SOME NOTES ON DIRECTIONAL POSITIONING THRUSTER FOR MOHOLE PROJECT

For Avondale Shipyards, Inc.

Project Director: Professor R. B. Couch

H. C. Kim

R. Zitner

Ann Arbor, Michigan

May, 1963

SOME NOTES ON DIRECTIONAL
POSITIONING THRUSTER
FOR
MOHOLE PROJECT

For Avondale Shipyards, Inc.

Project director: R. B. Couch
H. C. Kim
R. Zitner

Ann Arbor, Michigan
May, 1963

ACKNOWLEDGEMENT

We would like to thank Professor F. C. Michelsen for his assistance in carrying out the analysis.

H. C. Kim

R. Zitner

ABSTRACT

One-dimensional momentum theory with empirical data on losses is applied to a thrust delivering device designed to be used for Mohole Project. The methods of estimating the maximum thrust and finding the best nozzle diameter including step by step calculations is presented. Design data on propeller pumps is obtained in the course of the investigation. Finally, the present system is evaluated and recommendations forwarded.

INTRODUCTION

The thrust delivering device per Avondale Shipyards, Inc. Drawing No. H4801 D-4 [1] has a vertical propeller pump of 9'-0" diameter operating in a 9'-1" diameter tube. Water is pumped up to the four-branch tee and is ejected through either one or up to four nozzles that are set in four different directions fixed 90 degrees apart. The required thrust is obtained by momentum exchange in the system, the lift on the propeller blades not being utilized. The combined magnitude and direction of the thrust is governed by the gate-valves located in each exit tube. The pump assembly driving motor, 2200 Hp at 870 maximum rpm, is situated above and the pump is driven by a vertical shaft through a variable reduction gear of from 1 to 1 to 3½ to 1.

The system may be schematically shown as in Fig. 1.

This paper shows an analytical treatment of estimating the maximum thrust and outlines a method of selecting a suitable pitch angle of the propeller pump at the operating revolutions.

Empirical data are extensively used throughout to compensate for the ideal case treated in the theory. Whenever required data were not available directly, extrapolated data from the published sources were used.

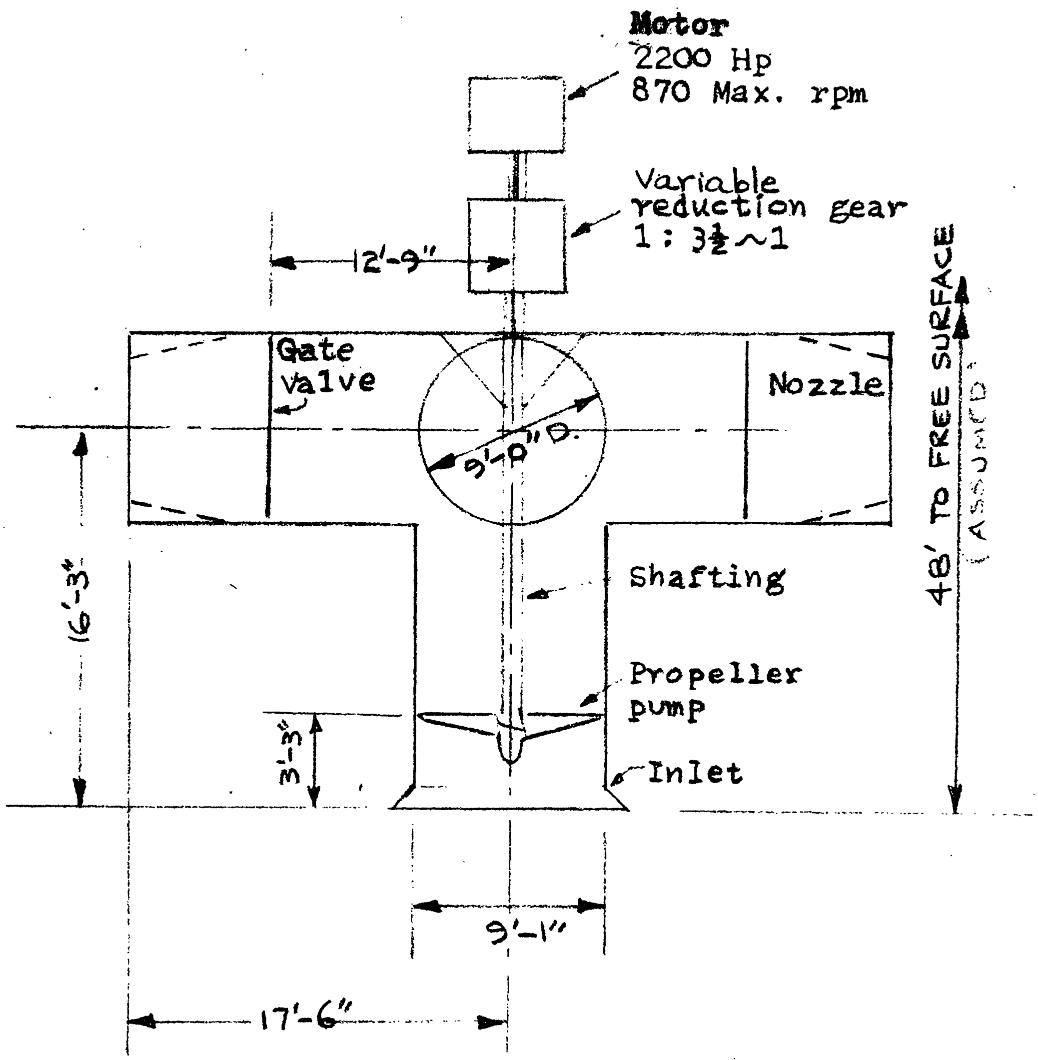


Fig. 1. Schematic Drawing of the System

The treatment is quite simple and general. This method is, therefore, in principle applicable to all preliminary design problems dealing with pump jet propulsion. With better design data, especially those on inlet and nozzle, the method could be greatly improved.

METHOD OF ANALYSIS

Consider an idealized system shown in Fig. 2. We will apply the concept of average velocity and one-dimensional momentum equation. Quantity of water, Q $[ft^3/sec]$, is pumped via the inlet (subscript 1); energy is added by the pump; and water is ejected through a nozzle (subscript 2) in the form of a jet. Total head loss in [feet] in the system is designated as h_L. Definitions of the terms are given in the appendix A.

Between control planes 1 and 2

1) Continuity Equation

$$Q = A_1 v_1 = A_2 v_2 = \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

2) Bernoulli's Equation

$$H = p_1/\gamma + v_1^2/2g + z_1 = p_p/\gamma + v_p^2/2g + z_p$$

$$= \text{constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where subscript p designates the lower side of pump plane.

Bernoulli's equation in this simple form cannot be applied across the pump plane. A similar equation for upper side of the pump may be written, however.

On the Fig. 2 are shown energy grade line of the system. The figure is self-explanatory, but a few comments may be in order. The energy carried away by the jet and the energy lost in the system must be supplied by the pump alone. Since the kinetic energy term in the uniform pipe section is constant except at the vicinity of the inlet and the nozzle, the energy supplied by the pump must be in the form of pressure increase at the pump plane. The function of a nozzle is to convert the pressure energy to kinetic energy, which gives the contribution to the thrust. The kinetic energy contained in the jet is eventually lost. It can be further noted that the friction losses in the system must be minimized.

3) Momentum Equation

$$\sum \text{External forces} = \frac{d}{dt} \sum_i (\rho Q v)_i$$

$$p_1 \bar{A}_1 - (\bar{T} - \bar{F}) - p_2 \bar{A}_2 = \rho Q(\bar{v}_2 - \bar{v}_1) \qquad (3)$$

where $\bar{F}$, $\bar{T}$ are the forces exerted on the fluid by the pipe and by the disc propeller respectively. $(\bar{T} - \bar{F})$, the reaction, is the force on the system. See Fig. 2.

If we assume the control plane 2 to be at the "vena contracta", which will be explained later, p_2 is the ambient pressure which can be put to zero.

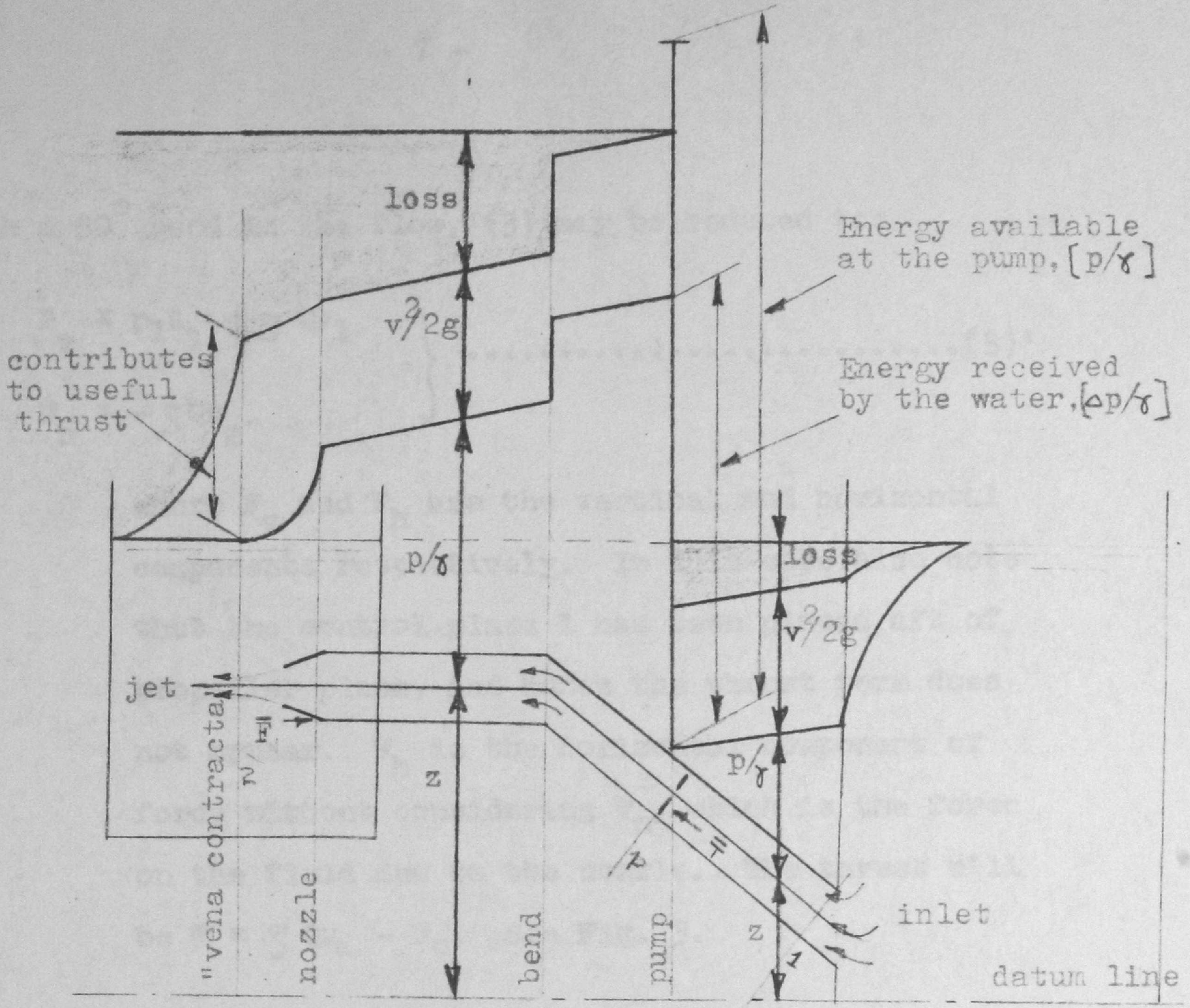


Fig. 2. Idealized system with Energy grade line [ft]

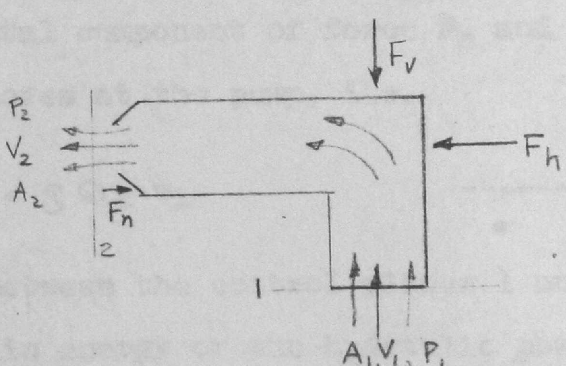


Fig. 3. System with a 90° bend

With a 90° bend in the flow, (3) may be reduced to:

$$\left.\begin{aligned} F_v &= p_1A_1 + \rho Qv_1 \\ F_h &= -\rho Qv_2 \end{aligned}\right\} \qquad (3)'$$

where F_v and F_h are the vertical and horizontal components respectively. In this case also note that the control plane 1 has been placed aft of propeller plane, and hence the thrust term does not appear. F_h is the horizontal component of force without considering F_n, which is the force on the fluid due to the nozzle. The thrust will be $T = \rho Qv_2 - F_n$. See Fig. 3.

4) Energy Equations

The rate of work-done is defined to be the product of horizontal component of force F_h and the rate at which the fluid moves at the pump, i.e.

$$\dot{W} = \rho Qv_2 \cdot v_1 \qquad (4)$$

Between the control planes 1 and 2, the rate of change in energy or the hydraulic power in $\left[\frac{\text{ft-lbs}}{\text{sec}}\right]$ is given by:

$$Q\gamma\left[\frac{v_2^{\,2}-v_1^{\,2}}{2g}+h_L\right] \qquad \text{---------------} \qquad (5)$$

5) Efficiency

The jet efficiency is defined as the ratio of the rate of work-done to the rate of energy change in the water, i.e.

$$\eta_j = \frac{\rho Q v_2 v_1}{Q\gamma\left[\frac{v_2^{\,2}-v_1^{\,2}}{2g}+h_L\right]} \qquad \text{---------------} \qquad (6)$$

The hydraulic power $Q\gamma\left[\frac{v_2^{\,2}-v_1^{\,2}}{2g}+h_L\right]$

$\left[\frac{ft\text{-}lb}{sec}\right]$ is the rate of change in energy in the water. With a given power at the pump only a portion of its power is actually transmitted into the water, which is to say that the pump is operating at a certain efficiency ηp.
From the driving motor a certain portion of power is lost in the transmission system such as shafting and reduction gears. We will designate the transmission efficiency as ηm. We now have

$$\gamma Q\left[\frac{v_2^{\,2}-v_1^{\,2}}{2g}+h_L\right] = \text{BHP at the engine}\cdot\eta_m\cdot\eta_P \qquad \text{----------} \qquad (7)$$

therefore,

$$\eta_j = \frac{\rho Q V_2 V_1}{BHp \times \eta_m \times \eta_p} \quad \ldots\ldots\ldots\ldots (8)$$

which becomes:

$$\eta_j \times \eta_m \times \eta_p = \frac{\rho Q V_2 V_1}{BHP} \quad \ldots\ldots\ldots\ldots (9)$$

The term on the right hand side is the ratio of the rate of work done to the horsepower input at the engine, which we will define as the overall system efficiency.

$$\eta_{overall} = \eta_m \times \eta_{pump} \times \eta_j \quad \ldots\ldots\ldots\ldots (10)$$

Written in this form, the above equation idicates that the system must not only be efficient in converting the power into thrust (jet efficiency), but also it must be able to absorb the given power efficiently (pump efficiency). The transmission efficiency η_m may be taken close to 95%.

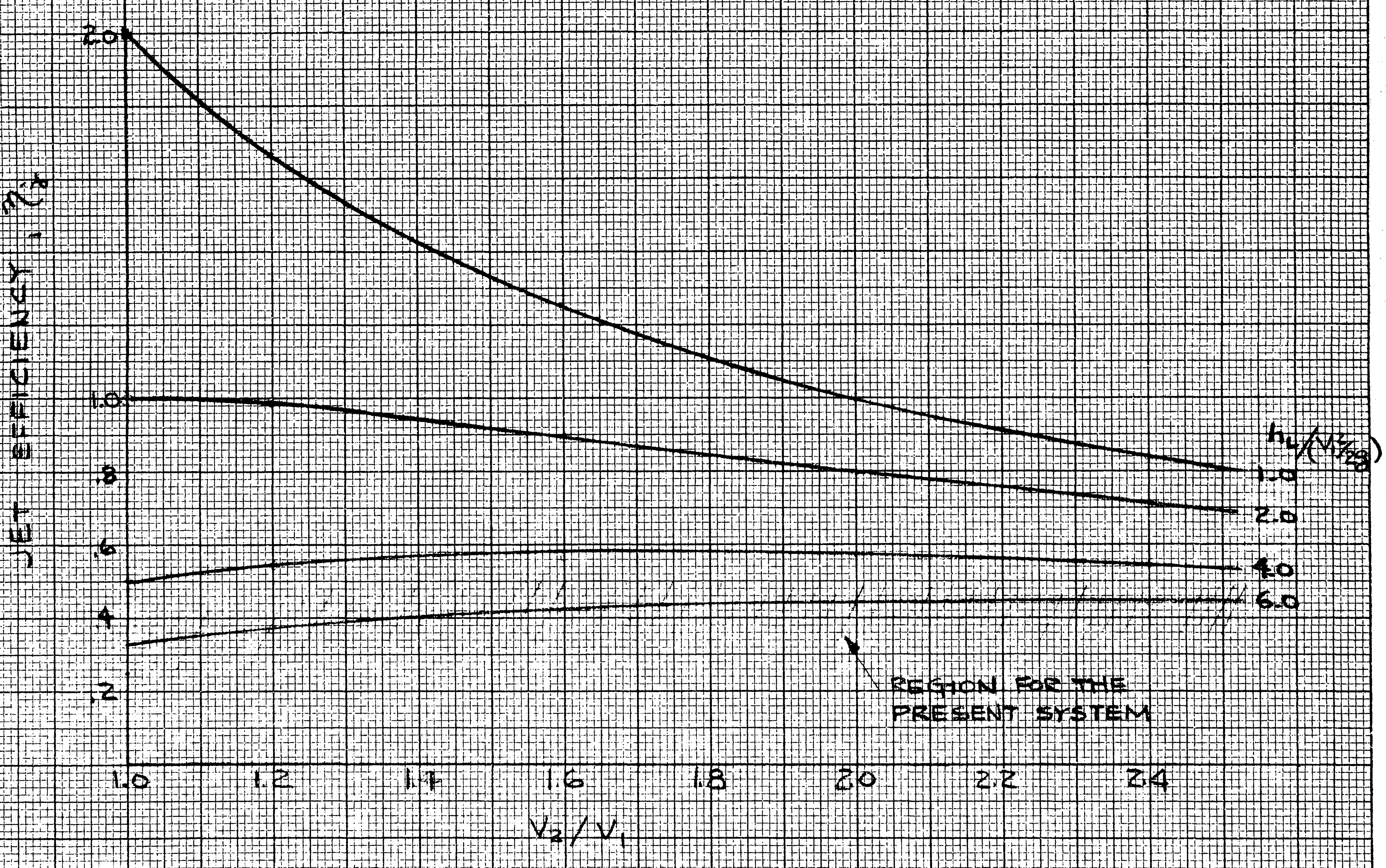
In order to carry out a simple explanation, (6) is non-dimensionalized as follows.

$$\eta_j = \frac{2(V_2/V_1)}{\left[\left(\frac{V_2}{V_1}\right)^2 - 1.0\right] - h_L/(V_1^2/2g)} \quad \ldots\ldots (11)$$

where $h_L/(V_1^2/2g)$ is nondimensionalized loss in the system.

A plot of equation (11) is shown in Fig. 4.[(2)] It is noted that:

FIG. 4. JET EFF. vs LOSSES & V_2/V_1



1) The term jet efficiency is somewhat misleading. It is quite evident that there is really no work done as a system, and $\dot{W}$ in (4) is defined so only for our convenience. "Jet efficiency", therefore, is a fictitious term which gives us a measure of judgement. It is not an efficiency used in normal sense which varies from 0% to 100%.

For example, in Eq. (11) when $h_L = 0$ and $V_2 = V_1$, we note that the η_j becomes infinity. In actual case, however, we cannot think of a system with a finite velocity and no loss. Loss term in reality is large enough to offset the values in denominater. Also V_2 is always greater than V_1 due to the "vena contracta".

2) With an actual system with finite loss, the attainable maximum efficiency is far less than the ideal efficiency.

3) With a given value of loss, there is a corresponding optimum v_2/v_1. This v_2/v_1 may not be easily found since any change in v_2/v_1 must be accompanied by the corresponding change in loss. But for a practical system, it is quite evident that the system should have a throttle or a reducing nozzle. An actual experiment carried out by NACA proves this.[3] Although the said experiment was for the compressible gas, at low Mach numbers the result is qualitatively applicable to

the case of incompressible fluid.

The major portion of current investigation includes the steps of finding the best nozzle diameter. The method of analysis is summarized as follows:

1). Assume a number of flow velocities v_1 or quantity flows Q. Assume a number of throttling nozzles either in terms of area ratio or diameter ratio.

2). For each of these combinations, calculate thrust from (3)', hydraulic power from (5), jet efficiency from (6), head increase at the pump plane which is equal to the total change in Bernoulli head in the system from (2), and the specific speed of the pump.

3). Select the pump from pump curves with the specific speed calculated. Net positive suction head must be calculated and the possibility of cavitation must be checked at some stage of this process. A curve such as given in Ref. 10, p. 221, is used.

4). From the product of pump efficiency and the jet efficiency and with the available brake horsepower, v_1 or Q, v_2/v_1 and nozzle diameter giving the best system efficiency are selected. System efficiency is calculated from (10). The transmission efficiency may be considered independent of these combinations.

5). Final result is interpolated for the v_2/v_1 and nozzle dimension.

6). Pitch distribution of propeller pump is from:

$$\tan = \frac{v_1}{2 \ n \ r} \quad \ldots\ldots\ldots\ldots (12)$$

If a bisymmetrical section is used it is desirable to increase the angle slightly. Then, the geometric pitch angle ϕ is:

$$\phi = \beta + (0 \sim 3.0)^{\circ} \quad \ldots\ldots\ldots\ldots (13)$$

The above is based on the one-dimensional analysis. To consider the non-uniformity of velocity distribution, the pitch may be slightly reduced toward the tip and increased toward the center so as to conform to the velocity profile for the axisymmetric case at the given Reynolds number.

In the above, by far the most important and laborious item to analyze is various losses due to friction in the system. The success of the method presented here depends much on how well the losses for each segment are evaluated.

The following shows how these estimates were made for the present problem. We will follow step by step from the inlet to the nozzle.

LOSSES IN THE SYSTEM

The losses may be expressed in either length h_L or pressure drop Δp. If h_L is used it is to be added to the actual length of the pipe. In order to better estimate the losses, Fig. 1 is reproduced as follows. Subscripts 1, 2,... designate the approximate locations in the diagram as shown. See Fig. 5.

A. Inlet

The point 0 is far away from the inlet but the elevation from the datum line is assumed to be the same as that of point 1. Apply the Bernoulli's equation between 0 and 1 (See Fig. 6).

$$H = \frac{p_1}{\gamma} + v_1^2/2g = p_0/\gamma + v_0^2/2g \quad \text{. . . . .(14)}$$

But $v_0 = 0$. If the datum line is taken at the free surface level, p_0 is the hydrostatic pressure, and we can calculate the inlet pressure p_1 at the point 1.

Inlet loss may be expressed by the following Darcy's formulas:[4]

$$\left.\begin{aligned} \Delta h_{L,1} &= f(L/D)v_1^2/2g \; [ft] \\ \Delta p_1 &= \frac{\rho f L v_1^2}{D2g} \; [psf] \end{aligned}\right\} \quad \text{. . . . .(15)}$$

$f(L/D)$ is sometimes expressed by K and are shown in Fig. 7 for various mouth configurations.[4] For the case we have, K may be taken as approximately equal to 0.15. If a better estimate is desired the following method is suggested.

The general discharge equation through an orifice is given by:

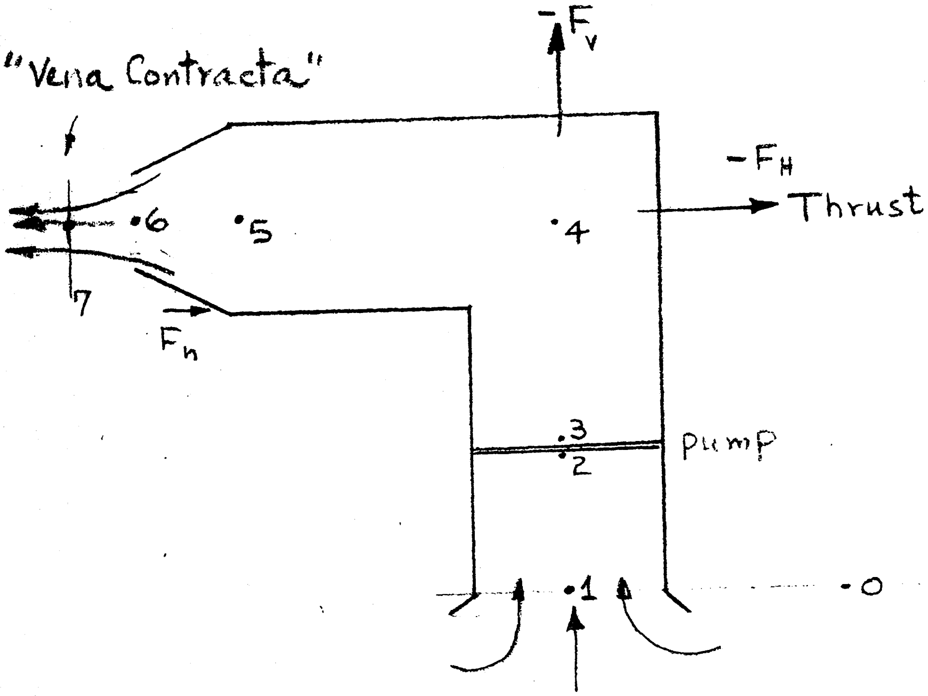


Fig. 5 Scheme of analysis of losses

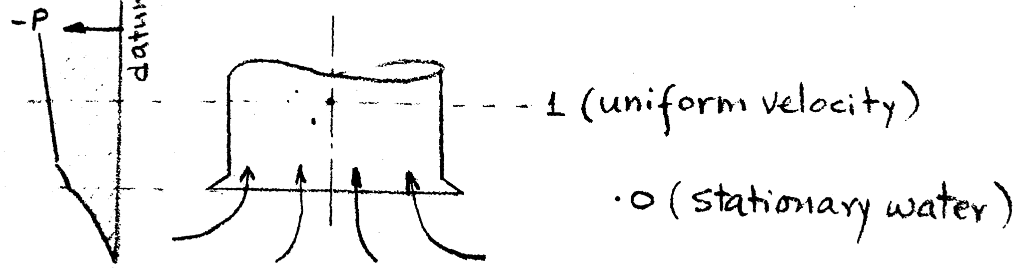


Fig. 6 Pressure Drop At the Inlet

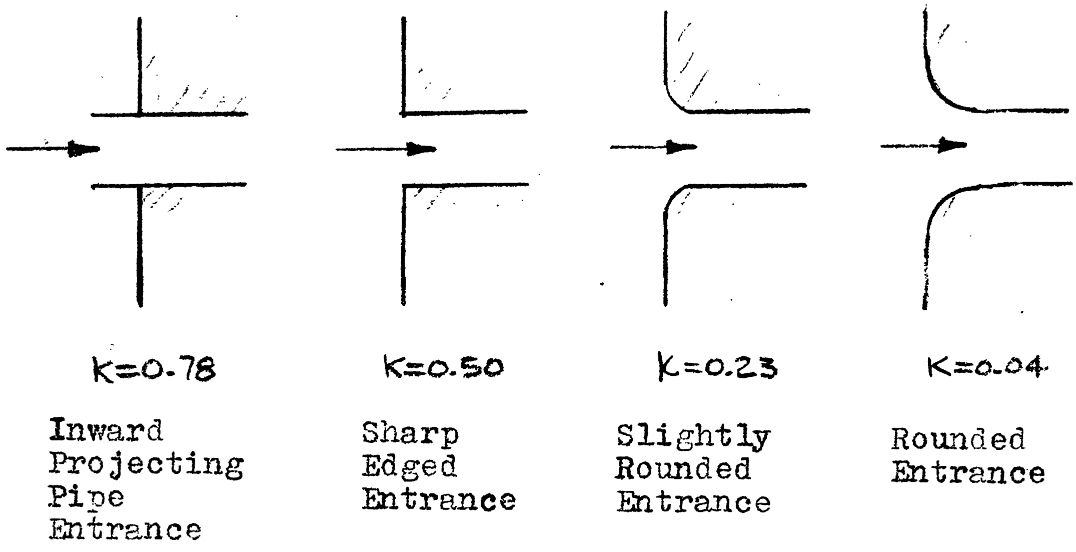


Fig. 7 Resistance Factor K for Pipe Entrance

$$Q = C_o A \left(\frac{2g\,\Delta p}{\gamma}\right)^{\frac{1}{2}} \qquad \ldots\ldots\ldots\ldots (16)$$

where Δp is the pressure drop across the contraction and C_o is the orifice coefficient.

The orifice coefficient is also the ratio of the actual discharge to the theoretical discharge. It is a function of geometry and Reynolds number. (It increases with increasing Reynolds number.) Theoretical as well as empirical data are available.(5,6,7) Figure 8 is reproduced from the references to be used for the present analysis.

For the loss calculation equation (13) is converted to read the pressure drop as follows.

$$\Delta p = \frac{Q^2 \times \gamma}{2g \times A^2 \times C_o^2} \; [\text{psf}] \qquad \ldots\ldots\ldots\ldots (17)$$

The results from (12) and (14) check well with each other for the present problem.

B. Net Positive Suction Head

Besides the pressure drop calculated by (17) we must add the pressure drop due to the friction in the straight pipe and the pressure drop due to the change in elevation. The pressure drop in the straight pipe is calculated from (15) using a roughness factor of $\epsilon/D = 0.000001$. The friction factor f is read from Moody diagram as a function of Reynolds number. The sum of the above is the NPSH (Net Positive Suction Head). In order to avoid cavitation we must not allow the pressure at the suction side of the pump to become lower than the vapor pressure of the water. A a matter of precaution we will further allow some 25% to the calculated NPSH.

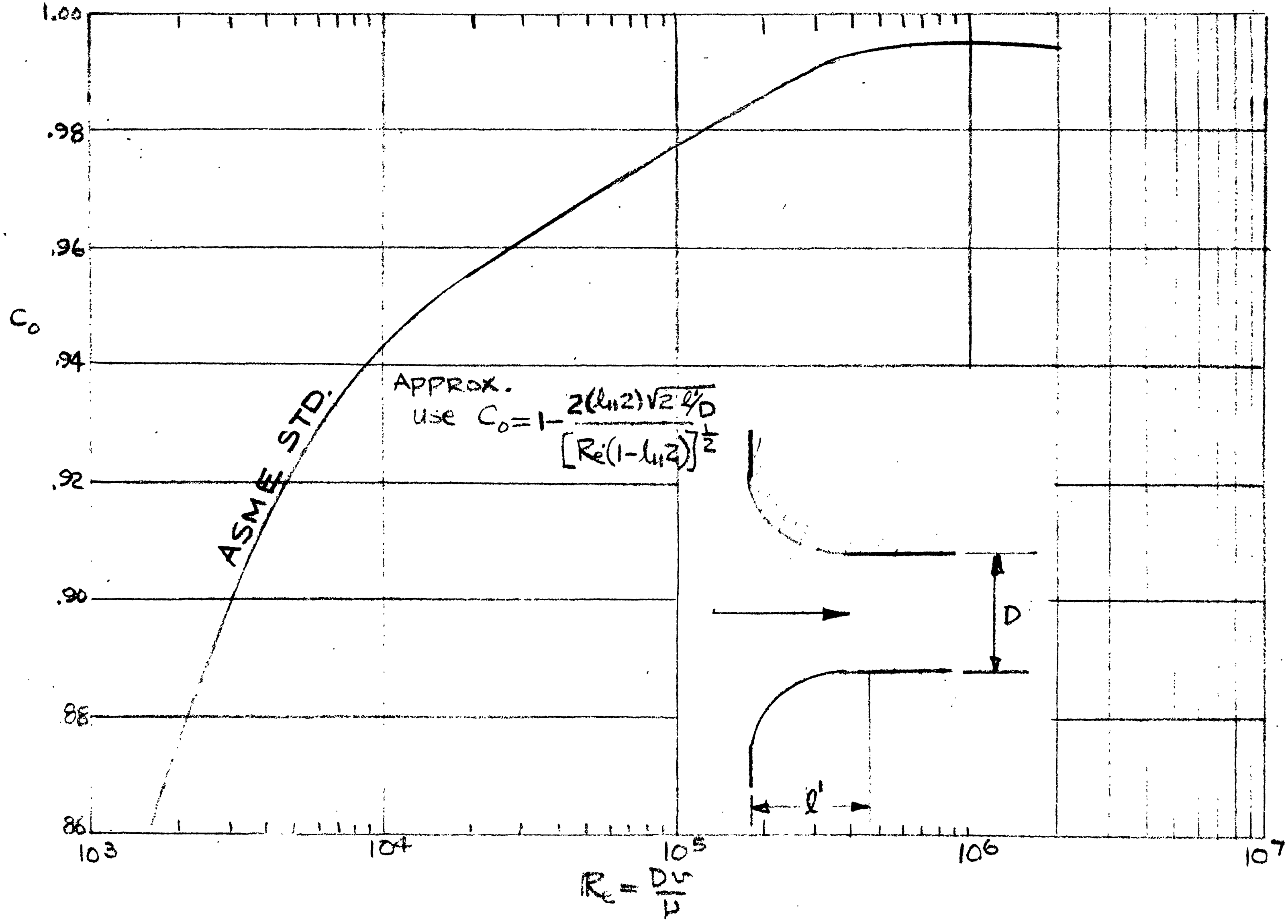


Fig. 6 Orifice Coefficient for Inlet

C. Pressure Increase at the Pump

Energy delivered by the pump must appear as an increase in pressure since the fluid velocity remains the same. FN(1) This pressure increase must be such that it would cancel the pressure losses prior to and aft of the pump, and that at the jet "vena contracta", the pressure must become the ambient pressure. The pressure increase multiplied by the total area is equal to the force on the disc, or the thrust. The propeller design may be carried out by knowing the pressure increase, the flow velocity and the revolutions.

D. Four-Branch Tee

The equivalent length of "Tee" section with cross-flow may be estimated from the extrapolated data such as shown in Fig. 9.[8] The data were available up to 24" diameter, but they can be plotted as a straight line on a log-log paper. Hence, the extrapolation may be justified.

Shafting and other obstructions are taken into consideration by allowing 15% to the above. Another 10% is added to account for four-branch "Tee" since the available data is good only for two-branch "Tee".

FN(1) The continuity equation for the axially symmetrical case is given by:

$$\partial V_r/\partial r + \partial V_z/\partial z = 0$$

V_r component after a while will disappear to become pressure head. The axial velocity V_z is actually smaller in the pump plane then what one-dimensional analysis would indicate.

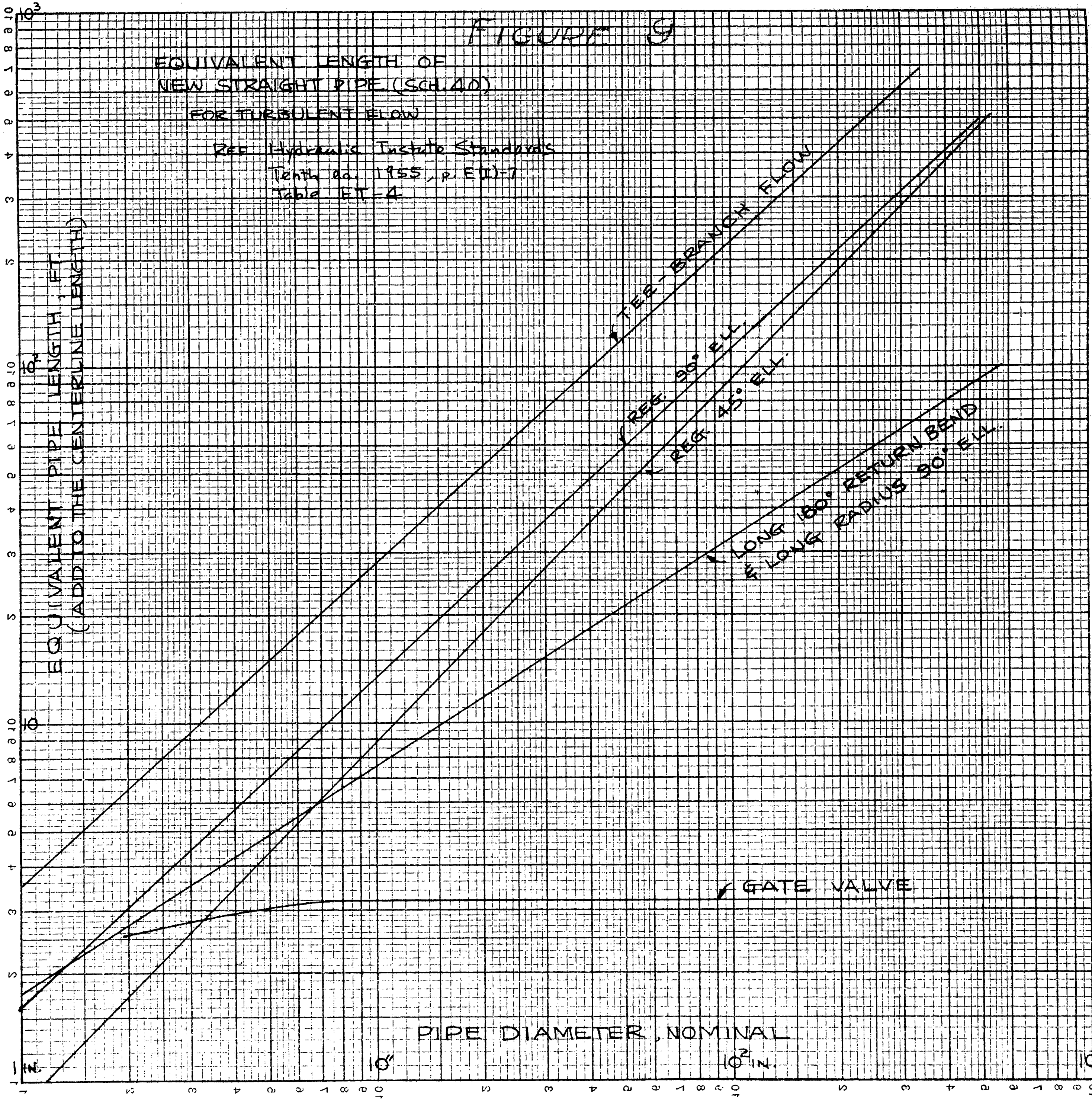


FIGURE 9
EQUIVALENT LENGTH OF
NEW STRAIGHT PIPE (SCH. 40)
FOR TURBULENT FLOW
REF Hydraulic Institute Standards
Tenth ed. 1955, p. EII-7
Table FT-4
TEE - BRANCH FLOW
REG. 90° ELL
REG. 45° ELL
LONG 180° RETURN BEND
& LONG RADIUS 90° ELL
GATE VALVE
EQUIVALENT PIPE LENGTH, FT.
(ADD TO THE CENTERLINE LENGTH)
PIPE DIAMETER, NOMINAL
1 IN.
10'
10² IN.
10³ IN.
10
10²
10³

E. Gate-Valve

The equivalent length of a gate-valve when fully open may be taken as 3.2 ft, irrespective of the size. No attempt was made to estimate the pressure drop when one of the other valves is half-open. It is expected that data on square orifices would apply to the latter.

F. Elevation

Use the centerline dimensions.

G. Throttling Nozzle

As has already been discussed, the optimum nozzle will not be a straight pipe. In order to find the optimum nozzle to tube diameter ratio, we will simply take several different combinations and carry through the calculations previously outlined. Presently we have taken:

1) straight nozzle d/D = 1.0; m = 1.0
2) reducing nozzle d/D = 0.92; m = .85
3) d/D = 0.805; m = .65
4) d/D = 0.706; m = .50
5) d/d = 0.547; m = .30

where d is the nozzle diameter.

For each of these nozzles, the pressure drop is calculated from (17). In using (17) the discharge coefficient for an outlet is found from the data given in Fig. 10 [9] ASME standard nozzle data were used. This type of nozzle is not of the optimum shape. A good nozzle avoids a sudden reduction. Data for gradually reduced nozzles with small cone-angles are not available at present.

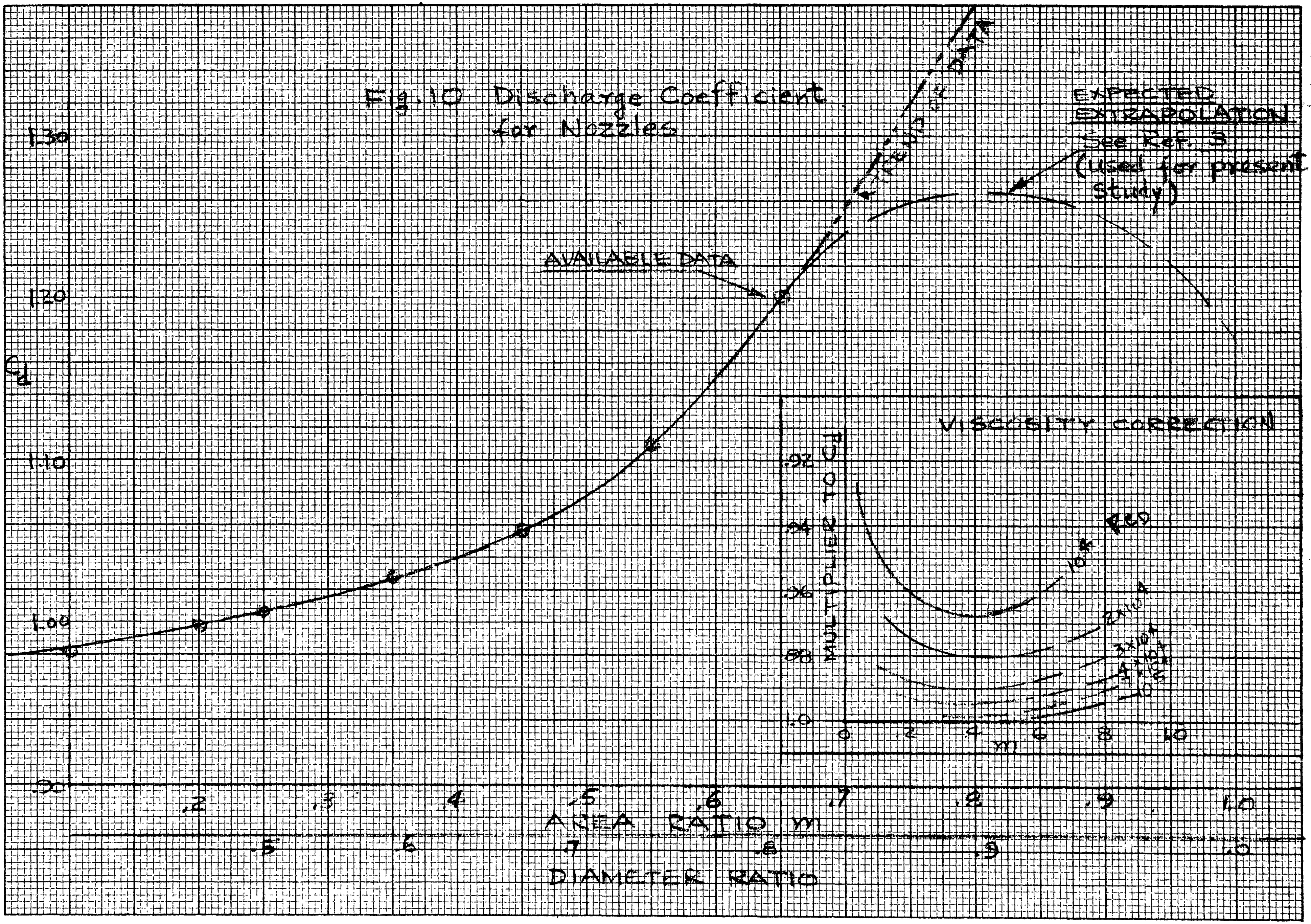


Fig. 10 Discharge Coefficient for Nozzles

Since the ambient pressure outside of the nozzle is known, we are able to find the pressure inside the nozzle.

For a jet ejected, the point where the streamlines are perfectly parallel to the centerline of the nozzle is slightly beyond the exit end of the nozzle. This point, as previously refered, is called "vena contracta". Cross-sectional area is normally slightly smaller at this point compared to the nozzle cross-sectional area and the pressures inside and outside of jet must necessarily become the same. In using momentum theory we must use the velocity at this "vena contracta". It poses certain difficulties, however, since area is unknown and, hence, the velocity is also unknown. In actual flow, nozzle geometry and viscosity play roles but the orifice coefficient empirically determined takes account of the combined effect of these two. In general the effect of viscosity is predominant, and when the data is insufficient the cross-sectional area of "vena contracta" due to geometry alone may be taken as approximately 0.98 to 1.0 times the nozzle outlet area.[9] Reduction in flow area due to the boundary layer thickness may be analytically calculated. Exit velocity is obtained from the following formulas:

Between three points 5, 6 and 7,

1) Continuity equation

$$A_5 v_5 = A_7 v_7 = A_6 v_6 \qquad \text{.............(18)}$$

2) Bernoulli's equation

$$\frac{p_5 - p_7}{\gamma} = \frac{1}{2g}\left(v_7^2 - v_5^2\right) \qquad \text{.............(19)}$$

Introduce a "vena contracta" contraction factor ξ such that

$$A_7 = \xi A_6 \qquad \text{.............(20)}$$

and a friction and viscosity factor such that

$$v_7 = \zeta v_6 \qquad \text{.............(21)}$$

With these, the volumetric rate of flow is expressed as

$$Q = \frac{\xi}{\left[1 - \frac{m^2}{\zeta^2}\right]^{\frac{1}{2}}} A_6 \left[\frac{2g}{\gamma}(p_5 - p_7)\right]^{\frac{1}{2}} \qquad \text{..........(22)}$$

$$= C_d A_6 \left[\frac{2g}{\gamma}(p_5 - p_7)\right]^{\frac{1}{2}}$$

where C_d is the discharge coefficient defined and as given in Fig. 10, and $m = A_6/A_5$.

From equation (19) we have:

$$\xi = C_d \left[1 - \frac{m^2}{\zeta^2}\right]^{\frac{1}{2}} \qquad \text{.............(23)}$$

Also from (15),

$$V_7 = A_6 v_6 / A_7 \qquad \text{.............(24)}$$

$$= \frac{v_6}{\xi}$$

Substituting (20) into (15)', we obtain:

$$v_7 = (v_6/C_d)\left[1 - \frac{m^2}{\zeta^2}\right]^{-\frac{1}{2}}$$

Since $\zeta = v_7/v_6$, solving for v_7,

$$v_7 = v_6 \left[\frac{1 + C_d^2 m^2}{C_d^2}\right]^{\frac{1}{2}} \qquad \text{.............(25)}$$

RESULTS

The result of caluclations are presented in graphical forms. Calculations pertaining to these are found in Appendix B.

1) Fig. 11.....Thrust vs. flow velocity for various nozzle area ratios. The arrows indicate the obtainable thrust with 2200 hp @ 3½ to 1 gear ratio.
2) Fig. 12.....Hydraulic horsepower vs. flow velocity for various nozzle area ratios.
3) Fig. 13.....Get efficiency vs. flow velocity for various nozzle area ratios.
4) Fig. 14.....Efficiency of commercial pumps (Extrapolated data used for the study) vs. Pump Specific Speed.
5) Fig. 15.....Specific speeds of pump propellers vs. flow velocity for various nozzle area ratios.
6) Fig. 16.....Estimated System efficiency
7) Fig. 17.....Brakehorsepower required at the propeller pump vs. area ratio and flow velocity. The arrows indicate the flow velocities obtainable with 2200 BHP @ 3½ to 1 gear ratio and $\eta_m = 0.95$.

Entering Fig. 17 with an available horsepower at the pump, we obtain the attainable flow velocity for given nozzle area ratios. With these flow velocities the obtainable thrusts are read off from Fig. 11. Other pertinent results are interpreted in a similar manner.

The result obtained in the above is correct only for non-cavitating conditions. When cavitation is considered the flow velocity should not exceed about 12 ft/sec at 250 revolutions as indicated on Fig. 15. The propeller revolutions, therefore, must be reduced to overcome the cavitation. The attainable velocities are now imposed by two lines, the maximum horsepower line (horizontal line at 2100 hp) and the cavitation limit line as indicated on Fig. 17.

With the above considerations the following table has been constructed for convenience.

Table I Summary of Result

	m=.50	m=.65	m=.85	m=1.0
v_1 [ft/sec]	11.00	12.3	12.5	12.5
Q [ft^3/sec]	700	783	795	795
HHP	1500	1100	1000	1000
Thrust [lbs]	19000	25800	26500	27500
η_j	.44	.66	.63	.63
η_p	.71	.69	.66	.66
η_m	.95	.95	.95	.95
η_s	.30	.43	.40	.40
N_s	22500	25500	30000	30000
Force on Disc Area [lbs]	49000	42000	34000	34000
F_v [lbs]	17000	20000	22000	22000
Heaving Force (Down) [lbs]	32000	22000	12000	12000
β @.7r	7°35'	Engine Hp Curve Needed to Calculate β and ϕ		
ϕ @.7r	appr. 9°			

DISCUSSION ON RESULTS

The result indicates that the present system would generate approximately from 26,000 to 27,000 pounds lateral thrust. The optimum nozzle area ratio according to the result in Fig. 11 is a straight nozzle. This is contrary to what was expected on the basis of preliminary studies which made use of extrapolated discharge coefficients.

In estimating the outlet velocity from the nozzle the discharge coefficient as given in Fig. 10 was used. Data are available in the literature for the range of m = .20~.65. These were extrapolated for m = .85 and m = 1.00. The extrapolation was based on another data source which gave the discharge coefficient for the case of m = 1.0.[3] The values used may have perhaps overestimated the exit velocity and, hence, influenced the final result in favor of the straight nozzle. Had we used the other extreme case (indicated by the "trend of data" on Fig. 10) the result would have shown the optimum case to be close to m = .65. We believe with some reason that the true discharge coefficients lie in between these two extreme cases, and that the reducing nozzle would improve the thrust in the actual system,[3] although the present analysis indicates otherwise.

It would, however, be doubtful whether the obtainable maximum thrust would be noticeably changed one way or the other. For the proposed system the maximum obtainable thrust is expected to lie in the range of 26,000-27,000 lbs.

The specific speeds of the propeller pumps are much greater than what are available commercially. The pump efficiencies and the cavitation criteria used in the present study have been reasonably extrapolated from the available data. ($N_s \leq 20,000$). For a better analysis the pump manufacturer should be able to supply much more concrete data on this criteria. It is, however, reasonable to assume that the system is very susceptable to cavitation (the depth to pump was taken as 45'0" below the free surface). The pitch angle is extremely small (for normal open propellers it would be almost three times as great), that is, the propeller revolutions are high compared to the optimum inflow velocity at the pump plane. It would seem logical to decrease the propeller revolutions such that the advance angle is more reasonable. The strength of the propeller shafting then would have to be increased for the latter case to transmit the same horsepower.

The force on the disc area is some 40,000 pounds for $m = .65$. The propeller blade thickness would have to be designed to withstand this force.

A force of approximately 20,000 pounds is exerted in the direction of submerging the ship (for $m \approx .65$). Whether this is acceptable to the ship is not known presently, but it is a force to be accounted for.

Finally, the total thrust of some 26,000 pounds must be borne by the opposite gate-valve arrangement. The gate-valve mechanism must withstand this large force and the force due the dynamic fluctuations. It appears from the drawings that the mechanism is underdesigned. The same opinion is given for the laterally restraining eye-bolts arrangement for the propeller.

It will be interesting to compare the present system with some other form of propulsion devices. Kort nozzle propeller would seem to be desirable. Because of the angle-drive which must be installed for obtaining multi-directional thrust, a large hub must be needed. For such a large hub the data is not readily available. We may, therefore, reduce the thrust by a certain fraction from calculated values.

Using vanManen's data[(11)], we obtain, for a B4-55 propeller in nozzle no. 7 at J = 0, as follows.

Assume N = 200 R.P.M., S.W., 9'-0" diameter and we calculate from the available horsepower at the propeller:

Torque coefficient K_M = .0408, F. W.

With this value we obtain from the curves

Pitch Diameter Ratio = 1.00

Thrust Coefficient K_s = .500, F.W.

Hence:

Thrust = 72,600 lbs.

We will reduce this to 80% for large hub to accommodate the gearing. Thus,

Thrust = 58,000 lbs.

It gives about 220% greater thrust than the present system.

RECOMMENDATIONS

The present analysis is limited in many respects. What we have presented is a method by which a reasonable engineering estimate can be made. The recommendations in the following, therefore, must not be construed as final.

1) Use Kort nozzle or some other arrangement for maximum thrust.

2) If 1) is not acceptable

   a) Analyze the system with lower propeller revolutions, thereby lowering specific speed and therefore improve cavitation characteristics of the system.

   b) Analyze and design the structure.

   c) Design the propeller using inflow velocity and the pressure increase at the propeller plane based on the present data and data from the Recommendation 2),a).

3) On the basis of the present study.

   a) Install a nozzle with a area ratio of approximately 70%. The reduction should be as gradual as possible to minimize the losses.

   b) Reduce the propeller revolutions appreciably to reduce the chance of cavitation.

   c) Redesign the structures.

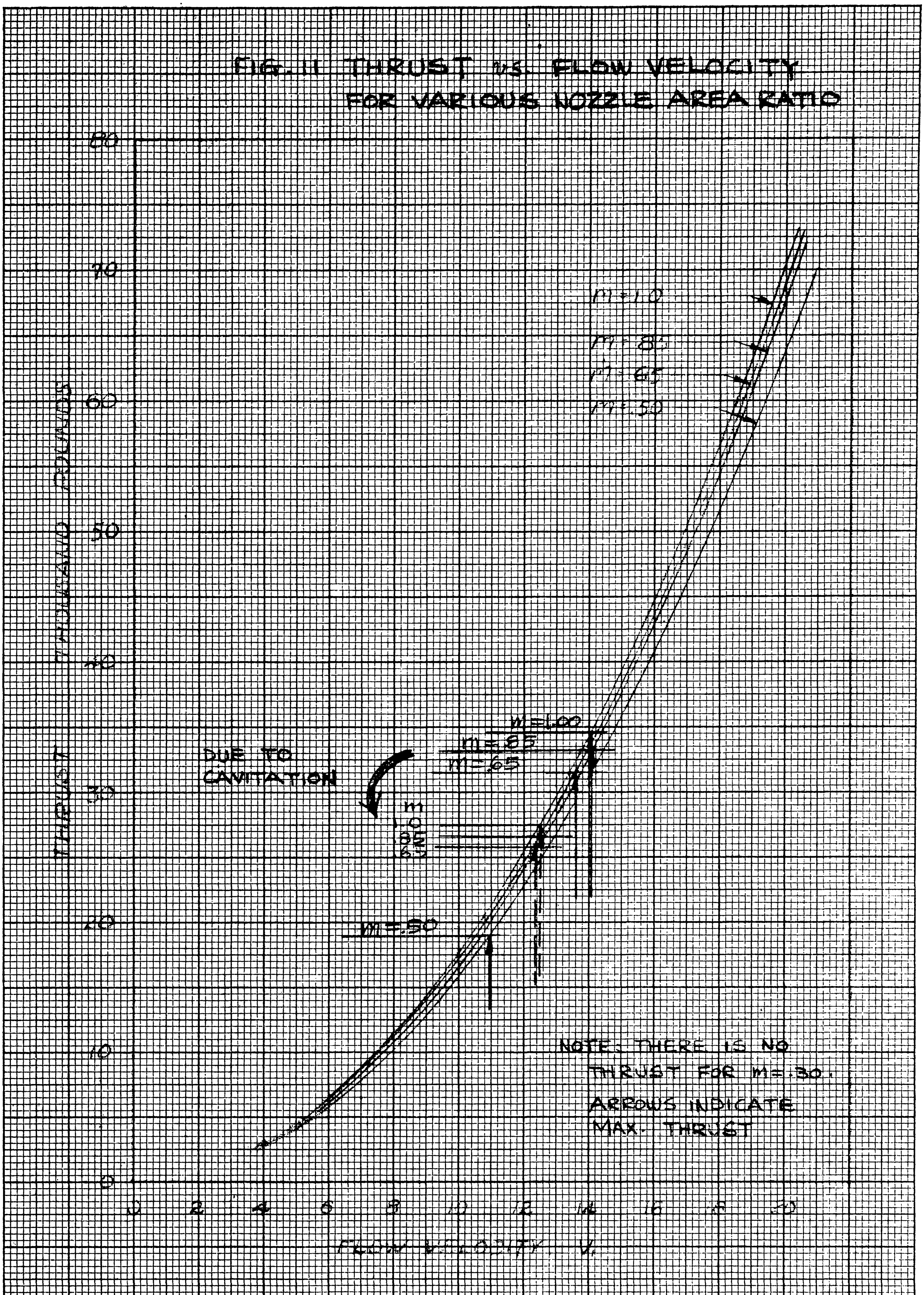


FIG. 11 THRUST vs. FLOW VELOCITY
FOR VARIOUS NOZZLE AREA RATIO
THRUST THOUSAND POUNDS
FLOW VELOCITY V.
m=1.0
m=.85
m=.65
m=.50
m=1.00
m=.85
m=.65
m=.50
DUE TO CAVITATION
NOTE: THERE IS NO THRUST FOR m=.30.
ARROWS INDICATE MAX. THRUST
0
10
20
30
40
50
60
70
80
2
4
6
8
10
12
14
16
18
20

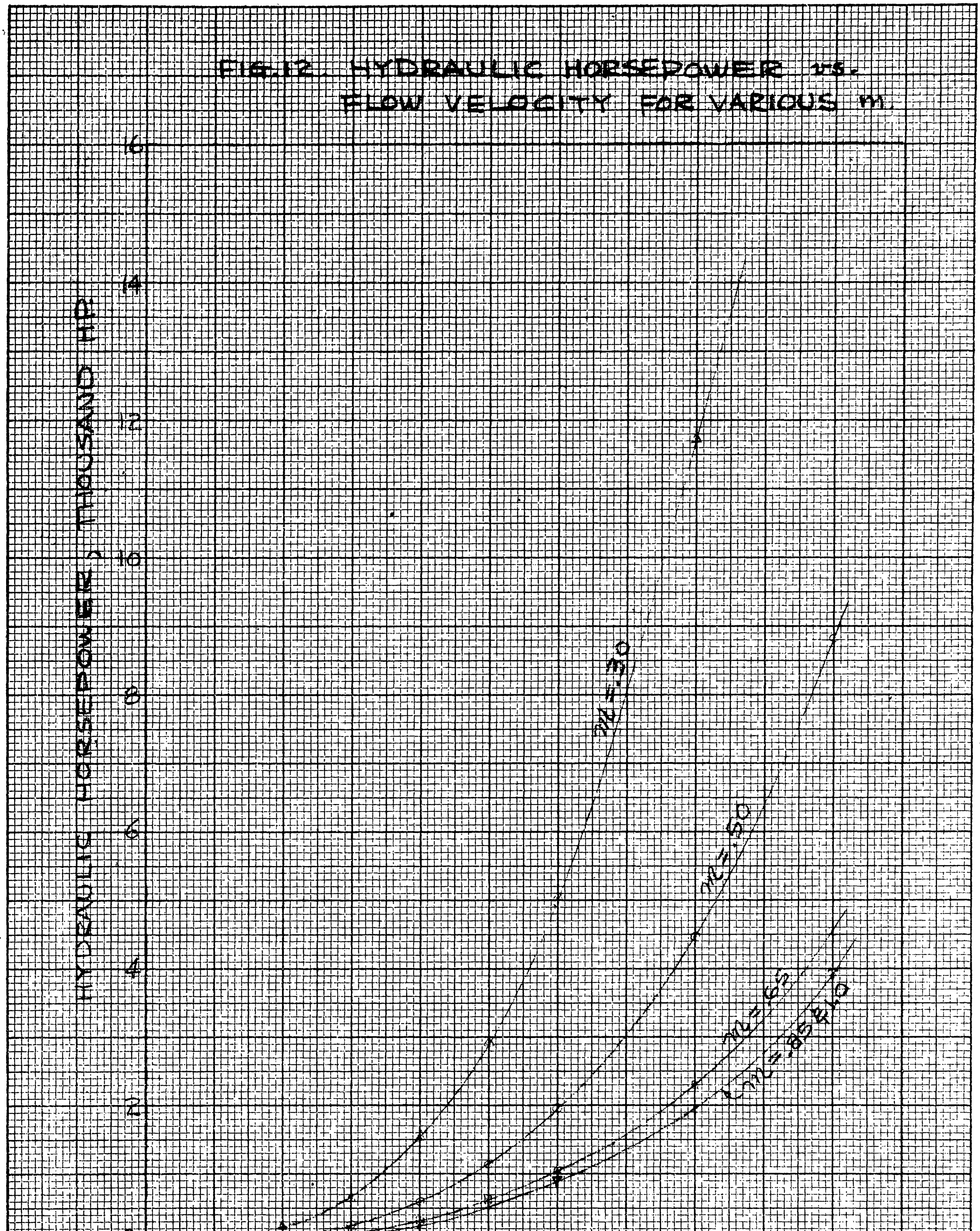


FIG. 12. HYDRAULIC HORSEPOWER vs. FLOW VELOCITY FOR VARIOUS m.
HYDRAULIC HORSEPOWER, THOUSAND HP.
FLOW VELOCITY, V1
0
2
4
6
8
10
12
14
16
0
4
8
12
16
20
m=.30
m=.50
m=.65
m=.85 & 1.0

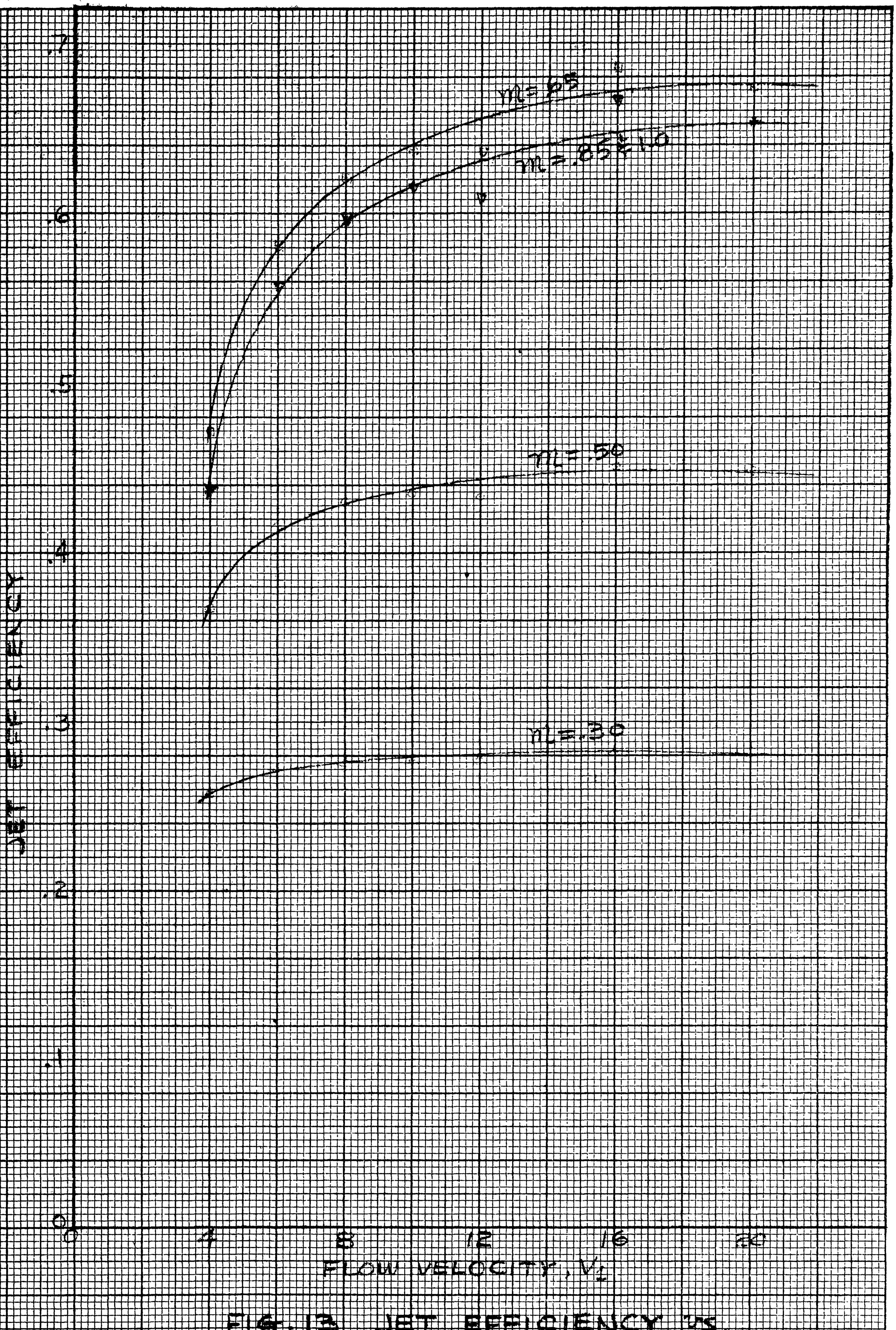


FIG. 13 JET EFFICIENCY vs. FLOW VELOCITY FOR VARIOUS m.

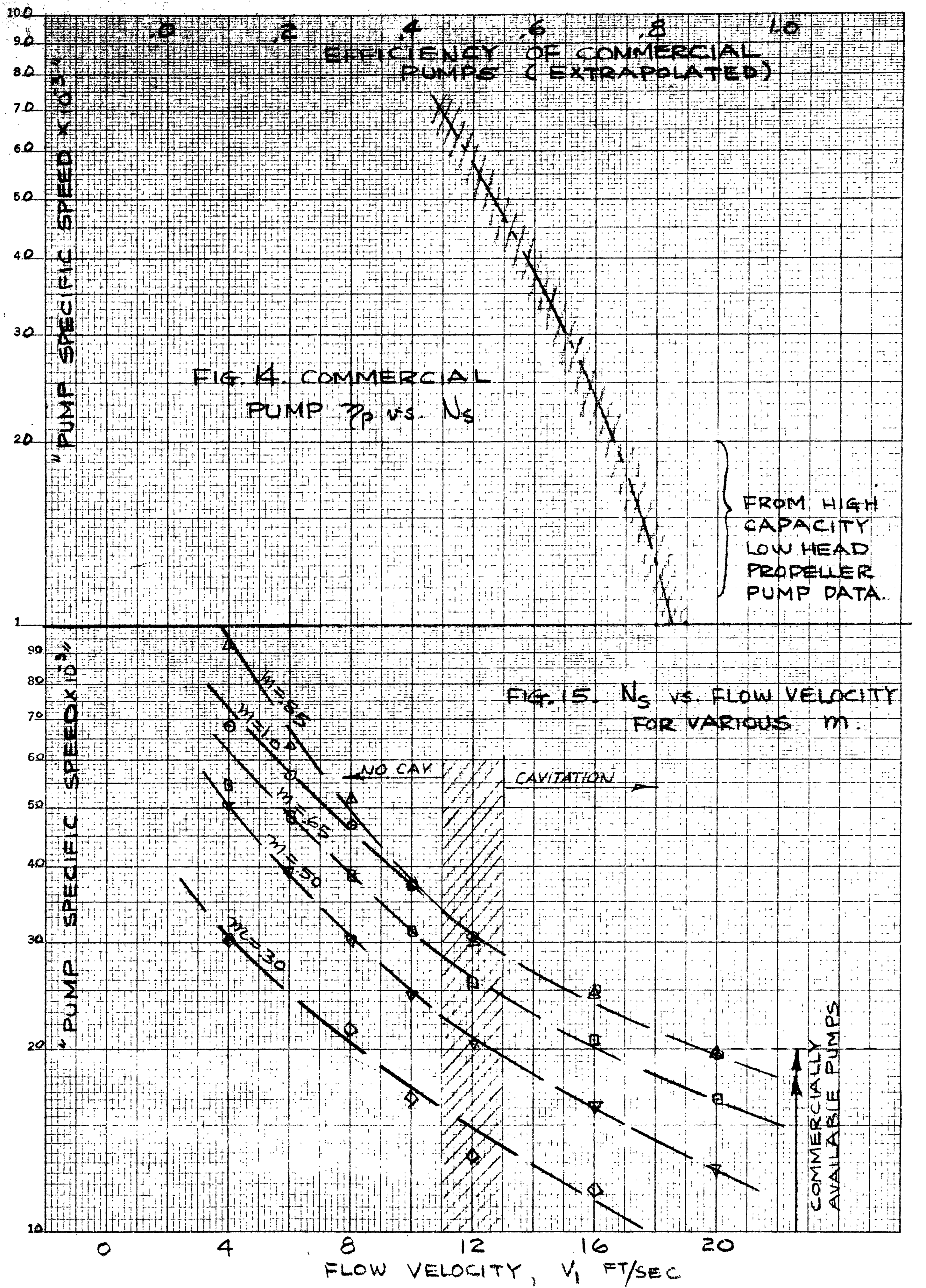
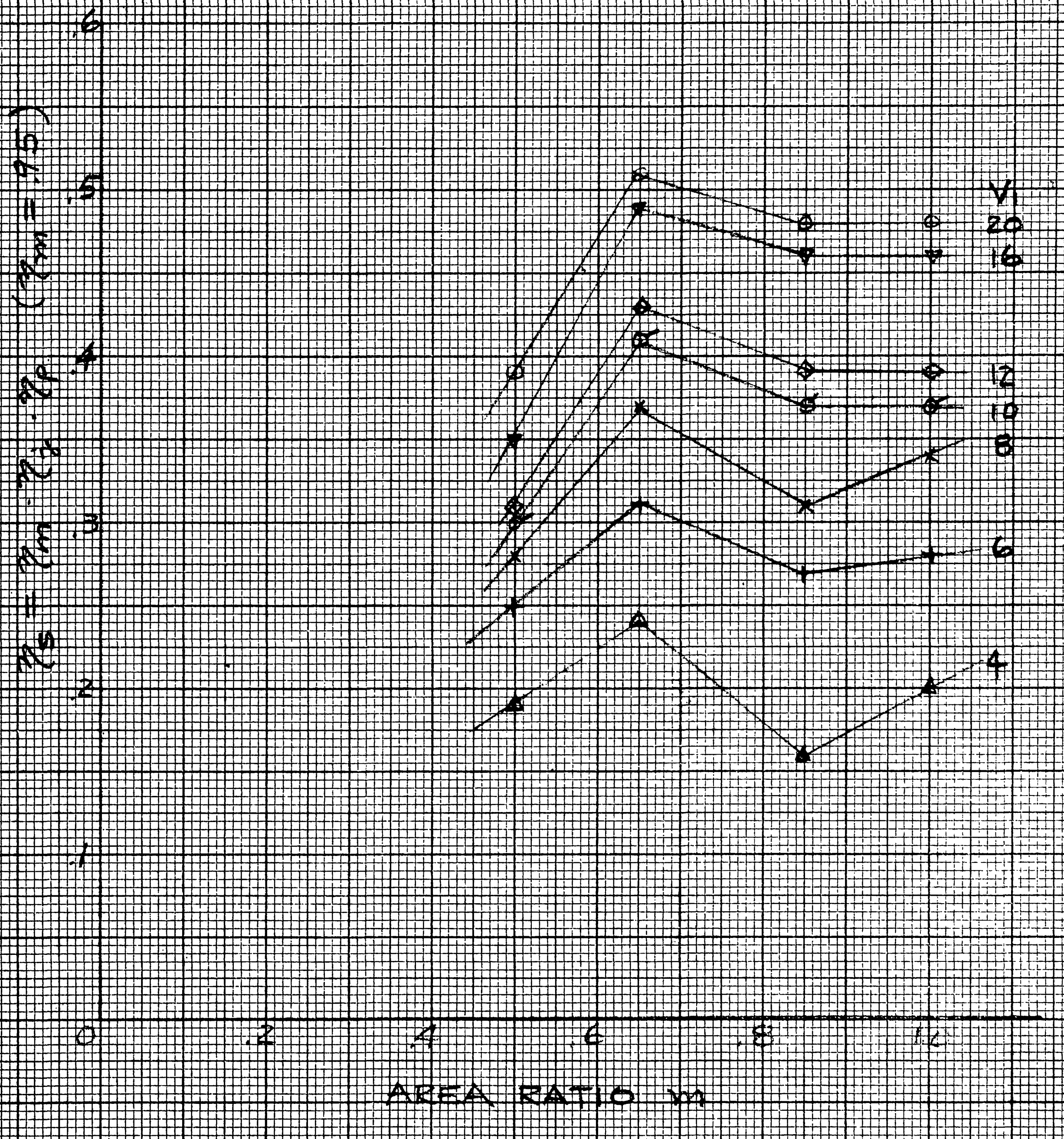


FIG. 14. COMMERCIAL PUMP η_p vs. N_s

FIG. 15. N_s vs. FLOW VELOCITY FOR VARIOUS m.

FIG. 16 SYSTEM EFFICIENCY (ESTIMATED)



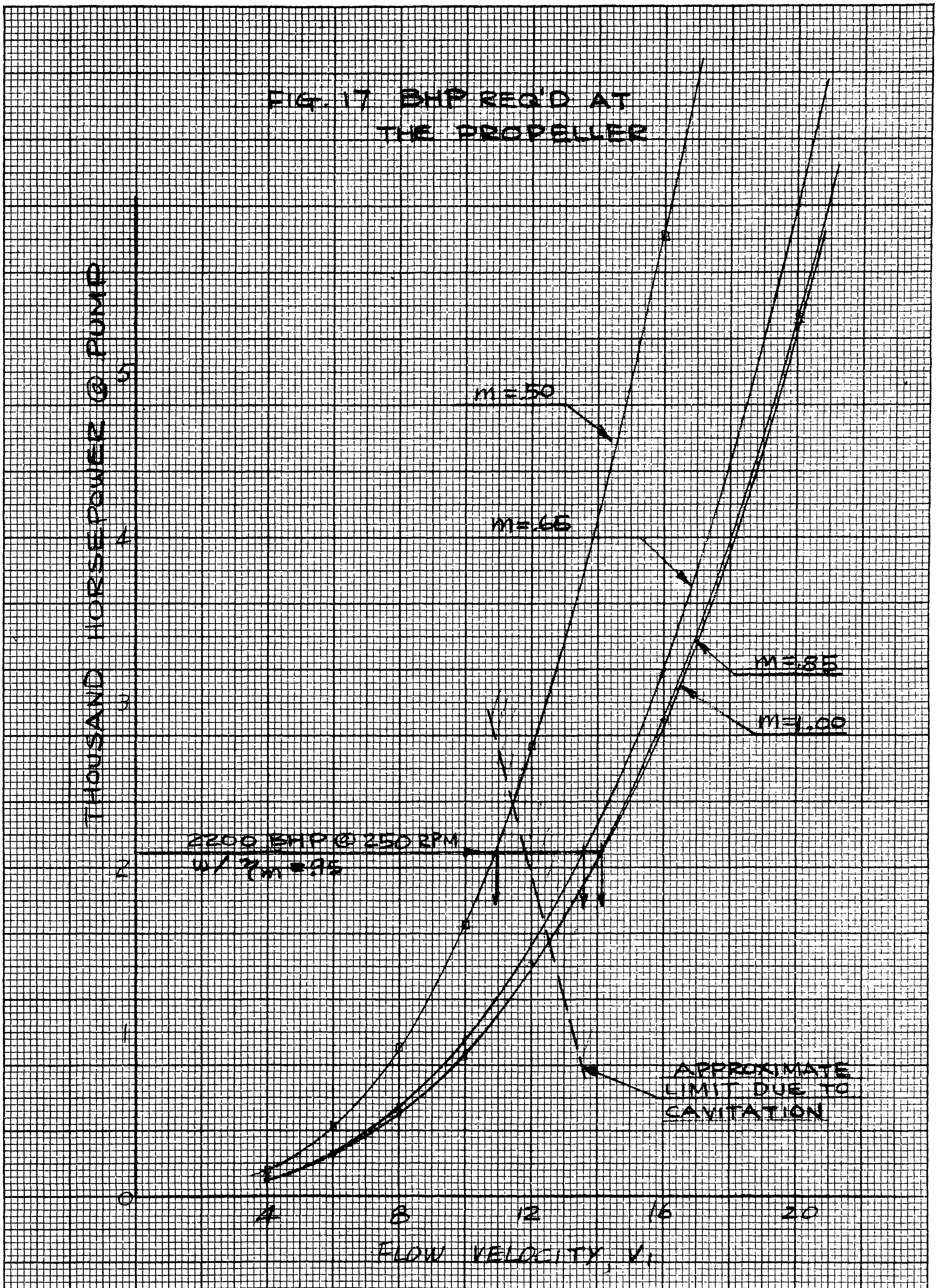


FIG. 17 BHP REQ'D AT
THE PROPELLER
THOUSAND HORSEPOWER @ PUMP
m=.50
m=.65
m=.85
m=1.00
2200 BHP @ 250 RPM
w/ ηm=.95
APPROXIMATE
LIMIT DUE TO
CAVITATION
0
1
2
3
4
5
4
8
12
16
20
FLOW VELOCITY, V₁

REFERENCE

1. Avondale Shipyards, Inc. Dwg. No. H4801D-A Mohole Project - Directional Positioning Thruster 2200 Hp General Arrangement for Brown and Root, Inc.

2. Thurston, S. and S. Huniu, "A Survey of Marine Propulsion Devices," A paper presented to The Southern California Section of The Society of Naval Architects and Marine Engineers, March, 1962

3. Grey and Wilsted, "Performance of Conical Jet Nozzles in Terms of Flow and Velocity Coefficients," NACA Technical Note No. 1757, November, 1948

4. Flow of Fluids (through Valves, Fittings and Pipe), Crane Technical Paper No. 410

5. Shapiro and Smith, "Friction Coefficients in the Inlet Length of Smooth Round Tubes," NACA TN No. 1785, November, 1948

6. Simmons, "Analytic Determination of the Discharge Coefficients of Flow Nozzles," NACA TN No. 3447, April, 1955

7. Mueller and Peters, "Coefficients of Flow of Standard Nozzles," NACA Technical Memorandums No. 549, January, 1930

8. Hydraulic Institute Standards, Tenth ed.

9. Standards for Discharge Measurement with Standardized Nozzles and Orifices, German Industrial Standard 1952, Fourth ed., NACA TM No. 952, September, 1940

10. Seward, H. L., Marine Engineering, vol. II, The Society of Naval Architects and Marine Engineers, New York, 1944

11. van Manen, J. D., "Recent Research on Propellers in Nozzles," Journal of Ship Research, vol. I, no. 2, July, 1957, p. 17

APPENDIX

A. Notations

A: Sectional area $[ft^2]$

C_d: Nozzle discharge coefficient

C_o: Orifice coefficient

D: Pipe diameter $[ft]$

d: Nozzle outlet diameter $[ft]$

d/D: Diameter ratio for reducing nozzle

ϵ/D: Pipe roughness factor , .000, 001

$\vec{F}$: Vector force exerted on the fluid by the pipe $[\vec{lb}]$

$F_{v,h}$: Vertical and horizontal component of $\vec{F}$ respectively

f: Pipe friction coefficient

HHP: Hydraulic Horsepower

g: Gravitational constant, 32.2 $[ft/sec^2]$

h_L: Head loss $[ft]$ or $[psf]$

$h_L/(v_1^2/2g)$: Nondimensional loss

K: $K = f(\frac{L}{D})$

m: Area ratio for reducing nozzle

N: rpm of pump

n: rps of pump

p: Static pressure $[lb/ft^2]$

Δp: Pressure drop $[psf]$

Q: Volumetric flow $[ft^3/sec]$

R_e: Reynolds number $\frac{Dv}{\nu}$

r: Radius $[ft]$

N_s: Pump Specific Speed

NPSH: Net Positive Suction Head

$\bar{T}$ vector force exerted on the fluid by the propeller pump $[\vec{Lb}]$

$\bar{v}$, v flow velocity [ft/sec]

V_r radial component of fluid velocity in 2-D flow

V_z axial component of fluid velocity in 2-D flow

$\dot{W}$ rate of work done $\dot{W} = \rho Q \eta_7 v_1$ $\left[\frac{ft-Lb}{sec}\right]$

z elevation from datum line [ft]

β advance angle [°ARC]

γ specific weight [lb/ft^3]

ξ "vena contracta" area contration factor

η "vena contracta" friction and viscosity factor

η_j jet efficiency

η_m transmission efficiencies

η_{pump} pump efficiency

$\eta_{overall}$, η_s : overall system efficiency

ν kinematic viscosity, 1.2817×10^{-5} [ft^2/sec]

ρ mass density, 1.9905 [$lb\text{-}sec^2/ft^4$]

ϕ pitch angle [°ARC]

σ cavitation number NPSH/H

B. <u>Calculation Sheets</u> (Appendix)

C. <u>Calculation Format</u> (Appendix)

INLET	V_1 (ft/sec) Assumed	4	6	8	10	12	16	20
	Q (ft^3/sec) = A_1V_1	259.0	388.6	518.1	647.6	777.1	1036.2	1295.2
	Re x 10^{-4}; $R = \frac{DV_1}{\mu}$; 59°F SW	2.835	4.252	5.670	7.087	8.504	11.340	14.174
	f+Δf; Moody Diag. w/Δf=0.0004	.024	.022	.020	.019	.0187	.0177	.0166
	Cd ; Fig.8	.960	.966	.970	.973	.975	.977	.982
	Cd^2	.9216	.9330	.941	.947	.951	.955	.964
	Q^2	67100	151,000	268,400	419,400	603,900	1,073,700	1,677,500
	$Q^2\gamma$; γ = 64.04 $\frac{LBM}{FT^3}$	4,299,800	9,674,700	17,209,900	26,874,300	38,696,900	68,803,400	107,496,900
	$2gA^2Cd^2$	249,000	252,000	254,160	255,800	256,860	257,940	260,400
	$\Delta P = Q^2\gamma/2gA^2Cd^2$ (psf)	-17.27	-38.40	-67.68	-105.10	-150.65	-266.70	-412.86
	h_L(pipe alone) $f(L/D)\cdot(v^2/2g)$	.0028	.0044	.0073	.0108	.0151	.0255	.0374
	h_L (psf)	-.18	-.28	-.47	-.692	-.967	-1.63	-2.40
	elevation (psf) for 3'-3"	-210	-210	-210	-210	-210	-210	-210
	hydrostatic pressure 48'	3090	3090	3090	3090	3090	3090	3090
	P_1/γ(ft) = 48' − $V_1^2/64$	47.8	47.4	47.0	46.4	45.8	43.6	41.8
	P_1 (psf)	3060	3050	3020	2980	2940	2860	2680
	NPSH (psf)	2833	2801	2742	2665	2578	2382	2055
	vapor pressure (psf)	53	53	53	53	53	53	53
	atmospheric pressure (psf)	2116	2116	2116	2116	2116	2116	2116

PIPE to TEE	L/D	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	h_L (FT)	.006	.0123	.0198	.0295	.0619	.0703	.1031
	h_L (psf)	-.384	-.79	-1.27	-1.89	-3.96	-4.50	-6.60
ELEV.	h_L (psf) for 13'-0"	−837 ←						→ −837

TEE	equivalent length							
	L/D							
	h_L							
	$\cdot h_L$							
	obstruction h_L x1.15							
TEE to OUTLET	length							
	L/D							
	h_L (ft)							
	h_L (psf)							
OUTLET	d/D = 1.0 ; m = 1.0 — Cd	1.08 ←						→ 1.08
	Cd corrected for viscosity	× .985 1.06	.992 1.07	.994 1.07	.995 1.075	.997 1.08	.998 1.08	.999 1.08
	Cd^2	1.12	1.14	1.14	1.16	1.17	1.17	1.17
	$A_3^2 = 63.6^2$	4045 ←						→ 4045
	$Q^2\gamma$	4,299,800	9,674,700	17,209,900	26,874,300	38,696,900	68,803,400	107,496,900
	$2gA_3^2Cd^2$	291,760	296,970	296,970	302,180	304,785	304,785	304,785
	ΔP (psf)	-14.74	-32.58	-57.95	-88.93	-126.96	-225.74	-352.70
	P_7 = 2060 psf hydrostatic Pressure after pump (psf)	2919	2945	2978	3020	3096	3205	3370
	Pressure before pump (psf)	2833	2801	2742	2665	2578	2382	2055
	Pressure drop across pump (psf)	86	144	236	355	518	823	1315
	Force on Disk Area ($\Delta P \times A_3$) (LBS)	5,470	9158	15,010	22,578	32,945	52,343	83,634

$m = 1.0$

NOZZLE VELOCITY	V_6 (ft/sec) $= V_1/m$	4.0	6.0	8.0	10.0	12.0	16.0	20.0
	$C_d^2 m^2$	1.12	1.14	1.14	1.16	1.17	1.17	1.17
	$[1+C_d^2 m^2]^{\frac{1}{2}}$	1.456	1.463	1.463	1.470	1.473	1.473	1.473
	$[1+C_d^2 m^2]^{\frac{1}{2}}/C_d \equiv \beta$	1.374	1.367	1.367	1.367	1.364	1.364	1.364
	$V_7 = \beta V_6$	5.496	8.202	10.936	13.670	16.368	21.824	27.28
FORCES	ρQ	515.7	773.6	1031.5	1289.4	1547.2	2063.1	2578.7
	$-F_H = \rho Q V_7$ (LBS)	2834.29	6,345.1	11,280.5	17,626.1	25,324.5	45,025.1	70,346.9
	$-F_V = \rho Q V_1$ (LBS)	2,062.8	4,641.6	8,252.0	12,894.0	18,566.4	33,009.6	51,574.0
	$T =$ (LBS)	2834	6,345	11,280	17,626	25,325	45,025	70,347
EFFICIENCY	V_7/V_1	1.374	1.367	1.367	1.367	1.364	1.364	1.364
	$(V_7/V_1)^2$	1.888	1.869	1.869	1.869	1.860	1.860	1.860
	$\Delta P \cdot 2g/\gamma V_1^2$: ΔP across pump	5.401	4.019	3.705	3.550	3.614	3.230	3.303
	$(V_7/V_1)^2 - 1 + \Delta P \cdot 2g/\gamma V_1^2 \equiv \Sigma$	6.289	4.888	4.574	4.419	4.474	4.09	4.163
	$\eta_{jet} = 2(V_7/V_1)/\Sigma$	.437	.559	.598	.619	.610	.667	.655
	$\eta_{thr} = 2(V_7/V_1 - 1)/\Sigma$	.119	.150	.160	.166	.163	.178	.175
HYDRAULIC H.P.								
	$\rho Q V_1^2/2 \times 550$	7.501	25.320	60.015	117.218	202.54	480.139	937.709
	$HHP = \Sigma \times \rho Q V_1^2/2 \times 550$	47.174	123.764	274.51	517.986	906.160	1,963.77	3,903.68
	REQ. BHP $= HHP/\eta_p \cdot \eta_m$	131	247.5	499.0	863.3	1415.7	2846	5348

m = 1.0

	TOTAL PUMPING HEAD (H)		1.342	2.246	3.682	5.538	8.081	12.839	20.514
	$H^{3/4}$		1.247	1.833	2.578	3.610	4.770	6.78	9.640
	gpm		115,988.7	174,027.4	232,021.4	290,015.7	348,010.1	464,043.2	580,097.7
	$gpm^{\frac{1}{2}}$		339.9	416.4	481.5	538.0	590.0	680.0	761.0
PUMP SELECTION	N= 250 rpm	SPECIFIC SPEED: N_s	68,143	56,793	46,693	37,258	30,920	25,074	19,736
		EFFICIENCY, η_p	.36	.50	.55	.60	.64	.69	.73
		$\sigma = NPSH/H$	32.94	19.45	11.62	7.51	4.98	2.89	1.56
		CAVITATION ?	NO CAV.	NO CAV	NO CAV	NO CAV	BORDER	CAV.	CAV.
		$\eta_m = 1.00$ $\eta_{SYSTEM} = \eta_i \cdot \eta_p \cdot \eta_m$	.20	.28	.34	.37	.39	.46	.48
	N= 500	N_s	136,285	113,585	93,385	74,515	61,840	50,148	39,471
		η_p							
		σ	32.94	19.45	11.62	7.51	4.98	2.89	1.56
		CAVITATION?							
		η_{SYSTEM}							
	N= 870	N_s	237,136	197,638	162,490	129,656	107,602	81,257	68,679
		η_p							
		σ	32.94	19.45	11.62	7.51	4.98	2.89	1.65
		CAVITATION?							
		η_{SYSTEM}							
	N=	N_s							
		η_p							
		σ							
		CAVITATION?							
		η_{SYSTEM}							

TEE	equivalent length							
	L/D							
	h_L							
	h_L							
	obstruction h_L x1.15							
TEE to OUTLET	length							
	L/D							
	h_L (ft)							
	h_L (psf)							
OUTLET	d/D = .92 ; m = .85 — Cd	1.26 ←					→	1.26
	Cd corrected for viscosity	×.985 1.241	.992 1.250	.994 1.252	.995 1.254	.997 1.256	.998 1.257	.999 1.259
	Cd^2	1.540	1.563	1.568	1.573	1.578	1.580	1.585
	$A_0^2 = 54.06^2$	2921.4 ←					→	2924.1
	$Q^2\gamma$	4,299,800	9,674,700	17,209,900	26,874,300	38,696,900	68,803,400	107,496,900
	$2gA_0^2Cd^2$	289,740	294,060	295,000	295,944	296,884	297,260	298,200
	ΔP (psf)	-14.84	-32.90	-58.34	-90.81	-130.34	-231.46	-360.50
	P_7 = 2060 psf hydrostatic Pressure after pump (psf)	2919	2946	2742	3021	3099	3210	3378
	Pressure before pump (psf)	2833	2801	2742	2665	2578	2382	2055
	Pressure drop across pump (psf)	86	145	236	356	521	828	1323
	Force on Disk Area ($\Delta P \times A_3$) (LBS)	5,470	9,222	15,010	22,642	33,136	52,661	84,143

$\dot{m} = .85$

NOZZLE VELOCITY	V_6 (ft/sec) $= V_1/m$	4.706	7.059	9.412	11.765	14.118	18.824	23.530
	$C_d^2 m^2$	1.113	1.129	1.133	1.136	1.140	1.142	1.145
	$[1+C_d^2 m^2]^{\frac{1}{2}}$	1.453	1.459	1.460	1.460	1.463	1.463	1.464
	$[1+C_d^2 m^2]^{\frac{1}{2}}/C_d \equiv \beta$	1.171	1.167	1.166	1,164	1,165	1,164	1.163
	$V_7 = \beta V_6$	5.511	8.238	10,974	13.694	16.447	21.911	27.365
FORCES	ρQ	515.7	773.6	1031.5	1289.4	1547.2	2063.1	2578.7
	$-F_H = \rho Q V_7$ (LBS)	2,842.02	6,372.92	11,319.7	17,657.0	25,446.8	45,204.6	70,566.1
	$-F_V = \rho Q V_1$ (LBS)							
	T = (LBS)	2,709.98	6,059.8	10,763	16,802	24,227	43,036	67,177
EFFICIENCY	V_7/V_1	1.378	1.373	1.372	1.369	1.371	1.369	1.368
	$(V_7/V_1)^2$	1.900	1.885	1.882	1.874	1.880	1.874	1.871
	$\Delta P \cdot 2g/\gamma V_1^2$: ΔP across pump	5.401	4.047	3.705	3.577	3.632	3.250	3.323
	$(V_7/V_1)^2 - 1 + \Delta P \cdot 2g/\gamma V_1^2 \equiv \Sigma$	6.301	4.932	4.587	4.451	4.512	4.124	4.194
	$\eta_{jet} = 2(V_7/V_1)/\Sigma$	.437	.557	.598	.615	.608	.664	.652
	$\eta_{thr} = 2(V_7/V_1 - 1)/\Sigma$	.120	.151	.162	.166	.164	.179	.175
HYDRAULIC HP								
	$\rho Q V_1^2/2 \times 550$	7.501	25.320	60.015	117.218	202.54	480.139	937.709
	HHP $= \Sigma \cdot \rho Q V_1^2/2 \times 550$	47.264	124.88	275.29	521.74	913.86	1,980.10	3,932.75
	REQ. BHP $=$ HHP$/\eta_p \cdot \eta_m$	131	260	529	870	1428	2869	5314
	$(P_7 - P_5)A_1(1-m)$	141.65	313.09	556.80	854.62	1220.09	2,169.36	3,389.45

$\eta_m = .85$

	TOTAL PUMPING HEAD, H		1.342	2.262	3.682	5.554	8.128	12.917	20.639
	$H^{3/4}$		.905	1.635	2.320	3.61	4.810	6.810	9.66
	gpm								
	$gpm^{\frac{1}{2}}$		339.9	416.4	481.5	538.0	590.0	680.0	761.0
PUMP SELECTION	n= 250 rpm	SPECIFIC SPEED: N_s	93,895	63,675	51,876	37,258	30,665	24,963	19,694
		EFFICIENCY, η_P	.36	.48	.52	.60	.64	.69	.74
		$\sigma = NPSH/H$	32.94	19.32	11.62	7.49	4.95	2.88	1.55
		CAVITATION ?	NO CAV.	NO CAV.	NO CAV	NO CAV.	BORDERL	CAV.	CAV.
		$\eta_m = 1.00$ $\eta_{SYSTEM} = \eta_i \cdot \eta_P \cdot \eta_m$	.16	.27	.31	.37	.39	.46	.48
	N= 500	N_s	187,790	127,35	103,752	74,515	61,330	49,926	39,389
		η_P							
		σ	32.94	19.32	11.62	7.49	4.95	2.88	1.55
		CAVITATION?							
		η_{SYSTEM}							
	N= 870	N_s	326,755	22,159	180,528	129,656	106,714	86,872	68,537
		η_P							
		σ	32.94	19.32	11.62	7.49	4.95	2.88	1.55
		CAVITATION?							
		η_{SYSTEM}							
	N=	N_s							
		η_P							
		σ							
		CAVITATION?							
		η_{SYSTEM}							

TEE	equivalent length (FT)	110	110	110	110	110	110	110
	L/D	12.2 ←						→ 12.2
	h_L (FT)	.073	.150	.242	.360	.755	.858	1.26
	h_L (Psf)	-4.67	-9.61	-15.50	-23.05	-48.35	-54.95	-80.56
	obstruction h_L x1.26	-5.88	-12.11	-19.53	-29.04	-60.92	-69.24	-101.51
TEE to OUTLET	length	16.5 ←						→ 16.5
	L/D	1.83 ←						→ 1.83
	h_L (ft)	.011	.034	.036	.054	.113	.129	.189
	h_L (psf)	-.70	-2.16	-2.33	-3.48	-7.28	-8.31	-12.11
OUTLET	d/D = .805 ; m = .65 Cd	1.18 ←						→ 1.18
	Cd corrected for viscosity	× .985 1.16	.992 1.17	.994 1.17	.995 1.17	.997 1.18	.998 1.18	.999 1.18
	Cd^2	1.346	1.369	1.369	1.369	1.392	1.392	1.392
	$A_B^2 = (41.4)^2$	1713.9						1713.9
	$Q^2 \gamma$	4,299,800	9,674,700	17,209,900	26,874,300	38,696,900	68,803,400	107,446,900
	$2gA_B^2Cd^2$	148,600	151,100	151,100	151,100	153,600	153,600	153,600
	ΔP (psf)	-28.94	-64.03	-113.90	-177.86	-251.93	-447.94	-699.85
PUMP	P_7 = 2060 psf hydrostatic Pressure after pump (psf)	2933	2976	3034	3109	3221	3427	3717
	Pressure before pump (psf)	2833	2801	2742	2665	2578	2382	2055
	Pressure drop across pump (psf)	100	175	292	444	643	1045	1662
	Force on Disk Area ($\Delta P \times A_3$) (LBS)	6,360	11,130	18,571	28,238	40,895	66,462	105,703

$m = .65$

NOZZLE VELOCITY	V_6 (ft/sec) $= V_1/m$	6.154	9.231	12.308	15.385	18.462	24.616	30.770
	$C_d^2 m^2$	.569	.578	.578	.578	.588	.588	.588
	$[1 + C_d^2 m^2]^{\frac{1}{2}}$	1.252	1.256	1.256	1.256	1.256	1.256	1.256
	$[1 + C_d^2 m^2]^{\frac{1}{2}} / C_d \equiv \beta$	1.079	1.074	1.074	1.074	1.064	1.064	1.064
	$V_7 = \beta V_6$	6.640	9.914	13.219	16.523	19.644	26.191	32.739
FORCES	ρQ	515.7	773.6	1031.5	1289.4	1547.2	2063.1	2578.7
	$-F_H = \rho Q V_7$ (LBS)	3,424.2	7,669.5	13,635.4	21,304.7	30,393.2	54,034.7	84,424.0
	$-F_V = \rho Q V_1$ (LBS)	2,062.8	4,641.6	8,252.0	12,894.0	18,566.4	33,096.0	51,574.0
	$T =$ (LBS)	2,780	6245	11,100	17,346	24,785	44,065	68,846
EFFICIENCY	V_7/V_1	1.660	1.652	1.652	1.652	1.637	1.637	1.637
	$(V_7/V_1)^2$	2.756	2.729	2.729	2.729	2.680	2.680	2.680
	$\Delta P \cdot 2g / \gamma V_1^2$: ΔP across pump	6.280	4.960	4.584	4.461	4.487	4.101	4.175
	$(V_7/V_1)^2 - 1 + \Delta P \cdot 2g / \gamma V_1^2 \equiv \Sigma$	7.036	5.689	5.313	5.190	5.167	4.781	4.855
	$\eta_{ideal} = 2(V_7/V_1)/\Sigma$	.472	.581	.622	.637	.634	.685	.674
	$\eta_{thr} = 2(V_7/V_1 - 1)/\Sigma$	.188	.229	.245	.251	.247	.266	.262
HYDRAULIC HP								
	$\rho Q V_1^2 / 2 \times 550$	7.501	25.320	60.015	117.218	202.54	480.139	937.709
	$HHP = \Sigma \cdot \rho Q V_1^2 / 2 \times 550$	52.778	144.045	318.860	608.36	1,046.52	2,295.5	4,552.6
	REQ. BHP = $HHP / \eta_p \cdot \eta_m$	103	267	541	950	1539	3188	5990
	$(P_7 - P_5) A_5 (1 - m)$	644.2	1425.0	2535.0	3959.0	5608	9,970	15,578

$m = .65$

		TOTAL PUMPING HEAD, H	1.560	2.730	4.555	6.926	10.031	16.302	25.927
		$H^{3/4}$	1.396	2.135	3.115	4.260	5.750	8.100	11.500
		gpm							
		$gpm^{\frac{1}{2}}$	339.9	416.4	481.5	538.0	590.0	680.0	761.0
PUMP SELECTION	N= 250	SPECIFIC SPEED: N_s	54,470	48,760	38,643	31,573	25,652	20,988	16,544
		EFFICIENCY, η_P	.51	.54	.59	.64	.68	.72	.76
		$\sigma = NPSH/H$	28.33	16.01	9.39	6.00	4.01	2.28	1.24
		CAVITATION ?	NO CAV	NO CAV	NO CAV	NO CAV	BORDER	CAV.	CAV.
		$\eta_m = 1.00$ $\eta_{SYSTEM} = \eta_c \cdot \eta_P \cdot \eta_m$	.24	.31	.37	.41	.43	.49	.51
	N= 500	N_s	108,94	97,520	77,285	63,145	51,304	41,975	33,087
		η_P							
		σ	28.33	16.01	9.39	6.00	4.01	2.28	1.24
		CAVITATION?			CAVITATE				
		η_{SYSTEM}							
	N= 870	N_s	211,83	169,68	134,476	109,872	89,269	73,037	57,571
		η_P							
		σ	28.33	16.01	9.39	6.00	4.01	2.28	1.24
		CAVITATION?							
		η_{SYSTEM}							
	N=	N_s							
		η_P							
		σ							
		CAVITATION?							
		η_{SYSTEM}							

TEE	equivalent length							
	L/D							
	h_L							
	h_L							
	obstruction h_L x1.15							
TEE to OUTLET	length							
	L/D							
	h_L (ft)							
	h_L (psf)							
OUTLET	d/D = .706 ; m = .50 — C_d	1.082 ←					→	1.082
	Cd corrected for viscosity	x .985 1.066	.992 1.073	.994 1.076	.995 1.077	.997 1.079	.998 1.080	.999 1.081
	Cd^2	1.136	1.151	1.158	1.160	1.164	1.166	1.169
	$A_3^2 = (31.8)^2$	1011.2 ←					→	1011.2
	$Q^2\gamma$	4,299,800	9,674,700	17,209,900	26,874,300	38,696,900	68,803,400	107,446,900
	$2gA_3^2Cd^2$	73,980	74,950	75,410	75,540	75,802	75,931	76,127
	ΔP (psf)	−58.12	−129.08	−228.22	−355.76	−510.50	−906.13	−1412.07
	P_T = 2060 psf hydrostatic Pressure after pump (psf)	2962	3041	3148	3287	3479	3885	4430.
	Pressure before pump (psf)	2833	2801	2742	2665	2578	2382	2055
	Pressure drop across pump (psf)	129.	240	406	622	901	1503	2375
	Force on Disk Area ($\Delta P \times A_3$) (LBS)	8204.	15,264	25822	39,559	57,304	95,591	151,050

$m = .50$

NOZZLE VELOCITY	V_6 (ft/sec) $= V_1/m$	8.0	12.0	16.0	20.0	24.0	32.0	40.0
	$C_d^2 m^2$	.284	.288	.290	.290	.291	.292	.292
	$[1 + C_d^2 m^2]^{\frac{1}{2}}$	1.132	1.136	1.136	1.136	1.136	1.136	1.136
	$[1 + C_d^2 m^2]^{\frac{1}{2}}/C_d \equiv \beta$	1.062	1.059	1.056	1.055	1.053	1.052	1.051
	$V_7 = \beta V_6$	8.496	12.708	16.896	21,100	25.272	33.664	42.040
FORCES	ρQ	515.7	773.6	1031.5	1289.4	1547.2	2063.1	2578.7
	$-F_H = \rho Q V_7$ (LBS)	4,381	9,831	17,428	27,206	39,100	69,452	108,408
	$-F_V = \rho Q V_1$ (LBS)							
	$T =$ (LBS)	2,533	5,726	10,139	15,893	22,866	40,637	63,506
EFFICIENCY	V_7/V_1	2.124	2.118	2.112	2,110	2.106	2.104	2.102
	$(V_7/V_1)^2$	4.511	4.486	4.461	4.452	4.435	4.427	4.418
	$\Delta P \cdot 2g/\gamma V_1^2$: ΔP across pump	8.109	6.704	6.380	6.255	6.292	5.904	5.971
	$(V_7/V_1)^2 - 1 + \Delta P \cdot 2g/\gamma V_1^2 \equiv \Sigma$	11.620	10.190	9.841	9.707	9.727	9.331	9.389
	$\eta_{jet} = 2(V_7/V_1)/\Sigma$	.366	.416	.429	.435	.433	.451	.448
	$\eta_{thr} = 2(V_7/V_1 - 1)/\Sigma$	.193	.219	.226	.229	.227	.237	.235
HYDRAULIC HP	$\rho Q V_1^2/2 \times 550$	7.501	25.319	60.013	117.22	202.54	480.13	937.72
	$HHP = \Sigma \cdot \rho Q V_1^2/2 \times 550$	87.16	258.00	590.59	1,137.85	1,970.11	4480.10	8,804.25
	REQ. BHP = $HHP/\eta_P \cdot \eta_m$	164	437	909	1649	2736	5818	11,005
	$(P_7 - P_5)A_1(1-m)$	1,848	4,105	7,289	11,313	16,234	28,815	44,902

m = .50

PUMP SELECTION		TOTAL PUMPING HEAD, H	2.012	3.744	6.334	9.703	14.055	23.447	37.050
		$H^{3/4}$	1.686	2.635	3.990	5.500	7.250	10.60	15.00
		gpm							
		$gpm^{\frac{1}{2}}$	339.9	416.4	481.5	538.0	590.0	680.0	761.0
	N = 250	SPECIFIC SPEED: N_s	50,400	39,507	30,169	24,455	20,345	16,050	12,683
		EFFICIENCY, η_P	.53	.59	.65	.69	.72	.77	.80
		$\sigma = NPSH/H$	21.96	11.67	6.75	4.28	2.86	1.58	.87
		CAVITATION ?	NO CAV	NO CAV	NO CAV	BORDER	CAV.	CAV.	CAV.
		$\eta_m = 1.00$ $\eta_{SYSTEM} = \eta_i \cdot \eta_P \cdot \eta_m$	.19	.25	.28	.30	.31	.35	.39
	N = 500	N_s	100,800	79,013	60,339	48,909	40,690	27,359	25,366
		η_P							
		σ	21.96	11.67	6.75	4.28	2.86	1.58	.87
		CAVITATION?							
		η_{SYSTEM}							
	N = 870	N_s	175,392	137,483	104,988	85,101	70,801	47,604	44,138
		η_P							
		σ	21.96	11.67	6.75	4.28	2.86	1.58	.87
		CAVITATION?							
		η_{SYSTEM}							
	N =	N_s							
		η_P							
		σ							
		CAVITATION?							
		η_{SYSTEM}							

TEE	equivalent length							
	L/D							
	h_L							
	h_L							
	obstruction h_L x1.15							
TEE to OUTLET	length							
	L/D							
	h_L (ft)							
	h_L (psf)							
OUTLET	d/D = .547 ; m = .30 — C_d	1.015 ←					→ 1.015
	Cd corrected for viscosity	x.985 1.000	x.992 1.007	.994 1.009	.995 1.010	.997 1.012	.998 1.013	.999 1.014
	Cd^2	1.000	1.014	1.018	1.020	1.024	1.026	1.028
	$A_B^2 = (19.08)^2$	364.05 ←					→ 364.05
	$Q^2\gamma$	4,299,800	9,674,700	17,209,900	26,874,300	38,696,900	68,803,400	107,496,900
	$2gA_B^2Cd^2$	23,440	23,773	23,867	23,914	24,008	24,055	24,101
	ΔP (psf)	-183.44	-406.96	-721.08	-1,123.80	-1,611.83	-2,860.25	-4,460.27
	P_7 = 2060 psf hydrostatic; Pressure after pump (psf)	3087.	3319	3641.	4055	4581	5839.	7477
	Pressure before pump (psf)	2833	2801	2742	2665	2578	2382	2055
	Pressure drop across pump (psf)	254	518	899	1390	2003	3457	5422
	Force on Disk Area ($\Delta P \times A_3$) (LBS)	16,154	32,145	51,176	88,404	127,391	219,865	344,859

$m = .30$

NOZZLE VELOCITY	V_6 (ft/sec) = V_1/m	13.333	20.000	26.666	33.333	40.000	53.333	66.666
	$C_d^2 m^2$	.090	.0913	.0916	.0918	.0922	.0923	.0925
	$[1 + C_d^2 m^2]^{\frac{1}{2}}$	1.044	1.044	1.045	1.045	1.045	1.045	1.045
	$[1 + C_d^2 m^2]^{\frac{1}{2}}/C_d \equiv \beta$	1.044	1.037	1.036	1.035	1.033	1.032	1.031
	$V_7 = \beta V_6$ (fps)	13.920	20.740	27.626	34.500	41.320	55.040	68.733
FORCES	ρQ	515.7	773.6	1,031.5	1289.4	1547.2	2063.1	2578.7
	$-F_H = \rho Q V_7$ (LBS)	7,178.5	16,044.5	28,496.2	44,484.3	63,930.3	113,553.0	177,241.8
	$-F_V = \rho Q V_1$ (LBS)							
	T = (LBS)				NEGATIVE THRUST			
EFFICIENCY	V_7/V_1	3.48	3.457	3.453	3.450	3.443	3.440	3.437
	$(V_7/V_1)^2$	12.110	11.951	11.923	11.902	11.854	11.834	11.813
	$\Delta P \cdot 2g/\gamma V_1^2$: ΔP across pump	15.950	14.457	14.114	13.966	13.976	13.568	13.620
	$(V_7/V_1)^2 - 1 + \Delta P \cdot 2g/\gamma V_1^2 \equiv \Sigma$	27.060	25.408	25.037	24.868	24.830	24.400	24.433
	$\eta_{jet} = 2(V_7/V_1)/\Sigma$	.257	.272	.276	.277	.277	.282	.281
	$\eta_{thr} = 2(V_7/V_1 - 1)/\Sigma$	.183	.193	.196	.197	.197	.200	.199
HYDRAULIC HP.								
	$\rho Q V_1^2/2 \times 550$	7.501	25.320	60.015	117.218	202.54	480.139	937.709
	$HHP = \Sigma \cdot \rho Q V_1^2/2 \times 550$	202.98	643.33	1502.60	2915.00	5,029.1	11,715.40	22,911.04
	$(P_7 - P_5)A_1(1-m)$	8,167	18120	32100	50,030	71760	127,340	198,560

m = .30

		TOTAL PUMPING HEAD, H	3.962	8.080	14.024	21.684	31.247	53.930	84.583
		$H^{3/4}$	2.800	4.790	7.24	10.00	13.20	19.70	28.00
		gpm							
		$gpm^{1/2}$	339.9	416.4	481.5	538.0	590.0	680.0	761.0
PUMP SELECTION	N = 250	SPECIFIC SPEED: N_s	30,348	21,733	16,626.	13,450	11,174	8,629.	6,795.
		EFFICIENCY, η_P			NOT CARRIED THROUGH				
		$\sigma = NPSH/H$	11.00	5.41	3.05	1.92	1.29	.69	.38
		CAVITATION ?							cav
		$\eta_m =$ $\eta_{SYSTEM} = \eta_d \cdot \eta_P \cdot \eta_m$							
	N = 500	N_s	60,695	43,465.	33,253	26,900	22,348	17,259	13,590
		η_P							
		σ	11.00	5.41	3.05	1.92	1.29	.69	.38
		CAVITATION?							
		η_{SYSTEM}							
	N = 870	N_s	105,609	75,630	57,859.	46,806	38,885	30,031	23,646
		η_P							
		σ	11.00	5.41	3.05	1.92	1.29	.69	.38
		CAVITATION?							
		η_{SYSTEM}							
	N =	N_s							
		η_P							
		σ							
		CAVITATION?							
		η_{SYSTEM}							

INLET	V (ft/sec)	(1)	assumed						
	Q (ft^3/sec)	(2)	$A_1 x V_1$						
	Re x 10^{-4}	(3)	Re=DV_1/ν; 59^0F SW						
	f+Δf	(4)	Ref. 4						
	Cd	(5)	Fig. 8						
	Cd^2	(6)	(5) x (5)						
	Q^2	(7)	(2) x (2)						
	$Q^2\gamma$	(8)	(7) x γ						
	$2gA^2Cd^2$	(9)	(2) x A_1^2 x (6) x 2g						
	$\Delta P = Q^2\gamma/2gA^2Cd^2$ (psf)	(10)	(8)/(9)						
	h_L(pipe alone) $f(L/D)(v^2/2g)$	(11)	(4) x (L/D) x (1) x (1)/2g						
	h_L (psf)	(12)	(11) x γ						
	elevation	(13)	psf for 3'-3"						
	hydrostatic pressure	(14)	48'-0"						
	P_1/γ(ft)	(15)	48'-0" − (1) x (1)/64.						
	P_1 (psf)	(16)	(15) x γ						
	NPSH (psf)	(17)	(16) - (10) - (13)						
	vapor pressure (psf)	(18)	53.0						
	atmospheric pressure (psf)	(19)	14.7 x 144						
PIPE to TEE	L/D	(20)							
	h_L	(21)	same as (11)						
	h_L (psf)	(22)	same as (12)						
ELEV.	h_L (psf) for 13'-0"	(23)	13' x 64.094						

TEE	equivalent length	24	Ref. Fig 9.						
	L/D	25							
	h_L (ft)	26	same as (11)						
	h_L (psf)	27	(26) x γ						
	obstruction h_L x1.15	28	1.15 x (27)						
TEE to OUTLET	length	29							
	L/D	30							
	h_L (ft)	31	same as (11)						
	h_L (psf)	32	(31) x γ						
OUTLET	d/D = m = C_d =	33	d/D and m are the parameters which were varied, C_d as before						
	Cd corrected for viscosity	34	C_d x .985, etc.						
	Cd^2	35							
	A_5^2	36							
	$Q^2 \gamma$	37	(8) x γ						
	$2gA_5^2Cd^2$	38							
	P (psf)	39	same as (10)						
	P 2060 psf hydrostatic Pressure after pump (psf)		2060 + (23) + (28) + (29) + (32)						
	Pressure before pump (psf)		(17)						
	Pressure drop across pump (psf)		(40) - (41)						
	Force on Disk Area (ΔP x A_3)	43	(42) x A_1						

$m =$

NOZZLE VELOCITY	V_6 (ft/sec) = V_1/m	(44)	(1) / m							
	$C_d^2 m^2$	(45)	C_d as in (34)							
	$[1 + C_d^2 m^2]^{\frac{1}{2}}$	(46)								
	$[1 + C_d^2 m^2]^{\frac{1}{2}} / C_d \equiv \beta$	(47)								
	$V_7 = \beta V_6$	(48)	(44) x (47)							
FORCES	ρQ	(49)	(2) x ρ							
	$-F_H = \rho Q V_7$ (lbs)	(50)	(48) x (49)							
	$-F_V = \rho Q V_1$ (lbs)	(51)	(48) x (1)							
THRUST	T (lbs)	(52)	(50) - (62)							
EFFICIENCY	V_7/V_1	(53)	(47) / (1)							
	$(V_7/V_1)^2$	(54)	(53) x (53)							
	$\Delta P \cdot 2g/\gamma V_1^2$: ΔP across pump	(55)	(42) x 2g / γ((1))^2							
	$(V_7/V_1)^2 - 1 + \Delta P \cdot 2g/\gamma V_1^2 \equiv \Sigma$	(56)	(54) - 1. + (55)							
	$\eta_{jet} = 2(V_7/V_1)/\Sigma$	(57)	2 x (53) / (56)							
	$\eta_{thr} = 2(V_7/V_1 - 1)/\Sigma$	(58)	2 x ((53) -1.)/(56)							
HYDRAULIC H.P.										
	$\rho Q V_1^2 / 2 \times 550$	(59)	(49) x (1) x (1) /1100							
	$HHP = \Sigma \cdot \rho Q V_1^2 / 2 \times 550$	(60)	(59) x (56)							
	Req. BHP HHP/$\eta_j \eta_p$	(61)	η_j and η_p ; (57) AND (58)							
	$(P_7 - P_5) A_5 (1-m)$	(62)	Force on nozzle							

$m =$

PUMP SELECTION		TOTAL PUMPING HEAD		(42) / γ
		$H^{3/4}$	(64)	(63)$^{.75}$
		gpm	(65)	
		$gpm^{\frac{1}{2}}$	(66)	
	n = ASSUME	SPECIFIC SPEED: N_s		rpm x (66) / (64)
	RPM	EFFICIENCY	(68)	Ref. Fig 14
		$\sigma = NPSH/H$	(69)	(17) / (63)
		CAVITATION ?		Ref. 10
		$\eta_m = 1.00$ $\eta_{SYSTEM} = \eta_j \cdot \eta_p \cdot \eta_m$		(68) x $\eta_J \eta_p$ (same efficiencies as in (61))
	n =	N_s	(72)	
		η_p	(73)	
		σ	(74)	
		CAVITATION?	(75)	
		η_{SYSTEM}	(76)	
	n =	N_s	(77)	
		η_p	(78)	
		σ	(79)	
		CAVITATION?	(80)	
		η_{SYSTEM}	(81)	
	n =	N_s	(82)	
		η_p	(83)	
		σ	(84)	
		CAVITATION?	85	
		η_{SYSTEM}	(86)	