# Reconciling prediction algorithms for *Dst*

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[1] The Burton et al. equation and the Ring Current Atmosphere Interaction Model (RAM) code give about equally good fits to *Dst* profiles of strong magnetic storms, yet they are driven by different functions of the interplanetary electric field, one rising linearly and the other eventually saturating. We show that by reformulating the Burton et al. equation such that a quadratic form of the driving term replaces the original linear form, the new prediction algorithm driven by the saturation form of the driving electric field produces about equally good fits to *Dst* as the original Burton et al. equation and the RAM code. The form of the quadratic driving term is constructed by specializing a general form given by dimensional analysis.

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## 1. Issue for Resolution: Differing Assumptions About *Dst* Driving Function Give About Equally Good Results

[2] Two familiar and relatively successful approaches to predicting the *Dst* index of magnetic storms use electric field measurements to drive (in whole or in large part) their prediction algorithms. In one of these approaches the electric field is measured in the free-stream solar wind (the interplanetary electric field or IEF), and in the other approach it is measured as a transpolar potential in the ionosphere (using the assimilated mapping of ionospheric electrodynamics (AMIE) technique) or just above the ionosphere with a Defense Meteorological Satellite Program (DMSP) satellite. This paper calls attention to what we think is an interesting situation that the electric fields so measured, the IEF and the transpolar potential, are in general not proportionally related in the high value range that characterizes magnetic storms, and so the *Dst* prediction algorithms that distinguish the two approaches appear to be driven by functions with different behaviors. Specifically, when the interplanetary magnetic field is southward, generally a precondition for the occurrence of a magnetic storm, the transpolar potential does indeed increase nearly linearly with increasing IEF in the nonstorm range of this parameter [*Reiff and Luhmann*, 1986]. However, in the range that characterizes magnetic storms, the transpolar potential often tends to saturate as the IEF increases [e.g., *Ober et al.*, 2003]. This apparent inconsistency in the behavior of the two types of driving functions poses an interesting problem in magnetospheric theory. In addressing this problem here, we restrict ourselves to the question: Is it possible to reformulate the IEF approach so that it uses a form of the driving function that has the same saturation type of behavior as the transpolar potential?

[3] To represent the IEF approach to *Dst* prediction, we focus on the Burton et al. equation [*Burton et al.*, 1975] as revised by *O'Brien and McPherron* [2000, 2002] (given below). It assumes that the time derivative of $-Dst$ is linearly proportional to the IEF. To represent the transpolar potential approach to *Dst* prediction, we use the Ring Current Atmosphere Interaction Model (RAM) [*Liemohn et al.*, 2001]. This model transports particles from the plasma sheet into the inner magnetosphere (ring current) using the drift equations of motion in which the electric field drift is parametrized in terms of the transpolar potential in the ionosphere, which, as stated, often tends to saturate when the IEF enters the range of magnetic storms.

[4] In this note we convert the linear IEF (Burton et al.) form of *Dst* prediction algorithm into a form that uses the saturation-transpolar-potential (RAM) form. It is not, however, our aim to present a precise *Dst* prediction algorithm to compete with or replace operational prediction algorithms. Instead this is a "proof of concept" exercise.

## 2. Burton et al. (BMR) Equation

[5] On the basis of fitting observations, *Burton et al.* [1975] formulated a *Dst* prediction algorithm that took the time derivative of $-Dst$ (i.e., storm time *Dst*) to be linearly proportional to magnetic merging component of the motional electric field of the solar wind ($-\mathbf{V} \times \mathbf{B_z}$), where $\mathbf{V}$ is solar wind velocity and $\mathbf{B_z}$ is the vector that results from projecting the interplanetary magnetic field vector onto the GSM z axis. More specifically, storm time *Dst* was found to increase in absolute value at a rate proportional

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to the rectified (i.e., negative values are set to zero) GSM y-component of the IEF (denote by $E$ below), thus

$$dDst^*/dt(nT/h) = -\alpha E(mV/m) - Dst^*(nT)/\tau(hour). \quad (1)$$

By fitting the formula to seven magnetic storms, the constants $\alpha$ and $\tau$ were determined to be 4.5 nT/hr/mV/m and 7.7 hours, respectively. The asterisk on *Dst* signifies that the value of *Dst* has been corrected for a solar wind pressure term and a small offset. Since neither of these enters into the present discussion, we will ignore the distinction between *Dst* and *Dst**. For brevity, we refer to equation (1) as the BMR equation, signifying the authors of the original 1975 paper, Burton, McPherron, and Russell.

[6] More recently, *O'Brien and McPherron* [2000] and *McPherron and O'Brien* [2001] tested whether the coefficients $\alpha$ and $\tau$ in (1), instead of being constants, might depend on $E$. They found that $\alpha$ remains reasonably constant and nearly equals the value 4.5 given in the original BMR equation as the IEF ranged from 0 to 14 mV/m, which was the limit within which sample size allowed statistically valid results. By contrast, they found that the decay constant, $\tau$, decreases markedly over the same range of $E$. *O'Brien and McPherron* [2002] have extended their study to include also seasonal and diurnal effect as parameterized by the geomagnetic colatitude ($\psi$) of the Sun at noon. In the application below, we take values for $\alpha$ and $\tau$ from this 2002 reference.

## 3. Ring Current Atmosphere Interaction Model (RAM)

[7] We use the Michigan version of the RAM code, as described by *Liemohn et al.* [2001], to represent transpolar-potential algorithms for predicting *Dst*. RAM solves the time-dependent, gyration-and-bounced-averaged kinetic equation for the distribution function of any chosen particle species in the ring current. Particles drift under the action of electrical, curvature, and magnetic-gradient forces. The energy of the particles changes to conserve the first and second adiabatic invariants. To "drive" the particles, the code uses a modified version of an analytical expression for the electric field in the equatorial plane of the ring current given by *McIlwain* [1986]. RAM can run with any specification of the equatorial electric field, but in tests the McIlwain formula has given best results. As boundary conditions, the code uses the transpolar potential (extracted from measurements) to specify the equatorial electric field (via the McIlwain formula, as mentioned) and particle measurements taken by geosynchronous satellites to specify the distribution function at the outer boundary of the solution domain. All of the RAM simulations whose results are shown here use 5-min cadence AMIE transpolar potential values. Each AMIE potential pattern, from which the transpolar potentials are extracted, is based on available ground-based and satellite data along with a "background" potential pattern from the *Weimer* [1996] model. Whereas the BMR equation explicitly assumes a linear dependence on the driving electric field, the RAM algorithm obtains its driving electric field from measurements taken in or above the ionosphere, which is known to saturate at high values [see, e.g., *Russell et al.*, 2000, 2001; *Liemohn et al.*, 2002; *Shepherd et al.*, 2002; *Tsutomu*, 2002; *Hairston et al.*, 2003; *Ober et al.*, 2003].

## 4. Bastille Day Storm as a Test Case

[8] During the Bastille Day storm (15 July 2000) the transpolar potential became strongly saturated, as indicated by the potential based on linear extrapolation exceeding 1000 kV, whereas the measured potential remained below 300 kV [*Liemohn et al.*, 2002; J. Raeder and G. Lu, Polar cap potential saturation during large geomagnetic storms, submitted to *Advanced Space Research*, 2005]. Thus more than a factor of three separates transpolar potentials that are associated with *Dst* predictions based on the BMR equation and the RAM algorithm.

[9] Figure 1 compares *Dst* for the Bastille Day storm with predictions of it made by the BMR equation, the revision of it by O'Brien and McPherron, and the Michigan RAM code. The magnitude of *Dst* for this storm reached slightly over 300 nT. It had a double maximum followed by a relatively slow decay. The original BMR equation overpredicts *Dst* for this storm by about 50%. The O'Brien-McPherron revision of it, which for the Bastille Day conditions has a smaller energy input rate and a faster decay time, predicts a main phase growth and magnitude that closely matches *Dst*. Down to the first main phase minimum, the two curves are indistinguishable. However, it fails to reproduce the second surge of the main phase. The RAM code generates the best overall fit, which apparently is owing to the influence of the plasma sheet density (a factor that is not part of the O'Brien-McPherron algorithm) rather than the transpolar potential [*Liemohn et al.*, 2002]. Another possible contributor to the mismatch after the first main phase between the Burton-O'Brien-McPherron (BOM) predicted *Dst* and the measured *Dst* is that the IMF did not weaken then; it merely rotated into the y-direction. The BOM algorithm assumes that only the z-component of the IMF is effective in generating *Dst*. Perhaps this assumption is too restrictive.

[10] The comparisons in Figure 1 make the central point that the BOM and RAM algorithms succeed about equally in predicting *Dst* magnitude and profile (except for the decay phase in the BOM case) despite the BOM algorithm being driven by the unsaturated solar wind electric field and the RAM algorithm being driven by the saturated ionospheric electric field. To quantify the magnitude of the inconsistency between the two codes' dependence on the driving electric fields, we show in the next section that if one drives the modified BOM with a saturated IEF, it significantly underpredicts *Dst* for the Bastille Day storm.

## 5. Failure of a Saturated Input Term to the BMR Equation to Generate a Realistic Storm Main Phase

[11] We integrate the BMR equation with a saturation-like form of the driving electric field while holding $\tau$ constant at the original BMR value of 7.7 hours. The point is to see whether this simple modification brings the BMR over-prediction of *Dst* seen in Figure 1 more in line with the observation. If so, the desired reconciliation would appear to have been achieved.

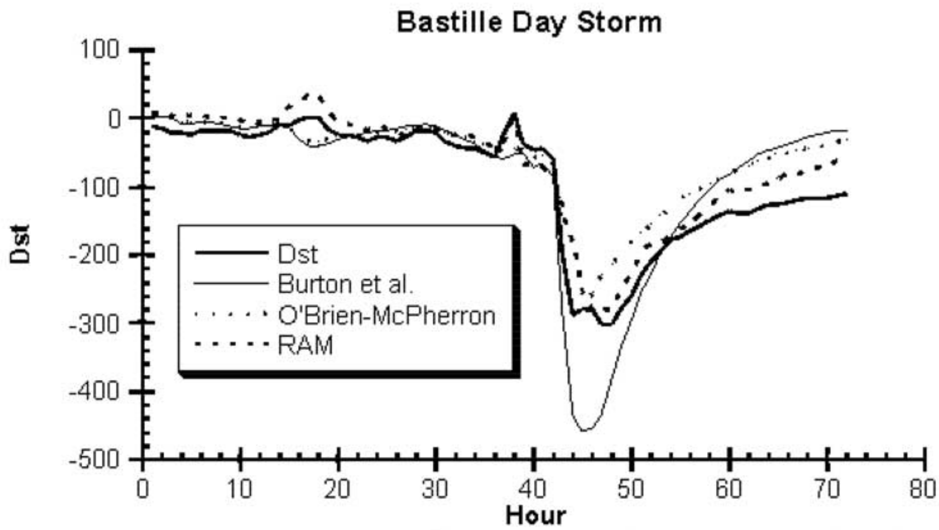


**Figure 1.** The *Dst* profile for the Bastille Day storm (15 July 2000) together with three predictions as labeled.

[12] To represent saturation of the transpolar potential, we use the Hill form of the saturation function [*Hill et al.*, 1976], which has been shown to work well in comparisons with transpolar potentials measured at the ionospheric level. (For a discussion of the Hill form of the saturation function and its ability to model transpolar potential saturation see *Siscoe et al.* [2002a, 2002b, 2004]). Then the source term in (1) becomes

$$\alpha E_H = \alpha E/(1 + E/E_S), \tag{2}$$

where $E_H$ is the Hill form of the saturation electric field, and $E_S$ is the saturation value of $E_H$, which is reached when $E \gg E_S$. Another way to think of $E_S$ is that it is the value of $E$ at which $E_H$ reaches one-half of its saturation value. The value of $E_S$ is not well determined empirically, varying from about 3 mV/m [*Russell et al.*, 2000, 2003] to something closer to 10 mV/m [*Liemohn and Ridley*, 2002]. In the Hill formulation, $E_S$ depends on ionospheric conductance and solar wind ram pressure, so one would expect it to vary from event to event. To illustrate the point that is relevant here, which is to show that the BMR equation with a saturated electric field significantly underpredicts the *Dst* main phase, we pick $E_S$ = 10 mV/m, which minimizes the effect of saturation and thus minimizes the underprediction.

[13] Figure 2 compares *Dst* predicted by the linear and saturated forms of the *E* field in the original BMR equation. Their peak magnitudes differ by more than a factor of three. Each differs from the actual *Dst* by about 50%, in one case too big and in the other case too small. This shows that

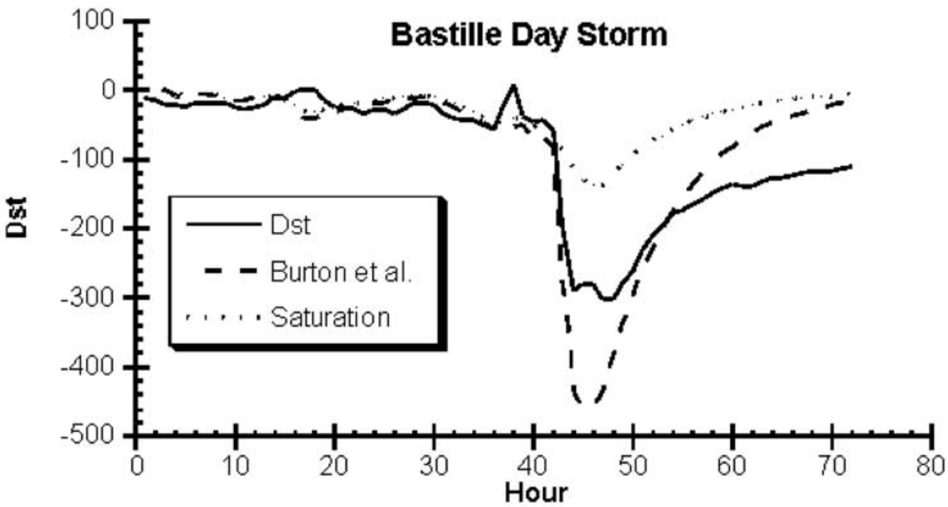


**Figure 2.** Same as Figure 2 showing the Burton equation *Dst* prediction given with a saturation form of the electric field.

simply modifying the BMR equation by substituting a saturated form for the electric field seems not to solve the problem of reconciling the two approaches to *Dst* prediction. We conclude therefore that if the BMR equation is to predict *Dst* using a saturation form of the driving electric field (and thereby become consistent in this regard with the RAM approach), the basic form of the source term in the BMR equation should be more sensitive to the driving electric field in order to compensate for the smaller driving strength that saturation entails. A quadratic dependence on the driving electric field instead of the original linear dependence, for example, might compensate for the smaller driving potential.

## 6. Allowable Linear and Quadratic Forms of the Source Term in a BMR-Like Equation

[14] This section treats the problem of finding a physically meaningful BMR-like equation in which the source term depends quadratically on the driving electric field instead of linearly. By BMR-like equation, we mean an equation in which the time rate of change of *Dst* equals a source term minus a decay term. Since the linear form of the source term in the original BMR equation was chosen primarily because it fit the data and not because it was physically mandated, any generalization, such as we seek here, that contains both the original linear form of the source term and an alternative quadratic form as options must emerge from an explicit interpretation of the physics behind the BMR equation. One possible interpretation is that the BMR equation expresses the operation of Faraday's induction law on a global magnetospheric scale in which the time rate of change of *Dst* is a proxy for the time rate of change of magnetic flux within the magnetosphere and the term linear in the electric field is a proxy for the associated EMF. This interpretation will not serve the present purpose, however, since by its nature the EMF term must be linear in the electric field and thus cannot be generalized to a quadratic form.

[15] A different interpretation, which as shown below does allow a quadratic source term, is that the BMR equation expresses conservation of energy into and out of the ring current. To pursue this interpretation, we apply the Dessler-Parker-Sckopke (DPS) equation, which relates the ring current's total energy to *Dst* [*Dessler and Parker*, 1959; *Sckopke*, 1966]

$$dDst^*/dt(nT/h) = -\beta(dW/dt)_{in}(watts) - Dst^*(nT)/\tau(hour), \tag{3}$$

where $\beta = 1.76 \times 10^{-10}$ nT/hr/watt is given by the DPS relation (with a 30% Earth-current amplification of the disturbance field), $(dW/dt)_{in}$ is the energy input rate to the ring current, and $\tau$ is the ring current decay rate. (The form of the DPS equation used in (3) omits a contribution from the magnetic energy of the ring current and from certain surface integrals as described by *Carovillano and Siscoe* [1973]. We proceed, however, under the usual assumption that the original DPS formulation is an adequate approximation, especially for an exploratory investigation such as this. The Carovillano and Siscoe reference also shows that the generalized DPS equation does not require the assumption of an axially symmetric ring current as in the original derivations. There is, however, a live issue on this topic that has not been explicitly addressed and resolved even in the cited general treatment. This is the issue of ionospheric closure of the partial ring current, as originally noted by *Burton et al.* [1975] and more recently by *Liemohn et al.* [2001]. That said, we continue nonetheless with the assumption that the original form of the DPS relation is adequate for the present purpose.)

[16] *Vasyliunas et al.* [1982] give a general form for the expression for the rate of energy input to the magnetosphere. This general form is based on the method of dimensional analysis, which says that any physically acceptable form for the energy input rate must be expressible as a dimensionally correct form for the input rate times some function of all the dimensionless variables that can be constructed out of the independent variables that might possibly affect the input rate. What form the function of dimensionless variables takes must be determined by the requirement that the final expression for the input rate should represent the physical mechanism that the author of the expression has in mind. By following this procedure, anyone having an idea about what the energy input mechanism is can write down an expression that, first of all, represents the idea and, second, can in principle be derived "from scratch" on the basis of a physically explicit argument. By contrast, any guessed-at expression for the energy input rate that cannot be obtained from the general dimensional considerations just stated also cannot be derived on the basis of a physically explicit argument. That is, such an expression is not a physical possibility.

[17] A general form for the energy input rate that applies to the present discussion is the following:

$$(dW/dt)_{in} = \rho V^3 \lambda_{CF}^2 Q(M_A, H). \tag{4}$$

Here $\rho$ and $V$ are mass density and speed of the solar wind; $\lambda_{CF}$ is the Chapman-Ferraro scale length (the characteristic size of the magnetosphere); $Q$ is the arbitrary function of dimensionless variables that, as mentioned, is needed in a dimensional analysis. The dimensionless variables are $M_A$, the Alfvén Mach number, and $H$, the ratio of the ionospheric Pedersen conductance to the Alfvén conductance of the solar wind. The following expressions give these variables in terms of solar wind and ionospheric parameters:

$$\lambda_{CF}^6 = M_E^2/\mu_o \rho V^2 \tag{5}$$

$$M_A^2 = \mu_o \rho V^2/B^2 \tag{6}$$

$$H = \mu_o \Sigma_P V. \tag{7}$$

$M_E$ is the magnetic moment of the geomagnetic dipole, and $\Sigma_P$ is ionospheric Pedersen conductance. The general form of (4) given by *Vasyliunas et al.* [1982] also includes a dimensionless variable for viscosity and an angle that specifies the orientation of the IMF. These we omit since viscous coupling has not been implicated in the production of magnetic storms, and the orientation of the IMF is implicitly included when we take the driving electric field to be the y-component of the IEF. (As noted above, however, this assumption might be too restrictive.)

[18] Equation (4) has the dimensions of energy per unit time as can be seen since $\rho V^3$ has the dimensions of energy per unit area per unit time, and $\lambda_{CF}^2$ has the dimension of area. This is therefore a dimensionally correct form for energy input. Any other form must be obtained by specifying the function $Q$. *Vasyliunas et al.* [1982] point out that the form of $Q$ that comes closest to matching the BMR equation's original input term ($\alpha E$) is $Q = 1/M_A$, which gives

$$(\mathrm{dW/dt})_{\mathrm{in}} = c_1 \gamma_p^{1/6} E, \tag{8}$$

where $\gamma_p$ denotes the solar wind ram pressure $\rho V^2$. As *Vasyliunas et al.* [1982] note, the source term in the original BMR equation and all subsequent variations of it have missed the sixth root of ram pressure as a multiplicative factor. This factor is necessary for the term to be derivable from a physically explicit mechanism. If the ram pressure factor were included, the coefficient $\alpha$ in the original equations would need to be reevaluated, which is why in (8) we have used a new constant $c_1$ in place of $\alpha$ in (1).

[19] The omission of the ram pressure factor in all previous applications of the BMR equation impacts the issue under discussion here (that the driving electric fields used by the BMR equation and the RAM algorithm are inconsistent under conditions of transpolar potential saturation) in the following way. If under conditions of transpolar potential saturation $\gamma_p$ and $E$ are positively correlated (which seems likely since saturation usually occurs during storm conditions when both factors are elevated), the energy input term given by (8) would increase faster than linearly with $E$, which, as noted at the end of section 5, is in the right direction for reconciling the Burton et al. and RAM approaches. Nonetheless, we proceed under the assumption that the weak, one-sixth power dependence on $\gamma_p$ renders this possibility for reconciling the two approaches implausible. To illustrate the point with the Bastille Day example, the ram pressure would have to increase more than 700 times to compensate for the factor of three underprediction of *Dst* found above using a saturated $E$ field in the BMR equation.

[20] Therefore it seems sensible to look beyond the possibility that the $\gamma_p$ factor might reconcile the two approaches and find instead a physically valid form of the energy input term in which the electric field enters quadratically instead of linearly. For this we take $Q = H/M_A^2 f(H)$, where $f(H)$ is a new arbitrary function of $H$, which turns (4) into

$$(\mathrm{dW/dt})_{\mathrm{in}} = c_2 \Sigma_P \Phi_r^2 f(H), \tag{9}$$

where $\Phi_r$ is the reconnection potential ($\sim VB\lambda_{CF}$) and $c_2$ is a dimensionless constant. Equation (9) is less general than (4) since we have specified an explicit dependence on $M_A$ to force a quadratic dependence on the driving electric field. An energy-input term similar to (9) in which the quadratic variable is taken to be the magnetic field (IMF) exists in the form of the much-used epsilon parameter of *Perrault and Akasofu* [1978]. One obtains a dimensionally correct form of the epsilon parameter from (9) by setting $f(H) = 1/H$ [*Vasyliunas et al.*, 1982].

[21] To show that the quadratic term in (9) is sufficient to achieve the sought-for reconciliation, we set $f(H) = 1$ to obtain the simplest, dimensionally correct form of the energy input term that is quadratic in the driving electric field

$$(\mathrm{dW/dt})_{\mathrm{in}} = c_2 \Sigma_P \Phi_r^2. \tag{10}$$

[22] Now we may ask: If we use (10) in the BMR equation and replace $\Phi_r$ with the saturation form of the transpolar potential, will we reproduce the empirical *Dst* curve? The saturation form of the transpolar potential as given by the Hill model, $\Phi_H$, can be written as

$$\Phi_H = \Phi_r/(1 + \Phi_r/\Phi_s), \tag{11}$$

where $\Phi_s$ is the value of the saturation potential. Since the denominator in (11) is dimensionless, it is clear that replacing $\Phi_r$ with $\Phi_H$ in (10) gives an energy input term that is consistent with the general form required by dimensional analysis (4). In fact, the ratio of potentials in the denominator can be written in terms of the dimensionless variables $M_A$ (equation (6)) and $H$ (equation (7)) as

$$\Phi_r/\Phi_s = a\, H/M_A, \tag{12}$$

where the dimensionless constant $a$ works out to be very close to 1.0 in the parameterization given by *Siscoe et al.* [2002a, 2002b].

[23] Having found a source term that is quadratic in the saturation-limited form of the driving electric field that is allowable under the general consideration of dimensional analysis, it remains to test whether as an energy input term in a BMR-like equation it can reproduce the empirical *Dst*. If so, a BMR-like equation thus reformulated represents a possible reconciliation between the IEF and transpolar-potential approaches to *Dst* prediction, in that both would be using the same saturation-limited form of the driving electric field.

## 7. Test of the Quadratic, Saturation-Limited Form of the Source Term

[24] Since our analysis does not determine the value of the multiplicative constant $c_2$ in (10), we carry out the test in the spirit of the original *Burton et al.* [1975] analysis; that is, we show that the $c_2$ chosen to fit one storm gives a good fit to other storms as well. To simplify the analysis, we ignore interstorm differences in ionospheric conductance and solar wind ram pressure. Moreover, as in our discussion in section 5, we assume that $\Phi_H$ in (11) is proportional to $E_H$ in (3) so that the energy input term may be written more conveniently in terms of the measured IEF

$$(\mathrm{dW/dt})_{\mathrm{in}} = \alpha' E_H^2, \tag{13}$$

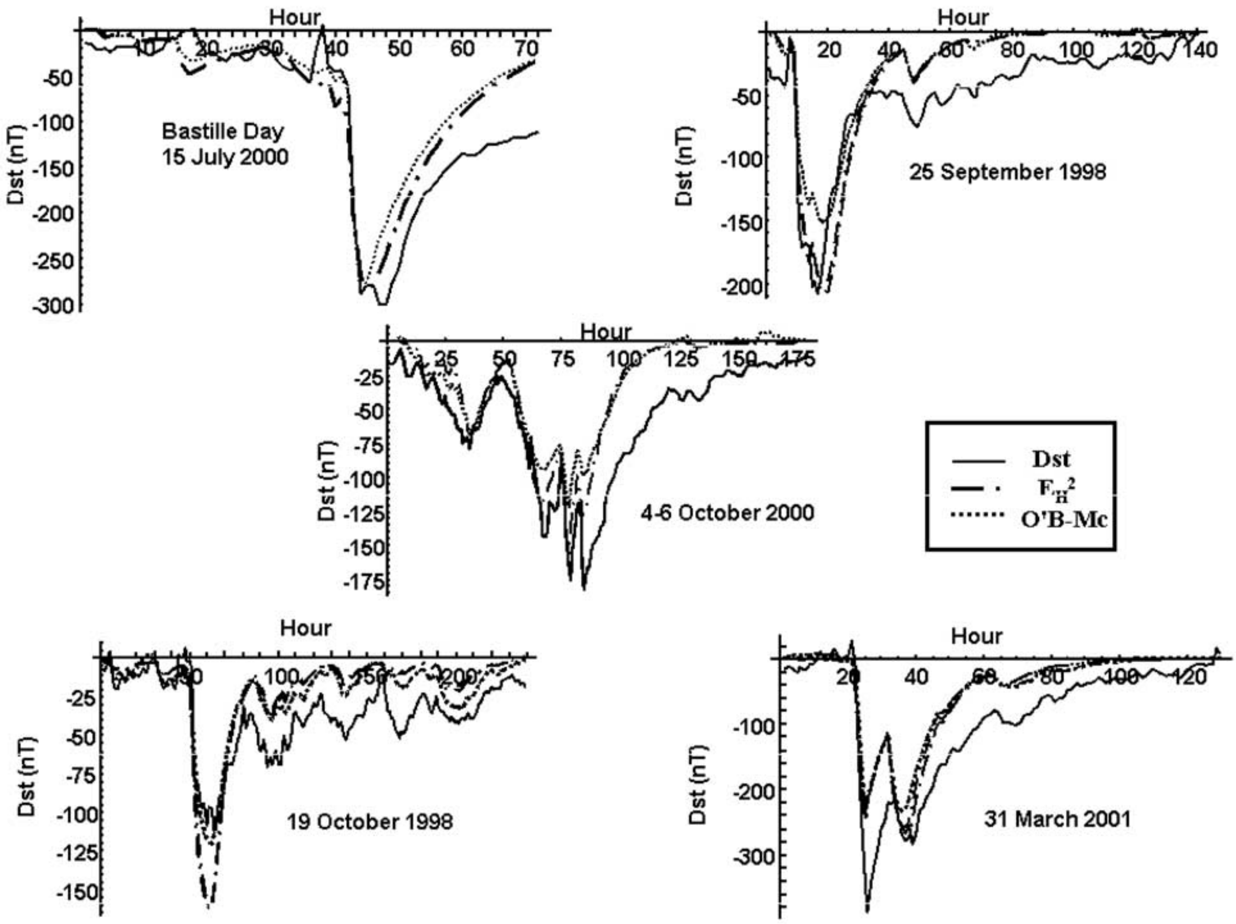


**Figure 3.** *Dst* profiles for five magnetic storms fit by the reformulated Burton et al. equation using the quadratic form of the energy input term (labeled $E_H^2$) and by the most advanced version of the Burton et al. equation using the linear form of the energy input term (labeled O'B-Mc).

where $\alpha'$ is a new constant corresponding to $c_2$, which must be determined by fitting to the *Dst* profile of some storm.

[25] Other preliminary remarks are needed. Whereas in section 5 we chose $E_S$ = 10 mV/m to minimize the effect of saturation on the result, here we choose $E_S$ = 7.5 mV/m to be more representative of the observed range of $E_S$ (roughly 3 mV/m to 10 mV/m). Recall that $E_S$ is expected to vary from one storm to another since it is a function of ionospheric conductance and solar wind ram pressure [*Siscoe et al.*, 2002a]. To make a meaningful comparison with *O'Brien and McPherron* [2002] (the most advanced formulation of the Burton et al. type), we replace the $\alpha E$ term in their formulation by $\alpha' E_H^2$. For consistency, we use the O'Brien-McPherron dependence of the decay constant, $\tau$, as a function of $E_H$ instead of a constant 7.7 hours as in the original Burton et al. formulation.

[26] Figure 3 shows the result of the test in which $\alpha'$ was chosen to give a good fit to the Bastille Day storm: $\alpha'$ = 2.9 nT/hr/(mV/m)$^2$. By itself, a good fit to the Bastille Day storm means that the $E_H^2$ formulation proposed here passes the entry-level test of fitting at least one *Dst* profile as well as the linear $E_H$ formulation of O'Brien-McPherron. Figure 3 shows further that the $E_H^2$ formulation (using the same value for $\alpha'$) also fits three of the four other storms as well as (or even better than) the O'Brien-McPherron formulation. The exception is the 19 October 1998 storm where the $E_H^2$ formulation overshoots the *Dst* profile. Apropos of poor fitting, neither the $E_H^2$ formulation nor the O'Brien-McPherron formulation correctly reproduces the decay phase of the storms. Possibly, this is owing to an ignored contribution from the y-component of the IMF (as previously mentioned) or to fluctuations in IMF $B_z$ that are faster than the 1-hour average values used here catch, but this has yet to be investigated.

[27] The decay-phase issue notwithstanding, Figure 3 demonstrates that, based on a source term that uses a saturation-limited form of the driving electric field and that satisfies the formal requirements of dimensional analysis (i.e., a BMR-like equation as reformulated here), it is possible for the energy-input approach to *Dst* prediction to give generally good fits to the main phase *Dst* of magnetic storms. This result demonstrates that there is at least one reconciliation possible (of course, there might also be others) between the IEF and transpolar-potential approaches

to *Dst* prediction. This particular reconciliation has interesting implications, which we shall point out in section 8 but otherwise pursue no further.

## 8. Implications of a Quadratic Source Term in a BMR-Like Equation

[28] The form of the quadratic energy input term (10) in which one substitutes the saturation form of the electric field for the IEF, namely, $c_2\Sigma_P\,\Phi_H^2$, can be converted by using the ionospheric Ohm's law to $c_3\ I_1\Phi_{TP}$, where $c_3$ is another constant, $I_1$ is the total current flowing in the region 1 current system, and $\Phi_{TP}$ is the transpolar potential (= $\Phi_H$ by assumption). However, $I_1\Phi_{TP}$ is the energy input rate to the ionosphere via the region 1 current system. Thus at face value, the result seems to imply that the rate of energy input to the ring current is proportional to the rate of electromagnetic energy input to the ionosphere. If this is the case, it requires a separate explanation. One possibility lies in the region 2 current system taking energy from the region 1 system and delivering it to the ring current during the storm main phase, as suggested by *Siscoe* [1982]. However, the rate of energy transfer to the ring current by this process was measured using the AMIE technique for the storm of 17 April 1994 and found to account for no more than 20% of the energy in the ring current [*Siscoe and Lu*, 1998]. A very similar result was found for the Bastille Day storm in connection with this study. If these examples are general, then some other cause of proportionality between energy flowing into the ionosphere and energy flowing into the ring current must be found.

[29] A second implication of the present reconciliation is no less unexpected. As Figure 3 shows, the original BMR equation and its modifications by O'Brien and McPherron achieve good fits to *Dst* with a source term that is linear in the driving electric field. If one ignores the result presented here, one might interpret the success of the BMR and O'Brien-McPherron algorithms to mean that a linear energy input rate to the ring current, as given by equation (8), actually applies. Indeed, a linear energy input rate has been suggested independently of the Burton et al. study [*Siscoe and Crooker*, 1974]. However, if one does not ignore the result presented here, that the energy input rate is, instead, quadratic in the driving electric field, the success of the BMR and O'Brien-McPherron algorithms must be expressing the operation of some principle other than the conservation of ring current energy. One possibility, as noted in section 6, is conservation of magnetic flux, since the BMR equation has the general form of Faraday's induction law, namely, the time rate of change of magnetic field is linearly related to an electric field.

## 9. Further Issues

[30] This study has focused on finding a formulation of the IEF approach to *Dst* prediction that is consistent with the transpolar-potential approach. The search appears to have been successful, but it raises other issues for further investigation. Two rather deep issues were described in the previous section. There are more prosaic issues as well, such as improving the quadratic form of the energy input term by including negative values of the driving electric field. Figure 3 shows the need for such an extension in that, whereas the RAM algorithm shows a dependence of the injection and decay rates on the IEF, the BMR and O'Brien-McPherron equations do not.

[31] Although we have focused on the term that represent the rate of energy injection into the ring current, the term representing the rate of decay is equally important. For example, one can understand the improvement that the O'Brien-McPherron equation makes over the original BMR equation in terms of a better representation of the rate of decay. This and the other issues mentioned are subjects for further investigations.

## 10. Summary

[32] The IEF and transpolar-potential approaches (as we have dubbed them) to *Dst* prediction, as epitomized by the BMR equation and the RAM algorithm, respectively, use different functions of the interplanetary electric field to depress *Dst*. The BMR equation uses a linear function whereas the RAM algorithm uses (implicitly) a function that saturates at high values, characteristic of storm times. To illustrate the difference, if the saturation form of the electric field that (implicitly) drives the RAM algorithm is used to drive the BMR equation, it significantly underpredicts *Dst*. This paper has shown that the IEF approach can be made consistent with the transpolar-potential approach by reformulating the BMR equation such that the source term is quadratic in the driving electric field instead of linear. Then the two approaches use the same functional form of the driving electric field and give about the same goodness of fit to storm time *Dst*. The paper constructs a form for the quadratic energy input term by specializing a general expression given by *Vasyliunas et al.* [1982] for all such terms that are physically allowable.

[33] Accepting the proposed reconciliation, with its particular quadratic form for the energy input term, has two interesting implications. (1) The rate at which energy enters the ring current is proportional to the rate at which energy enters the ionosphere via the region 1 current system. (2) The original BMR equation with its linear source term reveals the presence of a principle other than conservation of ring current energy connecting the IEF and *Dst*, possibly magnetic flux conservation. These are topics to be explored.

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