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Under contract with:

Maritime Administration
U.S. Department of Commerce
Contract No. MA-2564, Task 6
Washington, D. C.

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OFFICE OF RESEARCH ADMINISTRATION · ANN ARBOR

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1. Introduction.

It has been one of the objectives of recent research undertaken at The University of Michigan under contract with Maritime Administration to develop suitable hull forms for high-speed cargo liners (Speed range: $1.0 \leq V_K/\sqrt{L} \leq 1.2$). This task has been attacked experimentally and theoretically.

The present report summarizes the theoretical work, which was done under Task 5 and chiefly under Task 6. By the end of Task 6 the project had not yet reached its final aims, but it was carried to the point of yielding the first optimized hull form results.

The theoretical work was undertaken as an independent enterprise, but with the same practical purpose as the experimental work. This purpose was the design of technically feasible cargo liner hulls of good performance in the speed range of $1.0 \leq V_K/\sqrt{L} \leq 1.2$. This is the range of the so-called second hump in the wave resistance coefficient curve which was in general avoided by ship designers in the past.

A fresh approach to this problem was encouraged by the success reached in recent years in the application of wave resistance theory to ship design by such scientists as Inui and Pien, ref. (1), (2), (3) and (4). In view of their results, it appears feasible now to find practical low wave resistance hull forms in this and other "unfavorable" speed ranges by direct use of the theory. It seems, in fact, most important to seek the guidance of the theory when the speed range is "unfavorable".

The application of theoretical methods is further encouraged because it allows the hull shape selection to be made in a comprehensive and systematical way. As a result a much greater variety of possible hull forms is explored than could practically be investigated by experiment.

When at the beginning of our work the possibilities of hull form optimization were examined and the existing techniques were reviewed the conclusion was reached that the given practical task could best be solved by means of singularity systems located outside the centerplane similar to those applied by Pien in his recent work, ref. (2), (3) and (4). The main reason for this is that methods based on centerplane distributions of singularities, or similar simple concepts, find it impracticable to generate beamy and full enough hull shapes within the restraints that are imposed on normal ship forms.

It is true that the off-centerplane distribution technique as developed by Pien has met with some fundamental objections, the most serious being raised against the use of linearized wave resistance theory for "fat" ships, and with regard to the uniqueness of the solution. (Cf. the discussions of ref. (3) by Newman and by Eggers).

The question whether nonlinear effects are of such magnitude and type that their neglect seriously impairs the optimization of "fat hulls" cannot be answered on a purely theoretical basis at the present state of scientific knowledge. It seems plausible at least, that the wave pattern of some nearly optimal hull forms is in better conformity with the linearized free surface condition than that of a hull form that is not optimal in wave resistance.

The uniqueness problem can also not be resolved before satisfactory nonlinear solutions to the wave resistance are found. It should be no surprise indeed that there are several linear approximations, i.e. several ways of representing a hull by singularity systems such that the linear free surface condition is satisfied. These distributions are associated with different approximation errors so that they lead to different wave resistance predictions.

But the working assumption usually made in optimization work is that the classes of singularity systems that are used, while they may differ in wave resistance prediction, still lead to equivalent and practically acceptable hull forms. This certainly does not hold for some misconstrued singularity systems; whether any particular system is suitable for optimization work or not, can presently be judged only by its practical success.

In conclusion, although some of the objections raised deserve further scientific attention they do not give any cogent reason why the off-centerline optimization method should fail. This is a sufficient pragmatic justification for examining the usefulness of the method as an engineering tool. No other consistent optimization technique by which to generate full and beamy hulls is in existence. Optimization based on nonlinear wave resistance theory is not yet feasible at present. The off-centerplane distribution method on the other hand has already led to some encouraging results (4).

Since the purpose and approach of this study are closely related to Pien's work the differences are mainly in scope and emphasis. The project was more limited here in its objectives and financial support. The work was begun with just one specific design task in mind. Computing time was an essential cost factor and much attention had to be devoted to the organization of computer programs in the most time-saving way without sacrificing accuracy.

There are also a few basic differences from Pien's approach in procedural respects as will be discussed in more detail in sections 2.2 and 2.3 below.

2. Mathematical Formulation.

A brief outline of the general procedure of hull optimization is given by the block diagram in figure 1. The assumptions and steps will be explained in more detail in the following, but with no emphasis on derivations since most of these can be found in the work of Havelock (6), Inui (1), (5), Pien (2), (3), H. C. Kim (7) and others. The original steps will be described more elaborately. Wherever possible Pien's notation has been used.

2.1 Singularity Distributions for the Main Hull.

The singularities are arranged outside the centerplane and somewhat inside the hull surface so that sufficiently full and beamy shapes can be generated. In order to deal with a relatively simple configuration it can be assumed that the singularities, i.e. the sources and sinks, are spread continuously over the four vertical side planes of a rhombical body as shown in figure 2. The coordinate system is Cartesian with the origin amidships in the load waterline. The rhomb is symmetrical forward and aft, and the angle of inclination of each side plane relative to the centerplane is $\tan^{-1}\eta_0$. The coordinates are nondimensionalized, using one half of the length of the rhomb:

$$\xi = \frac{X}{L/2} \; ; \quad \eta = \frac{Y}{L/2} \; ; \quad \zeta = \frac{Z}{L/2} \qquad (1)$$

The situation is illustrated in figure 2. The coordinates x, y, z of a field point, when used subsequently, are normalized in the same manner.

BLOCK DIAGRAM OF OPTIMIZATION METHOD

ASSUMED SINGULARITY DISTRIBUTION (MAIN HULL):

$$M(\xi,\zeta)=\sum_i\sum_j a_{ij}\cdot\xi^i\zeta^j$$

WAVE RESISTANCE COEFFICIENT:

$$C_w=\sum_i\sum_j\sum_k\sum_l a_{ij}a_{kl}\cdot C_{ijkl}$$

STEP 1: CALCULATION OF C_{ijkl}

STEP 2: OPTIMIZATION OF MAIN HULL

Conditions:

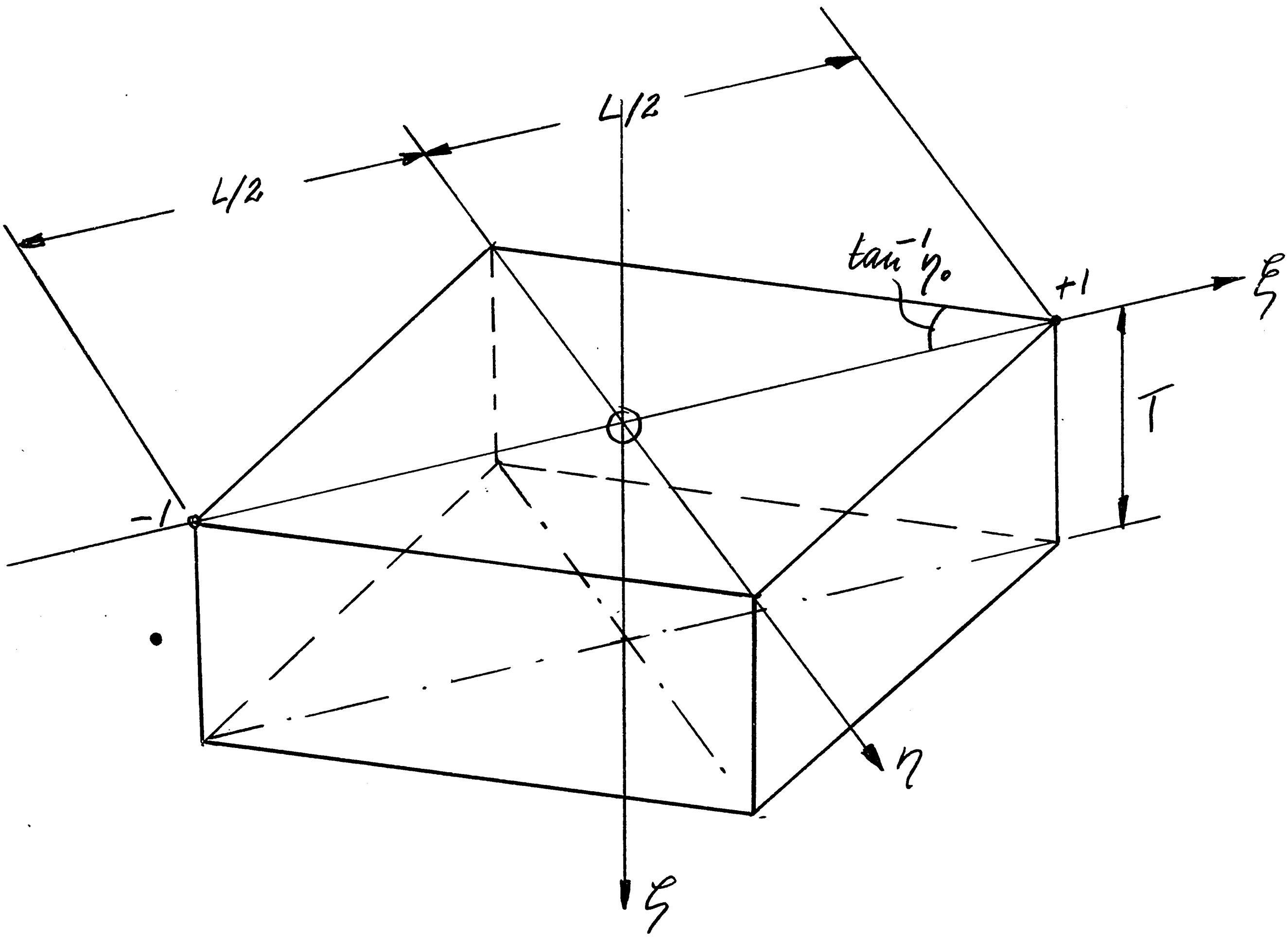
$$\frac{\partial C_w}{\partial a_{ij}}=0 \qquad \text{(Ritz conditions)}$$

$$\sum_j a_{ij}B_{ij}=r_i \qquad \text{(Restraints)}$$

STEP 3: FEASIBILITY CHECK OF BULB SINGULARITIES ON THE BASIS OF HAVELOCK'S FREE WAVE AMPLITUDE EXPRESSIONS.

STEP 4: TRACING OF STREAMLINES BY RUNGE-KUTTA METHOD

Figure 1



<u>Figure 2</u>

Coordinates and rhombic body on whose sides the singularity distribution is arranged.

The singularity distribution is thus located on the surface:

$$\eta = \pm \eta_0 \cdot (1 - |\xi|), \quad \text{for} \quad -1 \leq \xi \leq +1$$

and its intensity is expressed by the polynomial:

$$M(\xi, \zeta) = \sum \sum a_{ij} \cdot \xi^i \cdot \zeta^j \tag{2}$$

which is normalized by unit ship speed, and is therefore dimensionless. The total singularity strength must vanish (body closure condition):

$$\int_{-\frac{2T}{L}}^{0} \int_{-1}^{+1} M(\xi, \zeta) \, d\xi \, d\zeta = 0 \tag{2a}$$

The magnitude of the coefficients a_{ij} is to be determined from the condition of minimal wave resistance within the given restraints. For this purpose the wave resistance coefficient must first be expressed in terms of the a_{ij}.

2.2 Wave Resistance Expressions.

The wave resistance associated with the source distribution of eq. (2) can be written as follows, ref. (2)

$$R_W = \frac{\rho g k_0 L^4}{16\pi} \int_0^{\pi/2} (P^2 + Q^2) \cdot \sec^3\theta \, d\theta \tag{3}$$

where
$$P = \sum_i \sum_j a_{ij} \iint_\eta \xi^i \zeta^j \cdot e^{\frac{\sec^2\theta}{2F^2}\zeta} \cdot \Big\{ \cos\big[k_0 \tfrac{L}{2}(\xi \cdot \cos\theta + \eta \sin\theta) \sec^2\theta\big] + \cos\big[k_0 \tfrac{L}{2}(\xi \cos\theta - \eta \sin\theta) \cdot \sec^2\theta\big] \Big\} \, d\xi \, d\zeta \tag{3a}$$

$$Q = \sum_i \sum_j a_{ij} \iint_\eta \xi^i \zeta^j \, e^{\frac{\sec^2\theta}{2F^2}\zeta} \cdot \Big\{ \sin\big[k_0 \tfrac{L}{2}(\xi \cos\theta + \eta \sin\theta) \sec^2\theta\big] + \sin\big[k_0 \tfrac{L}{2}(\xi \cos\theta - \eta \sin\theta) \sec^2\theta\big] \Big\} \, d\xi \, d\zeta \tag{3b}$$

with $$k_0 = \frac{g}{V^2} \quad \text{and} \quad F = \frac{V}{\sqrt{gL}}$$

These expressions are practically the same as in ref. (2) except that P, Q, ξ, η are truly dimensionless which has resulted in the factor $L^4/16$ in eq. (3). The integrals in (3a) and (3b) must be taken over the positive half $\eta = +\eta_0(1-|\xi|)$ of the rhomb only.

The wave resistance coefficient is correspondingly

$$C_W = \frac{R_W}{\rho/2 \cdot V^2 L^2} = \frac{1}{8\pi F^4} \int_0^{\pi/2} (P^2 + Q^2) \cdot \sec^3\theta \, d\theta \tag{4}$$

Let

$$P = \sum_i \sum_j a_{ij} P_{ij}, \quad \text{and} \quad Q = \sum_i \sum_j a_{ij} Q_{ij} \tag{5}$$

where

$$P_{ij} = \int_{-t}^{0} \zeta^j \cdot e^{\frac{\sec^2\theta}{2F^2}\zeta} \cdot \int_{-1}^{+1} \xi^i \cdot \cos\left(\frac{\sec\theta}{2F^2}\xi\right) \cdot \cos\left[\eta_0 \frac{\tan\theta \sec\theta}{2F^2}(1-|\xi|)\right] d\xi \, d\zeta \tag{5a}$$

$$Q_{ij} = \int_{-t}^{0} \zeta^j \cdot e^{\frac{\sec^2\theta}{2F^2}\zeta} \cdot \int_{-1}^{+1} \xi^i \cdot \sin\left(\frac{\sec\theta}{2F^2}\xi\right) \cdot \cos\left[\eta_0 \frac{\tan\theta \sec\theta}{2F^2}(1-|\xi|)\right] d\xi \, d\zeta$$

$$\text{with } t = \frac{2T}{L} \tag{5b}$$

Let further

$$Z_j = \int_{-t}^{0} \zeta^j \cdot e^{\frac{\sec^2\theta}{2F^2}\zeta} \, d\zeta \tag{6}$$

$$X_{Pi} = \int_{-1}^{+1} \xi^{i} \cdot \cos\left(\frac{\sec\theta}{2F^2}\xi\right) \cdot \cos\left[\eta_0 \frac{\tan\theta \sec\theta}{2F^2}(1-|\xi|)\right] d\xi \qquad \text{(6a)}$$

$$X_{Qi} = \int_{-1}^{+1} \xi^{i} \cdot \sin\left(\frac{\sec\theta}{2F^2}\xi\right) \cdot \cos\left[\eta_0 \frac{\tan\theta \sec\theta}{2F^2}(1-|\xi|)\right] d\xi \qquad \text{(6b)}$$

so that:

$$P_{ij} = Z_j \cdot X_{Pi} \, , \quad \text{and} \quad Q_{ij} = Z_j \cdot X_{Qi} \qquad \text{(6c)}$$

Then, if the eqs. (5) through (6d) are substituted in eq. (4):

$$C_W = \sum_i \sum_j \sum_k \sum_l a_{ij} \cdot a_{kl} \cdot C_{ijkl} \qquad \text{(7)}$$

where

$$C_{ijkl} = \frac{1}{8\pi F^4} \int_0^{\pi/2} \left(P_{ij} \cdot P_{kl} + Q_{ij} \cdot Q_{kl}\right) \cdot \sec^3\theta \, d\theta \qquad \text{(8)}$$

$$= \frac{1}{8\pi F^4} \int_0^{\pi/2} Z_j \cdot Z_l \cdot \left(X_{Pi} \cdot X_{Pk} + X_{Qi} \cdot X_{Qk}\right) \cdot \sec^3\theta \, d\theta \qquad \text{(8a)}$$

The integral (6a) can be evaluated by the following closed expressions:

$$Z_j = \int_{-t}^{0} \zeta^{j} \cdot e^{\frac{\sec^2\theta}{2F^2}\zeta} d\zeta =$$

$$= t^{j+1} \int_{-1}^{0} \zeta^{j} e^{\frac{\sec^2\theta}{2F^2} t\zeta} d\zeta = t^{j+1} \cdot \bar{Z}_j = \qquad \text{(9)}$$

$$= (-1)^{j} \cdot t^{j+1} \cdot \left[\frac{j!}{a^{j+1}}\left(1 - \sum_{n=1}^{j+1} \frac{a^n}{n!} e^{-a}\right)\right]$$

The integrals X_{Pi} and X_{Qi} can also be evaluated in closed form. Pien's eqs. (44) and (45) in ref. (2) are immediately applicable since his integrals X_i^S and X_i^C are identical to X_{Qi}' and X_{Pi}', respectively.

It should be noted that X_{Pi}' vanishes whenever i is an odd number because in that event the integrand is an odd function of with respect to the origin. X_{Qi}' vanishes for even values of i. Consequently, whenever an odd value i occurs in combination with an even value of k in eq. (8), or vice versa, so that the sum of the two subscripts i and k is odd, the integrand of (8a) vanishes and C_{ijkl} is zero in this event.

This result is not in conformity with Pien's tabulated wave resistance coefficients C_{ijkl} published in ref. (2), Table 1. There, all coefficients differ from zero.

This was a puzzling discrepancy at first since the equations used here to compute the coefficients C_{ijkl} were identical to those given by Pien in the same reference. But it could be clarified in a later discussion with Pien that the C_{ijkl} tabulated in ref. (2) are only forebody wave resistance coefficients. This means, as we presume, that the integrations for X_{Pi}' and X_{Qi}', eqs. (6b) and (6c), were carried from $\xi = 0$ to $\xi = 1.0$ only.

The motivation in applying forebody wave resistance coefficients was clarified in Pien's subsequent paper, ref. (3). We quote:

> "The most frequent use made of the theory in ship design problems is to optimize the wavemaking resistance of a whole ship without checking the forebody free-surface disturbance alone. It is conceivable that the optimum

value so obtained might be attributable not to the fact that both the bow and stern produce very small free waves but rather to the favorable theoretical interference effect of large bow and stern free wave systems. Due to the viscosity effect, the existing theory cannot accurately predict either the amplitude or the phase of the stern free waves, so that the favorable interference effect as predicted by the theory may not always be realized in practice, thus leading to a large wavemaking resistance. Therefore, it is rather important to minimize the forebody free-surface disturbance."

Thus, if we interpret Pien's statements correctly, he has apparently used only the forebody singularity system and forebody wave resistance coefficients C_{ijkl} when searching for a singularity system producing minimal wave resistance. The afterbody singularity system was disregarded in this optimization step. The same procedure had also been used and recommended by Inui, ref. (5), but with the warning that it must be limited to singularity distributions with moderate interference effects. Generally, the wave resistance can be represented by a term due to the bow half singularity system, one due to the stern half singularity system, and an interference term. Optimizing the forebody separately can lead to consistent results only if the interference term is negligible.

This condition may have been satisfied in the cases treated by Pien and Inui, but it becomes too restrictive when other types of singularity distributions are used.

It was therefore decided here to follow, for the time being, the approach that bases the optimization on the wave resistance coefficients of the total singularity system.

2.3 Optimization of the Main Hull.

Every singularity distribution of some assumed type describes a great number of possible hull shapes which can be generated by varying the free coefficients a_{ij} of the singularity strength function, eq. (2). It is true that the variation in shape is limited by the number of terms assumed and by the location of the singularity system. But there are normally some hull shapes of low wave resistance even within the most limited family of singularity distributions.

Wave resistance optimization techniques have the aim of selecting a singularity distribution within the family that results in the lowest wavemaking resistance compatible with all practical restraints of the design.

This problem can be formulated more rigorously in the following manner: The wave resistance is a function of the whole set of parameters a_{ij} of the singularity function:

$$C_w = f(a_{10}, a_{11}, a_{12}, \ldots, a_{20}, \ldots, a_{mn})$$

This function describes a "surface" in the multidimensional parameter space, and we want to find a minimum on this surface that complies with the given restraints. The absolute minimum exists, but it is trivial because C_w is zero, of course, when all parameters a_{ij} vanish, but then the displacement is zero, too. Since C_w is a continuous function of every parameter, however, there must also exist at least one relative minimum at which the displacement differs from zero, and the restraints are satisfied.

The restraints are related to certain prescribed properties of the singularity distribution or of the hull shape. They are sometimes simple, sometimes complicated functions of the parameters a_{ij}. The following restraints e.g. are of linear, i.e. relatively simple type:

$$V = \int_{-t}^{0}\int_{0}^{1} M(\xi,\zeta)\cdot\xi \, d\xi \, d\zeta = \sum_i \sum_j \frac{a_{ij}}{(i+2)(j+1)} \cdot \left(-(-t)^{j+1}\right)$$

$$B = \int_{-t}^{0}\int_{0}^{1} M(\xi,\zeta) \, d\xi \, d\zeta = \sum_i \sum_j \frac{a_{ij}}{(i+1)(j+1)} \left(-(-t)^{j+1}\right) \qquad (10)$$

$$T = M(\xi=1,\zeta) = \sum_i \sum_j a_{ij} \zeta^j$$

These restraints are expressed in terms of the singularity distribution, but they bear a certain physical meaning by their relation to displacement, midship area, and entrance angle of the waterlines. The scale of these relations must be established by calibration from case to case.

In order to obtain a straight keel or a flat bottom and similar features, more complicated restraints must be introduced.

Two equivalent solution techniques exist for solving the optimization problem with restraints. The <u>elimination method</u> makes use of the restraints by substituting them into the C_W - function so that the number of free parameters is reduced by one per resraint. The minimum of C_W is then sought in terms of the remaining unrestricted variables in the usual manner of an extreme problem or free variational problem. The conditions for a minimum are

$$\frac{\partial C_W}{\partial a_{pq}} = 0 = 2 \cdot \sum_i \sum_j a_{ij} \cdot C_{pijq} \tag{11}$$

This results in a linear system of equations for the unknowns a_{ij}. The parameters previously eliminated can be found by substitution into the restraints. The approach outlined above was used by Pien, ref. (2).

An alternative solution technique is the <u>method of Lagrangian multipliers</u>. According to this method, ref. (9), the restraints are written the form

$$\phi_i(a_{10}, a_{11}, a_{12}, \dots, a_{20}, \dots, a_{mn}) = 0 \tag{11a}$$

and one undetermined multiplier λ_i is introduced for each restraint. The optimization problem with restraints can then be transformed into a free variational problem of the modified function

$$\overline{C_W} = C_W + \sum_i \lambda_i \phi_i \tag{11b}$$

The minimum of this function is sought by means of

$$\frac{\partial \overline{C_W}}{\partial a_{pq}} = 0 = \frac{\partial C_W}{\partial a_{pq}} + \sum_i \lambda_i \frac{\partial \phi_i}{\partial a_{pq}} \tag{11c}$$

This yields as many equations as there are a_{ij}, and the restraints, eq. (11a), furnish the missing equations allowing to find all unknowns including the λ_i.

The two methods may differ somewhat from numerical points of view, but they should both lead to equivalent solutions of the optimization problem. Although their use is recommendable for direct optimization purposes none of the two methods was used here because a somewhat different question was posed in the exploratory stage of our work that is reported here.

Whenever the attention is fixed upon finding the optimal hull one tends to overlook the variety of other favorable hull shapes that exist within a certain family. The pure optimization methods do not reveal the full picture. But this would be desirable, for among the second-best shapes there may be some that are superior to the others from the standpoint of seagoing ability, propulsive performance, ballast performance or the like. If the wave resistance of these hulls is still acceptably low one may choose the most suitable shape from these secondary aspects.

There are certainly many systematic ways of exploring the wave resistance properties of a hull shape family under given restraints. The establishment of a comprehensive evaluation method should be given some more thought in the future.

In order to generate just a few other hull shapes, satisfying the restraints, but maybe somewhat less than optimal, the following procedure was applied here: Only one restraint was used, the displacement restraint of eq. (10). Then in eq. (11a) all minimum conditions but one were satisfied; the disregarded condition was replaced with the restraint so that a determinate system of equations for the a_{ij} was obtained.

The decision which minimum condition to ignore is of course arbitrary, and in order to exhaust the possibilities, the condition being replaced was varied in a cyclic manner. In this way a whole set of distributions was obtained all satisfying the same displacement restraint, but resulting in quite distinct shapes. The procedure and the results are discussed further by an example below.

It is not claimed that the hull shapes so obtained have to be anywhere close to optimal. By disregarding certain minimum conditions we have ignored the influence of the associated coefficients upon the wave resistance in our "optimization" method. But our primary purpose is only to generate a set of distinct shapes that differ in a systematic way.

The wave resistance properties of these hulls have been evaluated for only a few examples, which are discussed below. The resistance seems to be favorably low for a variety of different shapes. But this must be interpreted with caution because the hulls have not been traced yet, and even though the displacement restraint is the same the displacement may differ.

2.4 The Selection of Bulbous Bows.

Although the singularities for the main hull are selected on the basis of optimum considerations there is sometimes room for improvement because the assumed type and location of the singularities cover only a limited scope of variations. It can in particular be checked whether the results become better if a bulbous bow is fitted to the main hull.

It was Inui's original idea to apply Havelock's concept of the far rear free wave pattern to answer this question and, in fact, to design bulbs, refs. (1) and (5). Pien has developed this scheme further, and it is along these lines that we proceeded, ref. (2).

Havelock, ref. (6), has shown that the wave resistance of a hull can be expressed as

$$R_W = \pi \rho V^2 \int_0^{\pi/2} \left\{ [A_C(\theta)]^2 + [A_S(\theta)]^2 \right\} \cos^3\theta \, d\theta \qquad (12)$$

where $A_C(\theta)$ and $A_S(\theta)$ represent the cosine and sine components of the "elementary" free wave amplitudes in the far rear. The objective in low wave resistance hull design is to minimize these amplitude functions in the important range of angles θ, i.e. where the factor $\cos^3\theta$ is still of significant magnitude. This can be achieved by adding bulb singularities to the main hull singularity system.

Pien has derived expressions for the amplitude functions produced by the main hull singularities, and the following are the equivalent equations in dimensionless form

$$\frac{A_C(\theta)}{L/2} = \sum_i \sum_j a_{ij} \frac{\sec^3\theta}{2\pi F^2} \int_{-t}^{0} \int_{-1}^{+1} \xi^i \zeta^j e^{\frac{\sec^2\theta}{2F^2}\zeta} \cdot \cos b\eta \cdot \cos a\xi \, d\xi \, d\zeta \qquad (13)$$

$$\frac{A_S(\theta)}{L/2} = \sum_i \sum_j a_{ij} \frac{\sec^3\theta}{2\pi F^2} \int_{-t}^{0} \int_{-1}^{+1} \xi^i \zeta^j e^{\frac{\sec^2\theta}{2F^2}\zeta} \cdot \cos b\eta \cdot \sin a\xi \, d\xi \, d\zeta$$

with $a = \frac{\sec\theta}{2F^2}$; $b = \frac{\sec\theta \cdot \tan\theta}{2F^2}$

Let

$$\bar{Z}_j = \frac{\sec^3\theta}{2\pi F^2} \cdot \int_{-t}^{0} \zeta^j \cdot e^{\frac{\sec^2\theta}{2F^2}\zeta} \, d\zeta = \frac{\sec^3\theta}{2\pi F^2} \cdot Z_j \qquad (14)$$

and for the case of the rhombic body $\eta = \pm\eta_0 \cdot (1-|\xi|)$:

$$X_i^C = \int_{-1}^{+1} \xi^i \cdot \cos(b\eta_0(1-|\xi|)) \cdot \cos a\xi \, d\xi = X_{P_i} \qquad (15)$$

$$X_i^S = \int_{-1}^{+1} \xi^i \cdot \cos(b\gamma_0 (1-|\xi|)) \cdot \sin a\xi \, d\xi = \bar{X}_{Qi} \tag{16}$$

Then

$$\frac{A_C(\theta)}{L/2} = \sum_i \sum_j a_{ij} \cdot \overline{\overline{Z}}_j \cdot X_i^C \tag{17}$$

$$\frac{A_S(\theta)}{L/2} = \sum_i \sum_j a_{ij} \cdot \overline{\overline{Z}}_j \cdot X_i^S \tag{18}$$

The integrals (15) and (16) can be written so that it becomes apparent which contributions are caused by the bow, midship, and stern wave systems respectively. It is the idea of Inui's and Pien's work to use the bulb singularities to cancel out only those free waves that are caused by the forebody of the main hull. The rest is ignored because its relation to the bulb waves is less immediate, and because it seems to be the practically less significant part.

If therefore the functions X_i^C and X_i^S are replaced with the part due to the forebody only one obtains

$$\frac{A_S^b(\theta)}{L/2} = -\sum_i \sum_j a_{ij} \cdot \overline{\overline{Z}}_j \cdot C_i(1) = \sum_i \sum_j a_{ij} \cdot A_{S_{ij}}^b \tag{19}$$

$$\frac{A_C^b(\theta)}{L/2} = \sum_i \sum_j a_{ij} \overline{\overline{Z}}_j \cdot S_i(1) = \sum_i \sum_j a_{ij} \cdot A_{C_{ij}}^b \tag{20}$$

which conforms with eq. (48) and (49) of ref. (2). The functions $C_i(1)$ and $S_i(1)$ are defined there too by a series expression, eq.(42).

$$C_i^{\pm}(\xi) = \frac{1}{a \pm b}\left[\xi^i - \frac{i(i-1)\xi^{i-2}}{(a \pm b)^2} + \dots\right]$$

$$S_i^{\pm}(\xi) = \frac{1}{a \pm b}\left[\frac{i \cdot \xi^{i-1}}{a \pm b} - \frac{i(i-1)(i-2) \cdot \xi^{i-3}}{(a \pm b)^3} + \dots\right] \tag{20a}$$

$$\text{and: } C_i(1) = \frac{1}{2}\left[C_i^{+}(1) + C_i^{-}(1)\right]$$

$$S_i(1) = \frac{1}{2}\left[S_i^{+}(1) + S_i^{-}(1)\right] \tag{20b}$$

It is now assumed that vertical lines of sources or alternatively of doublets are arranged along the front edge of the rhombic body inside the hull in order to generate bulbs. Their strength is

Source line:

$$S(\zeta) = \sum_j s_j \zeta_j$$

Doublet line: (21)

$$D(\zeta) = \sum_j d_j \zeta_j$$

The corresponding free wave amplitudes are

Source line, cosine component:

$$\frac{A_c^{\ell}(\theta)}{L/2} = \sum_j s_j \cdot \overline{\overline{Z}}_j = \sum_j s_j \cdot a_{Sj} \tag{22}$$

Doublet line, sine component:

$$\frac{A_s^{\ell}(\theta)}{L/2} = -\frac{\sec\theta}{2F^2} \sum_j d_j \overline{\overline{Z}}_j = \sum_j d_j \cdot a_{Dj} \tag{23}$$

The sine component of the source line, and the cosine component of the doublet line vanish.

In many cases it is either the sine or the cosine component of the main hull free wave amplitude function that is predominant in the wave resistance expression, eq. (12). The resistance can then be minimized simply by using only a source or only a doublet line at the bulb. If e.g. the cosine component of the main hull system is negligible it is sufficient to minimize the integral:

$$I = \int_0^{\pi/2} \left[A_S^b(\theta) - A_S^l(\theta)\right]^2 \cdot \cos^3\theta \, d\theta \tag{24}$$

This can be achieved by substituting eqs. (20) and (23) into (24), and by solving for those doublet strength coefficients d_j that make (24) a minimum. One equation is obtained for every value p of the subscript j:

$$\frac{\partial I}{\partial d_p} = 0 = \frac{\partial}{\partial d_p} \int_0^{\pi/2} \left\{\left[A_S^b(\theta)\right]^2 - 2A_S^b(\theta) \cdot A_S^l(\theta) + \left[A_S^l(\theta)\right]^2\right\} \cdot \cos^3\theta \, d\theta = \tag{25}$$

$$= -2 \cdot \int_0^{\pi/2} A_S^b(\theta) \cdot a_{D_p}(\theta) \cdot \cos^3\theta \, d\theta + 2 \sum_j d_j \int_0^{\pi/2} a_{D_j}(\theta) \cdot a_{D_p}(\theta) \cdot \cos^3\theta \, d\theta$$

Let

$$D_{jp} = \int_0^{\pi/2} a_{D_j}(\theta) \cdot a_{D_p}(\theta) \cdot \cos^3\theta \, d\theta \tag{26}$$

$$r_p = \int_0^{\pi/2} A_S(\theta) \cdot a_{D_p}(\theta) \cdot \cos^3\theta \, d\theta$$

The system of equations to be solved for the coefficients d_j becomes:

$$\sum_j D_{jp} \cdot d_j = r_p$$

In a practical case the amplitude functions $A_S^b(\theta)$ and $A_C^b(\theta)$ of the main hull are discussed first. If one of them is excessive the appropriate bulb singularity, either a source line or a doublet line, is selected, and its strength distribution is determined using the far rear wave amplitude concept as expressed in eq. (27).

It should be pointed out that this concept, while it does promise improvements over the main hull, differs from the optimization concept used in general in optimizing the main hull. The principal reason for using this alternative method is its simplicity. It would be tedious, although not impossible in principle, to optimize the bulb by the extreme value and restraint method as before.

Despite the practical merits of this bulb selection method it should be realized that by the type of singularity we choose we are limiting the changes to the neighborhood of the bow, and we are only finding such improvements that can be obtained by different bow configurations. We would not be led to such improved versions of the design that necessitate changes throughout the forebody, and might result in bulbless forms.

Generally speaking, the fact that two selection techniques are used, the second of which is of less generality, somewhat obscures the picture. The Inui and Pien approach favors bulbs because it uses them to correct for insufficiencies in the main hull, but this does not allow the conclusion that there are no equivalent bulbless hull shapes.

2.5 Streamline Tracing.

When the optimal singularity distribution is known the shape of the hull must be determined. There is no shortcut relationship between singularities and hull shape like for the Michell ship so that the contours of the body must be found by tracing the closing streamline around the singularities. The differential equation of a streamline is

$$\frac{dx}{U_0 + u} = \frac{dy}{v} = \frac{dz}{w} \tag{28}$$

From this the streamline itself can be traced by the Runge-Kutta method or similar approximate integration procedures. The streamlines inside the closing streamline end on the singularity surface, and it takes a few trial and error steps before the starting point of the closing streamlines can be estimated properly.

The velocities u, v, w are induced by the singularities on the rhombic body. The following velocity expressions can be derived (compare the elaborate derivation and discussion by H. C. Kim in ref. (7)).

$$u = \frac{1}{4\pi} \sum_i \sum_j a_{ij} \int_{-1}^{+1} \int_{-t}^{0} \frac{\zeta^j}{R^3} d\zeta \cdot \sqrt{1+\eta'^2}\,(x-\xi) \cdot \xi^i d\xi$$

$$v = \frac{1}{4\pi} \sum_i \sum_j a_{ij} \int_{-1}^{+1} \int_{-t}^{0} \frac{\zeta^j}{R^3} d\zeta \cdot \sqrt{1+\eta'^2}\,(y-\eta) \cdot \xi^i d\xi \tag{29}$$

$$w = \frac{1}{4\pi} \sum_i \sum_j a_{ij} \int_{-1}^{+1} \int_{-t}^{0} \frac{\zeta^j (z-\zeta)}{R^3} d\zeta \cdot \sqrt{1+\eta'^2} \cdot \xi^i d\xi$$

where

$R = [(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2]^{1/2} =$

= the distance from the source point (ξ, η, ζ) on the singularity surface to the field point (x, y, z) at which the velocities are to be determined and $\eta' = \frac{d\eta}{d\xi} =$

= the slope of the singularity surface at the source point with respect to the ξ- direction.

Let

$$I_j = \int_{-t}^{0} \frac{\xi^j}{R^3} d\xi = f(t, z, A, j), \text{ where } A = [(x-\xi)^2 + (y-\eta)^2]^{1/2} \quad (30)$$

and further

$$I_i = \frac{\sqrt{1+\eta'^2}}{4\pi} \cdot \sum_j a_{ij} I_j \quad (31)$$

Then:

$$\left. \begin{aligned} u &= \sum_i \int_{-1}^{+1} I_i \, \xi^i (x-\xi) d\xi \\ v &= \sum_i \int_{-1}^{+1} I_i \cdot \xi^i (y-\eta) d\xi \\ &\text{and, as shown in ref. (6)} \\ w &= \sum_i \int_{-1}^{+1} (z I_i - I_{i+1}) \cdot \xi^i d\xi \end{aligned} \right\} \quad (32)$$

The evaluation of these integral expressions and hence the tracing in general are most time-consuming processes, in the order of magnitude of ten to twenty minutes computing time per waterline with the IBM 7090 computer. Many measures have therefore been applied here to organize the computer programs in the most time-saving manner:

1. A fast and accurate integration subroutine based on the Romberg method (ref. (8)), was written.
2. Since the integrands in the three velocity expressions (32) are similar, the integration was organized in a parallel manner so that the common factor had to be

determined only once.

3. The inner integrals I_j of eq. (30) were calculated beforehand in tabular form and stored on magnetic tape. The interpolation of these values during the subsequent evaluation of the expressions (32) in tracing is about twice to three times faster than a direct computation. The time required to compute the tables and the access times to the tapes must be added as overheads. But when the number of coefficients a_{ij} was greater than 5, definite savings were obtained. Every optimized set of coefficients a_{ij} forms a case stored on tape separately. But the integrals I_j are also saved on tape so that parameter variations that result in changes of the a_{ij} can be executed conveniently at any later time.

Although these measures have resulted in appreciable reductions of computing time the present time requirements are still much higher than desirable. Further reductions can be achieved however. Instead of computing a great number of flow velocities along every streamline, it is possible, e.g. to compute only a few selected function values at important locations and to crossfair this array manually so that enough data are then available for the tracing routine to interpolate.

3. Results.

The general status of the project is that all basic programs have been completed and checked out, but the time was not sufficient to complete the given design task. This would require further systematic evaluation of the existing possibilities along with the introduction of proper restraints (flat bottom).

The examples for which calculations have been carried out so far have the basic properties dictated by the Maritime Administration design task, but they also happen to be in the range for which some data were published by Pien in ref. (2) so that the results could be compared and checked conveniently.

The design speed-length ratio was selected as $V_K/\sqrt{L} = 1.05$ for the test example (Froude number F = 0.32). Since, however, experience shows that the wave resistance curves computed by theory are shifted to somewhat higher speeds in comparison with tests, the actual calculations were carried out for $V_K/\sqrt{L} = 0.92$ (F = 0.28). The following parameters were selected in accordance with Pien's calculations to facilitate checking:

Draft - length ratio of the rhombic body

$t = 0.03$

Slope of sidewalls of rhomb

$\eta_0 = 0.12$

Number of terms provided in surface singularity polynomial

$j = 0, \ldots, 3$, $i = 1, \ldots, 5$ } Maximum of 20 terms: actually only up to 10 so far.

Number of terms in line singularity polynomials at bow

$j = 0, \ldots, 3$

For this set of parameters the wave resistance coefficients C_{ijkl} of the main hull, eq. (8) and (8a), were computed first. The integrals (6a), (6b), (6c) which are required for this purpose were tabulated on punched cards for the important range of θ - values, and an interpolation routine was written to use these tables in evaluating (8a). Table 1 shows a set of the coefficients C_{ijkl} obtained for the same case that Pien has published in ref. (2).

It has been mentioned in section 2.2 of this report that there are some differences between Pien's assumptions and ours, and that under our assumptions the C_{ijkl} must vanish whenever $(i + k)$ is odd. There are also some other differences in the results. The fact that some of the C_{ijkl} are negative here while all results are positive in Pien's work is of minor importance because he probably nondimensionalized his values differently. Some of the coefficients are in fairly good agreement, in particular for $i = 1$, where only a few percent difference occur which may be attributable to integration inaccuracies. Other results differ more substantially, e.g. when $i = k = 4$ (Pien up to about 45% lower). When this was discovered the results obtained here were checked very carefully by alternative methods and by increasing the accuracy of integration repeatedly. The checks confirmed the validity of the figures obtained, but gave no indication what the reason for the deviations in Pien's results may have been.

The optimal singularity distribution was determined for two examples of polynomial expressions:

CASE A: $M(\xi,\zeta) = a_{10}\cdot\xi + a_{20}\cdot\xi^2 + a_{30}\cdot\xi^3 + a_{40}\cdot\xi^4 + a_{50}\,\xi^5$ (33)

which is the case of a uniform draftwise singularity distribution and

CASE B: $$M(\xi,\zeta) = (a_{10}\,\xi + a_{20}\,\xi^2 + a_{30}\,\xi^3 + a_{40}\,\xi^4 + a_{50}\,\xi^5)\cdot\zeta^0 + (a_{11}\,\xi + a_{21}\,\xi^2 + a_{31}\,\xi^3 + a_{41}\,\xi^4 + a_{51}\,\xi^5)\,\zeta \quad (34)$$

In the first case, five Ritz conditions of form eq. (10) are obtained

$$\begin{aligned}
C_{1100}\cdot a_{10} + C_{2100}\cdot a_{20} + C_{3100}\cdot a_{30} + C_{4100}\cdot a_{40} + C_{5100}\cdot a_{50} &= 0\\
C_{1200}\cdot a_{10} + C_{2200}\cdot a_{20} + C_{3200}\cdot a_{30} + C_{4200}\cdot a_{40} + C_{5200}\cdot a_{50} &= 0\\
C_{1300}\cdot a_{10} + C_{2300}\cdot a_{20} + C_{3300}\cdot a_{30} + C_{4300}\cdot a_{40} + C_{5300}\cdot a_{50} &= 0\\
C_{1400}\cdot a_{10} + C_{2400}\cdot a_{20} + C_{3400}\, a_{30} + C_{4400}\, a_{40} + C_{5400}\cdot a_{50} &= 0\\
C_{1500}\cdot a_{10} + C_{2500}\cdot a_{20} + C_{3500}\cdot a_{30} + C_{4500}\, a_{40} + C_{5500}\cdot a_{50} &= 0
\end{aligned} \quad (35)$$

In matrix form with numerical values inserted, this becomes:

$$\begin{bmatrix}
0.111\cdot10^{-2} & 0 & 0.939\cdot10^{-3} & 0 & 0.692\cdot10^{-3}\\
0 & 0.996\cdot10^{-3} & 0 & 0.972\cdot10^{-3} & 0\\
0.939\cdot10^{-3} & 0 & 0.852\cdot10^{-3} & 0 & 0.680\cdot10^{-3}\\
0 & 0.972\cdot10^{-3} & 0 & 0.994\cdot10^{-3} & 0\\
0.692\cdot10^{-3} & 0 & 0.680\cdot10^{-3} & 0 & 0.596\cdot10^{-3}
\end{bmatrix}\cdot
\begin{bmatrix} a_{10}\\ a_{20}\\ a_{30}\\ a_{40}\\ a_{50}\end{bmatrix} = 0 \quad (36)$$

The coefficient matrix is symmetrical. It can also be seen that in the present case row 2 and row 4 are contradictory conditions. Hence, it is necessary here to eliminate one of these two equations and to replace it with a restraint.

The "displacement" restraint, eq. (10) is chosen for this purpose, e.g. V = 0.35:

$$V = \frac{1}{3}a_{10} + \frac{1}{4}a_{20} + \frac{1}{5}a_{30} + \frac{1}{6}a_{40} + \frac{1}{7}a_{50} = 0.35 \quad (37)$$

It now has to be decided whether eq. (37) shall replace the second or the fourth row of the system (35). Both alternatives were investigated. The solutions are:

When (37) replaces row 2 (Case A I):

$a_{10} = 4.86 \cdot 10^{-6}$; $a_{20} = 4.02$; $a_{30} = -1.00 \cdot 10^{-5}$; $a_{40} = -3.93$; $a_{50} = 5.79 \cdot 10^{-6}$

When (37) replaces row 4 (Case A II):

$a_{10} = 1.57 \cdot 10^{-6}$; $a_{20} = 4.42$; $a_{30} = -3.08 \cdot 10^{-6}$; $a_{40} = -4.53$; $a_{50} = 1.66 \cdot 10^{-6}$

The resulting singularity functions are plotted in figure 3. Only Case A I yields a positive value of the singularity function at the bow. Case A II is not feasible because the entrance angle of the waterlines would be negative.

This means that under the restraints assumed only one feasible solution remains. It is, however, likely that with more restraints, e.g. one for the entrance angle, we could have obtained more than one feasible solution, by using the same principle of cyclic permutation.

The hull of Case A I was traced, and is shown in figure 4. It has the following characteristics:

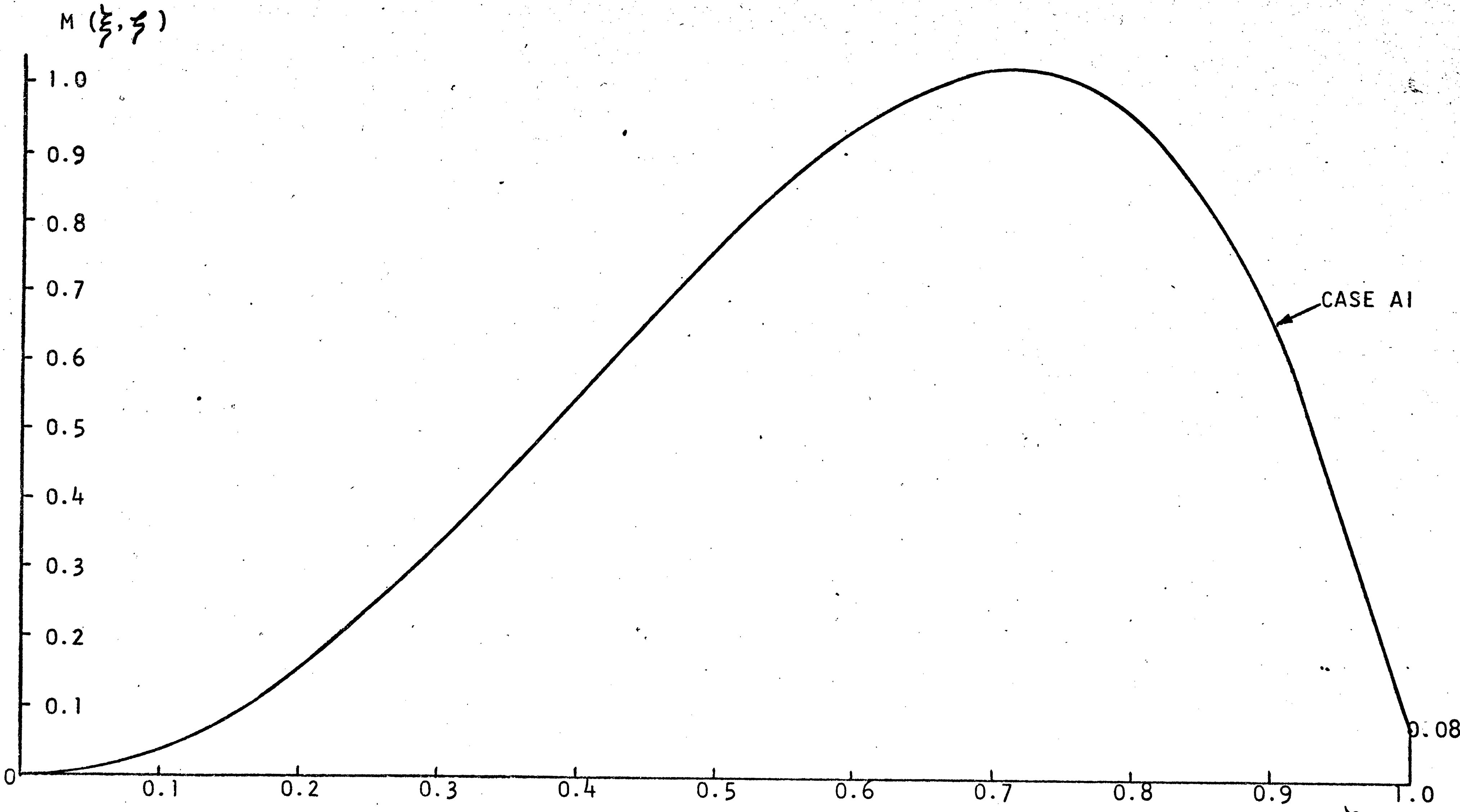
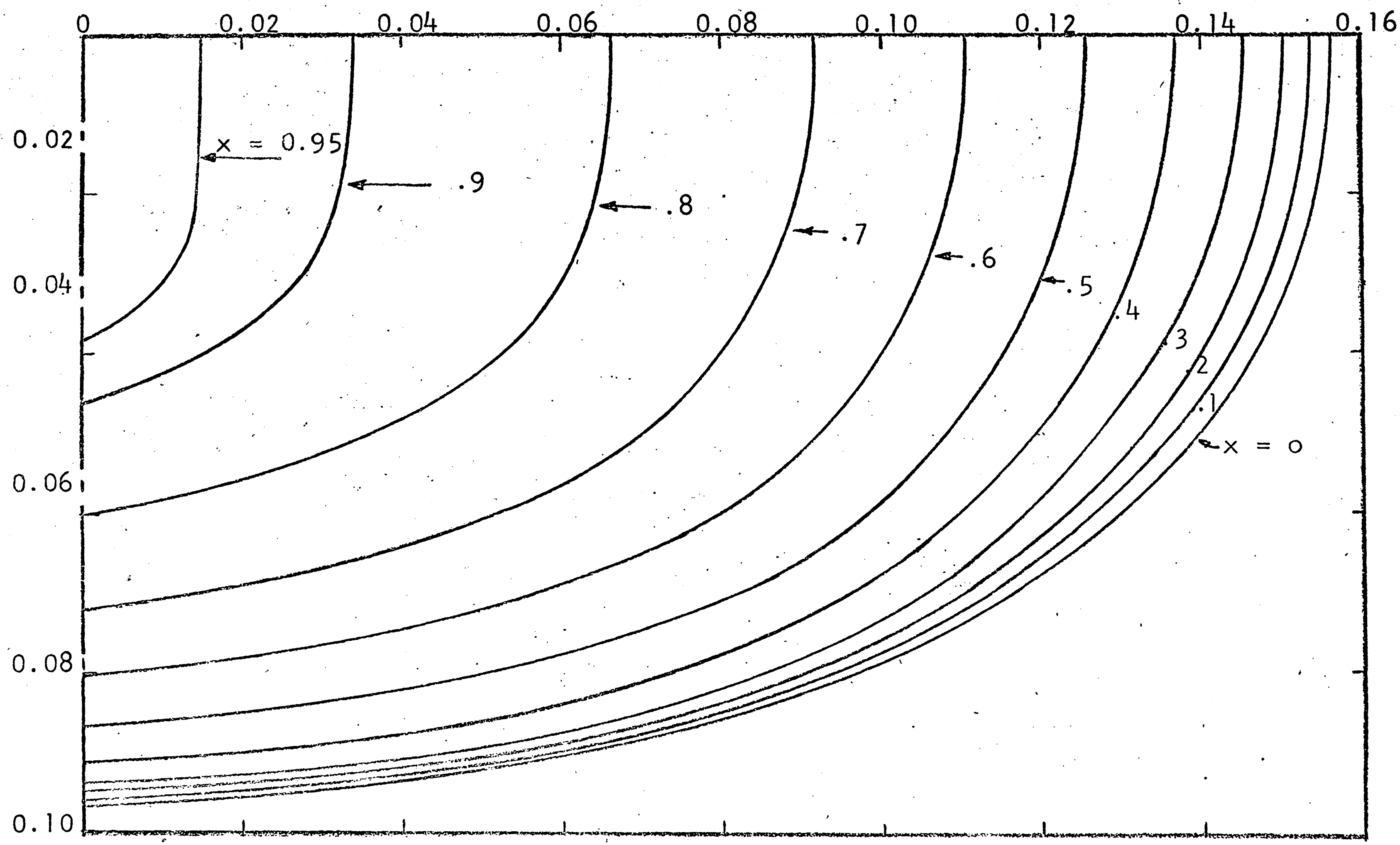


Figure 3 Singularity Strength, $M(\xi, \zeta)$ for Uniform Draftwise Distribution, $j = 0$, and five Polynomial Terms ($i = 1, \ldots, 5$).



- 31 -

Body Plan, Mathematical Lines, Case Al
(Five term polynomial on rhombic surface).

Figure 4

$L/B = 6.4$

$B/H = 3.25$

$C_B = 0.645$

$C = 0.766$

It is similar to Pien's first model 4946, ref. (2), in its overall appearance. But the differences in wave resistance coefficients C_{ijkl} have caused a distinct displacement distribution longitudinally and the cusp in the waterline at the bow is notable. The feasibility of adding a bulbous bow has not yet been investigated. It would likewise be of importance to test this hull shape experimentally to examine the success of the method. The next stage of development will also necessitate bottom singularities to generate still more practical shapes.

Case B with ten unknown polynomial terms has been treated in the same manner as Case A. The "displacement" restraint, eq. (10) with V = 0.35, obtains the following form:

$$V = \frac{1}{3}a_{10} + \frac{1}{4}a_{20} + \frac{1}{5}a_{30} + \frac{1}{6}a_{40} + \frac{1}{7}a_{50} + \frac{1}{6}a_{11} + \frac{1}{8}a_{21} + \frac{1}{10}a_{31} + \frac{1}{12}a_{41} + \frac{1}{14}a_{51} = 0.35 \quad (38)$$

This condition was substituted for each of the ten Ritz conditions consecutively and ten solutions were obtained. Only three of these, however, had positive singularity strength at the bow. These three promise to yield feasible hull shapes with positive entrance angles. The corresponding singularity distributions are plotted in figure 5 for the draft at the bottom of the rhombic body. The other drafts would look similar. It is interesting to note that the three singularity distributions differ greatly. Case B X has the most

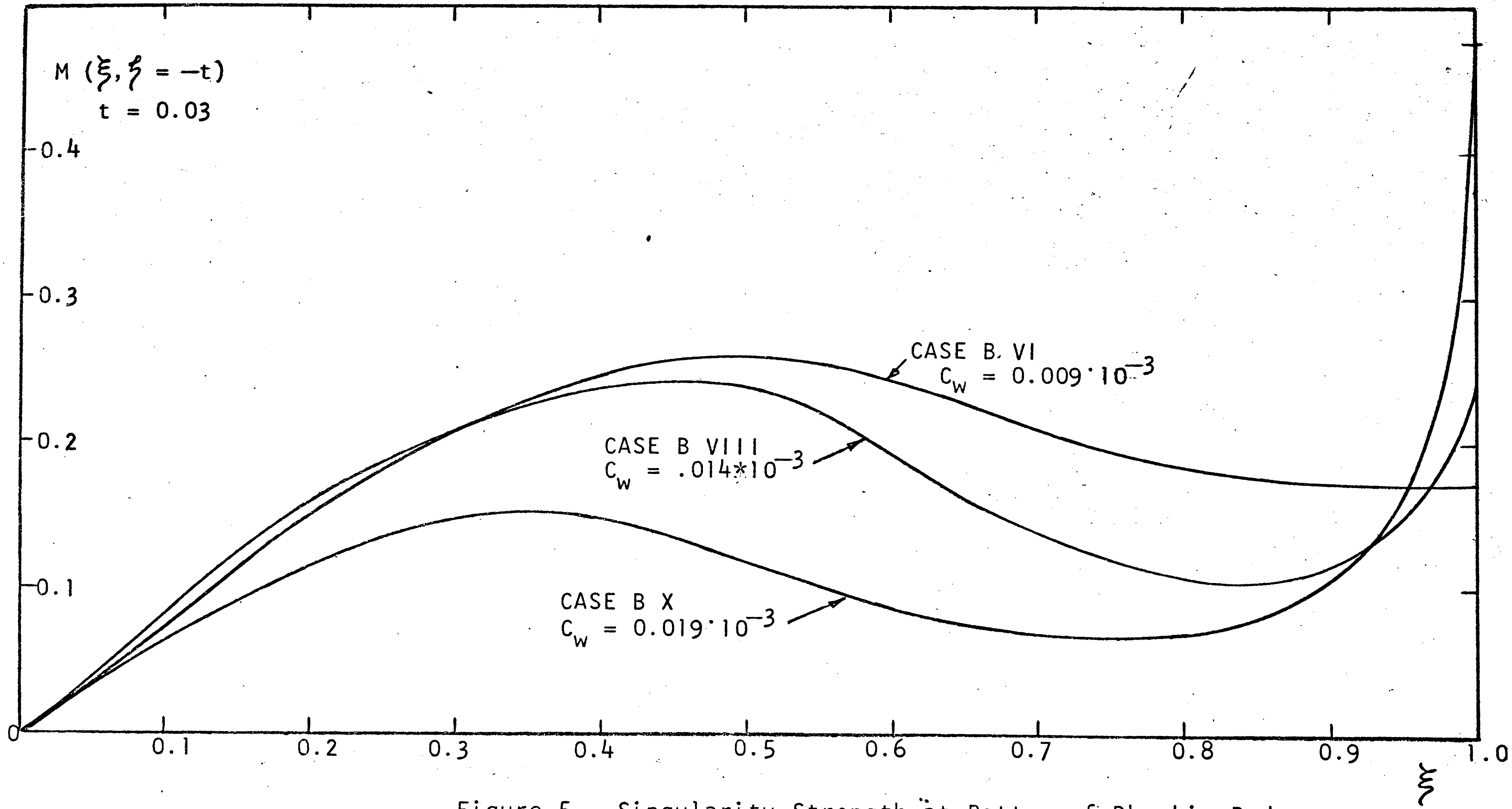


Figure 5 Singularity Strength at Bottom of Rhombic Body for Cases with Ten Polynomial Terms ($i = 1, \ldots, 5$, $j = 0$, and 1)

pronounced bulbous bow singularity peak, incorporated in the main hull system. Case B VIII probably has a medium or small size bulb while Case B VI is most likely associated with a cusp shaped waterline entrance like Case A I.

The wave resistance coefficients for these distributions are all low, but they differ, and surprisingly with a tendency in favor of the bulbless shapes. This, however, should not let somebody leap on the conclusion that the cusp shapes must be favored, for the hulls have not been traced yet; it is possible that they differ in displacement, too, although the "displacement" restraint in the same.

In any event, the question posed by these results is whether there exist a number of quite distinct shapes with good and almost equivalent wave resistance properties. We feel that this is a very important question from a practical point of view, and much attention should be devoted to it in the continuation of this work.

The far rear elementary wave amplitude functions both for the main hull, i.e. $A^b_{Sij}(\theta)$ and $A^b_{Cij}(\theta)$, eqs. (17) and (18), and for the source and dipole lines at the bow, i.e. $A_{Sj}(\theta)$ and $A_{Dj}(\theta)$, eqs. (22) and (23), were computed for a few cases. Table II shows an example of the results. These function values conform fully with Pien's results under corresponding conditions.

A few optimal bulb singularity distributions have been determined for some of the main hulls which were considered so far. But since these hulls have not been traced as yet, and no evaluation can be made it is considered too early to present the results.

4. Conclusion.

The task of designing practical low wavemaking resistance hull shapes in the speed range of $1. \leq V_K / \sqrt{L} \leq 1.2$ has been attacked by means of continuous off-centerplane distributions of sources and sinks located on the surface of a rhombic body. The possibility of adding special bulb singularities is provided.

The status of the project is that all basic programs have been completed and checked out; the time was, however, not sufficient for completing the systematic evaluation of the design possibilities. A flat bottom restraint still needs to be incorporated.

Our hull shape selection methods differed somewhat from those used by Pien. Consequently, the calculations carried out here lead to different hull shapes. One example that has been traced can be compared with Pien's model 4946 theoretically. It is hoped that tests will be carried out under future contracts so that the results can be examined experimentally.

It would be desirable to extend the work into the direction of more systematic exploration of favorable hull shapes. This should be done by means of a faster tracing procedure which can be developed along the lines suggested by current experience.

Acknowledgements:

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Table 1

TABLE OF WAVE RESISTANCE COEFFICIENTS C(I,J,K,L)

FROUDE-NO.= .2800,DRAFT-LENGTH RATIO OF RHOMBIC BODY 2T/L= .030,SLOPE OF RHOMBIC BODY ETA0= .1200

IKJL	C(I,J,K,L)	IKJL	C(I,J,K,L)	IKJL	C(I,J,K,L)	IKJL	C(I,J,K,L)	IKJL	C(I,J,K,L)
1100	.11138E-02	1200	0	1300	.93939E-03	1400	0	1500	.69214E-03
1101	-.46219E-03	1201	0	1301	-.38989E-03	1401	0	1501	-.28130E-03
1102	.28675E-03	1202	0	1302	.24092E-03	1402	0	1502	.17209E-03
1103	-.20677E-03	1203	0	1303	-.17328E-03	1403	0	1503	-.12309E-03
1111	.20291E-03	1211	0	1311	.16988E-03	1411	0	1511	.12047E-03
1112	-.12726E-03	1212	0	1312	-.10613E-03	1412	0	1512	-.74655E-04
1113	.83080E-04	1213	0	1313	.76666E-04	1413	0	1513	.53686E-04
1122	.72252E-04	1222	0	1322	.66534E-04	1422	0	1522	.46463E-04
1123	-.52449E-04	1223	0	1323	-.48145E-04	1423	0	1523	-.33482E-04
1133	.38105E-04	1233	0	1333	.34868E-04	1433	0	1533	.24153E-04
2200	.99590E-03	2300	0	2400	.97168E-03	2500	0		
2201	-.41370E-03	2301	0	2401	-.40803E-03	2501	0		
2202	.25551E-03	2302	0	2402	.25263E-03	2502	0		
2203	-.18368E-03	2303	0	2403	-.18182E-03	2503	0		
2211	.17999E-03	2311	0	2411	.17824E-03	2511	0		
2212	-.11232E-03	2312	0	2412	-.11143E-03	2512	0		
2213	.81079E-04	2313	0	2413	.80520E-04	2513	0		
2222	.70325E-04	2322	0	2422	.69887E-04	2522	0		
2223	-.50844E-04	2323	0	2423	-.50578E-04	2523	0		
2233	.36787E-04	2333	0	2433	.36631E-04	2533	0		
3300	.85153E-03	3400	0	3500	.68001E-03				
3301	-.35201E-03	3401	0	3501	-.27618E-03				
3302	.21664E-03	3402	0	3502	.16854E-03				
3303	-.15543E-03	3403	0	3503	-.12035E-03				
3311	.15225E-03	3411	0	3511	.11771E-03				
3312	-.94759E-04	3412	0	3512	-.72743E-04				
3313	.68306E-04	3413	0	3513	.52226E-04				
3322	.59200E-04	3422	0	3522	.45153E-04				
3323	-.42751E-04	3423	0	3523	-.32486E-04				
3333	.30900E-04	3433	0	3533	.23397E-04				
4400	.99379E-03	4500	0						
4401	-.42107E-03	4501	0						
4402	.26148E-03	4502	0						
4403	-.18845E-03	4503	0						
4411	.18481E-03	4511	0						
4412	-.11577E-03	4512	0						
4413	.83751E-04	4513	0						
4422	.72743E-04	4522	0						
4423	-.52701E-04	4523	0						
4433	.38209E-04	4533	0						
5500	.59577E-03								
5501	-.23796E-03								
5502	.14428E-03								
5503	-.10265E-03								
5511	.10028E-03								
5512	-.61628E-04								
5513	.44104E-04								
5522	.38055E-04								
5523	-.27296E-04								
5533	.19602E-04								

F = .28, 2T/L = .03, ETAO = .12

ANGLE=10.0 DEGREES

A(J)-VECTOR FOR SOURCE LINE DISTRIBUTION

J	0	1	2	3
F(J)	.1053753E 00	-.4923195E-01	.3171483E-01	-.2330168E-01

A(J)-VECTOR FOR DIPOLE LINE DISTRIBUTION

J	0	1	2	3
F(J)	-.6824035E 00	.3188229E 00	-.2053832E 00	.1509002E 00

AS(I,J)- AND AC(I,J)-MATRICES FOR MAIN SOURCE DISTRIBUTION ON DIAMOND

J	COMP.	F(I=1,J)	F(I=2,J)	F(I=3,J)	F(I=4,J)	F(I=5,J)
0	ASIN	.1627911E-01	.1550103E-01	.1394485E-01	.1183413E-01	.9615940E-02
0	ACOS	.2516037E-02	.5032075E-02	.7187016E-02	.8619764E-02	.9142277E-02
1	ASIN	-.7605697E-02	-.7242170E-02	-.6515118E-02	-.5528977E-02	-.4492623E-02
1	ACOS	-.1175507E-02	-.2351015E-02	-.3357816E-02	-.4027204E-02	-.4271325E-02
2	ASIN	.4899529E-02	.4665348E-02	.4196987E-02	.3561723E-02	.2894112E-02
2	ACOS	.7572525E-03	.1514505E-02	.2163078E-02	.2594293E-02	.2751554E-02
3	ASIN	-.3599807E-02	-.3427749E-02	-.3083632E-02	-.2616887E-02	-.2126377E-02
3	ACOS	-.5563724E-03	-.1112745E-02	-.1589268E-02	-.1906092E-02	-.2021636E-02