maturities. We model monetary policy as an interest-rate policy rule that reacts to economic conditions. The model can be seen as an extension of the standard New-Keynesian framework (see *Woodford* (2003), for instance) and is based on *Li and Palomino* (2012). It incorporates recursive preferences for households and is solved using a second-order perturbation to capture and analyze bond pricing dynamics.

2.4.1 Household

The representative agent in this economy chooses consumption C_t and labor N_t to maximize the *Epstein and Zin* (1989) recursive utility function

$$V_t = (1 - \beta)U(C_t, N_t^s)^{1-\psi} + \beta\mathbb{E}_t\left[V_{t+1}^{\frac{1-\gamma}{1-\psi}}\right]^{\frac{1-\psi}{1-\gamma}}, \tag{2.1}$$

where $0 < \beta < 1$ is the subjective discount factor, and ψ and γ determine the elasticity of intertemporal substitution and the coefficient of relative risk aversion, respectively. The recursive utility formulation allows us to relax the strong assumption of $\gamma = \psi$ implied by constant relative risk aversion. The intratemporal utility of consumption, C_t, and aggregate labor supply, N_t^s, is

$$U(C_t, N_t^s) = \left(\frac{C_t^{1-\psi}}{1-\psi} - \frac{(N_t^s)^{1+\omega}}{1+\omega}\right)^{\frac{1}{1-\psi}},$$

where ω^{-1} captures the Frisch elasticity of labor supply. The consumption good is a basket of differentiated goods produced in a continuum of firms. Specifically,

consumption of the final good is

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}}, \tag{2.2}$$

where $\theta > 1$ is the elasticity of substitution across differentiated goods, and $C_t(j)$ is the consumption of the differentiated good j. Labor supply is the aggregate of a continuum of different labor types supplied to the production sector. Specifically,

$$N_t^s = \int_0^1 N_t^s(k) \, dk,$$

where $N_t^s(k)$ is the supply of labor type k.

The representative agent is subject to the intertemporal budget constrain

$$\mathbb{E}_t \left[\sum_{s=0}^\infty M_{t,t+s}^\$ P_{t+s} C_{t+s} \right] \leq \mathbb{E}_t \left[\sum_{s=0}^\infty M_{t,t+s}^\$ P_{t+s} \left(LI_{t+s} + \Psi_{t+s} \right) \right], \tag{2.3}$$

where $M_{t,t+s}^\$$ is the nominal discount factor for cashflows at time $t + s$, P_t is the nominal price of a unit of the basket of goods, LI_t is the real labor income from supplying labor to the production sector and Ψ_t is the aggregate profits and other claims to the production sector.

Appendix B.1 shows that the household's optimality conditions imply that the one-period real and nominal stochastic discount factors are

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \left(\frac{V_{t+1}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]^{1/(1-\gamma)}} \right)^{\psi-\gamma}, \quad \text{and} \quad M_{t,t+1}^\$ = M_{t,t+1} \left(\frac{P_{t+1}}{P_t} \right)^{-1} \tag{2.4}$$

respectively. They allow us to price real and nominal default-free bonds. In particular, a one-period nominal bond has the price

$$e^{-i_t} = \mathbb{E}_t \left[M_{t,t+1}^\$ \right], \tag{2.5}$$

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where the one-period nominal interest rate i_t is the instrument of monetary policy.

Wage Setting

As in *Li and Palomino* (2012), we model an imperfectly competitive labor market where the representative household monopolistically provides a continuum of labor types indexed by $k \in [0, 1]^2$. Specifically, the supply of labor type k satisfies the demand equation

$$N_t^s(k) = \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w} N_t^d, \tag{2.6}$$

where N_t^d is the aggregate labor demand of the production sector, $W_t(k)$ is the wage for labor type k, and W_t is the aggregate wage index given by

$$W_t = \left[\int_0^1 W_t^{1-\theta_w}(k)\, dk \right]^{\frac{1}{1-\theta_w}}.$$

The labor demand equation (2.6) is derived in the production sector section below. The household chooses optimal wages $W_t(k)$ for all labor types k under *Calvo* (1983) staggered wage setting. Specifically, each period the household is only able to adjust wages optimally for a fraction $1-\alpha_w$ of labor types. A fraction α_w of labor types adjust their previous period wages by the wage indexation factor $\Lambda_{w,t-1,t}$. We choose the indexation factor to capture real growth in the economy and inflation. The optimal wage maximizes (2.1) subject to demand functions (2.6) and the budget constraint

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[2]This approach is different from the standard heterogeneous households approach to model wage rigidities as in *Erceg et al.* (2000), where each household supplies a differentiated type of labor. In the presence of recursive preferences, this approach introduces heterogeneity in the marginal rate of substitution of consumption across households since it depends on labor. We avoid this difficulty and obtain a unique marginal rate of substitution by modeling a representative agent who provides all different types of labor.

(2.3), where real labor income is given by

$$LI_t = \int\limits_0^1 \frac{W_t(k)}{P_t} N_t^s(k) dk \, .$$

Since both the demand curve and the cost of labor supply are identical across different labor types, the household chooses the same optimal wage W_t^* for all the labor types subject to a wage change at time t. Appendix B.1 shows that the optimal wage satisfies

$$\frac{W_t^*}{P_t} = \mu_w \kappa_t \left(N_t^s\right)^\omega C_t^\psi \frac{G_{w,t}}{H_{w,t}} , \qquad (2.7)$$

where $\mu_w \equiv \frac{\theta_w}{\theta_w - 1}$,

$$H_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{N_{t+1}^d}{N_t^d}\right) \left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w} H_{w,t+1} \right],$$

$$\text{and} \quad G_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{P_{t+1}}{P_t}\right) \left(\frac{C_{t+1}}{C_t}\right)^\psi \left(\frac{N_{t+1}^d}{N_t^d}\right) \left(\frac{\kappa_{t+1}}{\kappa_t}\right) \right.$$

$$\left. \times \left(\frac{N_{t+1}^s}{N_t^s}\right)^\omega \left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w} G_{w,t+1} \right] .$$

In the absence of wage rigidities ($\alpha_\omega = 0$), the optimal wage is given by the markup-adjusted marginal rate of substitution between labor and consumption, with optimal markup μ_w. Wage rigidities imply the time-varying markup $\mu_w \frac{G_{w,t}}{H_{w,t}}$.

2.4.2 Firms

The production of differentiated goods is characterized by monopolistic competition and price rigidities. Producers have market power to set the price of their differentiated goods in a *Calvo* (1983) staggered price setting. That is, a producer is able to change the product price optimally at each point of time, with a probability

$1 - \alpha$. If the producer is not able to adjust the price optimally, the price is the previous period price adjusted by the price indexation factor $\Lambda_{p,t,t+1}$. We choose an indexation factor that captures the previous period inflation. When the producer is able to adjust the price optimally, the price is set to maximize the present value of profits. The maximization problem can then be written as

$$\max_{\{P_t(j)\}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \alpha^s M_{t,t+s}^{\$} \left[\Lambda_{p,t,t+s} P_t(j) Y_{t+s|t}(j) - W_{t+s|t}(j) N_{t+s|t}^d(j) \right] \right\},$$

subject to the production function

$$Y_{t+s|t}(j) = A_{t+s} N_{t+s|t}^d(j),$$

and the demand function

$$Y_{t+s|t}(j) = \left(\frac{P_t(j) \left(\Lambda_{p,t,t+1} \right)^s}{P_{t+s}} \right)^{-\theta}.$$

The producer takes into account the probability of not being able to adjust the price optimally in the future. The price in this case is adjusted to incorporate the indexation $\Lambda_{p,t,t+s}$, for $s > 0$.

We assume that labor productivity contains permanent and transitory components. Specifically, $A_t = A_t^p Z_t$, where the permanent and transitory components follow processes

$$\Delta \log A_{t+1}^p = \phi_a \Delta \log A_t^p + \sigma_a \varepsilon_{a,t+1},$$

and

$$\log Z_{t+1} = \phi_z \log Z_t + \sigma_z \varepsilon_{z,t+1},$$

respectively, with Δ as the difference operator, and innovations $\varepsilon_{a,t}$ and $\varepsilon_{z,t} \sim \text{IID}\mathcal{N}(0,1)$.

Labor demand in production is a composite of a continuum of differentiated labor

types indexed by $k \in [0, 1]$ via the aggregator

$$N_t^d(j) = \left[\int_0^1 N_t^d(j, k)^{\frac{\theta_w - 1}{\theta_w}} \, dj \right]^{\frac{\theta_w}{\theta_w - 1}}, \tag{2.8}$$

where $\theta_w > 1$ is the elasticity of substitution across differentiated labor types.

The appendix shows that the solution to this maximization problem involves solving the equation

$$\left(\frac{P_t^*}{P_t} \right) H_t = \frac{\mu}{A_t} \frac{W_t}{P_t} G_t, \tag{2.9}$$

where $\mu = \frac{\theta}{\theta - 1}$, and the processes H_t and G_t are described by the recursive equations

$$H_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^\$ \Lambda_{p,t,t+1}^{1-\theta} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_{t+1}}{P_t} \right)^{-\theta} H_{t+1} \right],$$

$$\text{and} \quad G_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^\$ \Lambda_{p,t,t+1}^{-\theta} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t}{P_{t+1}} \right)^{-\theta} \left(\frac{W_{t+1}}{W_t} \right) \left(\frac{A_t}{A_{t+1}} \right) G_{t+1} \right],$$

respectively. The product price is the markup-adjusted marginal cost of production. In the absence of price rigidities, the markup is constant, given by μ. Price rigidities imply the time-varying markup $\mu \frac{G_t}{H_t}$.

2.4.3 Monetary Policy

We model a monetary authority that sets the level of a short-term nominal interest rate. Monetary policy is described by the policy rule

$$i_t = \rho i_{t-1} + (1 - \rho) \left[\bar{\imath} + \imath_\pi (\pi_t - \pi_{t-1}^\star) + \imath_x x_t \right] + u_t. \tag{2.10}$$

The one-period nominal interest rate, i_t, has an interest-rate smoothing component captured by ρ, and is set responding to aggregate inflation, the output gap, and a

policy shock u_t. The ouput gap x_t is defined as the log deviation of total output, Y_t from the output in an economy under flexible prices and wages, Y_t^f. That is, $X_t \equiv \frac{Y_t}{Y_t^f}$, and $x_t \equiv y_t - y_t^f$. The coefficients $\imath_x$, and $\imath_\pi$ capture the response of the monetary authority to the output gap and inflation, respectively. The process $\pi_t^\star$ denotes the time-varying inflation target. The inflation target is time-varying, as in *Ireland* (2007) and *Rudebusch and Swanson* (2010). Its process is

$$\pi_t^\star = (1 - \phi_{\pi^\star})g_\pi + \phi_{\pi^\star}\pi_{t-1}^\star + \sigma_{\pi^\star}\varepsilon_{\pi^\star,t},$$

where g_π is the unconditional mean of inflation, $\phi_{\pi^\star}$ is the autoregressive coefficient, $\sigma_{\pi^\star}$ is the conditional volatility of the inflation target shock, and $\varepsilon_{\pi^\star,t} \sim \text{IID}\mathcal{N}(0,1)$. The policy shocks u_t follow the process

$$u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u,t+1},$$

where $\varepsilon_{u,t} \sim \text{IID}\mathcal{N}(0,1)$.

2.4.4 The Term Structure of Interest Rates

The price of a real and nominal bonds with maturity at $t+n$ can be written as

$$\exp\left(-r_t^{(n)}\right) = \mathbb{E}_t[M_{t,t+n}], \quad \text{and} \quad \exp\left(-i_t^{(n)}\right) = \mathbb{E}_t[M_{t,t+n}^{\$}], \tag{2.11}$$

respectively, where $r_t^{(n)}$ and $i_t^{(n)}$ are the associated real and nominal bond yields, and $M_{t,t+n}$ and $M_{t,t+n}^{\$}$ are the real and nominal discount factors for payoffs at time $t+n$.

2.4.5 Equilibrium

2.5 Analysis

We study the bond pricing implications of the economic model in section 2.4. We analyze the short-term real rate, the term premia in real bonds, and the inflation risk premia in nominal bonds. We describe the baseline calibration and present different model specifications with and without nominal rigidities to understand their main effects. We complete the analysis with comparative statics for rigidity and monetary policy parameters, and present impulse responses to the model shocks to understand the implications of monetary policy on the real term structure and inflation risk premia. Finally, we provide an analysis of the effects of nominal rigidities on the volatility of real and nominal bond yields, the government borrowing costs associated to nominal vs. real bonds, and the diversification benefits of real bonds.

2.5.1 Calibration

We calibrate the model to match some properties of United States macroeconomic and financial quarterly data from 1987:Q1 to 2010:Q4. We focus on the Greenspan era to avoid potential changes in monetary policy and the interest-rate policy rule, as suggested by *Clarida et al.* (1999). The consumption growth series was constructed using quarterly data on real per-capita consumption of nondurables and services from the Bureau of Economic Analysis. The inflation series was constructed following the methodology in *Piazzesi and Schneider* (2006) to capture inflation related to nondurables and services consumption only. The three-month T-bill and the bond yield data series are from the Federal Reserve Economic Data (FRED II) published by the Federal Reserve Bank of St. Louis.

Table B.5 presents the parameter values used in the calibration. Panel A is a collection of preference and rigidity parameters. The values are standard in the

macroeconomic literature, except for the parameter γ which is chosen to capture the high Sharpe ratio of financial assets. Under recursive preferences on consumption and labor, the average coefficient of risk aversion can be computed as in *Swanson* (2012). It is

$$\frac{\psi}{1 + \frac{\psi}{\omega\mu}} + \frac{\gamma - \psi}{1 - \frac{1-\psi}{1+\omega}} \approx 32\,.$$

The elasticities θ and θ_ω are set such that the markups in prices and wages are 20%, which is within the range found in the literature. The rigidity parameters α and α_w capture the average duration of prices and wages, respectively, in the data.

[**Table** B.**5 Here**]

Panel B displays the parameters of the interest-rat monetary policy rule. The parameters satisfy the Taylor principle and then ensure a stable equilibrium. The baseline values of the Taylor rule coefficients are consistent with what *Clarida, Gali, and Gertler* (1999) find for the Greenspan era. Panel C shows the autoregressive coefficients and the conditional volatilities of the policy shock to the Taylor rule, the permanent productivity shock and the transitory productivity shock. Following *Li and Palomino* (2012), these numbers are chosen to match some of the empirical results presented in *Altig, Christiano, Eichenbaum, and Linde* (2011). Spcifically, the variance of inflation, de-trended consumption, and the nominal short-term rate attributed to productivity and policy shocks. Finally, panel D shows values for parameters controlling the time-varying inflation target. The unconditional mean of inflation is chosen to be roughly 2% annually. This helps us to match the level of the nominal short rate. The autoregressive coefficient and the conditional volatility of the shock to the inflation target are in line with values used by *Rudebusch and Swanson* (2010). The parameter σ_π is chosen to match the unconditional volatility of inflation.

Table B.6 shows selected moments implied by the data and the model. The model

does a good job matching the volatility of de-trended consumption, consumption growth, and inflation. The average levels of inflation and the short-term rate are also matched. The macroeconomic volatility in the model is not enough to capture the high volatility of the short-term nominal interest rate. While matching the average level of the short end of the nominal yield curve, the model fails to generate the observed average slope of the nominal curve between the 3-month yield and the 5-year yield. Increasing the risk aversion parameter, γ, does not seem to make a noticeable difference. The time-varying inflation target has the greatest impact on the average volatility of long-term yields.

[**Table** B.**6 Here**]

2.5.2   Model Specification

To understand how each component of the model contributes to the results, we reproduce summary statistics under different model specifications by shutting down exogenous shocks and nominal rigidities in turn. We focus our attention on unconditional means and volatilities of macroeconomic and financial variables implied by the model. The variable definitions are as follow: $\tilde{c}$ is de-trended real consumption, x is the output gap, w is the real wage, $log(\mu)$ is the price markup due to monopolistic power, Δc is real consumption growth, Δw is real wage growth, π is inflation, r is the real interest rate and i is the nominal interest rate. $IRP^{(n)}$ stands for the inflation risk premium calculated as the difference between the nominal yield and the real yield for n-quarter bonds and subtracting expected inflation.

Table B.7 compares the benchmark specification to the model specification when only one exogenous shock is turned on. Examining the unconditional inflation risk premium and the average slope of the nominal curve, the permanent productivity shock is the most important shock to bond risk premia because no other shock by itself can generate the same level of IRP. This is consistent with the fact the permanent

productivity shock contributes the most to the average volatility of real consumption growth, which leads to higher volatility on the pricing kernel. Looking at the effects of the inflation target shock, it contributes the most to the second moments of real output, inflation and interest rates. The shock to the inflation target has an especially strong impact on the volatility of the long-term nominal interest rate that no other shock can reproduce. On an interesting note, the transitory productivity shock seems to have a strong effect on the volatility of the real wage. This can be due to the presence of wage rigidities.

[**Table** B**.7 Here**]

Besides the different shock specifications, the model also allows for comparisons with and without price and wage rigidities. Table B.8 presents summary statistics of the model under four scenarios: no rigidities[3], only wage rigidity (WR)[4], only price rigidity (PR) and both rigidities (benchmark). The significance of WR on bond risk premia is striking. In the absence of WR, columns (2) and (4), the unconditional IRP is negative, and the nominal term structure is downward sloping, on average. A closer examination of the level of real yields shows that the average real term structure is also downward sloping when WR is turned off. This observation implies that WR makes long-term bonds risky instruments since their cashflows are low during periods of high marginal utility.

[**Table** B**.8 Here**]

Since the permanent productivity shock is the major source of bond risk premia, we want to see how the model would behave under different rigidity specifications in the absence of these shocks. Table B.9 presents summary statistics of the model under five scenarios with no permanent productivity shocks: no rigidities, only wage rigidity

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[3]In the absence of both price and wage rigidities, there is no output gap and markup.
[4]In the absence of price rigidities, there is no resulting markup for the firm.

(WR), only price rigidity (PR) , both WR and PR, and the benchmark specification. Similar to table B.8, we see that the unconditional IRP is negative without WR in columns (2) and (4). However, by turning the permanent productivity shock off, PR alone is enough to generate an upward sloping nominal yield curve, on average. This is consistent with what the existing literature using DSGE models to study the term structure has found[5]. The average real yield curve is almost flat across the five different specifications in table B.9, and then negligible term premia. This evidence suggests that the permanent productivity shock not only affects the magnitude of bond risk premia, as shown in table B.7, but it also drives the hedging properties of long-term bonds. Without WR, permanent shocks make long-term bonds hedging instruments against high marginal utility, resulting in negative risk premia.

[**Table** B.**9 Here**]

2.5.3 Dynamic Responses

We study the impulse response functions of the individual shocks to understand how the interaction between shocks and rigidities affects the dynamic behavior of the endogenous variables in the model. From a steady state, we perturb the system with a positive one standard deviation permanent productivity shock, transitory productivity shock, policy shock and inflation target shock, individually. For each shock, we examine the impulse response of the endogenous variables under four rigidity specification: the benchmark, no rigidities, wage rigidity only (No PR) and price rigidity only (No WR).

Figure B.3 presents the impulse response of macroeconomic variables and interest rates following a positive one standard deviation permanent productivity shock. When prices and wages are fully flexible, real output does not respond to the permanent productivity shock while inflation and interest rates show very strong reactions.

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[5]See *Rudebusch and Swanson* (2010) and *Hsu* (2012) for example

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The opposite is true when prices and wages are sticky. Interest rates in general increase immediately following the permanent shock in the absence of rigidities, while they decrease first when prices and wages are rigid.

Figure B.4 show the impulse responses following a positive one standard deviation transitory productivity shock. Inflation increase following shock and is very persistent in the presence of price rigidity. Interest rates generally decrease after the shock with the exception of the real short rate. When prices are rigid, the real short rate increase first then decrease before reverting back to the steady state. The slope of the nominal term structure has a strongly positive reaction to the transitory shock when wages are rigid. Finally, the response of the real yields is much weaker following the transitory productivity shock than following the permanent productivity shock.

The dynamic response of the endogenous variables due to a positive one standard deviation policy shock are displayed in figure B.5. Not surprisingly, when prices and wages are fully flexible, the policy shock solicit almost no reaction from the economy except inflation. With rigidities, the policy shock generates strong reactions in real and nominal yields, especially at the short-end of the yield curves. The is consistent with the fact that the nominal short rate is the policy instrument. In addition, the policy shock has a flattening effect on the nominal term structure under price and wage rigidities.

Finally, we examine the impulse responses following a positive one standard deviation shock to the inflation target in figure B.5. Similar to the policy shock, the inflation target shock does not generate much reaction from the macroeconomy in the absence of rigidities. It has a big impact on inflation, however, when prices and wages are fully flexible. Furthermore, nominal yields increase following the positive shock and stay elevated due to the persistence of the inflation target shock. This is purely due to the inflation effect since the shock has no impact on real yields. When prices and wages are sticky, the inflation target shock lowers productivity and consumption

growth, which leads to lower real interest rates after the shock. The inflation target shock also has a steepening effect on the nominal term structure in the presence of sticky price and sticky wage.

2.5.4   Comparative Statics

We conduct comparative statics on the model by perturbing selected parameters to gain further insight into the mechanism delivering the results. Table B.10 contains summary statistics of the model when we vary six parameters, one at a time, while keeping all other parameters at their baseline values. Under each parameter, the middle column shows the summary statistics under the benchmark specification. The column to the right of the benchmark shows how the model responds when the parameter value is increased, and the column to the left of the benchmark shows the response when the parameter value is decreased.

[**Table** B.**10 Here**]

In panel A, we focus on the parameters governing price rigidity (PR), α, and wage rigidity(WR), α_w. As α and α_w increase, the degree of rigidity goes up in the economy. This means price and wage, respectively, become stickier. Immediately, it is apparent that the impact of WR on bond risk premia is more pronounced than the impact of PR on bond risk premia, and the two frictions work in opposite directions. Higher PR leads to lower unconditional IRP and lower average slope of the nominal curve, while the opposite is true for WR. A 5% decrease in WR translates to a 15 basis points decrease in the unconditional IRP, or roughly 40% of the benchmark value.

In panel B, we vary the autoregressive coefficient and the conditional volatility of the shock to the inflation target. Not surprisingly, when the inflation target shock becomes more persistent or more volatile, the unconditional IRP increases, and the

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nominal term structure becomes steeper, on average. The impact of these parameters is particularly striking on the unconditional volatility of the long-term nominal interest rate. When $\sigma_{\pi^\star}$ is zero, equivalent to shutting down the inflation target shock, the long-term yield becomes very stable, and the model has trouble matching the data.

[**Table** B.11 **Here**]

Table 9 presents comparative statics on the Taylor rule coefficients. The coefficients $\imath_\pi$ and $\imath_x$ capture the reaction of the nominal short rate on inflation and output gap, respectively. The coefficient ρ captures interest-rate smoothing. Consistent with the existing literature on monetary policy, as the central bank tightens monetary policy by increasing $\imath_\pi$, the unconditional volatility of inflation decreases, leading to lower average IRP and a flatter nominal yield curve. In addition, a stronger response to inflation also has a stabilizing effect on the economy, evident from lower unconditional standard deviations of output gap, consumption growth, inflation and long-term nominal rates. On the other hand, the effects of a stronger response to the output gap by the central bank through increasing $\imath_x$ are completely opposite. It generates inflation and associated IRP while making the economy more volatile in general. Lastly, when the the nominal short rate is more persistent, high ρ, monetary policy is more stable. Therefore, even though inflation is higher, on average, the unconditional IRP is actually lower, and the nominal yield curve is flatter.

2.5.5   Applications

Term Premia

The average spread between long- and short-term bonds contains the average compensation for risk required by investors to hold long-term bonds over short-term

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bonds. It can be shown that average spreads for real bonds satisfy

$$n\mathbb{E}\left[r_t^{(n)} - r_t\right] + \frac{1}{2}\sum_{k=1}^{n-1}(n-k)^2\mathbb{E}\left[\text{var}_t\left(r_{t+1}^{(n-k)}\right)\right] = \sum_{k=1}^{n-1}(n-k)\mathbb{E}\left[\text{cov}_t\left(m_{t,t+1}, r_{t+1}^{(n-k)}\right)\right].$$

Therefore, long-term real bonds contain compensations for permanent and transitory productivity shocks, policy shocks and inflation target shocks. The average real term structure is upward sloping in the model, implying positive risk premia for long-term real bonds. Following a negative permanent productivity shock, consumption growth is low meaning marginal utility is high. In the presence of sticky wages, real interest rates increase after the negative shock, and prices of real bonds decrease. This means that in the bad state of the world when marginal utility is high, long-term real bonds are cheap, making them bad hedges against consumption risk. Therefore, ex ante, investors require positive term premia in exchange for holding real bonds. Wage rigidity is crucial to generate the positive covariance between the marginal utility and yields after a permanent productivity shock is realized. If prices and wages are fully flexible or if only prices are sticky, then this covariance is negative meaning risk premia on long-term real bonds are negative.

In the case of transitory productivity shocks, the dynamics between marginal utility and bond prices are similar but less quantitatively important. Figure B.4 shows that consumption growth is low following a negative transitory productivity shock resulting in higher real marginal utility. However, the response of real yields is weaker compared to the permanent productivity shock, and the price of long-term real bonds decreases regardless of the rigidity specification. The policy shock only affects the term premium in the presence of rigidities, price or wage. In figure B.5, a positive shock to the nominal short rate lowers consumption growth making the real marginal rate of consumption substitution higher. At the same time, real yields increase following the shock if price or wage or both are sticky. Therefore, the policy

shock by itself can generate the positive covariance between the real marginal utility and real interest rates, leading to positive term premia for long-term real bonds.

The impact of wage rigidity on bond risk premia in conjunction with the inflation target shock is actually negative. Like the policy shock, the inflation target shock does not affect real consumption growth in a fully flexible economy and has no impact on bond risk premium. From the impulse responses in figure B.6, when wages are sticky, a positive inflation target shock raises the marginal utility of consumption while decreasing real yields making real bonds more expensive. Thus, in the bad state of the world when consumption growth is low, long-term real bonds payoff higher and are good hedging instruments against consumption risk in the presence of wage rigidity. The resulting term premium is negative.

Inflation Risk Premia

Nominal bond yields can be decomposed into real bond yields and an inflation compensation as

$$ni_t^{(n)} = nr_t^{(n)} + \mathbb{E}_t[\pi_{t,t+n}] - \frac{1}{2}\text{var}_t(\pi_{t,t+n}) + \text{cov}_t(m_{t,t+n}, \pi_{t,t+n}),$$

where

$$\pi_{t,t+n} = \sum_{s=1}^{n} \pi_{t+s}$$

is the inflation observed between t and $t+n$ and

$$m_{t,t+n} = \sum_{s=1}^{n-1} \log M_{t+s,t+s+1}$$

is the marginal rate of substitution of consumption between t and $t+n$. The difference between nominal and real yields contains the expected inflation during the life of the bond and compensations for inflation risk or inflation risk premia. These premia capture the expected excess real return for investing in nominal n-period bonds over real

n-period bonds for *n* periods. Investors require a compensation for holding nominal bonds over real bonds because the marginal utility of consumption is correlated with inflation and the return of nominal bonds is affected by inflation. When consumption and inflation are uncorrelated, the expected real returns on real and nominal bonds with the same maturity are the same. Then, understanding the sources of covariance between consumption and inflation helps us to understand the determinants of the inflation risk premia. The covariance of the real discount factor and inflation, expressed as a function of their sensitivities to the exogenous shocks, determines the inflation risk premia.

Following a negative permanent productivity shock, consumption growth is low and the marginal rate of consumption substitution is high. Without rigidities, inflation is low, and low inflation increases the payoff on nominal bonds in real terms. The negative covariance between the marginal utility and inflation generates negative inflation risk premium for the investors who hold nominal bonds. However, when wages are sticky, inflation is high after the negative permanent productivity shock. High inflation erodes the payoff on nominal bonds in real terms making them bad hedges against inflation in the bad state of the world. As a result, investors demand a positive inflation risk premium in exchange for holding nominal bonds. Figure B.3 outlines the dynamics of consumption growth and inflation due to a permanent productivity shock.

The dynamics of consumption growth and inflation in the aftermath of a transitory productivity shock are shown in figure B.4. Following a negative transitory shock, marginal rate of consumption substitution is high while inflation is also high in the presence of wage rigidity. The positive covariance between the real marginal utility and inflation means nominal bonds are risky and inflation risk premium is positive. The mechanism is the same here as for the permanent productivity shock, but the magnitudes are different. A one standard deviation permanent productivity shock

generates a greater response in consumption growth than a one standard deviation transitory productivity shock does.

Figure B.5 helps us understand the impact of the policy rule shock on inflation risk premium. In a fully flexible economy, marginal utility is not affected by the positive policy shock. This means that even though inflation is low, inflation risk premium is actually zero. Price rigidity makes the policy shock non-neutral to the real economy, and the marginal utility increases after the positive shock is realized. However, inflation is lower following the same shock in the case of only sticky prices. This means nominal bonds are good hedging instruments, and inflation risk premium is negative. Sticky wages neutralize the reaction of inflation due to the policy shock, and the negative inflation risk premium is mitigated.

The inflation target shock and inflation risk premium are also connected through consumption growth and inflation. Similar to the policy shock, figure B.6 shows that the inflation target shock is neutral with respect to consumption growth when prices and wages are fully flexible leading to zero inflation risk premium. However, wage rigidity changes the dynamics by allowing the inflation target shock to affect consumption growth. Following a positive inflation target shock, marginal rate of consumption substitution is high when wages are sticky. Inflation is also high following the same shock, eroding the payoff on nominal bonds. The positive covariance between marginal utility and inflation means wage rigidity helps the inflation target shock to generate positive inflation risk premium.

We can link the inflation risk premium to government borrowing costs. When inflation and output are correlated the return required by investors to hold comparable real and nominal bonds differ. The government then can reduce the cost of issuing debt by choosing between real or nominal debt. Since the sign of inflation risk premium depends on the presence of wage rigidity in this economy, issuing nominal bonds involves a lower financing cost for the government than issuing comparable real

bonds if wages are fully flexible. The savings become larger for weaker responses to economic conditions in the policy rule.

Volatility of Real and Nominal Yields

Table B.1 shows that the volatility of TIPS has been similar or even higher than the volatility of comparable nominal government bonds. It is reasonable then to ask whether different monetary policies can have different implications on the volatility of real and nominal bonds. We compute the ratio of the unconditional volatilities of nominal and real bonds implied by the model. Table B.12 is the comparative statics of the ratios of unconditional volatilities of nominal yields over real yields at different maturities (top two rows in each panel) as well as the ratios of unconditional volatilities of long-term yields over short-term yields (bottom two rows in each panel). Panel A shows the effects of price and wage rigidities on the volatility ratios. Overall, the rigidities do not have a significant impact on the volatility ratios. Generally speaking, higher rigidities increase the ratios with the exception of wage rigidity on long-term interest rates. Higher wage rigidity decrease the ratio of volatilities slightly. There is minimal impact on the volatility ratios between long-term and short-term bonds from price and wage rigidities.

[**Table** B.**12 Here**]

Panel B in table B.12 presents the comparative statics by varying the coefficients governing the inflation target shock. Since the persistence of the inflation target shock as well as its conditional volatility both increase the volatility of inflation, it is straight forward to explain the increase in the volatility ratios at both ends of the term structure. Because inflation volatility only affects nominal yields and not real yields, the volatility ratios increase dramatically with $\phi_{\pi^\star}$ and $\sigma_{\pi^\star}$ at all maturities. An interesting observation is that the only incidence where the unconditional volatility ratio of nominal over real is less than one is in the absence of the inflation target shock

$(\sigma_{\pi^\star} = 0)$ for short-term rates. Without the shock to the inflation target, nominal interest rates become very stable. The model requires another potential source of variations to generate observed volatilities on real yields.

Panel B also demonstrates the importance of the inflation target shock in generating volatility on long-term nominal yields. When the conditional volatility of the inflation target shock is zero, the long-term nominal yield is not volatile compared to the short-term yield. However, when the inflation target shock is switched on, the volatility ratio of long-term over short-term nominal yields gets closer to one.

Panel C shows that more aggressive responses to economic conditions in the policy rule reduces the volatility of nominal yields with respect to real yields. The reason is that more aggressive policies reduce the volatility of output and inflation. As an extreme case, when inflation is constant, real and nominal rates move one to one. Monetary policy also affects the volatility of long-rates with respect to short-term rates. As the response of monetary policy to economic conditions increases the rate of decay in volatility across maturities increases for both nominal and real curves.

The Correlation Between Nominal and Real Bonds and the Diversification Benefits of Real Bonds

An interesting question to ask is whether real bonds provide investors with additional diversification benefits. The complete-market environment that characterizes the economic model in this paper does not allow us to obtain a satisfactory answer to this question. However, we can provide some insights into the benefits of diversification of a real bond and how it might be valuable to a constrained investor who only has access to a nominal bond with a particular maturity. Given that there are four sources of risk affecting the marginal utility of consumption, this investor faces an incomplete market and could be benefited by the existence of a real bond. We try to capture the risk sharing benefits of a real bond for this investor by computing the unconditional correlation between the realized real return of the nominal bond and

the realized return of the real bond with the same maturity.

[**Table** B.**13 Here**]

Table B.13 panel A presents the unconditional correlations of returns on real bonds and real returns (excess of inflation) on nominal bonds under different rigidity specifications. Realized returns are calculated over one quarter holding horizons. Return correlation is a decreasing function of time to maturity. The correlation is the weakest when only prices are sticky. On the other hand, the correlation is the strongest when only wages are sticky.

The returns of long-term real and nominal bonds are the least correlated when only prices are rigid. This implies greater benefits of diversification for the constrained investor. In the absence of price and wage rigidities, the return correlations are high even for long-term bonds. By examining the impulse responses, we see that both real and nominal interest rates react the strongest and in the same direction following a one standard deviation permanent productivity shock when prices and wages are flexible. Whereas the interest rate responses are mild or none following a one standard deviation transitory productivity shock, a policy shock or a inflation target shock when prices and wages are flexible. The permanent productivity shock seems to drive the high correlations in a fully flexible economy.

Panel B in table B.13 confirms this intuition by showing the return correlations under different rigidity specifications when the permanent productivity shock is switched off. Immediately, we notice that return correlations under no rigidities are much smaller in panel B than panel A, and they are monotonically increase with maturity. Wage rigidity still drives large portions of the correlations across maturities. Finally, in the absence of the permanent productivity shock, returns on real bonds and real returns on nominal bonds are the least correlated under no rigidities, making real bonds desirable for portfolio diversification purposes.

Table B.14 presents the comparative statics of the real and nominal return correlations as functions of monetary policy rule variables. Real returns for real and nominal bonds become more correlated, unconditionally, under tight monetary policy regimes when $\imath_\pi$ is high or when $\imath_x$ is low. This is consistent with the fact that inflation risk premium is low when monetary policy is tight, and nominal yields behave more like real yields once inflation is subtracted. Furthermore, when the nominal short rate is persistent, ρ is high, the correlations are also high. This is also due to the fact that inflation risk premium is low under high persistence, and nominal yields act like real yields.

2.6 Conclusion

This paper explores the implications of a monetary policy with real economic effects on the compensations for risk in long-term real bonds and the inflation risk premium in nominal bonds. The analysis shows that productivity growth shocks in the presence of wage rigidity implies positive compensations for risk in long-term real bonds and positive inflation risk premia. Stronger monetary policy reaction to inflation increases the consumption hedging properties of nominal bonds implying lower inflation risk premia, while decreases the consumption hedging properties of long-term real bonds implying higher real term premia. Stronger monetary policy response to the output gap has the opposite outcome, inflation risk premia go up and real term premia go down. Persistence in the nominal short rate decreases bond risk premia as a result of stable monetary policy.

In an economy where productivity shocks can be permanent, wage rigidity is crucial to generate positive average inflation risk premium and positive average real term premium in the model. A negative permanent productivity shock lowers consumption

growth and raises the marginal utility. When wages are rigid, inflation is high following the permanent productivity shock leading to positive inflation risk premium, and long-term TIPS are cheap after the same shock resulting in positive real term premium. If wages are flexible, then the direction of response of inflation and TIPS price is flipped generating negative average risk premia, regardless of price rigidity.

This implication of wage rigidity on bond risk premia does not hold when productivity shocks are always transitory. Without permanent productivity shocks, the unconditional real term premium is always positive regardless of the rigidity specification, while the unconditional inflation risk premium is positive in the presence of rigidities, wage or price. This is largely driven by the transitory productivity shock and the monetary policy shock, both of which have significant impact on inflation and long-term real yields in comparison to the inflation target shock.

Despite the friction presented by wage and price rigidities, the calibrated model fails to capture the average yield spreads of the real and nominal term structures. Specifically for nominal bonds, the unconditional slope of the model-implied yield curve is roughly one quarter of what is observed in the data from 1982 to 2010. This "bond premium puzzle" is particular prevalent in models with production economies where the representative agent can further smooth consumption through labor supply. As the next step, we aim to incorporate other source of friction, such as heterogeneous households, in the model to better capture the dynamics of the yield curve and resolve the bond premium puzzle.

CHAPTER III

Fiscal Policy Shocks and Bond Return Predictability

3.1 Abstract

I demonstrate that shocks to fiscal policy today can explain Treasury bond returns between today and a year from today, violating the expectations hypothesis of the term structure of interest rates. This finding holds for bonds of all maturities in the sample. Predictive regressions of one-year holding period excess returns on government spending and revenue shocks verify that innovations in government spending are statistically significant in explaining future bond returns. The result is robust after controlling for monetary policy as well as financial variables such as the *Cochrane and Piazzesi* (2005) "return-forecasting" factor. Stochastic volatility of the spending shocks is also statistically significant in a separate set of predictive regressions involving future returns. This finding is consistent with the implications of the dynamic stochastic general equilibrium model of *Hsu* (2012), in which the conditional variance of the government spending shock drives the time variation in bond risk premia and expected returns.

3.2 Introduction

Excess return predictability for Treasury bonds is well documented in the term structure literature. *Fama and Bliss* (1987) conclude that n-period forward rates have predictive power in explaining one-year excess holding period returns of n-period zero coupon bonds. Furthermore, *Campbell and Shiller* (1991) find that yield spreads between long-term and short-term zero coupon bonds forecast movements in the yield curve. They show that when spreads are high, the long-term rate tends to decrease while the short-term rate increases. More recently, *Cochrane and Piazzesi* (2005) construct a single "return-forecasting factor" from a linear combination of forward rates that describes expected bond returns of all maturities up to 35% R^2. *Cieslak and Povala* (2010) increase the predictive power of the *Cochrane and Piazzesi* (2005) results using a single risk premium factor by separating long-term yields into a persistent inflation component and a cyclic component.

While it is interesting from the finance perspective to find predictors of future bond returns, the ultimate aim of this particular area of research is to understand the macroeconomic drivers of bond risk premia and what causes the term structure to deviate from the expectations hypothesis, which states that long-term yields are the average of expected future short-term yields. Many authors have focused their efforts on associating the dynamics of the yield curve to various macroeconomic variables. *Ang and Piazzesi* (2003) is one of the earlier papers to use monetary policy variables to explain movements in the term structure using a no-arbitrage state-space model. In a similar setup, *Dai and Philippon* (2006) expands the *Ang and Piazzesi* (2003) factors to incorporate fiscal policy variables to fit the yield curve. However, these papers made only limited attempts to explain future excess returns using these macroeconomic factors; they do not directly test the expectations hypothesis, under which excess returns should not be predictable. Fiscal policy is overlooked as an implicit source of variations. The focus of this paper is to investigate the predictive power of fiscal

policy variables on future excess returns. The goal is to illustrate fiscal policy is one of the macroeconomic drivers of bond risk premia, which the existing literature has not necessarily captured.

The theoretical foundation underlying the relationship between fiscal policy and bond risk premia is sound. *Hsu* (2012) builds a dynamic stochastic general equilibrium model with heterogeneous agents to demonstrate shocks to government spending and revenue directly affect the representative pricer's consumption choice and marginal utility. In the absence of Ricardian equivalence, fiscal policy shocks help resolve the bond premia puzzle by permitting the model to generate high unconditional risk premium on long-term bonds that can be matched to the data.

The numerical solution to the *Hsu* (2012) benchmark model yields time-variation in expected returns, because risk premia are functions of exogenous and predetermined state variables. Time-varying expected returns on nominal bonds suggest that bond returns are predictable using the state variables as predictors. I empirically verify this testable implication by constructing fiscal policy shocks from macroeconomic data and regress excess bond returns on these fiscal policy shocks in a series of predictive regressions. In concurrence with the theoretical model that predicts positive spending shocks make nominal bonds risky, I find excess bond returns across maturities load positively on government spending shocks but negatively on taxation shocks, and the coefficients on spending shocks are statistically significant. This holds true when I use monetary policy related independent variables as controls in the predictive regression as well as using *Cochrane and Piazzesi* (2005) forward rates as controls: the effect of government spending shocks on future returns is never overwhelmed by the presence other regressors. This finding is important because it implies fiscal policy variables can possibly improve many of the existing macro-finance no-arbitrage models in the term structure literature if they are included in the state-space dynamics.

After establishing fiscal policy shocks have predictive power on future excess re-

turns, I construct stochastic volatility on government spending shocks for the return predictive regressions. Bond risk premium in the *Hsu* (2012) model is conditionally varying only in the presence of stochastic volatility on government spending. Therefore, I conjecture that stochastic volatility on government spending is the source behind the predictive power of the spending shocks on future returns. Running predictive regressions on excess holding period returns across the maturity structure and using stochastic volatility as the independent variable, I find stochastic volatility is positive and statistically significant in explaining future returns. Furthermore, the R^2 of the regressions involving stochastic volatility are higher than the R^2 resulting from the regressions employing fiscal shocks. The result suggests stochastic volatility is the more important factor in capturing the dynamics of excess returns, as the theory indicates. It should not be surprising that both the spending shocks themselves and the stochastic volatility on the same shocks can explain excess returns because the two series are affine transformations of each other.

The theoretical framework proposed by *Hsu* (2012) lays the foundations for conducting empirical tests establishing the link between fiscal policy variables and bond risk premia. Using the expected return as a proxy for bond risk premium, the results presented here provide validation to the hypothesis that fiscal policy has real impact on the macroeconomy, which in turn affects the fair compensation investors require for holding Treasury bonds.

3.3   Related Literature

Testing for violations of the expectations hypothesis in order to understand time-varying bond risk premia is a major area of research in the term structure literature. Typically, these tests are performed by using future realized returns as a proxy for expected returns, which is closely tied with risk premia. *Fama and Bliss* (1987) and *Campbell and Shiller* (1991) are among the earlier works to examine yield predictabil-

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ity using information obtained from the nominal curve. These authors find forward rates and yield spreads have substantial power in predicting future movements of the term structure. In a similar vein, *Cochrane and Piazzesi* (2005) extend the *Fama and Bliss* (1987) classic regression to incorporate a single linear combination of forward rates, and they find the same "return-forecasting factor" can explain expected bond returns of all maturities. Not only is the explanatory power, in terms of R^2, of this factor rarely matched in the literature, but it is also not very correlated with the purely statistical factors obtained from principal component analysis of the yields.

Building on the insights of these classics, a number of recent papers focus on constructing bond return predictors with inputs from macroeconomic variables. *Cooper and Priestley* (2009) attribute stock and bond return predictability directly to the output gap, which can be defined as the deviation of economic output (GDP) from its quadratic trend. The output gap is highly correlated with the *Cochrane and Piazzesi* (2005) forward rate factor, which helps to pin some economic intuition on the yield curve-based predictor. *Ludvigson and Ng* (2009) examine bond risk premia using principal components from the dynamic factor analysis of 132 macroeconomic and financial time series. The principal components are shown to be able to survive alongside the *Cochrane and Piazzesi* (2005) factor, and they greatly improve its predictability. *Cieslak and Povala* (2010) factor is manufactured from the decomposition of long-term yields into an inflation component and a cyclic component. The term premium factor has higher R^2 compared to the return-forecasting factor and it also renders other predictive factors insignificant in side-by-side tests. Although it is not directly tied to return predictability, *Kurmann and Otrok* (2012) demonstrate that news shocks about future total factor productivity (TFP) have substantial explanatory power in capturing shocks to the slope of the term structure.

This paper is also related to the macro-finance literature employing the no-arbitrage state-space framework to model the term structure of interest rates and bond risk pre-

mia. *Ang and Piazzesi* (2003) use monetary policy variables as well as latent factors to fit the yield curve. *Cochrane and Piazzesi* (2008) incorporate the return-forecasting factor into an affine term structure model together with the traditional level, slope, and curvature factors. Their key insight is that expected returns are largely compensation for holding level risk. *Joslin, Priebsch, and Singleton* (2010) builds a dynamic term structure model to quantify the relationship between the level, slope, and curvature factors and real and nominal risks in the economy,. *Duffee* (2011) finds a hidden factor not spanned by the cross-section of yields that has strong forecast power on bond excess returns.

On the theoretical front, the regressions specified here are based on the general equilibrium work of *Hsu* (2012). From the impulse responses, *Hsu* (2012) demonstrates that a one standard deviation shock to government spending generates as much term premium on the long-term bond as a one standard deviation shock to monetary policy. Furthermore, stochastic volatility on government spending drives the time variation of bond risk premia in the model, thus making future returns predictable. A number of theory papers jointly study the macroeconomic environment and the bond risk premium in a structural setting. I refer the readers to the literature review in *Hsu* (2012) for a complete survey.

3.4   Theoretical Framework

To understand how government spending and fiscal policy affect bond yields and associated risk premia in equilibrium, *Hsu* (2012) constructs a general equilibrium model with a production sector and a government. There are two types of households in the economy: savers and spenders. The savers are forward-looking optimizers who can save by purchasing government bonds, and the spenders are constrained consumers with no access to any financial instruments or savings technology. Both types of households provide labor to a representative firm in return for earning a

market clearing wage. The firm produces a single consumption good using labor as the only factor of production without capital. Furthermore, the savers and spenders have the same preferences, for consumption, labor dis-utility, and homogeneous preference parameters across types.

The reason for the presence of spenders in the economy is twofold. First, because the spenders are locked out of the financial market, they are non-Ricardian in the sense that they have no savings to increase consumption when taxes are high and government debt is low. Conversely, when taxes are low and government debt is high, they are forced to consume more, holding labor income constant. Their consumption is completely dictated by how taxes are determined, and this breaks Ricardian equivalence allowing fiscal policy to be non-neutral. Under Ricardian equivalence, government's financing decision of tax vs. borrowing is irrelevant to the real economy, such as output and consumption. The friction posed by the spenders affects consumption growth of the savers. The limited participation in the financial market of the spenders makes the savers the marginal pricers in the economy. Therefore, consumption growth of the representative saver uniquely determines the pricing kernel and the prices of all assets in the economy, including Treasury bonds. By not allowing the savers to fully optimize their consumption choice, the inclusion of spenders generates higher risk premia in the economy.

I focus on two equations taken from the benchmark model of *Hsu* (2012). The first is the market clearing condition of the economy with no investment:

$$Y_t = C_t + G_t, \tag{3.1}$$

where Y_t is output, C_t is consumption, and G_t is government spending. The second is the fiscal policy rule used by the government to determine the level of lump-sum

taxes to collect from both types of households:

$$\tau_t = \rho_b d_{t-1}(t) + \rho_g g_t + \epsilon_t^\tau, \tag{3.2}$$

where τ_t is log tax, $d_{t-1}(t)$ is the log of real debt outstanding at $t-1$ due at t, g_t is log government spending ($g_t = logG_t$), and $\epsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2)$ is the shock to tax revenue. ρ_b and ρ_g are reaction coefficients of taxes on debt and spending, respectively. The government sets fiscal policy by adjusting these two parameters. Finally, *Hsu* (2012) assumes government spending is exogenously specified to follow an $AR(1)$ process with mean θ_g:

$$g_{t+1} = (1 - \phi_g)\theta_g + \phi_g g_t + \epsilon_{t+1}^g, \tag{3.3}$$

where ϵ_{t+1}^g is a $\mathcal{N}(0, \sigma_g^2)$ random shock.

The term spread of a n-period to maturity nominal bond over the nominal short rate can be decomposed into the covariance between the nominal pricing kernel and its yield plus a Jensen's inequality term:

$$i_t^{(n)} - i_t = \frac{1}{n}\left\{\sum_{k=1}^{n-1}(n-k)cov_t\left(m_{t,t+1}^\$, i_{t+1}^{(n-k)}\right) - \frac{1}{2}\sum_{k=1}^{n-1}(n-k)^2 var_t\left(i_{t+1}^{(n-k)}\right)\right\},$$

where $m_{t,t+1}^\$$ is the nominal pricing kernel, inversely related to the saver's consumption growth between today and tomorrow. The covariance term represents the risk premium of the n-period bond. If the price of the long-term bond is low in the state of the world when consumption growth is low, then the bond is a bad hedge against consumption risk and investors demand a positive risk premium in return for holding the asset. This means the long-term yield is high when the marginal utility to consume (pricing kernel) is also high, and the covariance is positive.

To see how the shock to government spending, ϵ_t^g, and the shock to government revenue, ϵ_t^τ, affect this covariance term, I consider their impact separately on the

saver's consumption growth and long-term nominal yields. In equation (3.3), a positive government spending shock (ϵ^g_{t+1}) increases government spending (g_{t+1}). The market clearing condition, equation (3.1), shows higher government spending lowers consumption since the increase in output is not enough to compensate for the extra amount the government is consuming[1]. This means positive government spending shock creates a high marginal utility state of the world due to low C_{t+1}, and $m^{\$}_{t,t+1}$ is high. At the same time, higher government spending leads the government to issue more debt because the tax adjustment is insufficient ($\rho_g < 1$) according to the fiscal rule, equation (3.2), and the nominal interest rate rises. Therefore, the covariance between the marginal utility to consume and the nominal yield is positive, resulting in a positive risk premium on the long-term bond.

A positive government revenue shock (ϵ^τ_{t+1}) increases tax (τ_{t+1}) through the fiscal rule, equation (3.2). Higher tax lowers consumption growth and leads to higher marginal utility similar to the positive government spending shock. However, unlike the spending shock, higher tax means the government borrows less to finance the same amount of spending, and the nominal interest rate decreases. The result is a negative covariance between the marginal utility to consume and the nominal yield generating negative risk premium on the long-term bond.

Employing this simple general equilibrium model, *Hsu* (2012) demonstrates the mechanism underlying the relationship between government spending and revenue shocks and bond risk premia. The current paper examines two testable implications: positive shocks to government spending make long-term nominal bonds riskier, and positive shocks to government revenue make long-tern nominal bonds less risky. The empirical strategy is based upon predictive regressions of excess holding-period Treasury bond returns on government spending and revenue shocks.

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[1]The fiscal multiplier in the model is less than 1.

3.5 Constructing Fiscal Shocks

The regression analysis presented here consists of two stages. In the first stage, I construct fiscal policy shocks borrowing from the macroeconomic literature on the dynamic effects of fiscal policy changes on the real economy. In the second stage, I run predictive regressions of one-year excess holding period returns of zero coupon bonds on the fiscal policy shocks obtained from the first stage. The economic intuition behind this test is simple, expected bond returns should adequately compensate the investors for bearing any risk that may affect future bond prices and shocks to fiscal policy should be reflected in the expected returns if they have statistically significant impact on future yields. In the appendix, I discuss in detail the various econometric issues such as generated regressors and small sample time series to arrive at robust inference.

The procedure used in the first stage to construct the fiscal policy shocks is standard from macroeconomics. Following *Blanchard and Perotti* (2002), I use a 4-lag vector autoregressive (VAR) model at quarterly frequency such that:

$$Y_t = A(L, q)Y_{t-1} + U_t, \tag{3.4}$$

where $Y_t \equiv [Tax_t, Spending_t, GDP_t]'$. The coefficient $A(L, q)$ is a four-quarter distributed lag polynomial that is a function of both the lag and the quarter, q, which it is indexed on. This specification is the Almon polynomial distributed lag model. Because there are seasonal patterns in the responses of taxes to economic activity, the regression coefficient is quarter-dependent[2].

$U_t \equiv [\tau_t, g_t, x_t]'$ is a vector of residuals for the government revenue equation, the government spending equation, and the GDP equation in the VAR, respectively. Once

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[2]For a detailed explanation, *Blanchard and Perotti* (2002) provide some evidence of quarter-dependence as well as contributing features to the quarter-dependence in the tax code in their appendices.

U_t in the VAR is obtained, the task is to identify the orthogonal shocks that make up these residuals. Without loss of generality, the residuals can be expressed as the following:

$$\tau_t = a_1 x_t + a_2 \epsilon_t^g + \epsilon_t^\tau \tag{3.5}$$

$$g_t = b_1 x_t + b_2 \epsilon_t^\tau + \epsilon_t^g \tag{3.6}$$

$$x_t = c_1 \tau_t + c_2 g_t + \epsilon_t^x. \tag{3.7}$$

In other words, each residual is modeled as a linear combination of the other two residuals plus orthogonal shocks to revenue, spending, and GDP. The key to the identification strategy, according to *Blanchard and Perotti* (2002), is that using quarterly data, they eliminate any automatic feedback from economic activity to government spending because government budgets are set annually prior to the fiscal year so $b_1 = 0$. In addition, within the quarter, either $a_2 = 0$ or $b_2 = 0$ but not both. This means the contemporaneous feedback between revenue and spending is one way: either the spending shock drives revenue within the quarter or the revenue shock drives spending. *Blanchard and Perotti* (2002) find that the correlation between tax and spending is sufficiently small that ordering the two makes little difference.

Here, I assume $b_2 = 0$, taking the view that the spending shock affects revenue in the same quarter, but the revenue shock is ineffective on government spending, consistent with the idea that budgets are set ahead of the fiscal cycle for the entire year. This leads to $g_t = \epsilon_t^g$. In other words, the residuals from the government spending equation in the VAR are shocks to spending. To identify ϵ_t^τ, I run the following regression:

$$\tau_t - \widehat{a_1} x_t = a_2 \epsilon_t^g + \epsilon_t^\tau. \tag{3.8}$$

I take $\widehat{a_1} = 2$ from the prior study. *Blanchard and Perotti* (2002) estimate a_1 separately using OECD data. They find the average value to a_1 between $1947 : I$ to

1997 : IV to be about 2.08. This number is within the range from numerous other studies. It is straightforward to see that the innovations, ϵ_t^τ, are shocks to taxes orthogonal to shocks to spendng.

3.6 Data

The sample period spans from 1971 : IV to 2010 : III, or 156 quarterly observations. Government expenditure and revenue used in the first-stage VAR, equation (3.4), are constructed following definitions in *Blanchard and Perotti* (2002) using the National Income and Product Accounts (NIPA) tables[3]. Government Expenditure (spending) is made up of Federal Defense Consumption, Federal Non-Defense Consumption, State and Local Consumption, Federal Defense Investment, Federal Non-Defense Investment and State and Local Investment. Government Revenue is obtained by taking Total Government Receipts and subtracting Net Transfer Payments and Net Interest Paid. GDP is also obtained from the NIPA tables while Inflation and Fed Funds Rate are from the FRED II database courtesy of the St. Louis Federal Reserve Bank. All macroeconomic variables are on a detrended, per capita, real basis[4]. The Output Gap is the result of applying the Hodrick-Prescott Filter to GDP using a λ coefficient of 1600.

The term structure data is from *Gurkaynak, Sack, and Wright* (2006), which publishes daily Treasury yields from 1961 to the present[5]. The estimation methodology for constructing the zero-coupon yield curves everyday is an extension of the Nelson-Siegel approach. The maturity structure of the data starts at 1-year to maturity up to 30-years to maturity at 1-year intervals. Since the data is reported in daily frequency, I choose yields at the end of every March, June, September and December to

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[3]These definitions are widely followed, including *Dai and Philippon* (2006) and *Mountford and Uhlig* (2009).

[4]Variables are divided by the GDP implicit price deflator in 2005 dollars.

[5]The entire data set can be downloaded from
http://www.federalreserve.gov/econresdata/researchdata.htm.

construct my quarterly yield series. Moreover, I extend the maturity of the bonds in the regression to 15 years, compared with 5 years in *Cochrane and Piazzesi* (2005), to study the effects of fiscal policy shocks on long-term bonds. The 15-year yields go back as far as the fourth quarter of 1971 in the *Gurkaynak, Sack, and Wright* (2006) data series.

To calculate one-year holding period excess returns, I follow the notation in *Cochrane and Piazzesi* (2005). Define $p_t^{(n)}$ as the time t log price of a n-years to maturity zero-coupon bond. The relationship between log price and log yield is:

$$p_t^{(n)} \equiv -n y_t^{(n)}. \tag{3.9}$$

To calculate the log forward rates, I take the difference between log prices at time t of a n-year bond and a (n-1)-year bond:

$$f_t^{(n-1 \to n)} \equiv p_t^{(n-1)} - p_t^{(n)}, \tag{3.10}$$

while returns can be expressed as:

$$r_{t+4}^{(n)} \equiv p_{t+4}^{(n-1)} - p_t^{(n)}. \tag{3.11}$$

Since bond maturities are in years but observation frequency is in quarters, $t + 4$ denotes the return one year later. Excess log return is:

$$rx_{t+4}^{(n)} \equiv r_{t+4}^{(n)} - y_t^{(1)}. \tag{3.12}$$

3.6.1   Descriptive Statistics

Table C.1 presents the summary statistics of the raw data used in the empirical exercise. Averages and standard deviations are calculated over the sample period

from the fourth quarter of 1971 to the third quarter of 2010. GDP, Government Expenditure and Government Revenue are expressed in billions of dollar. On average, the U.S. government has been running a deficit during the sample period as evident by the shortfall in revenue. Yields and excess returns are expressed in percentages. The nominal term structure during the sample period is upward sloping. While the volatilities of the nominal yields decrease as maturity increases, the volatilities of the excess returns increase with maturity.

[**Table** C**.1 Here**]

3.7   Predictive Regressions

3.7.1   Fiscal Policy Shocks

Table C.2 reports the estimation results of regressing excess returns on government spending and tax shocks. Bonds with 2-, 6-, 10-, and 14-years to maturity are shown to represent the entire yield curve. For $n = 2$, this is the return, over the 1-year yield, of buying a 2-year bond at time t and selling the same bond a year later. The most striking finding is that government spending shocks are significant in explaining excess bond returns across all maturities while taxation shocks are not. Furthermore, the coefficient estimates load in the direction that one would expect: positive government spending shocks increase excess bond returns while positive tax shocks decrease excess returns. This finding is consistent with the theoretical model from the previous section in which higher government spending, or loose fiscal policy, leads to higher bond risk premia and higher returns. Finally, the t-statistics on g_t are monotonically decreasing in maturity, which indicates spending shocks are more significant in explaining returns at the short-end of the yield curve than the long-end. This result is in line with *Dai and Philippon* (2006), who find fiscal policy shocks contribute more to variations in the the long-end of the yield curve, but their statistical significance decreases with

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maturity. To see the economic significance of these results, for the U.S., a one standard deviation increase in government spending will lead to an increase of 43 basis points in the excess return of 2-year bonds.

[**Table** C.**2 Here**]

Table C.3 reports the excess return regression results using fiscal policy shocks and monetary policy proxies as independent variables. The monetary proxies are the Fed Funds Rate, output gap, and inflation. The Fed Funds Rate drives the dynamics of the short-term yields, while output gap and inflation are inputs to the simple Taylor rule implemented in the full theoretical model. After inflation, spending shock is the most significant factor in explaining excess returns across maturities except for the 2-year bond where Fed Funds Rate is more significant. Tax shocks, on the other hand, are insignificant across maturities.

[**Table** C.**3 Here**]

Not surprisingly, the R^2 in this regression are much higher than R^2 in the previous regression due to the presence of monetary policy variables. However, the economic significance of the government spending shock is lost alongside the monetary policy variables. Looking at the excess return of the short-term bond, a positive one standard deviation shock to government spending leads to an increase of 40 basis points in excess return, similar to before; whereas a one percentage increase in the Fed Funds Rate increases excess return by 22 basis points. The spending shock variable maintains its statistical and economic significance despite the inclusion of macroeconomic control variables.

To see how the fiscal policy shocks perform alongside financial variables in the excess return regression, I run the test using forward rates as control variables. Following *Cochrane and Piazzesi* (2005), only excess returns of bonds with maturities

of 2- to 5-years are used, and the independent variables include the 1-year yield and 1-year forward rates at one, two, three and four years in the future[6]. The results are shown in table C.4. Despite the presence of the collection of forward yields, the spending shock remains significant across maturities, although at the 10% level as opposed to the 5% level, as in previous regressions. Surprisingly, the tax shock remains insignificant while the forward rates are also insignificant.

[**Table** C.**4 Here**]

I verify these results by performing the same test using the Fama-Bliss zero-coupon yields from CRSP instead of the *Gurkaynak, Sack, and Wright* (2006) data. Table C.5 reports the regression estimates. The t-statistics of the spending shocks are fairly close to their counterparts in table C.4 and are significant at the 10% level. However, the t-statistics of the coefficient estimates on $f_t^{(2\to3)}$ and $f_t^{(4\to5)}$ are highly significant now, whereas they are statistically insignificant in table C.4. Furthermore, using the Fama-Bliss data, the coefficients on the 1-year yields and the forward rates display the "tent-shaped" linear relationships found by *Cochrane and Piazzesi* (2005). A closer examination of the R^2 of these predictive regressions provides further evidence that the spending shocks are orthogonal to whatever macroeconomic factors the forwards are supposedly capturing. The difference in R^2 between table C.5 and table 1 in *Cochrane and Piazzesi* (2005) for each of the regression equations is roughly 4%, which is in the range of the variations in excess returns that can be explained by spending shocks alone in table C.2. These findings combined confirm the intuition from the theoretical model that government spending shocks generate time-varying bond risk premia and drive expected returns on bonds.

[**Table** C.**5 Here**]

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[6]For example $f_t^{(2\to3)}$ denotes the 1-year forward rate between two and three years from today.

3.7.2 Stochastic Volatility on Government Spending

From *Hsu* (2012), the theoretical model generates time variation in bond risk premia through stochastic volatility on government spending shocks. Without stochastic volatility in spending shocks, bond risk premia are constant. Expected returns on Treasury bonds are also constant, and there is no return predictability. Therefore, a true empirical test of the theoretical model requires stochastic volatility as the independent variable in the predictive regressions.

In the literature, researchers have relied upon two methodologies to obtain stochastic volatility. One method is to conduct structural estimation on a Stochastic Autoregressive Volatility (SARV) model. The other method is to fit a Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) model to the mean-zero residuals. The resulting time-varying volatility should be implied by the true stochastic volatility of the data-generating process. For the sake of computational convenience, I obtain conditional volatilities on government spending shocks by estimating a GARCH model on the zero-mean innovations to government spending. The GARCH equation I estimate is of the following form:

$$\sigma_{g,t} = \omega^\sigma + \sum_{p=1}^{P} \alpha_p |\epsilon_{t-p}^g| + \sum_{q=1}^{Q} \beta_q |\sigma_{g,t-q}|, \tag{3.13}$$

where $\sigma_{g,t}$ is the conditional volatility on government spending shocks, ϵ_t^g is the mean-zero shock series. Furthermore, ω^σ, α_p and β_q are fitted coefficients through maximum likelihood estimation, assuming normally distributed errors.

I estimate the conditional volatility directly using lagged absolute values of shocks and volatility as opposed to estimating the conditional variance on lagged squared values. This idea is similar to the Threshold GARCH (TGARCH) model from Zokian (1994)[7] without the asymmetry in the ARCH process. I find the GARCH diagnos-

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[7]See Bollerslev (2007) for a glossary of various GARCH specifications.

tic tests and the predictive regressions have much higher power when I specify the model in terms of volatilities without having to square the innovations to government spending.

To determine the optimal number of lags to include in the GARCH estimation, two diagnostic tests are employed on ϵ_t^g to find P and Q. The Lagrange multiplier test pins down the number of ARCH elements, P, by checking the existence of an ARMA component in the autoregressive process on government spending. The Ljung-Box test, on the other hand, decides the number of GARCH elements, Q, by checking for serial correlation on ϵ_t^g. Given the government spending shocks I obtained in the previous section, I specify and estimate a TGARCH$(5,5)$ model to construct conditional volatility on spending shocks.

Table C.6 reports the results of predictive regressions of excess Treasury bond returns on time-varying government spending volatility. Recall the dependent variable under $n = 2$ is the return, over the 1-year riskfree rate, of buying a 2-year bond at time t and selling the same bond a year later. Similar to the regression results using government spending shocks as the independent variable, stochastic volatility on spending shocks is significant in explaining future excess bond returns across the maturity structure, although the significance level decreases as n increases. Furthermore, the slope coefficients are all positive and monotonically increasing. When conditional volatility on government spending is high, excess bond returns are also high implying higher risk premia on Treasury bonds. Looking at the R^2 values, conditional volatility on government spending has more explanatory power on excess returns when compared to the R^2 values reported in table C.2. This might serve as evidence that stochastic volatility on government spending shocks is the more relevant independent variable to use in excess bond return predictive regressions, as the theory suggests, than the level of shocks.

[**Table C.6 Here**]

3.8 Robustness

To control for cointegration, where the macro variables share a stochastic drift, I also run a Vector Error Correction Model (VECM) on $Y_t \equiv [Tax_t, Spending_t, GDP_t]'$. The resulting fiscal shocks are weaker in the baseline regressions but remain statistically significant in explaining excess 1-year holding period returns. The coefficient estimates from the predictive regression using fiscal shocks constructed from the VECM can be found in table C.7. The statistical significance of the spending shocks, g_t, drops from the 5% level to the 10% level for bonds of all maturities. At the same time, revenue shocks remain insignificant. The slope coefficients on g_t are all positive and monotonically increasing as a function of maturity, while the slope coefficients on τ_t are all negative and monotonically decreasing. Finally, the R^2 of the regressions are lower than the baseline specification. A possible explanation is a stochastic trend exists among output, government spending, and government revenue that contributes to bond return predictability.

[**Table** C**.7 Here**]

To ensure the results are not driven by the most recent stimulus package, enacted in February of 2009, I cut off the sample in the fourth quarter of 2008 and run the same tests. Government spending shocks remain significant at the 10% level in predicting excess bond returns across all maturities in the sample.

3.9 Conclusion

Understanding the relationship between bond risk premia and the macroeconomic environment is a central theme of the term structure literature. While a wealth of papers have been written documenting various financial and economic factors that can affect term premia and predict future returns, the role of fiscal policy on the

term premia has been largely neglected. In the current paper, results produced by regression analysis involving fiscal policy variables provide evidence that there is predictability in government spending shocks on future bond excess holding period returns.

Government spending shocks are positive and statistically significant in explaining future excess returns across the maturity structure. This finding is in agreement with the implication of the theoretical model. *Hsu* (2012) reports higher government spending lowers consumption at the same time it increases nominal interest rates. In the bad state of the world when a positive spending shock is realized, bond prices are low making them bad hedges against the consumption risk. Ex ante, investors will require higher expected returns in exchange for holding Treasury bonds. Government revenue shocks, on the other hand, are negative but statistically insignificant in the predictive regressions across maturities. The negative slope coefficients verify the insight from the theoretical model: when the government finances it is real obligations through higher taxes as opposed to issuing more debt, Treasury bonds become less risky, and bond risk premium decreases.

The regression results are robust to a number of controls and robustness tests. For government spending shocks, the coefficient loadings are always positive, and the statistical significance survives when I control for monetary policy conditions and the *Cochrane and Piazzesi* (2005) "tent-shaped" prediction factor. Of particular interest, for a given maturity, the difference in R^2 of the regression with both fiscal policy shocks and the return-forecasting factor and the original *Cochrane and Piazzesi* (2005) regression is roughly equal to the R^2 of the baseline specification with only fiscal policy shocks. This finding suggests that the fiscal policy shocks are somewhat orthogonal to the return-forecasting factor and can capture variations in expected returns beyond the previous results employing return-forecasting factor has shown. Although the government revenue shocks are never significant under any of

the regression specification, it should be noted that the coefficient loadings are always negative, consistent with theory.

I also perform return predictive regressions with stochastic volatility constructed from government spending shocks through TGARCH. In the theoretical model, conditional variance of the spending shocks drives the time variation of risk premia for bonds. I conjecture this conditional variance is the fundamental source of return predictability displayed by the government spending shocks documented previously. The regression results confirm the hypothesis: stochastic volatility is positive and significant in explaining future returns. The regression R^2 is higher than the R^2 using the level of shocks for all maturities, suggesting stochastic volatility has additional explanatory power to capture the dynamics of returns.

For future work, it would be interesting to test the explanatory power of the fiscal policy variables along side macroeconomic factors that recently have shown to be useful in capturing the dynamics of expected returns. The first variable that comes to mind is the main principal component resulting from the *Ludvigson and Ng* (2009) analysis of 132 series of macroeconomic and financial data. The second is the term premium factor proposed by *Cieslak and Povala* (2010) via decomposing long-term yields into a persistent component and a cyclic component. In addition, government spending and revenue shocks can be incorporated into a no-arbitrage macro-finance model to fit the yield curve. This would be a setup similar to that of *Dai and Philippon* (2006) but augmented by the presence of stochastic volatility on these shocks.

APPENDICES

APPENDIX A

Does Fiscal Policy Matter for Treasury Bond Risk Premia?

A.1 Solving the Simple Model

A.1.1 Government's Present Value Budget Constraint

Here I explicitly derive the present value government budget constraint starting with its flow budget constraint:

$$B_{t-1}(t) - \sum_{j=1}^{\infty} Q_t^{(j)} \left[B_t(t+j) - B_{t-1}(t+j) \right] = P_t S_t$$

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^{\gamma} \frac{P_t}{P_{t+j}} B_{t-1}(t+j) - \sum_{j=1}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^{\gamma} \frac{P_t}{P_{t+j}} B_t(t+j) \right] = P_t S_t$$

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^{\gamma} \frac{B_{t-1}(t+j)}{P_{t+j}} - \beta L^{-1} \sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^{\gamma} \frac{B_{t-1}(t+j)}{P_{t+j}} \right] = S_t$$

$$E_t \left[1 - \beta L^{-1} \right] E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^{\gamma} \frac{B_{t-1}(t+j)}{P_{t+j}} \right] = S_t,$$

and L is the lag operator. The present value equation is

$$\frac{B_{t-1}(t)}{P_t} + E_t \left[\sum_{j=1}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^\gamma \frac{B_{t-1}(t+j)}{P_{t+j}} \right] = E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^\gamma S_{t+j} \right]$$

A.1.2 Log-Linearization

In order to write the rule-of-thumb consumer's consumption equation in terms of aggregate variables, I substitute (1.26) and (1.29) into (1.25) for w_t and n_t^r:

$$
\begin{aligned}
c_t^r &= \frac{WN}{C}(w_t + n_t^r) - \frac{T}{C}\tau_t^r \\
c_t^r &= \frac{WN}{C}(\gamma c_t + \omega n_t + \omega^{-1}(w_t - \gamma c_t^r)) - \frac{T}{C}\tau_t^r \\
c_t^r &= \frac{WN}{C}(\gamma c_t + \omega n_t + \omega^{-1}(\gamma c_t + \omega n_t - \gamma c_t^r)) - \frac{T}{C}\tau_t^r \\
\omega c_t^r &= \frac{WN}{C}(\omega\gamma c_t + \omega^2 n_t + \gamma c_t + \omega n_t - \gamma c_t^r) - \frac{T}{C}\omega\tau_t^r \\
\left(\omega + \frac{WN}{C}\gamma\right) c_t^r &= \frac{WN}{C}[(1+\omega)\gamma c_t + (1+\omega)\omega n_t] - \frac{T}{C}\omega\tau_t,
\end{aligned}
$$

where the last equality applies the assumption that $T_t^r = T_t^0$, which means $\tau_t^r = \tau_t^o$ and $\tau_t^r = \tau_t$ under aggregation.

To log-linearize the government's budget constraint, start with the righthand-side

of the present value equation:

$$
E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^{\gamma} S_{t+j} \right]
$$

$$
= E_t \underbrace{\left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_t^o}{C_{t+j}^o} \right)^{\gamma} \frac{S_{t+j}}{S_t} \right]}_{H_t} S_t
$$

$$
= \left\{ 1 + E_t \left[\beta \left(\frac{C_t^o}{C_{t+1}^o} \right)^{\gamma} \frac{S_{t+1}}{S_t} \sum_{j=1}^{\infty} \beta^{j-1} \left(\frac{C_{t+1}^o}{C_{t+j}^o} \right)^{\gamma} \frac{S_{t+j}}{S_{t+1}} \right] \right\} S_t
$$

$$
= \left\{ 1 + \beta E_t \left[\left(\frac{C_t^o}{C_{t+1}^o} \right)^{\gamma} \frac{S_{t+1}}{S_t} E_{t+1} \underbrace{\left[\sum_{j=1}^{\infty} \beta^{j-1} \left(\frac{C_{t+1}^o}{C_{t+j}^o} \right)^{\gamma} \frac{S_{t+j}}{S_{t+1}} \right]}_{H_{t+1}} \right] \right\} S_t
$$

$$
= \left\{ 1 + \beta H e^{logE_t \left[e^{-\gamma \Delta c_{t+1}^o + s_{t+1} - s_t + h_{t+1}} \right]} \right\} S_t.
$$

Applying first order Taylor series expansion to H_t and denote

$\Upsilon_t = logE_t \left[e^{-\gamma \Delta c_{t+1}^o + s_{t+1} - s_t + h_{t+1}} \right]$,

$$
\begin{align}
h_t &= log(1 + \beta H e^{\Upsilon_t}) - log(H) \tag{A.1} \\
&\approx log(1 + \beta H e^{\bar{\Upsilon}}) + \frac{\beta H e^{\bar{\Upsilon}}}{1 + \beta H e^{\bar{\Upsilon}}}(\Upsilon_t - \bar{\Upsilon}) - log(H) \tag{A.2} \\
&= \underbrace{log(1 + \beta H e^{\bar{\Upsilon}}) - log(H) - \frac{\beta H e^{\bar{\Upsilon}}}{1 + \beta H e^{\bar{\Upsilon}}}\bar{\Upsilon}}_{\overline{\eta_h}} + \underbrace{\frac{\beta H e^{\bar{\Upsilon}}}{1 + \beta H e^{\bar{\Upsilon}}}}_{\eta_h} \Upsilon_t \tag{A.3} \\
&= \overline{\eta_h} + \eta_h logE_t \left[e^{-\gamma \Delta c_{t+1}^o + s_{t+1} - s_t + h_{t+1}} \right], \tag{A.4}
\end{align}
$$

where H is the deterministic steady state of H_t and $\bar{\Upsilon}$ is the unconditional expectation of Υ_t:

$$
\begin{align}
\bar{\Upsilon} &= h + (-\gamma c_d^o + s_d + h_d)d + [(\gamma c_d^o - s_d)(1 - d_d) + h_d d_d]\frac{d}{1 - d_d} + \\
&\quad \frac{1}{2}(-\gamma c_g^o + s_g + h_g)^2 \sigma_g^2 + \frac{1}{2}(-\gamma c_z^o + h_z)^2 \sigma_z^2.
\end{align}
$$

To linearize (1.19), the present value budget constraint is rewritten as

$$\left(1 + \frac{\varphi}{1-\varphi}\right)\frac{B_{t-1}(t)}{P_{t-1}}\frac{1}{\Pi_t} = \left[H_t + \frac{\varphi}{1-\varphi}\right]S_t$$

$$\left(1 + \frac{\varphi}{1-\varphi}\right)D_{t-1}(t)\frac{1}{\Pi_t S_t} = \left[H_t + \frac{\varphi}{1-\varphi}\right]$$

$$\left(1 + \frac{\varphi}{1-\varphi}\right)\frac{D}{\Pi^* S}e^{d_{t-1}(t)-\pi_t-s_t} = \left[He^{h_t} + \frac{\varphi}{1-\varphi}\right]$$

$$log\left(1 + \frac{\varphi}{1-\varphi}\right) + log\frac{D}{S} + d_{t-1}(t) - (\pi_t + \pi^*) - s_t = log\left[He^{h_t} + \frac{\varphi}{1-\varphi}\right].$$

Applying first order Taylor expansion on the righthand-side of the above equation, the fully linearized fiscal equation is

$$log\left(1 + \frac{\varphi}{1-\varphi}\right) + log\frac{D}{S} + d_{t-1}(t) - (\pi_t + \pi^*) - s_t$$

$$\approx log\left[He^{\bar{h}} + \frac{\varphi}{1-\varphi}\right] + \frac{He^{\bar{h}}}{He^{\bar{h}} + \frac{\varphi}{1-\varphi}}(h_t - \bar{h})$$

$$= \underbrace{log\left[He^{\bar{h}} + \frac{\varphi}{1-\varphi}\right] - \frac{He^{\bar{h}}}{He^{\bar{h}} + \frac{\varphi}{1-\varphi}}\bar{h}}_{\overline{\eta_\varphi}} + \underbrace{\frac{He^{\bar{h}}}{He^{\bar{h}} + \frac{\varphi}{1-\varphi}}}_{\eta_\varphi}h_t,$$

where $\bar{h}$ is the unconditional expectation of h_t:

$$\bar{h} = h + h_d\left(\frac{d}{1-d_d}\right).$$

A.1.3 The System of Equations

The simplified model with long term bonds presented in this section has twelve endogenous variables, namely $\{w_t, n_t, c_t, y_t, \pi_t, \tau_t, s_t, i_t, h_t, d_{t-1}(t), c_t^o, c_t^r\}$. I have a system of twelve equations resulting from log-linearization of the first order conditions

and policy rules:

$$w_t = \gamma c_t + \omega n_t, \tag{A.5}$$

$$-i_t = log\beta + logE_t\left[e^{-\gamma\Delta c^o_{t+1}-\pi_{t+1}}\right], \tag{A.6}$$

$$y_t = \alpha n_t, \tag{A.7}$$

$$w_t = log\alpha + (\alpha-1)n_t, \tag{A.8}$$

$$h_t = \overline{\eta_h} + \eta_h logE_t\left[e^{-\gamma\Delta c^o_{t+1}+s_{t+1}-s_t+h_{t+1}}\right], \tag{A.9}$$

$$\overline{\eta_\varphi} + \eta_\varphi h_t = log\left(1+\frac{\varphi}{1-\varphi}\right) + log\frac{D}{S} + d_{t-1}(t) - \pi_t - s_t, \tag{A.10}$$

$$s_t = \frac{T}{S}\tau_t - \frac{G}{S}g_t, \tag{A.11}$$

$$y_t = \frac{C}{Y}c_t + \frac{G}{Y}g_t, \tag{A.12}$$

$$i_t = \imath + \rho_\pi\pi_t, \tag{A.13}$$

$$\tau_t = \rho_b d_{t-1}(t) + \rho_g g_t \tag{A.14}$$

$$c_t = \mu c^r_t + (1-\mu)c^o_t \tag{A.15}$$

$$\left(\omega+\frac{WN}{C}\gamma\right)c^r_t = \frac{WN}{C}[(1+\omega)\gamma c_t + (1+\omega)\omega n_t] - \frac{T}{C}\omega\tau_t, \tag{A.16}$$

where g_t is the exogenous government spending process previously defined. To solve system of linear equations, I guess the solution form to be affine in real short term

debt maturing at time t and the government spending shock at time t, such that:

$$
\begin{bmatrix} w_t \\ n_t \\ c_t \\ y_t \\ \pi_t \\ \tau_t \\ s_t \\ i_t \\ h_t \\ d_t \\ c_t^0 \\ c_t^r \end{bmatrix} = \begin{bmatrix} w \\ n \\ c \\ y \\ \pi \\ \tau \\ s \\ i \\ h \\ d \\ c^0 \\ c^r \end{bmatrix} + \begin{bmatrix} w_d \\ n_d \\ c_d \\ y_d \\ \pi_d \\ \tau_d \\ s_d \\ i_d \\ h_d \\ d_d \\ c_d^0 \\ c_d^r \end{bmatrix} d_{t-1} + \begin{bmatrix} w_g \\ n_g \\ c_g \\ y_g \\ \pi_g \\ \tau_g \\ s_g \\ i_g \\ h_g \\ d_g \\ c_g^0 \\ c_g^r \end{bmatrix} g_t
$$

A.1.4   Steady State Analysis

Notice from the definition of primary surplus and the market clearing condition, respectively, that:

$$
\begin{aligned}
\frac{G}{S} &= \frac{T}{S} - 1 \\
\frac{G}{Y} &= 1 - \frac{C}{Y}.
\end{aligned}
$$

Using the recursive definition of H_t, the steady state value of H can be found by dropping the time subscript:

$$H = 1 + \beta E_t\left[\left(\frac{C^o}{C^o}\right)^\gamma \frac{S}{S} H\right]$$
$$H = 1 + \beta E_t[H]$$
$$H = \frac{1}{1-\beta}.$$

The same procedure can be applied to the definitions of F and J such that

$$F = J = \frac{1}{1-\alpha\beta}.$$

In steady state, (A.27) becomes

$$\left[\frac{1}{1-\alpha}\left(1 - \alpha\left(\frac{\Pi^*}{\Pi}\right)^{1-\theta}\right)\right]^{\frac{1}{1-\theta}} F = \nu\frac{W}{Z}J.$$

Given $\Pi = \Pi^*$, $F = J$ and $Z = 1$, the steady state wage is

$$W = \frac{1}{\nu}.$$

To find $\frac{D}{S}$, I use the fiscal valuation equation and drop all time subscript to find the steady state:

$$\left(1 + \frac{\varphi}{1-\varphi}\right)\frac{D}{\Pi^*} = \left[H + \frac{\varphi}{1-\varphi}\right]S$$
$$\left(1 + \frac{\varphi}{1-\varphi}\right)\frac{D}{S} = \left[\frac{1}{1-\beta} + \frac{\varphi}{1-\varphi}\right]\Pi^*$$
$$\frac{D}{S} = \left[\frac{\frac{1}{1-\beta} + \frac{\varphi}{1-\varphi}}{\left(1 + \frac{\varphi}{1-\varphi}\right)}\right]\Pi^*,$$

where $\Pi^* = e^{\pi^*}$.

Lastly, I need to find $\frac{WN}{C}$ and $\frac{T}{C}$ to evaluate (A.16). I notice the rule-of-thumb household's consumption equation can be written in term of steady state values:

$$C_t^r = W_t N_t^r - T_t^r \quad \Rightarrow \quad C = WN - T$$

$$or$$

$$\frac{T}{C} = \frac{WN}{C} - 1.$$

After substituting for $\frac{T}{C}$, take the unconditional expectation of (A.16) to find $\frac{WN}{C}$ in terms of the parameters and coefficients in the model:

$$E\left[\left(\omega + \frac{WN}{C}\gamma\right) c_t^r\right]$$

$$= E\left[\frac{WN}{C}(1+\omega)[\gamma c_t + \omega n_t] - \left(\frac{WN}{C} - 1\right)\omega(\rho_b d_{t-1}(t) + \rho_g g_t)\right]$$

$$\frac{WN}{C} = \frac{\omega[c^r + (c_d^r - \rho_b)\frac{d}{1-d_d}]}{(1+\omega)[\gamma c + \omega n + (\gamma c_d + \omega n_d)\frac{d}{1-d_d}] - \gamma c^r - (\gamma c_d^r + \omega \rho_b)\frac{d}{1-d_d}}.$$

A.1.5   Pricing Kernel

The nominal pricing kernel in this economy is:

$$M_{t,t+1}^{\$} = \beta \left(\frac{C_t^o}{C_{t+1}^o}\right)^{\gamma} \frac{P_t}{P_{t+1}},$$

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then

$$-logM_{t,t+1}^{\$}$$

$$= -log\beta + \gamma(c_{t+1}^{o} - c_{t}^{o}) + \pi_{t+1}$$

$$= -log\beta + \gamma c_{d}^{o}(d_{t} - d_{t-1}) + \gamma c_{g}^{o}(g_{t+1} - g_{t}) + \pi + \pi_{d}d_{t} + \pi_{g}g_{t+1}$$

$$= -log\beta + \gamma c_{d}^{o}(d + d_{d}d_{t-1} + d_{g}g_{t} - d_{t-1}) + \gamma c_{g}^{o}(\phi_{g}g_{t} + \sigma_{g}\epsilon_{g,t+1} - g_{t}) +$$

$$\quad \pi + \pi_{d}(d + d_{d}d_{t-1} + d_{g}g_{t}) + \pi_{g}(\phi_{g}g_{t} + \sigma_{g}\epsilon_{g,t+1})$$

$$= -log\beta + \gamma c_{d}^{o}d + \pi + [\gamma c_{d}^{o}(d_{d} - 1) + \pi_{d}d_{d}]d_{t-1} +$$

$$\quad [(\gamma c_{d}^{o} + \pi_{d})d_{g} + \gamma c_{g}^{o}(\phi_{g} - 1) + \pi_{g}\phi_{g}]g_{t} + (\gamma c_{g}^{o} + \pi_{g})\sigma_{g}\epsilon_{g,t+1}$$

$$= \Gamma_{0}^{\$} + \Gamma_{d}^{\$}d_{t-1} + \Gamma_{g}^{\$}g_{t} + \lambda_{g}^{\$}\sigma_{g}\epsilon_{g,t+1},$$

where

$$\Gamma_{0}^{\$} = -log\beta + \gamma c_{d}^{o}d + \pi$$

$$\Gamma_{d}^{\$} = \gamma c_{d}^{o}(d_{d} - 1) + \pi_{d}d_{d}$$

$$\Gamma_{g}^{\$} = (\gamma c_{d}^{o} + \pi_{d})d_{g} - \gamma c_{g}^{o}(1 - \phi_{g}) + \pi_{g}\phi_{g}$$

$$\lambda_{g}^{\$} = \gamma c_{g}^{o} + \pi_{g}.$$

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A.1.6 The Term Structure of Interest Rate

$$
\begin{aligned}
e^{-ny_t^{(n)}} &= E_t\left[e^{m_{t,t+1}-(n-1)y_{t+1}^{(n-1)}}\right] \\[1mm]
&= E_t\left[e^{-\Gamma_0-\Gamma_d d_{t-1}-\Gamma_g g_t-\lambda_g\sigma_g\epsilon_{g,t+1}}e^{-A_{n-1}-B_{d,n-1}d_t-B_{g,n-1}g_{t+1}}\right] \\[1mm]
&= e^{-\Gamma_0-\Gamma_d d_{t-1}-\Gamma_g g_t-A_{n-1}-B_{d,n-1}(d+d_d d_{t-1}+d_g g_t)} \\[1mm]
&\quad E_t\left[e^{-\lambda_g\sigma_g\epsilon_{g,t+1}-B_{g,n-1}(\phi_g g_t+\sigma_g\epsilon_{g,t+1})}\right] \\[1mm]
&= e^{-\Gamma_0-A_{n-1}-B_{d,n-1}d-(\Gamma_d+B_{d,n-1}d_d)d_{t-1}-(\Gamma_g+B_{d,n-1}d_g+B_{g,n-1}\phi_g)g_t} \\[1mm]
&\quad E_t\left[e^{-(\lambda_g+B_{g,n-1})\sigma_g\epsilon_{g,t+1}}\right] \\[1mm]
&= e^{-\Gamma_0-A_{n-1}-B_{d,n-1}d-(\Gamma_d+B_{d,n-1}d_d)d_{t-1}-(\Gamma_g+B_{d,n-1}d_g+B_{g,n-1}\phi_g)g_t+\frac{1}{2}(\lambda_g+B_{g,n-1})^2\sigma_g^2}
\end{aligned}
$$

Since $e^{-ny_t^{(n)}}=e^{-A_n-B_{d,n}d_{t-1}-B_{g,n}g_t}$, I we can match coefficients to find A_n, $B_{d,n}$ and $B_{g,n}$:

$$
\begin{aligned}
-A_n &= -\Gamma_0-A_{n-1}-B_{d,n-1}d+\frac{1}{2}(\lambda_g+B_{g,n-1})^2\sigma_g^2 \\[1mm]
-B_{d,n} &= -\Gamma_d-B_{d,n-1}d_d \\[1mm]
-B_{g,n} &= -\Gamma_g-B_{d,n-1}d_g-B_{g,n-1}\phi_g
\end{aligned}
$$

where $A_0=B_{d,0}=B_{g,0}=0$.

A.2 Solving the Extended Model

A.2.1 Households with Epstein-Zin Preference

The savers' optimization problem is:

$$
\max \quad V(C_t, N_t) = \left\{ (1-\beta)U(C_t, N_t)^{1-\psi} + \beta E_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1-\psi}{1-\gamma}} \right\}^{\frac{1}{1-\psi}}
$$

$$
\text{s.t.} \quad E_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} P_{t+s} C_{t+s} \right]
$$

$$
\leq \quad E_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} (W_{t+s} P_{t+s} N_{t+s} - P_{t+s} T_{t+s} + P_{t+s} \Psi_{t+s}) \right],
$$

where

$$
C_t = \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}
$$

and

$$
U(C_t, N_t) = \left[\frac{C_t^{1-\psi}}{1-\psi} - \frac{N_t^{1+\omega}}{1+\omega} \right]^{\frac{1}{1-\psi}}.
$$

The first order conditions are:

$$
\frac{\partial V_t}{\partial C_t} : \frac{1}{1-\psi} \left[V_t^{1-\psi} \right]^{\frac{1}{1-\psi}-1} (1-\beta) C_t^{-\psi} - \lambda M_{t,t}^{\$} P_t = 0 \tag{A.17}
$$

$$
\frac{\partial V_t}{\partial N_t} : \frac{1}{1-\psi} \left[V_t^{1-\psi} \right]^{\frac{1}{1-\psi}-1} (1-\beta)(-N_t^{\omega}) + \lambda M_{t,t}^{\$} W_t P_t = 0 \tag{A.18}
$$

$$
\frac{\partial V_t}{\partial C_{t+1}} : \frac{1}{1-\psi} \left[V_t^{1-\psi} \right]^{\frac{1}{1-\psi}-1} \beta \left(\frac{1-\psi}{1-\gamma} \right) E_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1-\psi}{1-\gamma}-1}
$$

$$
(1-\gamma) V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial C_{t+1}} - \lambda M_{t,t+1}^{\$} P_{t+1} = 0. \tag{A.19}
$$

Furthermore,

$$
\frac{\partial V_{t+1}}{\partial C_{t+1}} = \frac{1}{1-\psi} \left[V_{t+1}^{1-\psi} \right]^{\frac{1}{1-\psi}-1} (1-\beta) C_{t+1}^{-\psi}. \tag{A.20}
$$

Combining (A.17) and (A.18), I have the household's intratemporal consumption and labor supply optimality condition:

$$\frac{\lambda(1-\psi)}{V_t^\psi(1-\beta)} = \frac{C_t^{-\psi}}{P_t} = \frac{N_t^\omega}{W_t P_t} \Rightarrow W_t = C_t^\psi N_t^\omega.$$

Finally, combining (A.17) ,(A.19) and (A.20), I obtain the intertemporal consumption optimality condition:

$$\frac{\lambda(1-\psi)}{V_t^\psi(1-\beta)} = \frac{C_t^{-\psi}}{P_t} = \beta\left(\frac{C_{t+1}^{-\psi}}{P_{t+1}}\right)\left(\frac{V_{t+1}^{\psi-\gamma}}{M_{t,t+1}^\$}\right) E_t\left[V_{t+1}^{\frac{1}{1-\gamma}}\right]^{\frac{\gamma-\psi}{1-\gamma}}.$$

To get the nominal pricing kernel, I solve for $M_{t,t+1}^\$$,

$$M_{t,t+1}^\$ = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\psi}\left(\frac{P_{t+1}}{P_t}\right)^{-1}\left[\frac{V_{t+1}}{E_t[V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right]^{\psi-\gamma}. \tag{A.21}$$

A.2.2 Monopolistic Producers and Price Rigidities

There is a dispersion of firms, denoted by j, with identical production technology in the economy. With nominal price stickiness and monopolistic competition, each firm is faced with the following optimization problem:

$$\max_{P_t^*} \; E_t\left[\sum_{s=0}^\infty \alpha^s M_{t,t+s}^\$ \left(P_t^*(\Pi^*)^s Y_{t+s|t}(j) - W_{t+s|t}(j)P_{t+s}N_{t+s|t}(j)\right)\right] \tag{A.22}$$

$$\text{s.t.} \quad Y_{t+s|t}(j) = Z_{t+s}N_{t+s|t}(j) \tag{A.23}$$

$$Y_{t+s|t}(j) = \left(\frac{P_t^*(\Pi^*)^s}{P_{t+s}}\right)^{-\theta} Y_{t+s} \tag{A.24}$$

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta}dj\right]^{\frac{1}{1-\theta}} = \left[(1-\alpha)P_t^{*1-\theta} + \alpha(P_{t-1}\Pi^*)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{A.25}$$

Using Calvo (1983) pricing, a firm can choose to optimally adjust price to P_t^* with probability $(1-\alpha)$ each period independent of the time elapsed between adjustments.

Furthermore, $t+s|t$ denotes the value in period $t+s$ given that the firm last adjusted price in period t. Π^* is the natural level of inflation that firms use to adjust their prices to from period to period if they cannot optimally set the price, and Z_t is the productivity shock on output. Log productivity is an exogenous AR(1) process such that

$$z_{t+1} = ln(Z_{t+1}) = \phi_z z_t + \sigma_z \epsilon_{z,t+1}.$$

The first order condition for firm j is:

$$E_t \left[\sum_{s=0}^{\infty} \alpha^s M_{t,t+s}^{\$} Y_{t+s|t}(j) \left(P_t^*(\Pi^*)^s - \nu P_{t+s} \frac{W_{t+s|t}(j)}{Z_{t+s}} \right) \right] = 0, \qquad (A.26)$$

where $\nu = \frac{\theta}{\theta-1}$ is the frictionless markup in the absence of price adjustment constraint. Utilizing (A.24) and the fact that $W_{t+s|t}(j) = W_{t+s}$, (A.26) can be rewritten as:

$$\left(\frac{P_t^*}{P_t} \right) F_t = \nu \frac{W_t}{Z_t} J_t$$

or after manipulating (A.25):

$$\left[\frac{1}{1-\alpha} \left(1 - \alpha \left(\frac{\Pi^*}{\Pi_t} \right)^{1-\theta} \right) \right]^{\frac{1}{1-\theta}} F_t = \nu \frac{W_t}{Z_t} J_t. \qquad (A.27)$$

F_t can be recursively expressed as:

$$
\begin{aligned}
F_t &= 1 + E_t \left[\sum_{s=1}^{\infty} (\alpha \Pi^*)^s M_{t,t+1}^{\$} M_{t+1,t+s}^{\$} \left(\frac{Y_{t+s}}{Y_{t+1}} \right) \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t \Pi^*}{P_{t+1}} \right)^{-\theta} \left(\frac{P_{t+1}(\Pi^*)^{s-1}}{P_{t+s}} \right)^{-\theta} \right] \\
&= 1 + \alpha \Pi^* E_t \left[M_{t,t+1}^{\$} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t \Pi^*}{P_{t+1}} \right)^{-\theta} \right. \\
&\qquad \left. E_{t+1} \left[\sum_{s=1}^{\infty} (\alpha \Pi^*)^{s-1} M_{t+1,t+s}^{\$} \left(\frac{Y_{t+s}}{Y_{t+1}} \right) \left(\frac{P_{t+1}(\Pi^*)^{s-1}}{P_{t+s}} \right)^{-\theta} \right] \right] \\
&= 1 + \alpha \Pi^* E_t \left[M_{t,t+1}^{\$} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{\Pi^*}{\Pi_{t+1}} \right)^{-\theta} F_{t+1} \right],
\end{aligned}
$$

Similarly, J_t has the following recursive formulation:

$$J_t = 1 + \alpha \Pi^* E_t \left[M_{t,t+1}^\$ \left(\frac{Z_t}{Z_{t+1}} \right) \left(\frac{W_{t+1}}{W_t} \right) \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{\Pi^*}{\Pi_{t+1}} \right)^{-1-\theta} J_{t+1} \right].$$

A.2.3 Flexible Output and Output Gap

The Taylor rule is a function of inflation as well as the output gap in the economy. In order to define the output gap, one needs to compute the fully flexible output when firms are always able to optimally adjust their prices every period, or when $\alpha = 0$.

In the absence of price rigidities in the economy, the first order condition for the firms is:

$$W_t^F(j) = W_t^F = \frac{1}{\nu} Z_t.$$

This can be written as:

$$w_t^F = log(\frac{1}{\nu}) + z_t.$$

Then, plug in (1.29) to get

$$\psi c_t^F + \omega n_t^F = log(\frac{1}{\nu}) + z_t.$$

Furthermore, the production function and market clearing condition yield:

$$e^{y^F} = e^{c^F} + e^g,$$
$$y^F = z + n^F.$$

The output gap can be formulated as the difference between output under price rigidities and the fully flexible output:

$$y = y^F + x,$$

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and the modified Taylor rule becomes $i_t = \imath + \rho_\pi \pi_t + \rho_x x_t + u_t$, where u_t is the monetary policy shock following an AR(1) process.

A.2.4 The System of Equations

The full model with long term bonds presented in this section has twenty endogenous variables, $\{\xi_t, v_t, c_t^o, c_t^r, c_t, c_t^F, \pi_t, w_t, n_t, n_t^F, \tau_t, y_t, y_t^F, x_t, s_t, d_{t-1}(t), h_t, f_t, j_t, i_t\}$. I have a system of twenty equations resulting from the first order conditions and policy rules:

$$e^{(1-\gamma)\xi_t} = E_t \left[e^{(1-\gamma)v_{t+1}} \right], \tag{A.28}$$

$$e^{(1-\psi)v_t} = (1-\beta) \left[\frac{e^{(1-\psi)c_t^o}}{1-\psi} - \frac{e^{(1+\omega)(w_t - \psi c_t^o)/\omega}}{1+\omega} \right] + \beta e^{(1-\psi)\xi_t}, \tag{A.29}$$

$$e^{-i_t} = \beta \left(\frac{e^{c_{t+1}^o}}{e^{c_t^o}} \right)^{-\psi} \left(\frac{1}{e^{\pi^* + \pi_{t+1}}} \right) \left(\frac{e^{v_{t+1}}}{e^{\xi_t}} \right)^{\psi - \gamma}, \tag{A.30}$$

$$w_t = \psi c_t + \omega n_t, \tag{A.31}$$

$$y_t = z_t + n_t, \tag{A.32}$$

$$e^{c_t^r} = e^{(1+1/\omega)w_t - (\psi/\omega)c_t^r} - e^{\tau_t}, \tag{A.33}$$

$$e^{y_t^F} = e^{c_t^F} + e^{g_t}, \tag{A.34}$$

$$y_t^F = z_t + n_t^F, \tag{A.35}$$

$$log \left(\frac{1}{\nu} \right) + z_t = \psi c_t^F + \omega n_t^F, \tag{A.36}$$

$$y_t = y_t^F + x_t, \tag{A.37}$$

$$e^{h_t} = \left(1 + \frac{\varphi}{1-\varphi} \right) \frac{e^{d_{t-1}}}{e^{\pi^* + \pi_t + s_t}} - \frac{\varphi}{1-\varphi}, \tag{A.38}$$

$$e^{h_t} = 1 + \beta E_t \left[\left(\frac{e^{c_{t+1}^o}}{e^{c_t^o}} \right)^{-\psi} \left(\frac{e^{v_{t+1}}}{e^{\xi_t}} \right)^{\psi - \gamma} \left(\frac{e^{s_{t+1}}}{e^{s_t}} \right) e^{h_{t+1}} \right], \tag{A.39}$$

$$\nu \frac{e^{w_t}}{e^{z_t}} e^{j_t} = \left[\frac{1}{1-\alpha} \left(1 - \alpha \left(\frac{1}{e^{\pi_t}} \right)^{1-\eta} \right) \right]^{\frac{1}{1-\eta}} e^{f_t}, \tag{A.40}$$

$$e^{f_t} = 1 + \alpha \beta E_t \left[\left(\frac{e^{c_{t+1}^o}}{e^{c_t^o}} \right)^{-\psi} \left(\frac{e^{v_{t+1}}}{e^{\xi_t}} \right)^{\psi - \gamma} \right.$$

$$\left. \times \left(\frac{e^{y_{t+1}}}{e^{y_t}} \right) \left(\frac{1}{e^{\pi_{t+1}}} \right)^{1-\eta} e^{f_{t+1}} \right], \tag{A.41}$$

$$e^{j_t} = 1 + \alpha \beta E_t \left[\left(\frac{e^{c_{t+1}^o}}{e^{c_t^o}} \right)^{-\psi} \left(\frac{e^{v_{t+1}}}{e^{\xi_t}} \right)^{\psi - \gamma} \left(\frac{e^{z_t}}{e^{z_{t+1}}} \right) \right.$$

$$\left. \times \left(\frac{e^{w_{t+1}}}{e^{w_t}} \right) \left(\frac{e^{y_{t+1}}}{e^{y_t}} \right) \left(\frac{1}{e^{\pi_{t+1}}} \right)^{-\eta} e^{j_{t+1}} \right], \tag{A.42}$$

$$e^{s_t} = e^{\tau_t} - e^{g_t}, \tag{A.43}$$

$$e^{y_t} = e^{c_t} + e^{g_t}, \tag{A.44}$$

$$i_t = \imath + \rho_\pi \pi_t + \rho_x x_t + u_t, \tag{A.45}$$

$$\tau_t = \rho_b d_{t-1}(t) + \rho_g g_t, \tag{A.46}$$

$$e^{c_t} = \mu e^{c_t^\tau} + (1-\mu) e^{c_t^o}, \tag{A.47}$$

where g_t, u_t and z_t are exogenous shocks to government spending, monetary policy and productivity, respectively:

$$
\begin{aligned}
g_{t+1} &= (1-\phi_g)\theta_g + \phi_g g_t + \sigma_{g,t} \epsilon_{g,t+1} \\
\sigma_{g,t+1}^2 &= (1-\phi_\sigma^g)\theta_\sigma^g + \phi_\sigma^g \sigma_{g,t}^2 + \sigma_\sigma^g \epsilon_{\sigma,t+1} \\
u_{t+1} &= \phi_u u_t + \sigma_u \epsilon_{u,t+1} \\
z_{t+1} &= \phi_z z_t + \sigma_z \epsilon_{z,t+1}.
\end{aligned}
$$

A.3   Figures
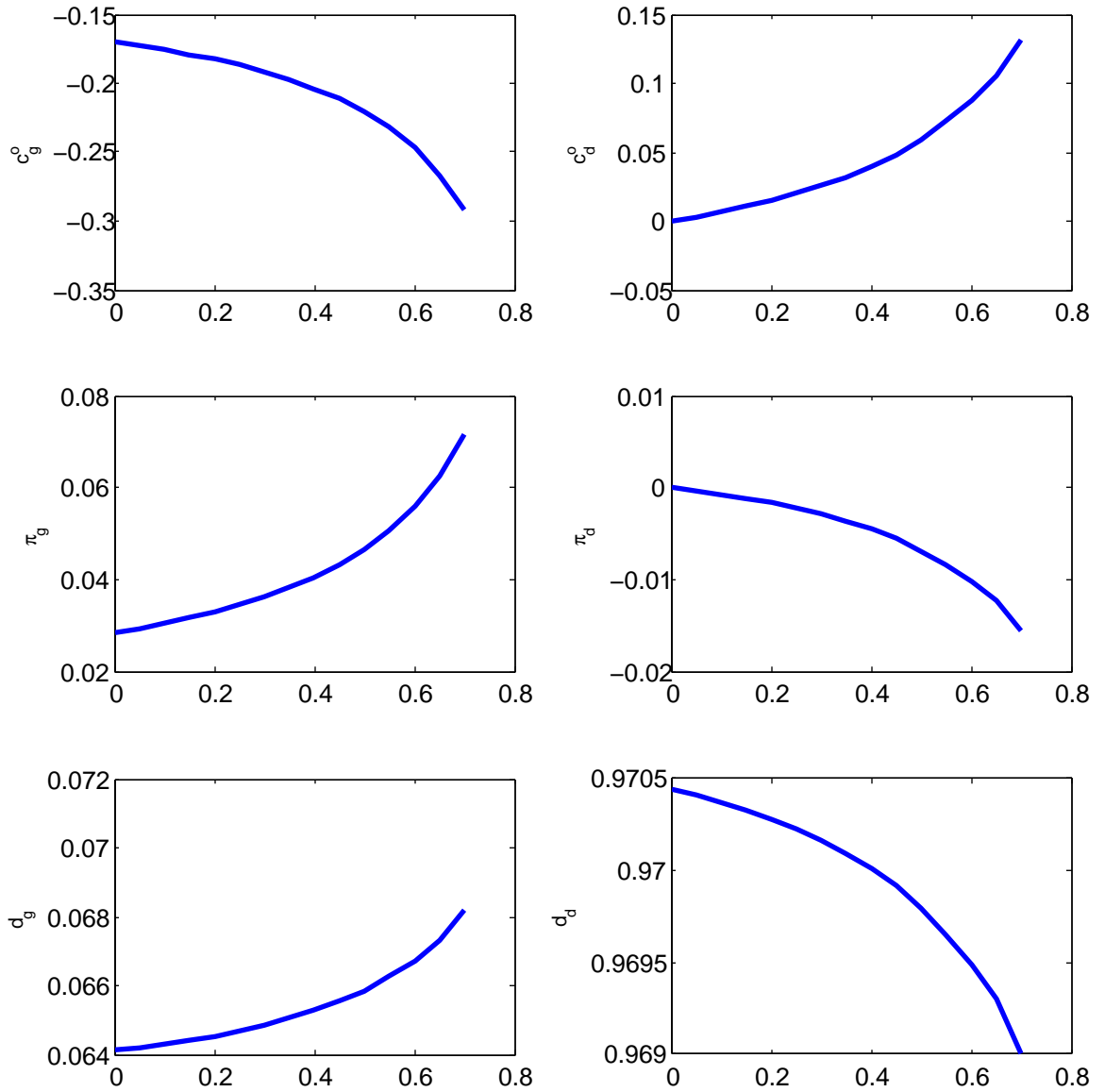
Figure A.1:
Comparative statics for μ, the percentage of spenders in the economy, from the simple model. The coefficient loadings for consumption of the savers, inflation, and short-term real debt are shown, in order from top to bottom. The left column is on the government spending shock, and the right column is on the maturing real debt level.
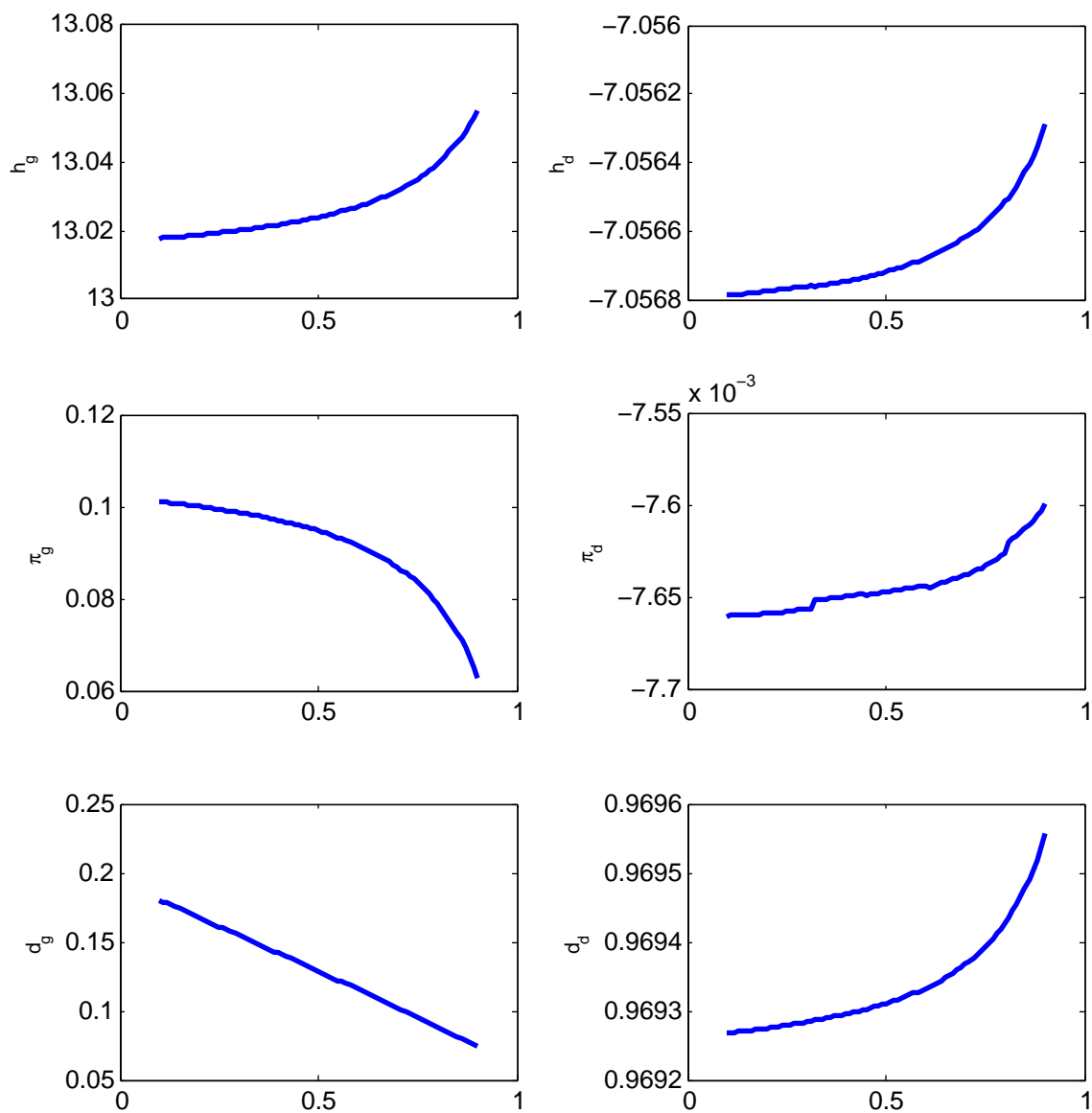
Figure A.2: Comparative statics for ϕ_g, the autoregressive coefficient for government spending shocks, from the simple model. The coefficient loadings for the present value of real surplus ratios, inflation, and short-term real debt are shown, in order from top to bottom. The left column is on the government spending shock, and the right column is on the maturing real debt level.
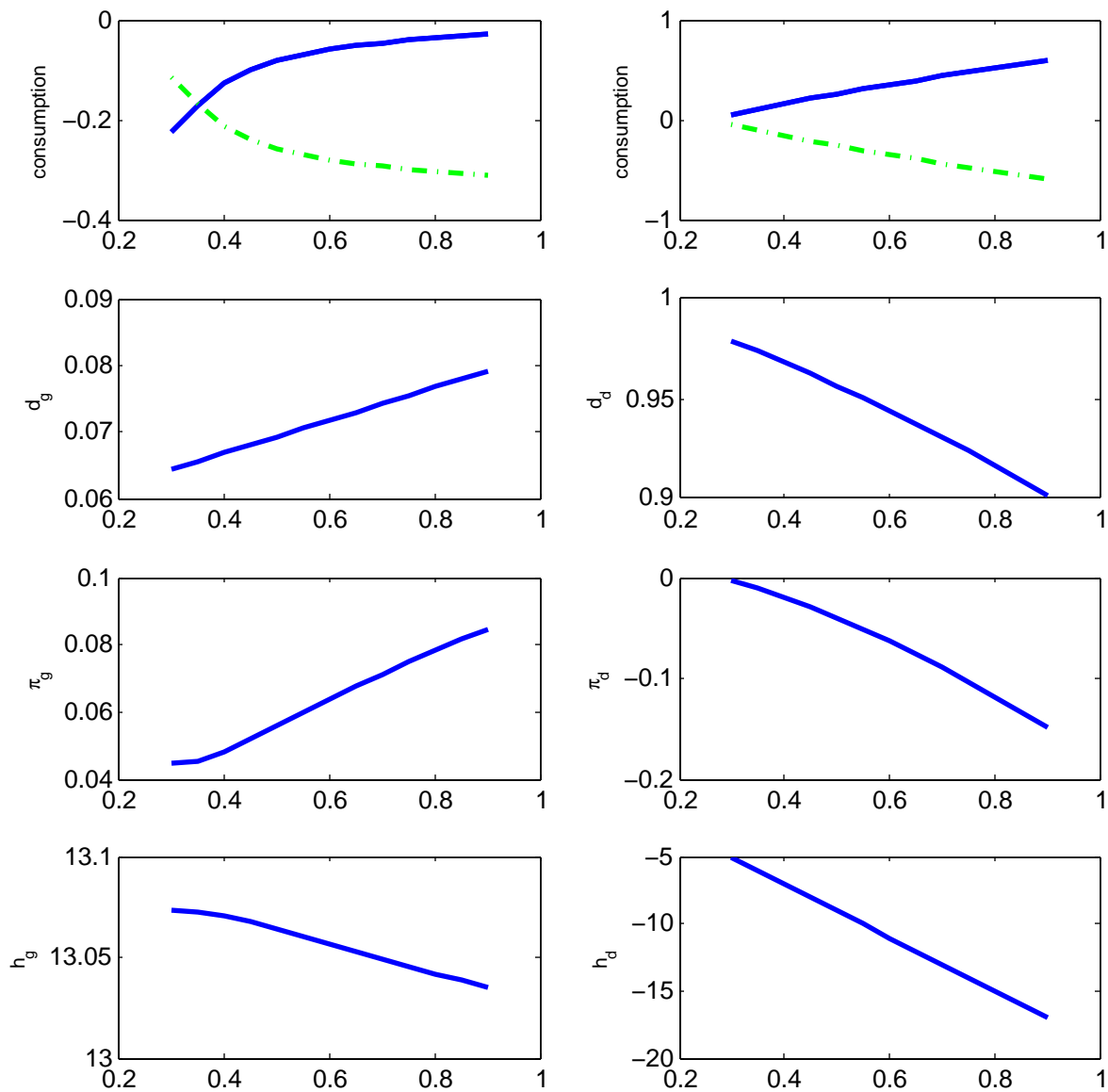
Figure A.3: Comparative statics for ρ_b, the fiscal policy response to maturing real debt, from the simple model. The coefficient loadings for consumption, short-term real debt, inflation, and the present value of real surplus ratios are shown, in order from top to bottom. The left column is on the government spending shock, and the right column is on the maturing real debt level. For consumption, the solid line denotes the comparative statics for the savers, and the dashed line denotes that for the spenders.
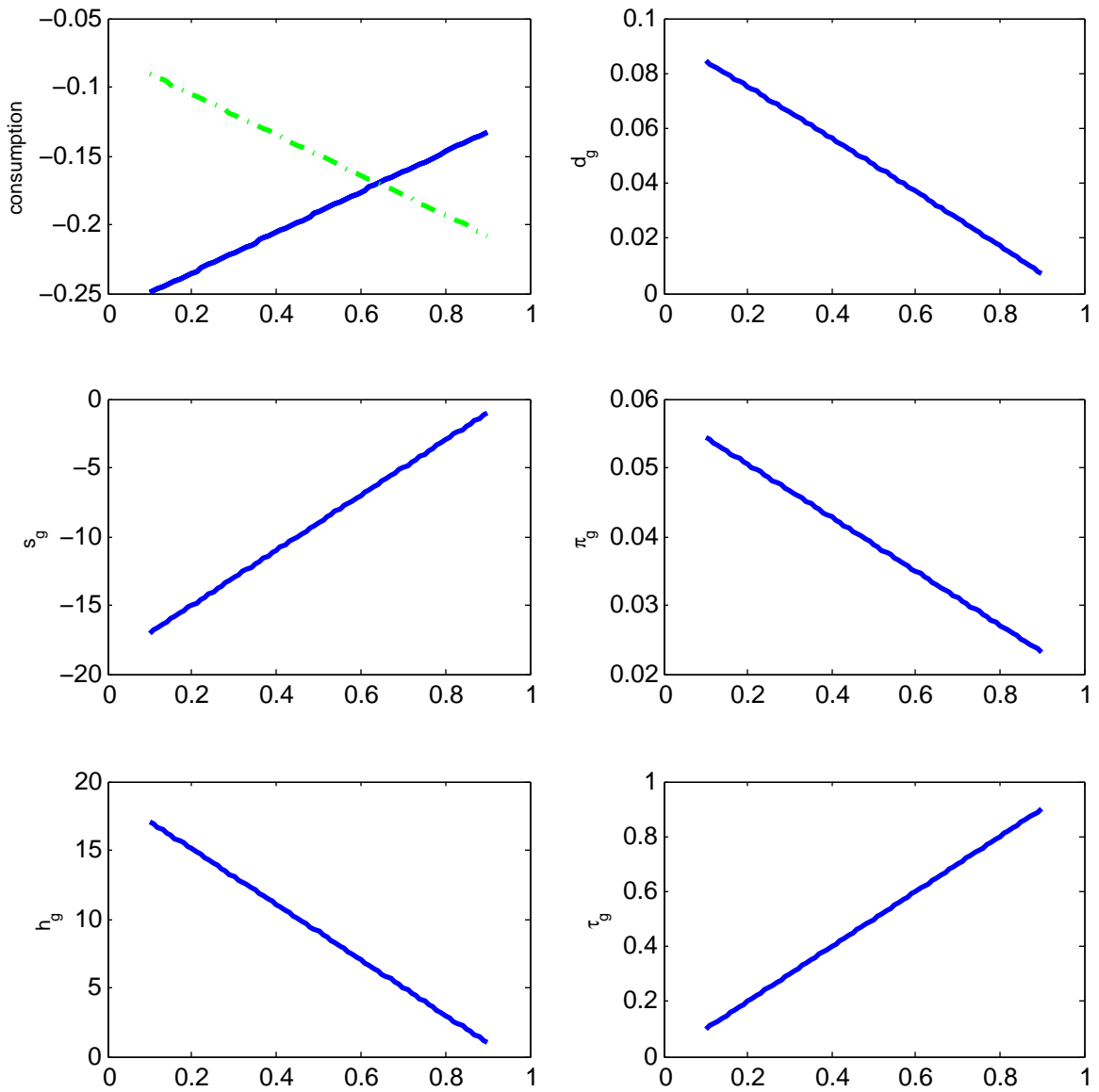
Figure A.4: Comparative statics for ρ_g, the fiscal policy response to government spending shocks, from the simple model. The coefficient loadings for consumption, short-term real debt, primary surplus, inflation, the present value of real surplus ratios, and taxes are shown. For consumption, the solid line denotes the comparative statics for the savers, and the dashed line denotes that for the spenders.
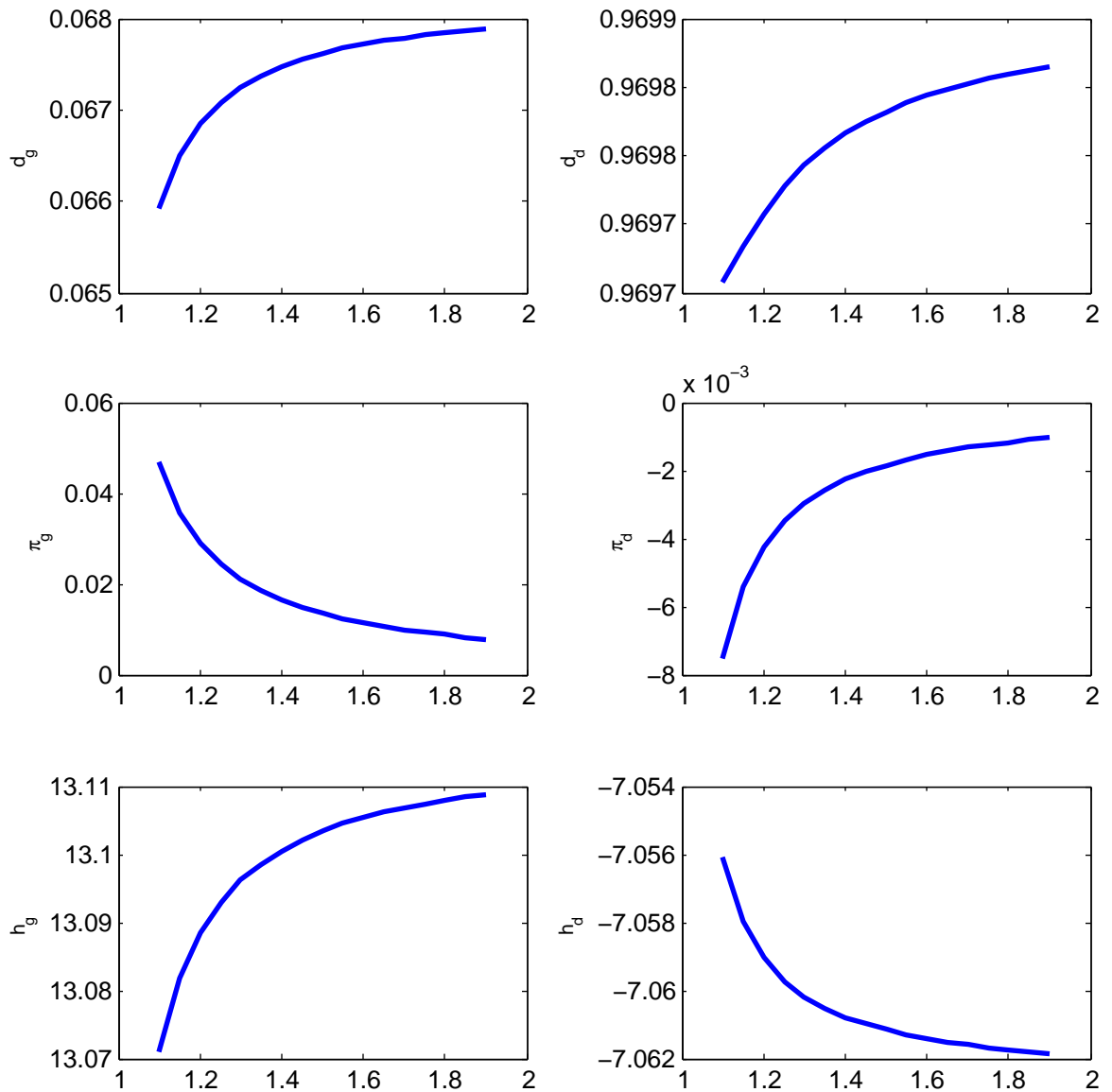
Figure A.5: Comparative statics for ρ_π, the monetary policy response to inflation, from the simple model. The coefficient loadings for short-term real debt, inflation, and the present value of real surplus ratios are shown, in order from top to bottom. The left column is on the government spending shock, and the right column is on the maturing real debt level.
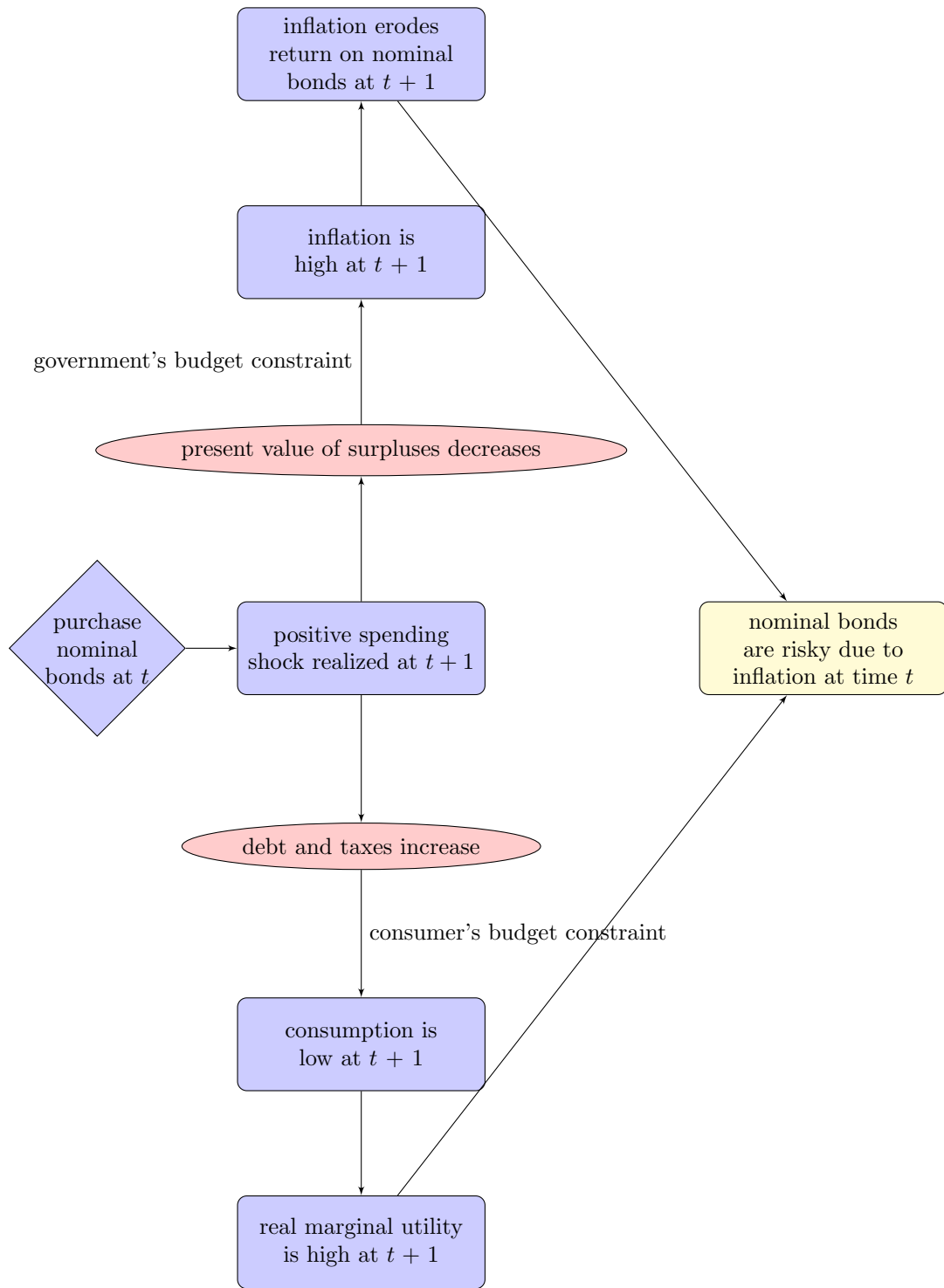
Figure A.6: Mechanism of Generating Positive Inflation Risk Premium
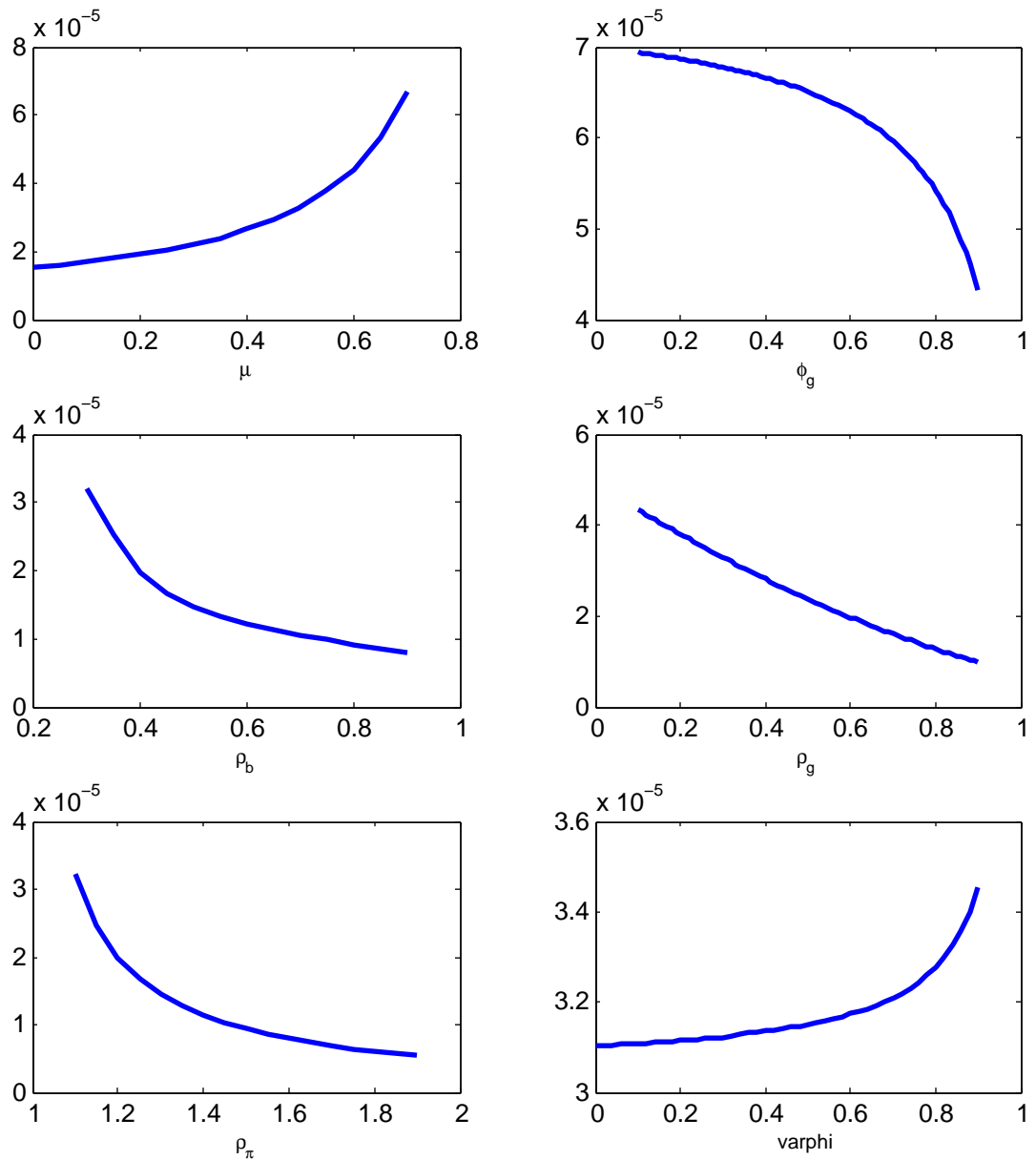
Figure A.7: Comparative statics for inflation risk premium from the simple model. The responses of unconditional inflation risk premium are shown by varying the percentage of spenders (μ), the autoregressive coefficient on spending shocks (ϕ_g), the fiscal policy response to maturing real debt (ρ_b), the fiscal policy response to government spending shocks (ρ_g), the monetary policy response to inflation (ρ_π), and the maturity structure parameter (φ).
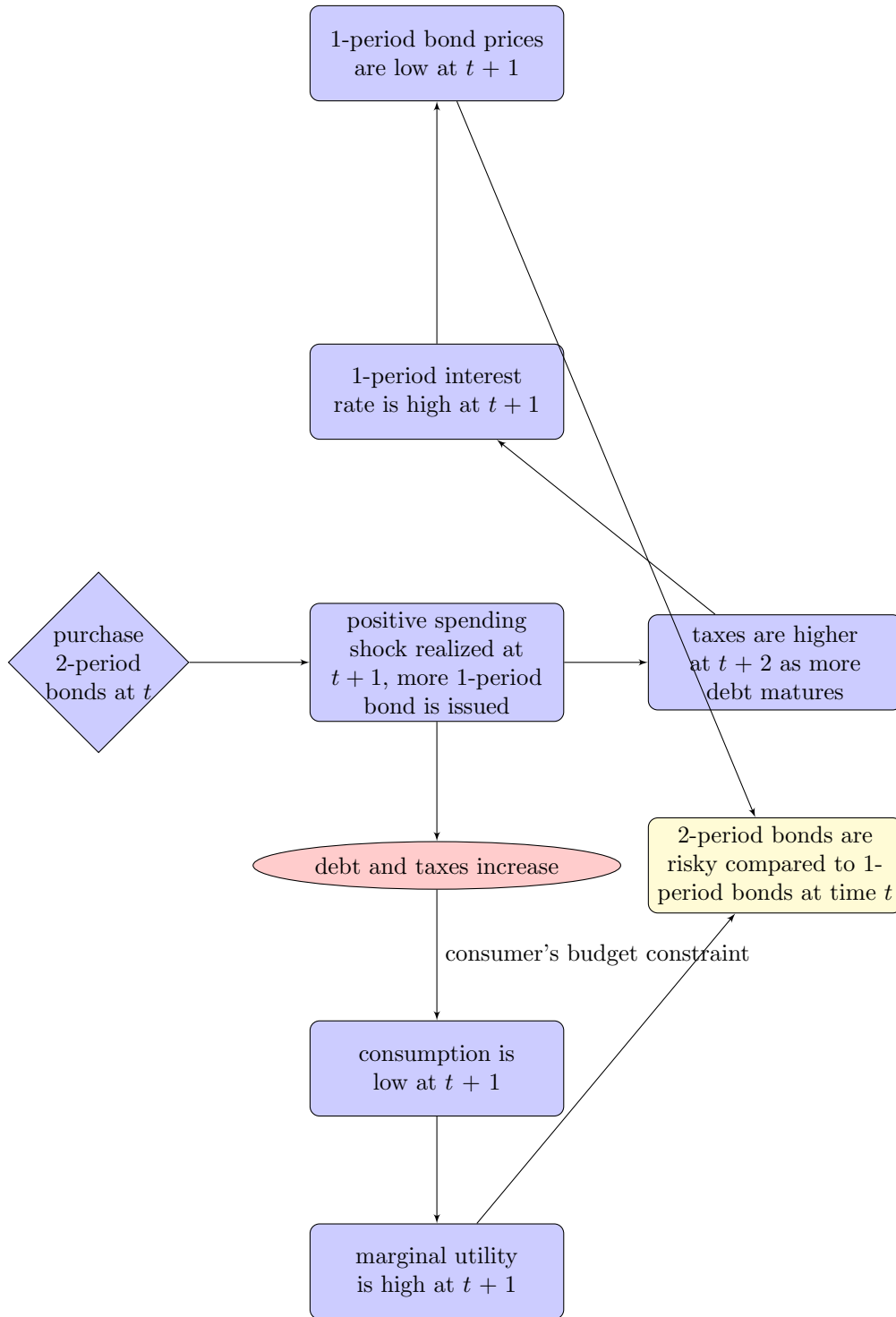
Figure A.8: Mechanism of Generating Positive Term Premium. 2-period bonds at time t become 1-period bonds at time $t + 1$. 1-period bonds at time t mature at time $t + 1$ and pay off face value without uncertainty.
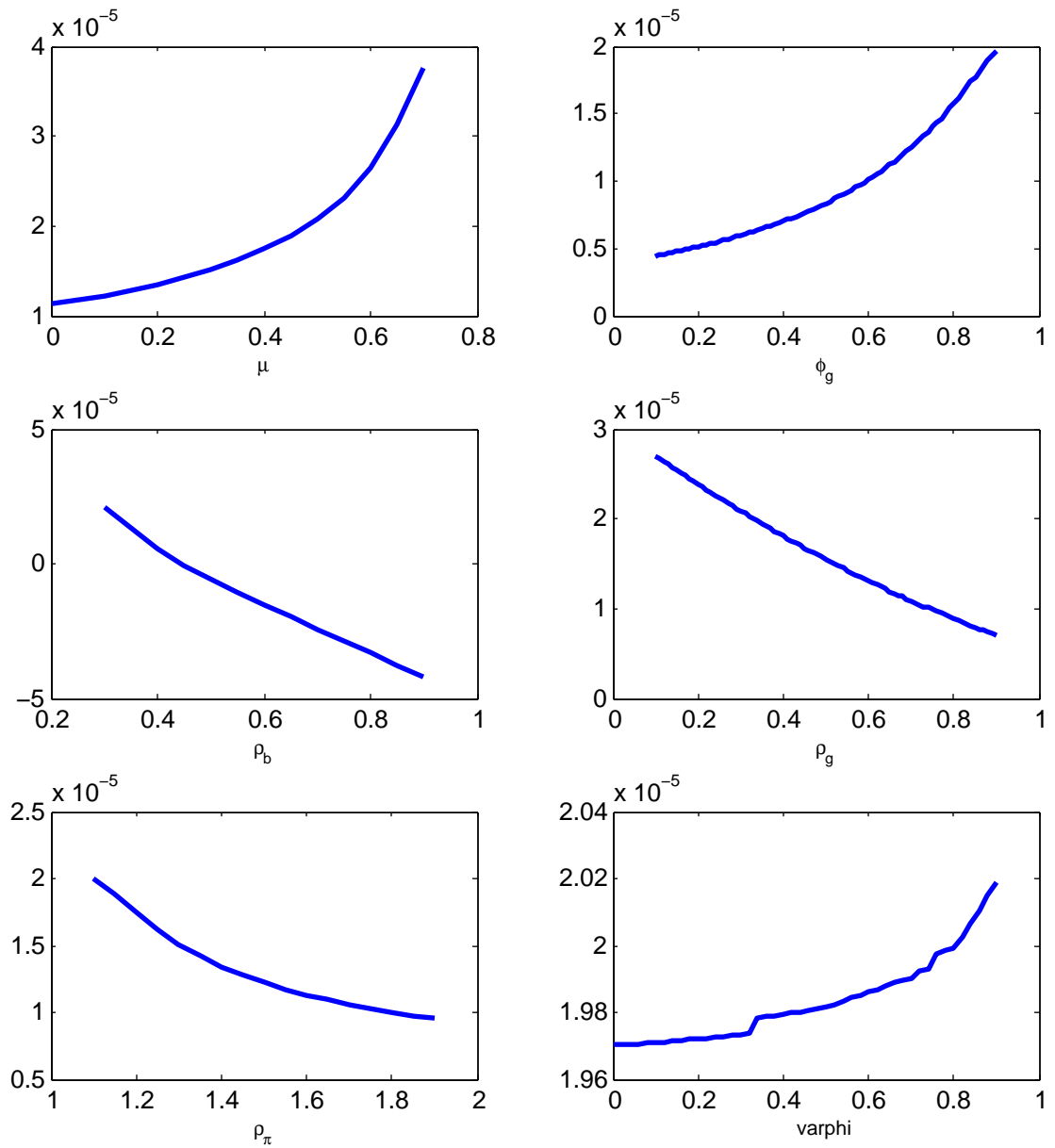
Figure A.9: Comparative statics for 2-period term premium from the simple model. The responses of unconditional term premium are shown by varying the percentage of spenders (μ), the autoregressive coefficient on spending shocks (ϕ_g), the fiscal policy response to maturing real debt (ρ_b), the fiscal policy response to government spending shocks (ρ_g), the monetary policy response to inflation (ρ_π), and the maturity structure parameter (φ).
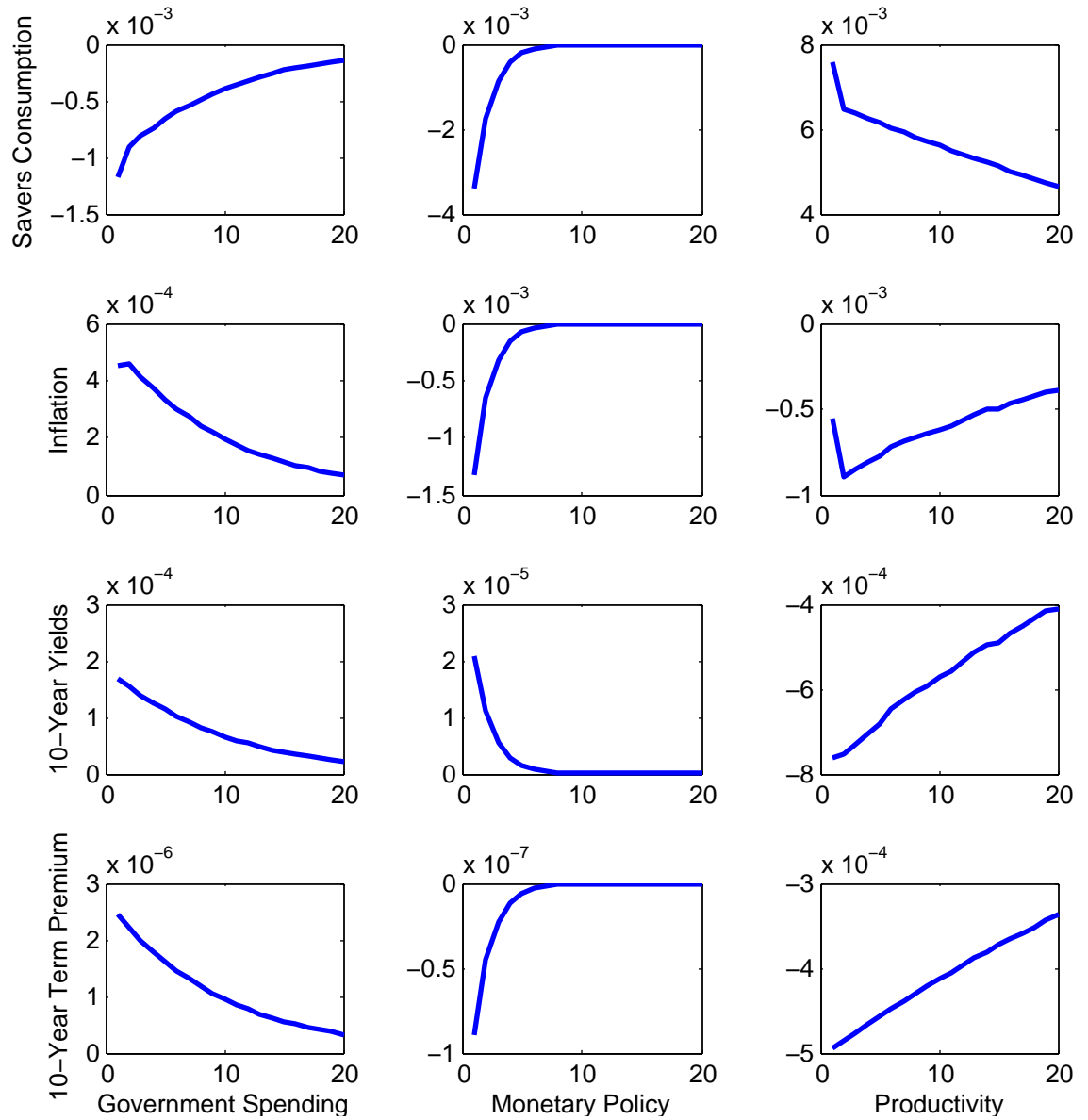
Figure A.10:
Impulse responses of endogenous macroeconomic and financial variables following one standard deviation structural shocks from the full model with Epstein-Zin preferences, price rigidities, and stochastic volatility in government spending shocks. The x-axis is the number of periods after the initial shock, and the y-axis is the deviation from its equilibrium staedy state.

A.4 Tables

Parameter	Description	Value
β	Subjective discount factor	0.995
ψ	Inverse of EIS	0.633
γ	Coefficient of risk aversion	32
ω	Inverse of labor supply elasticity	0.2
μ	Percentage of spenders in the economy	0.5
α	Price rigidity parameter	0.66
η	Price markup parameter	-0.9
$\imath$	Constant in Taylor rule	0.005
ρ_π	Taylor rule coefficient on inflation	1.25
ρ_x	Taylor rule coefficient on output gap	0.3
π^*	Inflation target	0.0022
φ	Maturity structure parameter	0.58
ρ_b	Fiscal policy rule coefficient on debt	0.33
ρ_g	Fiscal policy rule coefficient on spending	0.1
ϕ_g	Autocorrelation of spending shocks	0.9
θ_g	Unconditional mean of government spending	0.1
ϕ_z	Autocorrelation of productivity shocks	0.98
σ_z	Conditional volatility of productivity shocks	0.005
ϕ_u	Autocorrelation of monetary policy shocks	0.5
σ_u	Conditional volatility of monetary policy shocks	0.003
ϕ_σ^g	Autocorrelation of volatility on g_t	0.8
θ_σ^g	Unconditional mean of volatility on g_t	0.0092
σ_σ^g	Conditional volatility of volatility on g_t	0.00026

Table A.1: **Calibrated Parameter Values**

	(1)	(2)
	Data from 1971 to 2010	Benchmark
$\mathbb{E}[\frac{C}{Y}]$	66.03	66.78
$\sigma(\frac{C}{Y})$	2.93	2.62
$\mathbb{E}[\pi]$	3.84	2.23
$\sigma(\pi)$	2.56	1.44
$\mathbb{E}[i_t^{(4)}]$	6.28	4.44
$\sigma(i_t^{(4)})$	3.12	1.65
$\mathbb{E}[i_t^{(8)}]$	6.52	4.44
$\sigma(i_t^{(8)})$	3.04	1.41

Table A.2: **Model Calibration**

	U.S. Data	RS	RS/LRR	Hsu	Hsu/SV
$\mathbb{E}[TP^{(40)}]$	1.06	1.12	1.00	1.31	1.04
sd$[TP^{(40)}]$	0.54	0.47	0.29	0.085	0.43
memo:					
1/EIS		9.09	11.11	0.633	0.633
Risk Aversion		110	110	32	32
1/Frisch		3.57	3.57	0.2	0.2

Table A.3: **Model comparisons with Rudebusch and Swanson (2010) (RS).** TP is term premium taken from Kim and Wright (2005) and rx is 1-quarter holding period excess returns. LRR and SV denote long run risk and stochastic volatility, respectively. Expressed in annualized percentage basis. Sample period is 1961 to 2007.

What Do Nominal Rigidities and Monetary Policy Tell Us about the Real Yield Curve?

B.1 Household's Utility Maximization under Wage Rigidities (from *Li and Palomino* (2012))

The household's problem is

$$\max_{\{C_t, N_t^s, W_t^*\}} \quad V_t = U_t + \beta Q_t^{\frac{1-\psi}{1-\gamma}}$$

where

$$U_t = \frac{C_t^{1-\psi}}{1-\psi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega}, \quad \text{and} \quad Q_t = \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\psi}} \right],$$

subject to the budget constraint

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} C_{t+\tau} \right] \leq \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} \left(LI_{t+\tau} + D_{t+\tau} \right) \right],$$

where LI_t and D_t are aggregate labor income and firm profits, respectively. The Lagrangian associated with this problem is

$$\mathcal{L} = \frac{C_t^{1-\psi}}{1-\psi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega} + \beta Q_t^{\frac{1-\psi}{1-\gamma}} + \lambda \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} \left(LI_{t+\tau} + D_{t+\tau} - C_{t+\tau} \right) \right].$$

It can be shown that utility maximization implies $\lambda = \frac{C_t^{-\psi}}{P_t}$, and

$$
\begin{aligned}
M_{t,t+1}^{\$} &= \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} \frac{P_t}{P_{t+1}} = \beta \frac{\frac{\partial Q_t}{\partial C_{t+1}} \frac{\partial V_t}{\partial Q_t}}{C_t^{-\psi}} \frac{P_t}{P_{t+1}} \\
&= \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \left(\frac{V_{t+1}^{1/(1-\psi)}}{Q_t^{1/(1-\gamma)}} \right)^{\psi-\gamma} \frac{P_t}{P_{t+1}}.
\end{aligned}
$$

The τ-period nominal pricing kernel is

$$M_{t,t+\tau}^{\$} = \prod_{s=1}^{\tau} M_{t,t+s}^{\$}.$$

The household cannot change wages for α_w fraction of labor types. For the remaining $1 - \alpha_w$ fraction of labor types k, the household chooses wages $W_t^*(k)$ to maximize V_t. We assume that the wage choice for one labor type has negligible effects on the aggregate wage index and the aggregate labor demand. To see the impact of $W_t^*(k)$ on the household's utility, we rewrite the labor supply at $t + \tau$ as

$$N_{t+\tau}^s = \int_0^1 N_{t+\tau}^s(k)\, dk = N_{t+\tau}^d \int_0^1 \left(\frac{W_{t+\tau}(k)}{W_{t+\tau}} \right)^{-\theta_w} dk,$$

and the aggregate labor income at $t + \tau$ as

$$LI_{t+\tau} = \int_0^1 \frac{W_{t+\tau}}{P_{t+\tau}}(k) N_{t+\tau}^s(k)\, dk = \frac{N_{t+\tau}^d W_{t+\tau}}{P_{t+\tau}} \int_0^1 \left(\frac{W_{t+\tau}(k)}{W_{t+\tau}} \right)^{1-\theta_w} dk.$$

For the wage of type k labor at $t + \tau$, there are $\tau + 2$ possible values:

$$W_{t+\tau}(k) = \begin{cases} W^*_{t+\tau-s}(k), & \text{with prob} = (1 - \alpha_w)\alpha_w^s \text{ for } s = 0, 1, \cdots, \tau \\[2mm] W_{t-1}\Lambda_{w,t-1,t+\tau}, & \text{with prob} = \alpha_w^{\tau+1}. \end{cases}$$

We obtain derivatives

$$\frac{\partial N^s_{t+\tau}}{W^*_t(k)} = N^d_{t+\tau}(1 - \alpha_w)\alpha_w^\tau \left(\frac{-\theta_w}{W^*_t(k)}\right) \left(\frac{W^*_t(k)\Lambda_{w,t,t+\tau}}{W_{t+\tau}}\right)^{-\theta_w},$$

$$\frac{\partial LI_{t+\tau}}{\partial W^*_t(k)} = \frac{N^d_{t+\tau}}{P_{t+\tau}}(1 - \alpha_w)\alpha_w^\tau(1 - \theta_w) \left(\frac{W^*_t(k)\Lambda_{w,t,t+\tau}}{W_{t+\tau}}\right)^{-\theta_w}.$$

The first-order condition of the Lagrangian with respect to $W^*_t(k)$ is given by

$$\frac{\partial \mathcal{L}}{\partial W^*_t(k)} = \frac{\partial V_t}{\partial W^*_t(k)} + \lambda \mathbb{E}_t \left[\sum_{\tau=0}^\infty M^\$_{t,t+\tau} P_{t+\tau} \frac{\partial LI_{t+\tau}}{\partial W^*_t(k)}\right] = 0,$$

where

$$\frac{\partial V_t}{\partial W^*_t(k)} = -\mathbb{E}_t \left[\sum_{\tau=0}^\infty M^\$_{t,t+\tau} \frac{P_{t+\tau}}{P_t} \left(\frac{C_{t+\tau}}{C_t}\right)^\psi \kappa_{t+\tau}(N^s_{t+\tau})^\omega \frac{\partial N^s_{t+\tau}}{\partial W^*_t(k)}\right].$$

Rearranging terms, we get

$$\mathbb{E}_t \left[\sum_{\tau=0}^\infty M^\$_{t,t+\tau} \Lambda_{w,t,t+\tau} \alpha_w^\tau W_{t+\tau}^{\theta_w} N^d_{t+\tau} \frac{W^*_t(k)}{P_t} C_t^{-\psi}\right]$$

$$= \mathbb{E}_t \left[\sum_{\tau=0}^\infty M^\$_{t,t+\tau} \Lambda_{w,t,t+\tau} \alpha_w^\tau \left(\frac{P_{t+\tau}}{P_t}\right) W_{t+\tau}^{\theta_w} N^d_{t+\tau} \mu_w \kappa_{t+\tau}(N^s_{t+\tau})^\omega \left(\frac{C_{t+\tau}}{C_t}\right)^\psi\right].$$

Since all labor types face the same demand curve, we have $W^*_t(k) = W^*_t$ for all k.

We can write the left-hand side of the equation as

$$LHS = C_t^{-\psi} W_t^{\theta_w} N_t^d H_{w,t} \frac{W^*_t}{P_t},$$

where

$$H_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1} \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} H_{w,t+1} \right].$$

Similarly, the right-hand side of the first-order condition can be written as

$$RHS = \mu_w W_t^{\theta_w} N_t^d (N_t^s)^\omega G_{w,t} = \mu_w W_t^{\theta_w} N_t^d \kappa_t (N_t^s)^\omega G_{w,t}$$

where

$$\begin{aligned} G_{w,t} &= 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{C_{t+1}}{C_t} \right)^\psi \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\frac{\kappa_{t+1}}{\kappa_t} \right) \right. \\ &\quad \times \left. \left(\frac{N_{t+1}^s}{N_t^s} \right)^\omega \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} G_{w,t+1} \right]. \end{aligned}$$

The optimal real wage and the optimal wage markup $\mu_{w,t}$ are then given by

$$\frac{W_t^*}{P_t} = \mu_{w,t} C_t^\psi \kappa_t (N_t^s)^\omega \quad \text{and} \quad \mu_{w,t} = \mu_w \frac{G_{w,t}}{H_{w,t}}.$$

B.2 Profit Maximization under Price Rigidities (from *Li and Palomino* (2012))

Consider the Dixit-Stiglitz aggregate (2.2) as a production function, and a competitive "producer" of a differentiated good facing the problem

$$\max_{\{C_t(j)\}} P_t C_t - \int_0^1 P_t(j) C_t(j) dj$$

subject to (2.2). Solving the problem, we find the demand function

$$P_t(j) = P_t \left(\frac{C_t(j)}{C_t} \right)^{-1/\theta} \tag{B.1}$$

The zero-profit condition implies

$$P_t C_t = \int_0^1 P_t(j) C_t(j) dj = \int_0^1 P_t C_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta} dj.$$

Solving for P_t, it follows that

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \tag{B.2}$$

which can be written as the demand function for each differentiated good

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} C_t. \tag{B.3}$$

Therefore, when prices are flexible, prices of all differentiated goods are the same.

The profit maximization problem is

$$\max_{\{P_t(j)\}} \quad \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} \alpha^\tau \left[\Lambda_{p,t,t+\tau} P_t(j) Y_{t+\tau|t}(j) - W_{t+\tau|t}(j) N_{t+\tau|t}^d(j) \right] \right]$$

subject to

$$Y_{t+\tau|t}(j) = Y_{t+\tau} \left(\frac{P_t(j)}{P_{t+\tau}} \right)^{-\theta}, \quad \text{and} \quad Y_{t+\tau|t}(j) = A_t N_{t+\tau|t}^d(j).$$

The first-order condition of this problem with respect to $P_t(j)$ is

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} \alpha^\tau Y_{t+\tau|t}(j) \Lambda_{p,t,t+\tau} P_t^*(j) \right] = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} \alpha_I^\tau Y_{t+\tau|t}(j) \mu \frac{W_{t+\tau|t}(j)}{A_{t+\tau}} \right].$$

The left-hand side (LHS) of the equation can be written recursively as

$$LHS = P_t^* \left(\frac{P_t^*}{P_t} \right)^{-\theta} Y_t H_t,$$

where

$$H_t = 1 + \alpha \mathbb{E}_t \left[M^{\$}_{t,t+1} \Lambda^{1-\theta}_{p,t,t+\tau} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t}{P_{t+1}} \right)^{-\theta} H_{t+1} \right].$$

Similarly, the right-hand side (RHS) of the equation can be written as

$$RHS = \frac{\mu}{A_t} Y_t \left(\frac{P^*_t}{P_{I,t}} \right)^{-\theta} \frac{W_t}{P_t} P_t G_t$$

where

$$G_t = 1 + \alpha \mathbb{E}_t \left[M^{\$}_{t,t+1} \Lambda^{-\theta}_{p,t,t+\tau} \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t}{P_{t+1}} \right)^{-\theta} \left(\frac{W_{t+1}}{W_t} \right) \left(\frac{A_t}{A_{t+1}} \right) G_{t+1} \right].$$

The optimal price is hence given by

$$\left(\frac{P^*_t}{P_t} \right) H_t = \frac{\mu}{A_t} \frac{W_t}{P_t} G_t.$$

Here, $P^*_t(j) = P^*_t$ because all firms changing prices face the same demand curve and hence the same optimization problem. Based on the definition of markup, the optimal time-varying product markup is given by

$$\mu_t = \mu \frac{G_t}{H_t} \quad \text{and} \quad P^*_t = \mu_t \frac{W_t}{A_t}.$$

Price inflation is given by

$$1 = (1-\alpha) \left(\frac{P^*_t}{P_t} \right)^{1-\theta} + \alpha \Lambda^{1-\theta}_{p,t-1,t} \left(\frac{P_{I,t+1}}{P_{I,t}} \right)^{-(1-\theta)}.$$

B.3 Equilibrium Conditions

This appendix provides a summary of the equilibrium equations for the model. These conditions need to be expressed in terms of de-trended variables. In order to obtain balanced growth, we make $\kappa_t = (A^p_t)^{1-\psi}$. This condition ensures that Y_t, W_t,

and W_t^* are growing at the same rate. Therefore, the equations can be written in terms of $\hat{Y}_t = \frac{Y_t}{A_t^p}$, $\hat{W}_t = \frac{W_t}{A_t^p}$, and $\hat{W}_t^* = \frac{W_t^*}{A_t^p}$.

Wage Setting

$$
\frac{W_t^*}{P_t} = \mu_w \kappa_t \left(N_t^s\right)^\omega C_t^\psi \frac{G_{w,t}}{H_{w,t}}.
$$

$$
H_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^\$ \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{N_{t+1}^d}{N_t^d}\right) \left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w} H_{w,t+1} \right],
$$

$$
\text{and} \quad G_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^\$ \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{P_{t+1}}{P_t}\right) \left(\frac{C_{t+1}}{C_t}\right)^\psi \left(\frac{N_{t+1}^d}{N_t^d}\right) \left(\frac{\kappa_{t+1}}{\kappa_t}\right) \right.
$$
$$
\left. \times \left(\frac{N_{t+1}^s}{N_t^s}\right)^\omega \left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w} G_{w,t+1} \right].
$$

Price Dispersion

$$
F_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} dj = (1-\alpha) \left(\frac{P_t^*}{P_t}\right)^{-\theta} + \alpha \Lambda_{p,t-1,t}^{-\theta} \left(\frac{P_{I,t-1}}{P_{I,t}}\right)^{-\theta} F_{I,t-1}.
$$

Wage Dispersion

$$
F_{w,t} = \int_0^1 \left(\frac{W_t(k)}{W_t}\right)^{-\theta_w} dk = (1-\alpha_w) \left(\frac{W_t^*}{W_t}\right)^{-\theta_w} + \alpha_w \Lambda_{w,t-1,t}^{-\theta_w} \left(\frac{W_{t-1}}{W_t}\right)^{-\theta_w} F_{w,t-1}.
$$

Wage Aggregator

$$
\left(\frac{W_t}{P_t}\right)^{1-\theta_w} = \int_0^1 \left(\frac{W_t(k)}{P_t}\right)^{1-\theta_w} dk
$$
$$
= (1-\alpha_w) \left(\frac{W_t^*}{P_t}\right)^{1-\theta_w} + \alpha_w \Lambda_{w,t-1,t}^{1-\theta_w} \left(\frac{P_{t-1}}{P_t}\right)^{1-\theta_w} \left(\frac{W_{t-1}}{P_{t-1}}\right)^{1-\theta_w},
$$

Price Setting

$$\left(\frac{P_t^*}{P_t}\right) H_t = \frac{\mu}{A_t} \frac{W_t}{P_t} G_t,$$

$$H_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{p,t,t+1}^{1-\theta} \left(\frac{Y_{t+1}}{Y_t}\right) \left(\frac{P_{t+1}}{P_t}\right)^{-\theta} H_{t+1} \right],$$

$$\text{and} \quad G_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{p,t,t+1}^{-\theta} \left(\frac{Y_{t+1}}{Y_t}\right) \left(\frac{P_t}{P_{t+1}}\right)^{-\theta} \left(\frac{W_{t+1}}{W_t}\right) \left(\frac{A_t}{A_{t+1}}\right) G_{t+1} \right].$$

Price Aggregator

$$1 = (1-\alpha) \left(\frac{P_t^*}{P_t}\right)^{1-\theta} + \alpha \Lambda_{p,t-1,t}^{1-\theta} \left(\frac{P_{t-1}}{P_t}\right)^{1-\theta}.$$

Aggregate Labor Supply and Demand

$$N_t^s = F_{w,t} N_t^d, \qquad N_t^d = \frac{Y_t}{A_t} F_t.$$

Markup

$$\mu_t = \frac{Y_t}{LI_t} = \frac{A_t}{F_t} \left(\frac{W_t}{P_t}\right)^{-1}.$$

Pricing Kernel

$$M_{t,t+1} = \left[\beta \left(\frac{Y_{t+1}}{Y_t}\right)^{-\psi} \right]^{\frac{1-\gamma}{1-\psi}} \left(\frac{1}{R_{YL,t+1}}\right)^{1-\frac{1-\gamma}{1-\psi}},$$

$$R_{YL,t+1} = (1-\nu_t) R_{C,t+1} + \nu_t R_{LI^*,t+1},$$

$$R_{Y,t+1} = \frac{C_{t+1} + S_{Y,t+1}}{S_{Y,t}}, \qquad R_{LI^*,t+1} = \frac{LI_{t+1}^* + S_{LI^*,t+1}}{S_{LI^*,t}},$$

$$\nu_t = \frac{\bar{\nu} S_{LI^*,t}}{\bar{\nu} S_{LI^*,t} - S_{Y,t}}.$$

Real and Nominal Bond Yields

$$\exp\left(-nr_t^{(n)}\right) = \mathbb{E}_t\left[M_{t,t+1}\exp\left(-(n-1)r_{t+1}^{(n-1)}\right)\right],$$

$$\exp\left(-ni_t^{(n)}\right) = \mathbb{E}_t\left[M_{t,t+1}^{\$}\exp\left(-(n-1)i_{t+1}^{(n-1)}\right)\right],$$

Indexation

$$\Lambda_{p,t,t+1} = \Pi_t^*,$$

$$\Lambda_{w,t,t+1} = e_a^g\Pi_t^*,$$

B.4   Figures
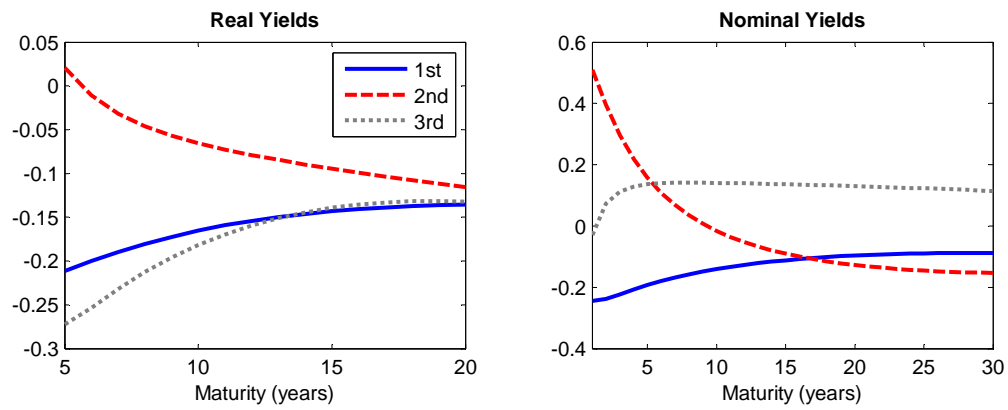
Figure B.1:
Loadings on the first three principal components for U.S. Government TIPs and Nominal Bond Yields. 1999 - 2008
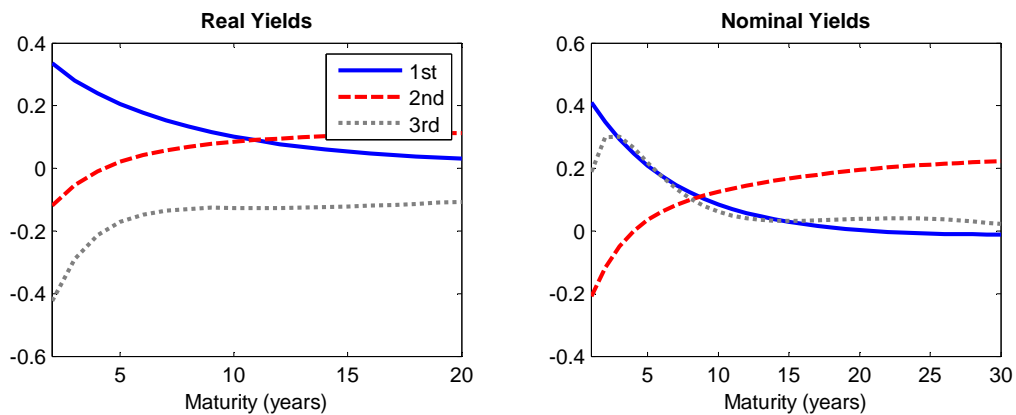
Figure B.2:
Loadings on the first three principal components for U.S. Government TIPs and Nominal Bond Yields. 2004 - 2008
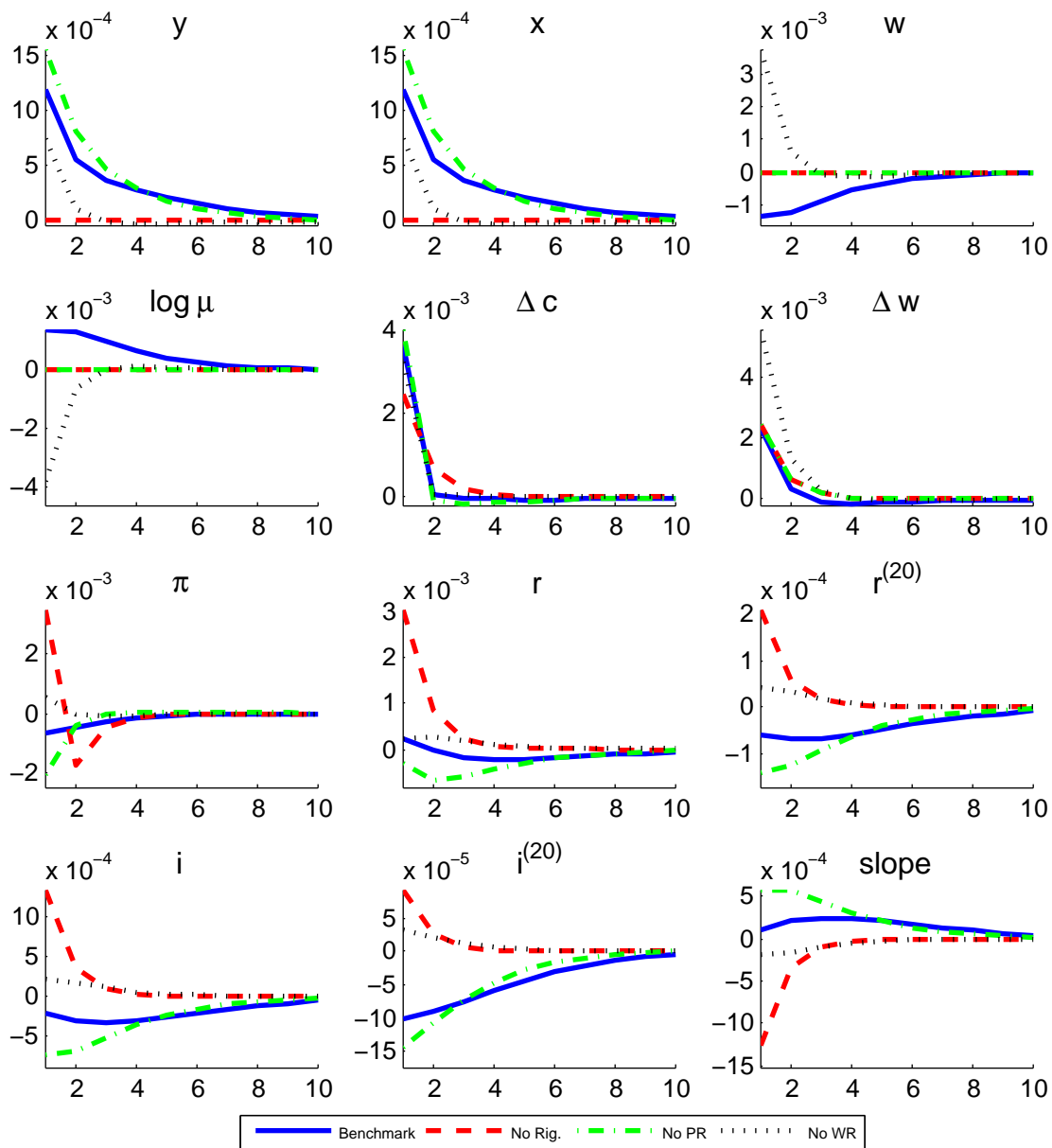
Figure B.3: Impulse responses to a one-standard deviation positive permanent productivity shock for different macroeconomic variables and interest rates. The parameter values are presented in Table B.5.
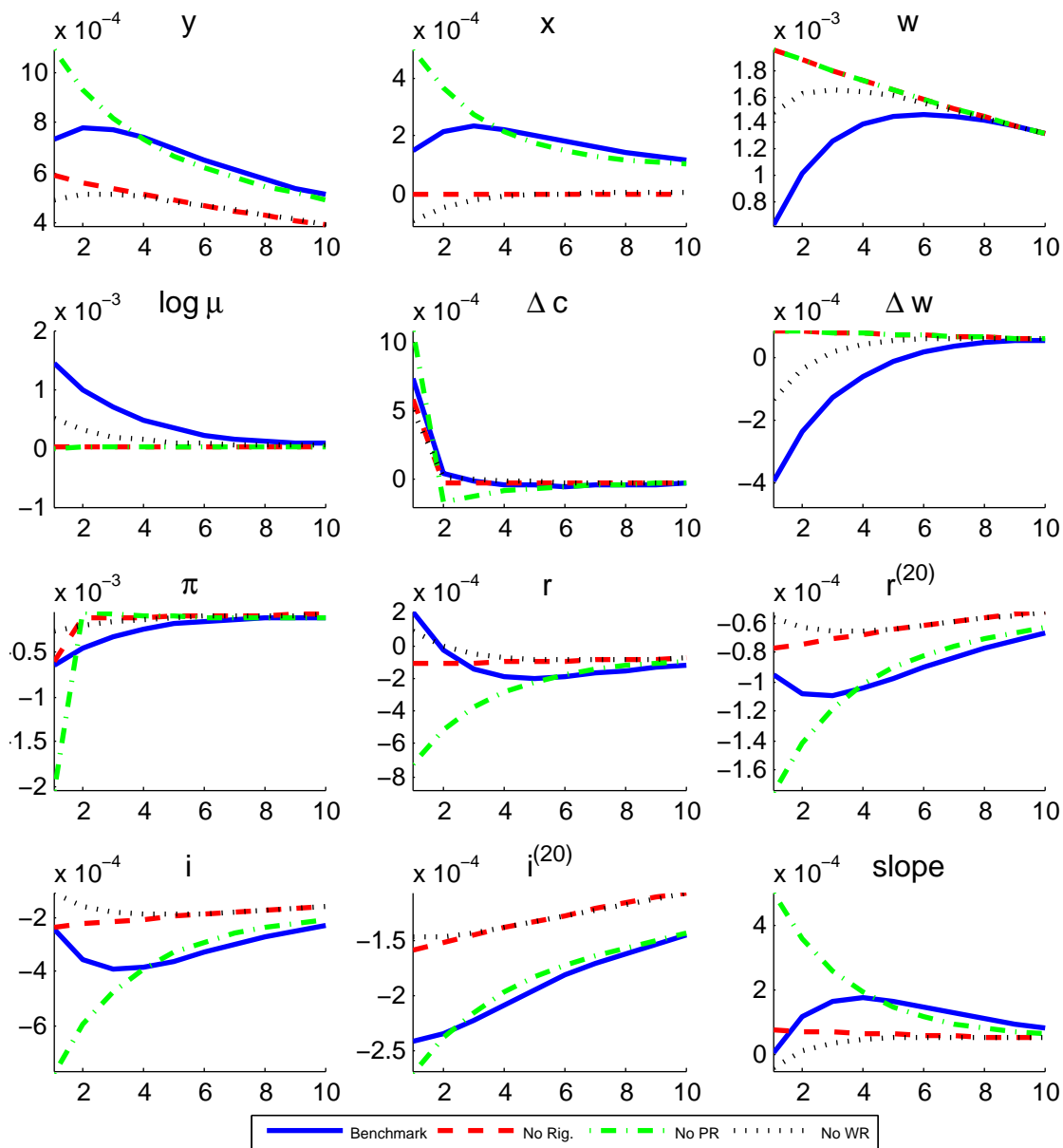
Figure B.4: Impulse responses to a one-standard deviation positive transitory productivity shock for different macroeconomic variables and interest rates. The parameter values are presented in Table B.5.
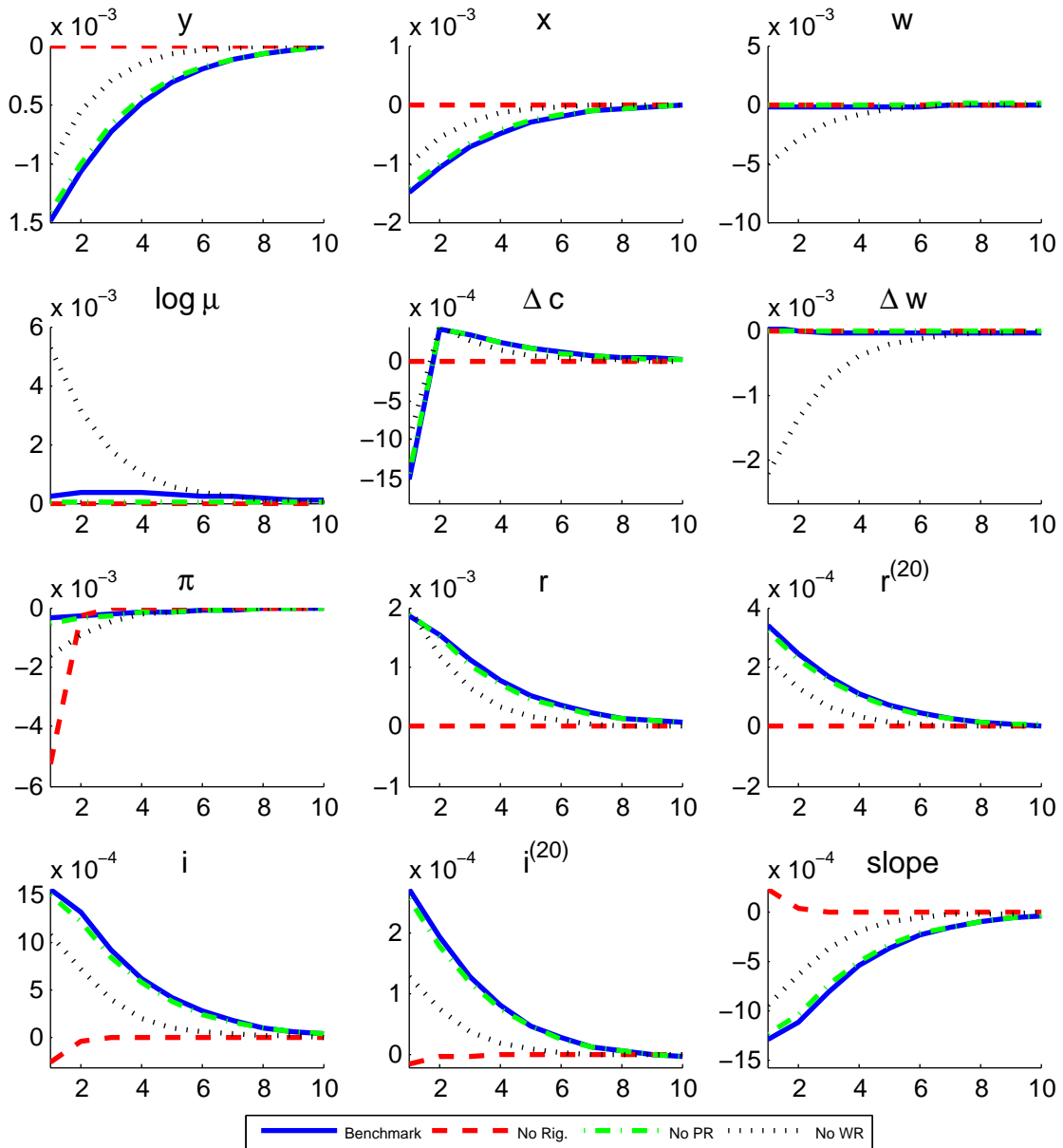
Figure B.5: Impulse responses to a one-standard deviation positive policy shock for different macroeconomic variables and interest rates. The parameter values are presented in Table B.5.
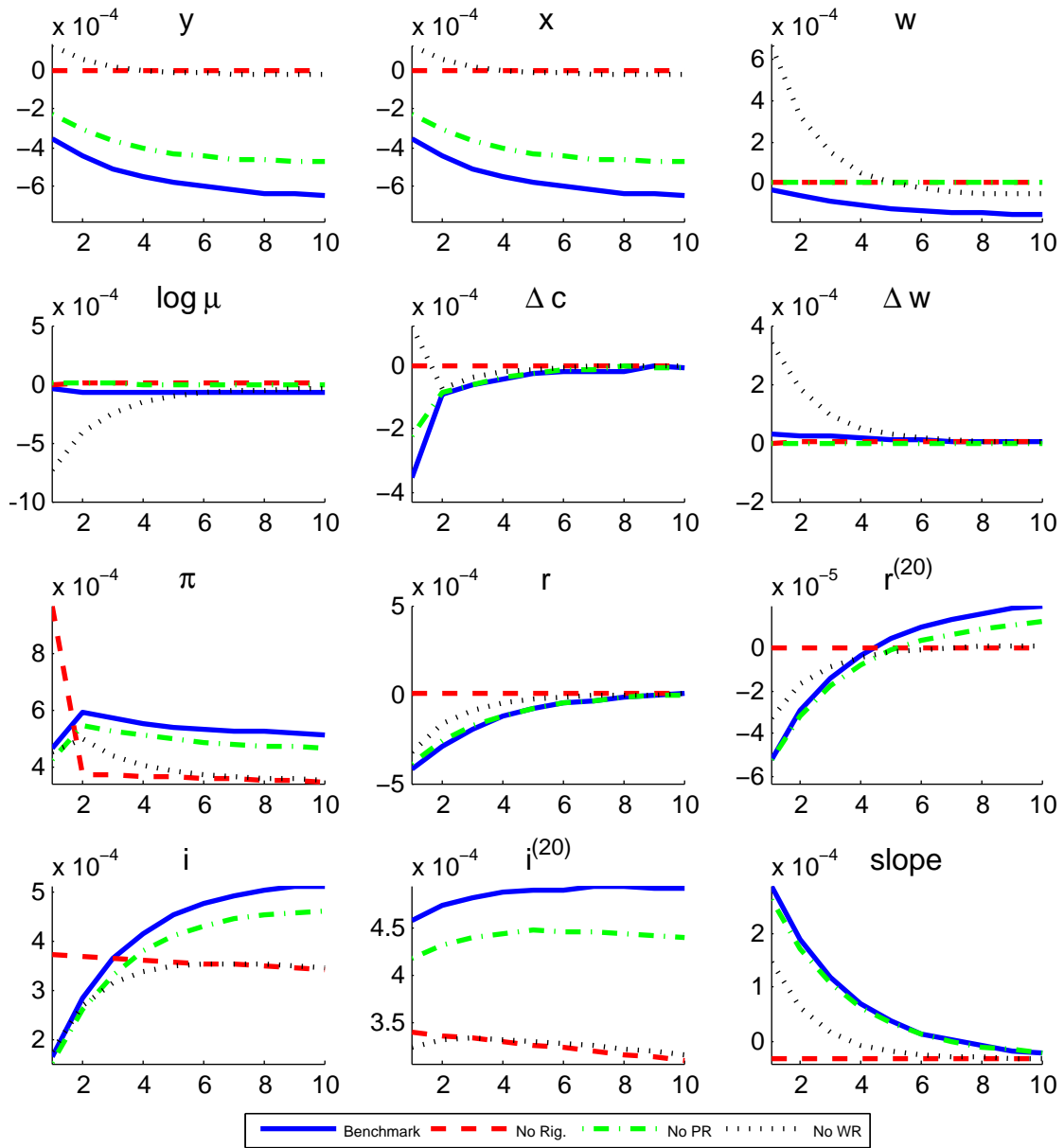
Figure B.6: Impulse responses to a one-standard deviation positive inflation target shock for different macroeconomic variables and interest rates. The parameter values are presented in Table B.5.

B.5   Tables

	1999 - 2008		2004 - 2008	
	Real	Nominal	Real	Nominal
Average Yields				
2 years		3.79	1.22	3.64
5 years	2.30	4.32	1.60	3.98
10 years	2.66	4.97	2.00	4.54
15 years	2.78	5.31	2.15	4.86
20 years	2.81	5.41	2.17	4.95
Standard Deviations				
2 years		1.50	1.06	1.09
5 years	1.11	1.05	0.63	0.65
10 years	0.87	0.73	0.34	0.33
15 years	0.77	0.62	0.25	0.29
20 years	0.75	0.58	0.22	0.32
Correlations				
2 years		N.A.		0.934
5 years		0.906		0.948
10 years		0.938		0.918
15 years		0.942		0.837
20 years		0.928		0.809

Table B.1:
Descriptive Statistics of U.S. Government TIPS and Nominal Bond Yields
Average levels (%), standard deviations (%), and correlations are reported.

	1999 - 2008			2004 - 2008		
	TIPS	Nominal	All	TIPS	Nominal	All
1st	98.04	78.46	81.88	90.41	70.63	75.63
2nd	1.92	20.89	15.50	9.09	28.33	21.2
3rd	0.03	0.44	2.05	0.44	0.53	2.19
Total	99.99	99.79	99.43	99.94	99.49	99.02

Table B.2:
Variability (%) explained by the first three principal components for U.S. Government TIPS and Nominal Bond Yields
"All" refers to columns when principal components are computed using both TIPS and nominal yields.

Maturity	1999 - 2008				2004 - 2008			
	1st	2nd	3rd	Adj. R^2	1st	2nd	3rd	Adj. R^2
1	-0.25	0.51	-0.03	0.986	0.41	-0.21	0.19	0.990
	-66.729	60.474	-1.283		69.43	-18.64	5.45	
2	-0.24	0.40	0.07	0.998	0.35	-0.12	0.30	0.999
	-219.232	158.798	10.583		211.56	-37.54	30.96	
5	-0.19	0.16	0.14	0.995	0.21	0.04	0.22	0.996
	-146.233	51.844	16.326		108.68	9.79	19.44	
7	-0.17	0.07	0.14	0.994	0.15	0.08	0.14	0.994
	-134.069	23.476	17.734		88.32	26.48	13.87	
10	-0.14	-0.02	0.14	0.994	0.09	0.13	0.06	0.981
	-134.394	-6.529	21.223		40.88	31.89	5.07	
15	-0.11	-0.09	0.14	0.996	0.03	0.17	0.03	0.975
	-148.815	-52.296	28.737		14.09	43.33	2.66	
20	-0.10	-0.13	0.13	0.996	0.00	0.20	0.04	0.992
	-147.321	-84.696	31.555		2.23	79.09	5.03	
25	-0.09	-0.15	0.12	0.992	-0.01	0.21	0.04	0.986
	-96.283	-68.27	20.97		-4.36	61.09	3.60	
30	-0.09	-0.15	0.12	0.974	-0.01	0.22	0.02	0.938
	-51.581	-39.313	10.683		-2.70	28.23	0.90	

Table B.3: **Nominal Yields Regression**

Regression of nominal yields on the three principal components for U.S. Government TIPS and nominal bond yields. T-statistics are reported below each regression coefficient.

Maturity	1999 - 2008				2004 - 2008			
	1st	2nd	3rd	Adj. R^2	1st	2nd	3rd	Adj. R^2
2	N.A.	N.A.	N.A.	N.A	0.34	-0.12	-0.42	0.991
	N.A.	N.A.	N.A.	N.A	75.06	-14.16	-16.17	
5	-0.21	0.02	-0.27	0.991	0.21	0.02	-0.17	0.995
					102.68	5.59	-14.56	
7	-0.19	-0.03	-0.23	0.995	0.15	0.06	-0.14	0.994
	-109.086	4.654	-22.219		87.77	17.27	-13.18	
10	-0.17	-0.07	-0.18	0.999	0.10	0.09	-0.13	0.994
	-149.327	-10.806	-28.902		82.81	36.57	-17.74	
15	-0.14	-0.09	-0.14	0.996	0.05	0.11	-0.12	0.969
	-420.088	-72.67	-73.086		27.13	28.58	-10.65	
20	-0.14	-0.12	-0.13	0.989	0.03	0.11	-0.11	0.907
	-164.719	-47.266	-25.316		10.10	19.60	-5.94	

Table B.4: **TIPS Yields Regression**

Regression of TIPS yields on the three principal components for U.S. Government TIPS and nominal bond yields. T-statistics are reported below each regression coefficient.

Parameter	Description	Value
Panel A: Preferences		
β	Subjective discount factor	0.992
ψ	Inverse of elasticity of intertemporal substitution	4.5
γ	Risk aversion parameter	120
ω	Inverse of Frisch labor elasticity	0.5
Panel B: Rigidities		
α	Price rigidity parameter	0.66
θ	Elasticity of substitution of differentiated goods	6
λ	Wage rigidity parameter	0.82
θ_w	Elasticity of substitution of labor types	6
Panel C: Interest Rate Rule		
ρ	Interest-rate smoothing coefficient in policy rule	0.76
$\bar{\imath} \times 10^2$	Constant in the policy rule	1.01
$\imath_\pi$	Response to inflation in the policy rule	1.6
$\imath_x$	Response to output gap in the policy rule	0.125
Panel D: Exogenous Shocks		
ϕ_u	Autocorrelation of policy shock	0.15
$\sigma_u \times 10^2$	Conditional vol. of policy shock	0.175
ϕ_a	Autocorrelation of permanent productivity shock	0.275
$\sigma_a \times 10^2$	Conditional vol. of permanent productivity shock	0.246
ϕ_z	Autocorrelation of transitory productivity shock	0.957
$\sigma_z \times 10^2$	Conditional vol. of transitory productivity shock	0.19625
Panel E: Inflation Target		
$g_{\pi^\star} \times 10^2$	Unconditional Mean of Inflation Target	0.55
$\phi_{\pi^\star}$	Autocorrelation of Inflation Target	0.99
$\sigma_{\pi^\star} \times 10^2$	Conditional vol. of Inflation Target	0.015

Table B.5: **Calibrated Parameter Values**

	(1)	(2)
	Data from 1982 to 2010	Benchmark
$\sigma(y)$	2.90	2.37
$\sigma(\Delta c)$	1.60	1.63
$\mathbb{E}[\pi_t]$	3.07	3.01
$\mathbb{E}[i_t]$	4.89	4.61
$\mathbb{E}[i_t^{(20)}]$	6.26	4.93
$\mathbb{E}[i_t^{(20)} - i_t]$	1.38	0.32
$\sigma(\pi_t)$	1.68	1.67
$\sigma(i_t)$	2.86	1.93
$\sigma(i_t^{(20)})$	2.86	1.52
$\sigma(i_t^{(20)} - i_t)$	0.94	0.88

Table B.6: **Model Calibration**

σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. y is output. Δc is consumption growth. π is inflation. r denotes the real yield while i denotes the nominal yield.

	(1)	(2)	(3)	(4)	(5)
	Benchmark	Only A^p	Only Z	Only u	Only $\pi^\star$
$\sigma(y)$	2.3730	0.5675	1.1126	0.8324	1.8380
$\sigma(x)$	2.1179	0.5675	0.3040	0.8324	1.8380
$\sigma(w)$	2.5944	0.8529	2.4037	0.2364	0.4119
$\sigma(log(\mu))$	1.3243	0.9614	0.8202	0.3328	0.2145
$\sigma(\Delta c)$	1.6299	1.4575	0.3041	0.6492	0.1352
$\sigma(\Delta w)$	0.9785	0.9545	0.2098	0.0450	0.0183
$\mathbb{E}[\pi_t]$	3.0134	2.9366	6.8019	6.8183	6.9473
$\mathbb{E}[IRP^{(20)}]$	0.3082	0.2360	0.0168	-0.0001	0.0555
$\mathbb{E}[rTP^{(20)}]$	0.1190	0.1183	0.0067	0.0056	-0.0116
$\mathbb{E}[r_t]$	1.4858	1.5251	3.1908	3.2068	3.2018
$\mathbb{E}[r_t^{(20)}]$	1.6045	1.6430	3.1971	3.2120	3.1898
$\mathbb{E}[i_t]$	4.6089	4.5621	9.9976	10.024	10.154
$\mathbb{E}[i_t^{(20)}]$	4.9257	4.8151	10.015	10.030	10.192
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3177	0.2539	0.0186	0.0063	0.0389
$\sigma(\pi_t)$	1.6701	0.3294	0.4266	0.2274	1.5643
$\sigma(r_t)$	1.2128	0.2074	0.2453	1.1457	0.2348
$\sigma(r_t^{(20)})$	0.2573	0.0583	0.1481	0.1884	0.0734
$\sigma(i_t)$	1.9268	0.2760	0.5044	0.9628	1.5668
$\sigma(i_t^{(20)})$	1.5189	0.0703	0.3266	0.1491	1.4742
$\sigma(i_t^{(20)} - i_t)$	0.8842	0.2131	0.1870	0.8152	0.1921

Table B.7: **Model Summary Statistics for Different Shock Specifications**
The baseline parameter values are presented in Table B.5. "Benchmark" indicates an economy with both price and wage rigidities plus all four exogenous shocks. "Only A^p" indicates only permanent productivity shocks ($\sigma_z = \sigma_u = \sigma_{\pi^\star} = 0$). "Only Z" indicates only transitory productivity shocks ($\sigma_a = \sigma_u = \sigma_{\pi^\star} = 0$). "Only u" indicates only policy shocks ($\sigma_a = \sigma_z = \sigma_{\pi^\star} = 0$). "Only $\pi^\star$" indicates only shocks to inflation target ($\sigma_a = \sigma_z = \sigma_u = 0$). σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. y is output. x is the output gap. w is wage. $log(\mu)$ is the markup charged by the producers. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. r denotes the real yield while i denotes the nominal yield. Annualized percentages are used in all appropriate cells.

	(1) Benchmark	(2) No Rig.	(3) Only WR	(4) Only PR
$\sigma(y)$	2.3730	0.8118	2.2158	0.9821
$\sigma(x)$	2.1179	$-$	1.9351	0.5760
$\sigma(w)$	2.5944	2.7061	2.7061	3.9183
$\sigma(log(\mu))$	1.3243	$-$	$-$	3.1058
$\sigma(\Delta c)$	1.6299	1.0508	1.7855	1.3785
$\sigma(\Delta w)$	0.9785	1.0301	1.0301	2.5638
$\mathbb{E}[\pi_t]$	3.0134	3.5176	2.9950	3.3688
$\mathbb{E}[IRP^{(20)}]$	0.3082	-0.2183	0.4106	-0.0607
$\mathbb{E}[rTP^{(20)}]$	0.1190	-0.6272	0.3900	-0.1117
$\mathbb{E}[r_t]$	1.4858	2.3714	1.2030	1.8276
$\mathbb{E}[r_t^{(20)}]$	1.6045	1.7438	1.5926	1.7154
$\mathbb{E}[i_t]$	4.6089	5.3210	4.5460	5.1082
$\mathbb{E}[i_t^{(20)}]$	4.9257	5.0434	4.9980	5.0238
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3177	-0.2775	0.4527	-0.0843
$\sigma(\pi_t)$	1.6701	2.8705	1.9288	1.3897
$\sigma(r_t)$	1.2128	1.2762	1.2955	1.0145
$\sigma(r_t^{(20)})$	0.2573	0.1380	0.2612	0.1520
$\sigma(i_t)$	1.9268	1.2464	1.8997	1.2403
$\sigma(i_t^{(20)})$	1.5189	0.9910	1.4434	1.0064
$\sigma(i_t^{(20)} - i_t)$	0.8842	0.5489	0.9330	0.5256

Table B.8: **Model Summary Statistics for Different Rigidity Specifications**
"No Rig." indicates no price and wage rigidities ($\alpha = \lambda = 0$). "Only WR" indicates no price rigidities ($\alpha = 0$). "Only PR" indicates no wage rigidities ($\lambda = 0$). σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. y is output. x is the output gap. w is wage. $log(\mu)$ is the markup charged by the producers. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. r denotes the real yield while i denotes the nominal yield. Annualized percentages are used in all appropriate cells.

	(1)	(2)	(3)	(4)	(5)
	Benchmark	No Rig.	Only WR	Only PR	WR and PR
$\sigma(y)$	2.3730	0.8118	2.0875	0.9343	2.3041
$\sigma(x)$	2.1179	–	1.7868	0.4901	2.0405
$\sigma(w)$	2.5944	2.7061	2.7061	3.6082	2.4502
$\sigma(log(\mu))$	1.3243	–	–	2.6875	0.9108
$\sigma(\Delta c)$	1.6299	0.2381	0.7711	0.5041	0.7296
$\sigma(\Delta w)$	0.9785	0.1164	0.1164	1.1060	0.2153
$\mathbb{E}[\pi_t]$	3.0134	5.8424	6.8007	5.8798	6.9071
$\mathbb{E}[IRP^{(20)}]$	0.3082	−0.0037	0.0602	0.0020	0.0722
$\mathbb{E}[rTP^{(20)}]$	0.1190	0.0044	0.0144	0.0057	0.0008
$\mathbb{E}[r_t]$	1.4858	3.2008	3.1677	3.1991	3.1736
$\mathbb{E}[r_t^{(20)}]$	1.6045	3.2047	3.1816	3.2045	3.1740
$\mathbb{E}[i_t]$	4.6089	9.0406	9.9867	9.0770	10.090
$\mathbb{E}[i_t^{(20)}]$	4.9257	9.0437	10.042	9.0865	10.153
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3177	0.0032	0.0562	0.0096	0.0638
$\sigma(\pi_t)$	1.6701	2.4034	1.7422	1.3727	1.6373
$\sigma(r_t)$	1.2128	0.1571	1.2167	1.0028	1.1950
$\sigma(r_t^{(20)})$	0.2573	0.1068	0.2451	0.1503	0.2506
$\sigma(i_t)$	1.9268	1.1137	1.8360	1.2337	1.9069
$\sigma(i_t^{(20)})$	1.5189	0.9902	1.4412	1.0063	1.5173
$\sigma(i_t^{(20)} - i_t)$	0.8842	0.1726	0.8389	0.5136	0.8582

Table B.9: **Model Summary Statistics under No Permanent Shocks**
Other than the Benchmark, columns (2) through (5) reports model output when $\sigma_a = 0$ jointly with different rigidity specifications. "No Rig." indicates no price and wage rigidities ($\alpha = \lambda = 0$). "Only WR" indicates no price rigidities ($\alpha = 0$). "Only PR" indicates no wage rigidities ($\lambda=0$). "WR and PR" indicates both rigidities are turned on. σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. y is output. x is the output gap. w is wage. $log(\mu)$ is the markup charged by the producers. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. r denotes the real yield while i denotes the nominal yield. Annualized percentages are used in all appropriate cells.

	Panel A: Rigidities					
Parameters:		α			λ	
	0.61	0.66	0.71	0.77	0.82	–
$\sigma(x)$	2.0396	2.1179	2.2762	1.2255	2.1179	–
$\sigma(\Delta c)$	1.6440	1.6299	1.6160	1.5794	1.6299	–
$\sigma(\Delta w)$	0.9581	0.9785	1.0035	0.9672	0.9785	–
$\mathbb{E}[\pi_t]$	3.0221	3.0134	2.9831	3.3114	3.0134	–
$\mathbb{E}[IRP^{(20)}]$	0.3261	0.3084	0.2892	0.1886	0.3083	–
$\mathbb{E}[rTP^{(20)}]$	0.1465	0.1190	0.0891	0.0931	0.1190	–
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3415	0.3174	0.2893	0.1994	0.3174	–
$\sigma(\pi_t)$	1.6673	1.6701	1.6967	1.4047	1.6701	–
$\sigma(i_t^{(20)})$	1.4928	1.5189	1.5692	1.2269	1.5190	–
	Panel B: Inflation Target					
Parameters:		$\phi_{\pi^\star}$			$\sigma_{\pi^\star} \times 10^2$	
	0.985	0.990	0.995	0	0.015	0.052
$\sigma(x)$	1.7446	2.1179	2.9703	1.0523	2.1179	6.4581
$\sigma(\Delta c)$	1.6287	1.6299	1.6313	1.6243	1.6299	1.6906
$\sigma(\Delta w)$	0.9784	0.9785	0.9785	0.9783	0.9785	0.9804
$\mathbb{E}[\pi_t]$	2.9737	3.0134	3.1302	2.8963	3.0134	4.3032
$\mathbb{E}[IRP^{(20)}]$	0.2924	0.3078	0.3341	0.2523	0.3078	0.9192
$\mathbb{E}[rTP^{(20)}]$	0.1223	0.1190	0.1132	0.1306	0.1190	−0.0084
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3059	0.3168	0.3352	0.2780	0.3168	0.7450
$\sigma(\pi_t)$	1.3930	1.6701	2.2961	0.5850	1.6701	5.4545
$\sigma(i_t^{(20)})$	1.2091	1.5189	2.1890	0.3658	1.5189	5.1236

Table B.10: **Comparative Statics I**

Model output by perturbing one parameter at a time while keeping all other parameters at the baseline values. Please see table B.5 for parameter definitions. Under each parameter, the middle column represents the baseline calibration. For λ, the wage rigidity parameter, the highest allowable value by the model is used for the basement calibration. σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. x is the output gap. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. i denotes the nominal yield.

	$\imath_\pi$		
	1.5	1.6	1.7
$\sigma(x)$	2.1179	2.1179	1.5378
$\sigma(\Delta c)$	1.6299	1.6299	1.6281
$\sigma(\Delta w)$	0.9785	0.9785	0.9726
$\mathbb{E}[\pi_t]$	3.0134	3.0134	3.0742
$\mathbb{E}[IRP^{(20)}]$	0.3082	0.3084	0.2638
$\mathbb{E}[rTP^{(20)}]$	0.1190	0.1190	0.1395
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3177	0.3174	0.2987
$\sigma(\pi_t)$	1.6701	1.6701	1.3662
$\sigma(i_t^{(20)})$	1.5189	1.5190	1.2112

	$\imath_x$		
	0	0.125	0.20
$\sigma(x)$	1.4916	2.1179	3.5325
$\sigma(\Delta c)$	1.6591	1.6299	1.6400
$\sigma(\Delta w)$	0.9708	0.9785	0.9849
$\mathbb{E}[\pi_t]$	3.0995	3.0134	2.3708
$\mathbb{E}[IRP^{(20)}]$	0.2442	0.3082	0.4278
$\mathbb{E}[rTP^{(20)}]$	0.1618	0.1190	0.0868
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3050	0.3172	0.3914
$\sigma(\pi_t)$	1.2694	1.6701	2.3196
$\sigma(i_t^{(20)})$	1.1103	1.5189	2.1805

	ρ		
	0.61	0.76	0.91
$\sigma(x)$	2.1118	2.1179	2.6696
$\sigma(\Delta c)$	1.5984	1.6299	1.9928
$\sigma(\Delta w)$	0.9791	0.9785	0.9863
$\mathbb{E}[\pi_t]$	2.9442	3.0134	3.2478
$\mathbb{E}[IRP^{(20)}]$	0.3525	0.3078	0.2283
$\mathbb{E}[rTP^{(20)}]$	0.1735	0.1190	0.0287
$\mathbb{E}[i_t^{(20)} - i_t]$	0.4083	0.3168	0.1618
$\sigma(\pi_t)$	1.7190	1.6701	1.8468
$\sigma(i_t^{(20)})$	1.5853	1.5189	1.2984

Table B.11: **Comparative Statics II**

Model output by perturbing one monetary policy parameter at a time while keeping all other parameters at the baseline values. Please see table B.5 for parameter definitions. σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. x is the output gap. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. i denotes the nominal yield.

Panel A: Rigidities						
Parameters:		α			λ	
	0.61	0.66	0.71	0.77	0.82	—
$\frac{\sigma(i_t)}{\sigma(r_t)}$	1.5649	1.5887	1.6312	1.3635	1.5887	—
$\frac{\sigma(i_t^{(20)})}{\sigma(r_t^{(20)})}$	5.8222	5.9032	6.0331	5.5541	5.9036	—
$\frac{\sigma(r_t^{(20)})}{\sigma(r_t)}$	0.2106	0.2122	0.2152	0.1859	0.2122	—
$\frac{\sigma(i_t^{(20)})}{\sigma(i_t)}$	0.7836	0.7883	0.7958	0.7571	0.7884	—
Panel B: Inflation Target						
Parameters:		$\phi_{\pi^\star}$			$\sigma_{\pi^\star} \times 10^2$	
	0.985	0.990	0.995	0	0.015	0.052
$\frac{\sigma(i_t)}{\sigma(r_t)}$	1.3926	1.5887	2.0542	0.9425	1.5887	3.8471
$\frac{\sigma(i_t^{(20)})}{\sigma(r_t^{(20)})}$	4.6828	5.9032	8.5877	1.4834	5.9032	14.461
$\frac{\sigma(r_t^{(20)})}{\sigma(r_t)}$	0.2130	0.2122	0.2101	0.2072	0.2122	0.2458
$\frac{\sigma(i_t^{(20)})}{\sigma(i_t)}$	0.7163	0.7883	0.8784	0.3262	0.7883	0.9238
Panel C: Policy Rule						
Parameters:		$\imath_\pi$			$\imath_x$	
	1.5	1.6	1.7	0	0.125	0.20
$\frac{\sigma(i_t)}{\sigma(r_t)}$	1.5887	1.5887	1.3855	1.2674	1.5887	2.0847
$\frac{\sigma(i_t^{(20)})}{\sigma(r_t^{(20)})}$	5.9032	5.9036	4.9905	4.3439	5.9032	7.6589
$\frac{\sigma(r_t^{(20)})}{\sigma(r_t)}$	0.2122	0.2122	0.2033	0.2034	0.2122	0.2337
$\frac{\sigma(i_t^{(20)})}{\sigma(i_t)}$	0.7883	0.7884	0.7324	0.6972	0.7883	0.8585

Table B.12: **Volatility Ratios**

The ratio of the unconditional standard deviations of real and nominal yields by perturbing one parameter at a time while keeping all other parameters at the baseline values. Please see table B.5 for parameter definitions. Under each parameter, the middle column represents the baseline calibration. For λ, the wage rigidity parameter, the highest allowable value by the model is used for the basement calibration. σ denotes the unconditional volatility. r denotes the real yield while i denotes the nominal yield.

	Panel A: With Permanent Productivity Shock			
	Benchmark	No Rig.	Only WR	Only PR
$\rho_r^{(4)}$	0.8174	0.7175	0.8233	0.5818
$\rho_r^{(8)}$	0.7493	0.6903	0.8050	0.4673
$\rho_r^{(12)}$	0.6543	0.6565	0.7383	0.3604
$\rho_r^{(16)}$	0.5666	0.6226	0.6704	0.2946
$\rho_r^{(20)}$	0.4982	0.5906	0.6122	0.2524
	Panel B: Without Permanent Productivity Shock			
	WR and PR	No Rig.	Only WR	Only PR
$\rho_r^{(4)}$	0.8503	0.1845	0.8352	0.5714
$\rho_r^{(8)}$	0.7571	0.2872	0.7784	0.4430
$\rho_r^{(12)}$	0.6458	0.3367	0.6919	0.3347
$\rho_r^{(16)}$	0.5515	0.3586	0.6148	0.2714
$\rho_r^{(20)}$	0.4814	0.3659	0.5540	0.2319

Table B.13: **Real and Nominal Return Correlations**

Unconditional correlations between returns on real bonds and real returns on nominal bonds (excess of inflation). Realized returns are calculated over 1-Quarter holding horizon. "No Rig." indicates no price and wage rigidities ($\alpha = \lambda = 0$). "Only WR" indicates no price rigidities ($\alpha = 0$). "Only PR" indicates no wage rigidities ($\lambda=0$). Values in the parentheses denote the number of quarters to maturity on the bond.

	$\imath_\pi$		
	1.55	1.6	1.65
$\rho_r^{(4)}$	0.7751	0.8174	0.8383
$\rho_r^{(8)}$	0.6816	0.7493	0.7836
$\rho_r^{(12)}$	0.5682	0.6543	0.7001
$\rho_r^{(16)}$	0.4732	0.5666	0.6186
$\rho_r^{(20)}$	0.4057	0.4982	0.5518
	$\imath_x$		
	0	0.125	0.175
$\rho_r^{(4)}$	0.8686	0.8174	0.7395
$\rho_r^{(8)}$	0.8332	0.7493	0.6252
$\rho_r^{(12)}$	0.7686	0.6543	0.4997
$\rho_r^{(16)}$	0.6997	0.5666	0.4024
$\rho_r^{(20)}$	0.6383	0.4982	0.3377
	ρ		
	0.61	0.76	0.91
$\rho_r^{(4)}$	0.7937	0.8174	0.7981
$\rho_r^{(8)}$	0.7259	0.7493	0.7859
$\rho_r^{(12)}$	0.6431	0.6543	0.7232
$\rho_r^{(16)}$	0.5745	0.5666	0.6427
$\rho_r^{(20)}$	0.5251	0.4982	0.5639

Table B.14: **Return Correlation Comparative Statics**
Unconditional correlations between returns on real bonds and real returns on nominal bonds (excess of inflation) calculated by varying monetary policy parameters. The middle column represents the benchmark calibration. Values in the parentheses denote the number of quarters to maturity on the bond.

APPENDIX C

Fiscal Policy Shocks and Bond Return Predictability

C.1   Small Sample Inference and Generated Regressors

The empirical exercise presented in this paper has two main challenges. First, using roughly 40 years of quarterly data, the sample size is relatively small for time series analysis where autocorrelation can be an issue. Second, the fiscal policy shocks used in the predictive regressions are residuals from a first-stage VAR, and *Pagan* (1984), among others, has shown that using generated regressors in a second-stage OLS regression can produce inconsistent estimates under certain conditions.

To treat these issues, I employ several different techniques to ensure the resulting standard errors from the predictive regressions are valid such that it is possible to conduct inference testing. To correct for the small sample size, I follow *Cochrane and Piazzesi* (2005) and augment the standard errors with the Newey-West weighting matrix with a lag of 6 quarters. In addition, I bootstrap the standard errors for further robustness. Because the predictive regressions involve generated regressors, I employ a block bootstrap with block size of 6 periods and 20,000 iterations. The

resulting t-statistics are under Newey-West t and Block Bootstrap t, respectively, in tables C.2, C.3, and C.7.

To deal with generated regressors in the second-stage, I resort to Monte Carlo simulations to find the standard errors. As pointed out by *Pagan* (1984), when the residuals from a first-stage regression and only those residuals are used in a second-stage regression, the OLS standard errors are actually consistent. Therefore, Monte Carlo simulations were not applied to the predictive regressions in tables C.2 and C.7. However, for the predictive regressions in tables C.3, C.4, and C.5, the regressors include not only residuals from a first-stage VAR but also other independent variables not included in the same VAR. To correct for the inconsistent standard errors, I perform Monte Carlo simulations by assuming the data generating process to be a 4-lag VAR which combines the bond yields and all the independent variables not included in the first-stage regression. For example, for the predictive regression in table C.3, the VAR is

$$Z_t = A_o + A_1 Z_{t-1} + \cdots + A_4 Z_{t-1} + E_t,$$

where $Z_t \equiv [y_t, g_t, \tau_t, Fun_t, Gap_t, Inf_t]'$, and E_t are innovations. y_t denotes yields, g_t denotes government spending shocks, and τ_t denotes taxation shocks. Fun_t, Gap_t and Inf_t are Fed Funds Rate, output gap, and inflation, respectively.

Once I obtain the coefficient estimates from the data generating VAR, I simulate all the relevant data and estimate the coefficients of the particular predictive regression as specified. Finally, I repeat the simulation 20,000 times and calculate the standard errors of the coefficient estimates from each simulation. These are the values I use as standard errors for the predictive regressions. The resulting t-statistics are reported under Monte Carlo t in tables C.3, C.4, and C.5.

C.2 Tables

	1971:IV - 2010:III	
	Averages	Standard Deviations
Macroeconomic Quantities, in billions		
GDP	6728.9	4175.4
Government Expenditure	1298.8	789.60
Government Revenue	921.30	540.73
Yields, in percentages		
2 year	6.3619	3.1170
6 years	6.9326	2.7498
10 years	7.2631	2.5298
14 years	7.4538	2.4258
Excess Returns, in percentages		
2 year	0.5841	1.9431
6 years	1.8029	7.3618
10 years	2.3092	11.7175
14 years	2.4901	15.8711

Table C.1: **Summary Statistics**

Averages and standard deviations of the raw data used in the study. Data starts in the fourth quarter of 1971 and ends in the third quarter of 2010, made up of 156 quarterly observations. Macroeconomic quantities are obtained from the National Income and Product Account tables, and yields are obtained from *Gurkaynak, Sack, and Wright* (2006). Returns are calculated using a one-year holding period, and excess returns are over the one-year nominal yields.

n	const.	g_t	τ_t	R^2
2	0.58	0.43	−0.017	0.044
Newey-West t	(1.89)	(2.29)	(−0.58)	
Block Bootstrap t	(2.11)	(2.29)	(−0.46)	
6	1.80	1.63	−0.145	0.049
Newey-West t	(1.70)	(2.11)	(−1.23)	
Block Bootstrap t	(1.79)	(2.07)	(−1.03)	
10	2.31	2.59	−0.22	0.048
Newey-West t	(1.44)	(1.98)	(−1.23)	
Block Bootstrap t	(1.47)	(1.97)	(−1.03)	
14	2.49	3.63	−0.24	0.049
Newey-West t	(1.17)	(1.96)	(−1.04)	
Block Bootstrap t	(1.18)	(1.97)	(−0.85)	

Table C.2: **Excess Return Regressions with Fiscal Shocks**

The regression equation is: $rx_{t+1}^{(n)} = \beta_0 + \beta_1 g_t + \beta_2 \tau_t + \epsilon_{t+1}^{(n)}$. rx_t is the excess 1-year holding period return, g_t is the shock to government spending and τ_t is the shock to government revenue. Newey-West t computes the t statistics using a weighting matrix with 6 lags. Block Bootstrap t uses a random regressor bootstrap with block size of 6 and 20,000 iterations. t-statistics are reported in the parentheses. n denotes n-year to maturity zero-coupon bonds are used in the regression. Regression R^2 are shown on the right hand side of the table.

n	const.	g_t	τ_t	Fun_t	Gap_t	Inf_t	R^2
2	0.60	0.40	-0.030	0.22	-0.25	-1.49	0.22
Newey-West t	(1.30)	(2.37)	(-1.08)	(2.74)	(-1.67)	(-2.86)	
Block Bootstrap t	(1.18)	(2.32)	(-0.96)	(2.40)	(-1.48)	(-2.96)	
Monte Carlo t	(0.21)	(2.75)	(-0.61)	(3.29)	(-1.46)	(-3.10)	
6	3.90	1.53	-0.15	0.50	-0.51	-5.48	0.20
Newey-West t	(2.38)	(2.26)	(-1.37)	(1.52)	(-0.96)	(-3.15)	
Block Bootstrap t	(2.00)	(2.15)	(-1.26)	(1.36)	(-0.83)	(-3.07)	
Monte Carlo t	(1.34)	(2.67)	(-0.82)	(2.39)	(-0.78)	(-3.33)	
10	5.97	2.43	-0.23	0.70	-0.71	-8.38	0.19
Newey-West t	(2.49)	(2.12)	(-1.32)	(1.28)	(-0.83)	(-3.11)	
Block Bootstrap t	(1.99)	(2.02)	(-1.23)	(1.15)	(-0.72)	(-2.99)	
Monte Carlo t	(1.45)	(2.66)	(-0.76)	(2.18)	(-0.68)	(-3.32)	
14	7.09	3.44	-0.25	0.96	-1.05	-11.12	0.18
Newey-West t	(2.27)	(2.08)	(-1.15)	(1.29)	(-0.88)	(-3.05)	
Block Bootstrap t	(1.80)	(2.02)	(-1.06)	(1.17)	(-0.76)	(-2.98)	
Monte Carlo t	(1.26)	(2.76)	(-0.62)	(2.17)	(-0.75)	(-3.29)	

Table C.3: **Excess Return Regressions with Fiscal Shocks and Controls**
The regression equation is: $rx_{t+1}^{(n)} = \beta_0 + \beta_1 g_t + \beta_2 \tau_t + \beta_3 Fun_t + \beta_4 Gap_t + \beta_5 Inf_t + \epsilon_{t+1}^{(n)}$.
rx_t is the excess 1-year holding period return, g_t is the shock to government spending and τ_t is the shock to government revenue. *Fun* is the Fed Funds Rate, *Gap* is the output gap, and *Inf* is inflation. Monte Carlo t reports t statistics from a 20,000 iteration simulated bootstrap assuming the data generating process is a 4-lag VAR combining bond yields and all the independent variables in the regression equation. t-statistics are reported in the parentheses. n denotes n-year to maturity zero-coupon bonds are used in the regression. Regression R^2 are shown on the right hand side of the table.

n	const.	g_t	τ_t	$y_t^{(1)}$	$f_t^{(1\to2)}$	$f_t^{(2\to3)}$	$f_t^{(3\to4)}$	$f_t^{(4\to5)}$	R^2
2	-1.75	0.29	-0.013	-1.09	2.18	-0.039	-3.66	2.87	0.27
	(-1.22)	(1.76)	(-0.42)	(-1.67)	(0.53)	(-0.0033)	(-0.26)	(0.46)	
3	-2.77	0.54	-0.038	-2.33	5.45	-4.40	-0.92	2.61	0.28
	(-1.06)	(1.81)	(-0.65)	(-1.98)	(0.73)	(-0.21)	(-0.035)	(0.23)	
4	-3.65	0.73	-0.059	-3.57	8.97	-10.15	3.77	1.50	0.29
	(-1.02)	(1.78)	(-0.73)	(-2.20)	(0.87)	(-0.34)	(0.10)	(0.095)	
5	-4.59	0.89	-0.076	-4.80	12.59	-16.12	8.36	0.60	0.30
	(-1.04)	(1.74)	(-0.76)	(-2.38)	(0.98)	(-0.44)	(0.19)	(0.031)	

Table C.4: **Excess Return Regressions with Fiscal Shocks and Forward Rates using Zero-Coupon Yields from *Gurkaynak, Sack, and Wright* (2006)**

The regression equation is: $rx_{t+1}^{(n)} = \beta_0 + \beta_1 g_t + \beta_2 \tau_t + \beta_3 y_t^{(1)} + \beta_4 f_t^{(1\to2)} + \ldots + \beta_7 f_t^{(4\to5)} + \epsilon_{t+1}^{(n)}$. rx_t is the excess 1-year holding period return, g_t is the shock to government spending and τ_t is the shock to government revenue. $f_t^{(n\to n+1)}$ is the forward rate between year n and $n+1$. This is a modified version of Cochrane and Piazzesi (2005) regression, and t statistics are constructed using Monte Carlo simulated bootstraps. t-statistics are reported in the parentheses. n denotes n-year to maturity zero-coupon bonds are used in the regression. Regression R^2 are shown on the right hand side of the table.

n	const.	g_t	τ_t	$y_t^{(1)}$	$f_t^{(1\to2)}$	$f_t^{(2\to3)}$	$f_t^{(3\to4)}$	$f_t^{(4\to5)}$	R^2
2	-1.18	0.25	0.00	-0.47	-0.62	2.08	0.12	-0.89	0.37
	(-1.20)	(1.65)	(-0.00)	(-1.33)	(-0.88)	(3.17)	(0.32)	(-2.64)	
3	-1.74	0.50	-0.014	-0.96	-1.50	4.53	0.066	-1.83	0.39
	(-0.95)	(1.81)	(-0.23)	(-1.47)	(-1.14)	(3.81)	(0.095)	(-2.92)	
4	-2.36	0.68	-0.029	-1.50	-1.89	5.69	0.79	-2.71	0.41
	(-0.95)	(1.78)	(-0.36)	(-1.68)	(-1.06)	(3.53)	(0.84)	(-3.15)	
5	-3.08	0.86	-0.034	-2.04	-1.89	6.44	0.70	-2.75	0.37
	(-0.99)	(1.77)	(-0.33)	(-1.82)	(-0.84)	(3.18)	(0.59)	(-2.52)	

Table C.5: **Excess Return Regressions with Fiscal Shocks and Forward Rates using Fama-Bliss Zero-Coupon Yields from CRSP**

The regression equation is: $rx_{t+1}^{(n)} = \beta_0 + \beta_1 g_t + \beta_2 \tau_t + \beta_3 y_t^{(1)} + \beta_4 f_t^{(1\to2)} + \ldots + \beta_7 f_t^{(4\to5)} + \epsilon_{t+1}^{(n)}$. t-statistics are reported in the parentheses. n denotes n-year to maturity zero-coupon bonds are used in the regression. Regression R^2 are shown on the right hand side of the table.

n	const.	$\sigma_{g,t}$	R^2
2	-2.15	2.43	0.078
Newey-West t	(-2.17)	(2.75)	
Block Bootstrap t	(-1.82)	(2.36)	
6	-7.12	7.92	0.058
Newey-West t	(-1.80)	(2.18)	
Block Bootstrap t	(-1.52)	(1.88)	
10	-11.83	12.55	0.057
Newey-West t	(-1.83)	(2.078)	
Block Bootstrap t	(-1.57)	(1.83)	
14	-16.76	17.074	0.058
Newey-West t	(-1.89)	(2.05)	
Block Bootstrap t	(-1.59)	(1.78)	

Table C.6: **Excess Return Regressions using Time-Varying Volatility on Government Spending**

The regression equation is: $rx_{t+1}^{(n)} = \beta_0 + \beta_1 \sigma_{g,t} + \epsilon_{t+1}^{(n)}$. $\sigma_{g,t}$ is the time-varying volatility of the government spending shock estimated by GARCH. Newey-West t computes the t statistics using a weighting matrix with 6 lags. Block Bootstrap t uses a random regressor bootstrap with block size of 6 and 20,000 iterations. t-statistics are reported in the parentheses. n denotes n-year to maturity zero-coupon bonds are used in the regression. Regression R^2 are shown on the right hand side of the table.

n	const.	g_t	τ_t	R^2
2	0.59	0.34	-0.017	0.025
Newey-West t	(1.92)	(1.99)	(-0.54)	
Block Bootstrap t	(1.99)	(1.99)	(-0.47)	
6	1.81	1.26	-0.15	0.029
Newey-West t	(1.68)	(1.80)	(-1.16)	
Block Bootstrap t	(1.72)	(1.80)	(-1.02)	
10	2.32	2.04	-0.22	0.030
Newey-West t	(1.41)	(1.76)	(-1.16)	
Block Bootstrap t	(1.42)	(1.79)	(-1.01)	
14	2.51	2.99	-0.25	0.031
Newey-West t	(1.14)	(1.84)	(-0.99)	
Block Bootstrap t	(1.16)	(1.86)	(-0.85)	

Table C.7: **Excess Return Regressions with Fiscal Shocks Constructed from the Vector Error Correction Model.**
The regression equation is: $rx_{t+1}^{(n)} = \beta_0 + \beta_1 g_t + \beta_2 \tau_t + \epsilon_{t+1}^{(n)}$. rx_t is the excess 1-year holding period return, g_t is the shock to government spending and τ_t is the shock to government revenue. Newey-West t computes the t statistics using a weighting matrix with 6 lags. Block Bootstrap t uses a random regressor bootstrap with block size of 6 and 20,000 iterations. t-statistics are reported in the parentheses. n denotes n-year to maturity zero-coupon bonds are used in the regression. Regression R^2 are shown on the right hand side of the table.

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