Methodological Considerations in the Analysis of Classroom Interaction in Community College Trigonometry

Vilma Mesa* – Elaine Lande

University of Michigan

Abstract

We report analyses of classroom interaction in trigonometry classes taught at an American community college focusing on two dimensions: the mathematical novelty of questions that instructors and students ask and the interactional moves that the instructors use to encourage student involvement in the lesson. The analyzed lessons were particularly challenging because existing frameworks that analyze classrooms did not account for the cases in which the delivery mode was lecture. We discuss the analytical strategies we used and show data to illustrate how they help us in capturing the complexity of classroom interaction and differences between instructors when lecture is the primary mode of instructional delivery. We conclude with suggestions for further work.

Keywords: Classroom Interaction, Lecture, Trigonometry, Teacher Moves, Questions,

Since the publication of the *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics, 1991), research in mathematics education has focused on how instructors and students manage interactions in the classroom. Attention to classroom interaction has long been of paramount importance in education, but the NCTM publication brought to light the need for creating different interaction dynamics in mathematics classes. Increased student participation in math classes has been heralded as an important element of good teaching practice at the post-secondary level (Blair, 2006) but the extent to which these practices are common in tertiary education is unknown.

Our work is contextualized in the American community college—a post-secondary institution that provides the first two years of baccalaureate degrees, vocational, technical, and enrichment education. These institutions are typically open access, non-residential, and they serve the community where they are located. In addition, they are less expensive relative to other post-secondary institutions. Community colleges are an attractive option for students for all these reasons. Courses are offered at many hours during the day and on weekends, allowing students to keep full-time jobs as they study. Community college classes are usually small (under 40 students), thereby allowing more opportunities for instructor-student interaction. Community colleges enroll a significant number of undergraduate students: in 2006, 50% of all the undergraduate population and nearly 49% of all undergraduate students taking mathematics were enrolled at a community college (Dowd et al., 2006; Rodi, 2007). An important feature of these mathematics classes is the prominence of lecture as a preferred mode of instructional delivery. In two-year colleges, for example, 74% of class sections in college algebra, 81% in trigonometry, and 93% in differential equations used predominantly lecture (Lutzer, Rodi, Kirkman, & Maxwell, 2007, p. 146).

The impetus of this paper comes from dissatisfaction with existing frameworks that investigate classroom interaction as those are mostly geared towards *Standards*-based mathematics education. These frameworks leave large portions of interaction unexamined, as they are grouped under a single label (e.g., lecture, Initiation-Response-Evaluation/Feedback [IRE/F]). In lessons in which the teacher lectures most of the time, the frameworks generally do not provide enough detail about what happens in any given class and are not able to differentiate across instructors. And although using open-ended activities that students explore in small groups and discuss as a large group may lead to the development of important mathematical competencies, our sense is that most of the mathematics classes in post-secondary education at community colleges are not of this nature. Our own experience working with instructors considered by their institutions as outstanding is that their students' participation is usually initiated by the teacher and is mostly around memorizing and practicing routine procedures (Mesa, 2010, 2011; Mesa, Celis, & Lande, 2013). If we want to change how students engage with mathematics and open up their opportunities to learn it, a better characterization of the complexity of classroom interactions that are happening in this setting is fundamental.

In this paper we describe what we have learned from analyses of 21 trigonometry lessons taught at a community college by five different instructors. In these analyses we describe the novelty of the mathematical questions that were posed and how the instructors managed the interaction in the classroom over time.

The chapter is organized into three sections. We start by briefly stating the theoretical grounding for this work and relevant prior research on classroom interaction. We then describe the methodology and the two analyses we conducted with the trigonometry lessons. In the final section we discuss the affordances and challenges of the analyses, ways in which they complement each other, and areas for further investigation.

Theoretical Background

We define instruction as the shared work on mathematical content between teachers and students within environments (the classroom, the school, and the community); this work changes over time (Cohen, Raudenbush, & Ball, 2003). This definition gives us an entry point into the complexity of any given classroom and also the ability to shift attention from the teacher, the student, or the content to the interactions between them, through which teaching, learning, and knowledge is manifested. By content we specifically refer to knowledge, skills, and dispositions that instructors, institutions, or society deem appropriate for students to learn. This definition acknowledges that learning happens all the time, even in cases in which an observer judges the teaching as leading to ‘impoverished’ learning.

Many scholars have established, both theoretically and empirically, that classroom interactions matter for student learning. Social constructivist theories of learning acknowledge that learning does not happen in a vacuum, that it is mediated by the interactions and the tools (e.g., language) available to learners at any given time (Bakhtin, 1981; Vygotsky, 1986). Seminal work in elementary mathematics classrooms in the late 80s and early 90s based on this theory, highlighted that the ways in which teachers and students interact with mathematical content in their classrooms shapes what children believe mathematics is about (Yackel & Cobb, 1996; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). In response to this change in thinking the community shifted its attention from teachers and students and their beliefs, attitudes, or knowledge (e.g., Fennema & Sherman, 1986; Secada, 1995; Thompson, 1992) to classroom processes (e.g., Moschkovich, 1996), and to the way in which those processes evolved and developed in real classrooms (Cobb, Stephan, McClain, & Gravemeijer, 2001). Driven also by a concern in reforming instruction, this work promoted a vision of classroom mathematical activity centered on complex mathematical tasks and questions, which would allow students to struggle with mathematics in the same way as mathematicians would (Schoenfeld, 1992).

A comparable interest exists in post-secondary education. Observational studies in college classrooms have characterized student learning as a ‘spectator sport’: “little participation occurs, few students are involved, and teacher questions focus on recall rather than critical thinking” (Nunn, 1996, p. 245). Scholars have advocated shifting from a ‘teaching’ to a ‘learning’ paradigm (Bass, 1999), in which student participation and engagement with the content during classrooms becomes central. Although there is an important body of work promoting the use of ‘active learning’ strategies in higher-education classrooms (e.g., Frederick, 1987; Johnson, Johnson, & Smith, 1998; Prince, 2004), very few faculty actually use them in their lectures (e.g., Dancy & Henderson, 2009). Likewise, teaching approaches such as Inquiry-Based Learning in mathematics (Coppin, Mahavier, May, & Parker, 2009), have been confined to few faculty within math departments.

In reviewing the literature, we found two areas that were important in analyzing the interaction between the teacher, the students, and the content. One was the nature of the questions asked by the teachers and students and the other was the extent to which instructors used strategies that opened the dialogue and invited students into the mathematical conversation.

The Nature of Questions Matters. There is a substantial agreement amongst scholars about the importance of questions and tasks for promoting students’ learning. Scholars in cognitive psychology (e.g., Anderson et al., 2001) highlight the importance of organizing instruction using questions that address different types of knowledge (factual, procedural, conceptual, and metacognitive) and that engage different levels of cognitive processes (remembering, understanding, applying, analyzing, creating, and evaluating). Ideally instruction is such that all types of knowledge and all different processes are addressed, yet some studies report that recall questions are the most common type used in college classrooms (Barnes, 1983; Nunn, 1996; Pollio, 1989).

A meta-analysis of parameters of discourse present in nine well-known strategies for text discussion in small-groups revealed that the quality of teacher and student questions (specifically novel questions), uptakes, the presence of elaborated explanations, and the density of reasoning words, were features of discussions that were deemed productive (i.e., they develop high-level thinking and comprehension) "despite the highly situated nature of small group discussions" (Soter et al., 2008, p. 373). Interestingly, Soter and colleagues' analysis of who spoke the most (either students or teachers, using number of words as a metric) did not support the idea that approaches dominated by teacher talk were less productive than approaches dominated by student talk, but that it was rather the nature of the questions that was more important.

Research on tasks used in mathematics classroom also indicates that tasks addressing novel mathematical questions are better than tasks focusing on routine or repetitive activities (Doyle, 1984, 1988). In addition, the way in which the tasks are enacted in the classroom matters. Students of teachers who tend to reduce the cognitive complexity of tasks by asking simpler more routine questions perform worse on standardized tests than students of teachers who tend to maintain or increase the cognitive complexity of the tasks they work on (e.g., Silver & Stein, 1996; Stein, Grover, & Henningsen, 1996).

How teachers invite students into the conversation matters. Work in linguistics, specifically with the engagement system (Martin & White, 2005), has highlighted that speakers use language to engage others by way of the interplay between two major discursive voices, *monogloss* and *heterogloss*. A monogloss voice seeks less to engage than to give facts that ostensibly concede no room for the negotiation of meaning. For example, to attempt to align an audience to an author's or speaker's side, she or he can manipulate assertive devices to elicit confidence in the statement she or he is making. On the other hand, a heterogloss voice seeks to engage the audience using a variety of linguistic resources, to open up or close down options for dialog, each of these conveying a varied strength of engagement (Martin & White, 2005).

Whereas one can perceive most lectures as mainly monoglossic, it is the case that instructors use certain language moves to open up the conversation, via heteroglossic devices that contract or expand the dialog. Some of these devices have been identified as having the potential for increasing students' involvement in the classroom and for changing their own positioning towards doers of mathematics (Males, Otten, & Herbel-Eisenmann, in press; Mesa & Chang, 2010). Smith and Higgins (2006) have proposed that more than the types of questions that teachers ask, the manner in which they react to and behave with students' responses can make a difference in the classroom, specifically in creating more interactive learning environments. In college classrooms Nunn (1996) found that behaviors that create a supportive atmosphere (i.e., praising students, indicating that an answer is right, and using students' names) were significantly correlated to the amount of time spent in student participation (p. 258). Thus, both the type of questions asked and the teacher's moves in response to the questions can signal that student participation is welcome and facilitate student engagement with the content.

Given the predominant role of lecture as a preferred delivery instructional method in post-secondary mathematics, and the scarcity of existing frameworks to capture what happens in lectures, we sought to use these two criteria, the Novelty of Mathematical Questions posed during class and the Teacher Moves observed during lecture, to characterize mathematics lectures. A main purpose for us was to see whether we could use these two attributes to characterize instruction (i.e., the interaction between students, teachers, and the content, over time, and within a particular environment) when the primary delivery mode is a lecture and also, to determine whether this process allowed us to differentiate across instructors who lecture.

Method

The setting for this study is a large suburban community college in Michigan with an approximate enrollment of 12,000 students and an average yearly retention rate of 50%. At the time of data collection (2008-2009), the mathematics department had 16 full-time and 75 part-time instructors and offered an average of 22 different courses per term, including remedial math courses (e.g., fundamental math, beginning and intermediate algebra), college courses for professional and liberal arts degrees (e.g., business, health, and education), STEM preparatory courses (college algebra, college trigonometry, and pre-calculus), and college courses for STEM degrees (e.g., calculus, linear algebra, and differential equations). Like other community colleges across the U.S., students may also obtain their general education diploma (GED). This particular college was chosen because the students' rating of teaching in the mathematics department was high (above 4.2 on a scale from 1 to 5), which suggests high student satisfaction with teaching. In addition, the department had recently appointed a very dynamic department chair, committed to investing time in improving teaching. Moreover, like other colleges in the state, the faculty felt pressure to increase passing rates in their courses. The department also received substantial support from the administration to engage in activities that would address the issue of the low passing rates (e.g., support for a faculty development group, time off for periodic evaluation of curriculum and syllabi, incentives for managing the coordination of the large number of part-time instructors, a college wide program to address students' orientations towards learning, and in general a carte-blanche for initiatives that would clearly focus on increasing passing rates).

As part of a larger study we had been interviewing and observing 12 instructors teaching College Algebra, Trigonometry, and Pre-Calculus (over 80 lessons). For this particular paper we analyzed 21 lessons that were on the same subject, Trigonometry, which gave us the most variation in terms of teachers' characteristics and therefore interesting differences to capture. During the 2008-2009 school year we observed seven trigonometry sections taught by five instructors (Table 1). After producing our initial descriptions, we met with each instructor to share our views of their interactions in the classroom (who participates, number of questions, use of IRE/F patterns, cognitive demand of tasks). In these meetings we sought to identify whether what we have captured was reflective of instructors' standard practice and their rationale for organizing the interaction in that way. All teachers confirmed that our descriptions were accurate and provided further insights about their practices.

---Insert Table 1 around here---

Each section met two days a week for 85 minutes per class. Each section was observed three times on consecutive days; we avoided exams in order to maximize the time that students and instructors interacted with each other with the content. Emmett was observed in three different sections, for a total of nine times. The lessons were audiotaped and extensive field notes were taken that included the work on the board and other observations not captured by the audiotape (e.g., students leaving or entering the room, overall student attitudes, teacher movement in the room, where the students sat, and who asked or answered questions). After each class, instructors commented on events that happened during the lesson, or that departed from other lessons observed (e.g., calling students by name to answer questions, sending students to the board, assigning seat-work) and about how representative of other lessons in the term the observed class was. The purpose was to determine what counted as ‘common’ or ‘standard’ practice and what was considered extraordinary. The audio recordings of the lessons were transcribed, with the length of pauses 3 seconds or more in speech noted; the transcripts were augmented with the work done on the board.

Analyses

The analyses of the classroom transcripts attended to the two dimensions of the interaction the Novelty of Mathematical Questions posed during class and the Teacher Moves we observed during the lecture. These afforded us several units of analysis: the questions posed, the turns in which teachers performed an interaction move, and the time that was allotted to certain Teacher Moves. We also contextualized these by the different types of activities (introducing New Material, doing Review, and Other—e.g., test-taking, discussing class logistics) that were evident in the lessons. Whereas Analysis 1, Novelty of Mathematical Questions, seeks to capture the novelty of the mathematical questions asked both by students and teachers, Analysis 2, Teacher Moves, attends solely to the way in which the teachers manage the dialogue that occurs in the classroom.

Analysis 1: Novelty of Mathematical Questions

We see mathematical questions as opportunities that instructors create to engage students in mathematical activity. With this framework, we sought to characterize the opportunities that are created by describing how novel the questions are. We developed this scheme by drawing from frameworks that analyze questions in classrooms (Nystrand, Wu, Gamoran, Zeiser, & Long, 2003; Wells & Arauz, 2006), specifically in mathematics (Nathan & Kim, 2009; Truxaw

& DeFranco, 2008), and using the data we collected. We started by synthesizing features of these various frameworks (e.g., cognitive demand, authenticity, uptake, teacher evaluation) and then created a categorization of questions that we applied to several of our transcripts, attending to content, intention, execution, and novelty. First we identified all questions that both instructors and students asked. Next, we took each teacher question and determined whether the question was mathematically oriented or not (content) and whether the instructor expected to obtain an answer from the students (intention). Those questions that were mathematically oriented and for which the instructor expected to obtain an answer became the focus of the analysis. We refer to these questions as the mathematical questions asked by the teacher. We further categorized these questions, indicating if the students answered the question or if the teacher paused and gave the students an opportunity to answer the question (execution) and if students were expected to know the answer or how to procedurally figure out the answer given what had been covered in the class (novelty).

The questions not included in the mathematical questions included questions about classroom procedures (e.g., “So how’s it going? Questions on the trig?”), discourse management (e.g., “Could you repeat that?”), and rhetorical questions, including a type of question that we called, statement-right (e.g., “Let’s see, from 0 to 180 degrees, that’s the window we’re looking for, right?”), which did not appear to be intended to be answered, but rather to have an agreement from the students. These questions were not usually open to discussion and were not included in this analysis.

In terms of execution, mathematical questions asked by the teachers could be aborted. This occurred when the question was not followed by a student response, either because the teacher did not provide enough pause time for the students (3 seconds or more) or he or she reworded or answered the question him or herself (e.g., “And what’s cosine of $-x$? That’s just cosine of x itself”). In terms of novelty, mathematical questions asked by teachers or students were *Routine*, when students were expected to know the answer or to know how to procedurally figure out the answer using information given in the class, or in previous classes or courses (e.g., “If I’m talking about negative pi over 2, which direction am I going first of all?,” “What’s the result?”) or *Novel,* when students were not expected to know the answer or the procedure to figure out the answer. Novel questions included those that required students to explain new connections between mathematical notions or connections to real-word scenarios (e.g., “Why is it sometimes that if the light is getting old that you’re able to see it flicker?”), to figure out something new using information that had not been discussed in the class (e.g., “And what’s cosine of $-x$?”), or that sought students’ thinking about a new mathematical notion (e.g., “b approaches 1, and a approaches 0. What do you think is going to happen to the ratio?”). Student questions that inquired about the how or why of the mathematics (e.g., “Doesn’t shifting affect whether it would be sine or cosine?”) were considered Novel, whereas student questions seeking for a specific, direct, answer (e.g., “It needs to be in radians right, not in degree mode?”) were considered Routine. We made the classification taking into account the talk and content that preceded and followed the question. A team of five researchers using NVivo coded the mathematical questions. Cohen’s κ, used to determine pair-wise agreement of the coding of teacher questions, ranged from .62 to .80.

Analysis 2: Teacher Moves

For the second analysis we used a framework for describing teacher moves in primary and secondary English and mathematics lessons developed by Mary Kay Stein and colleagues (Scherrer & Stein, 2012). This framework attends specifically to the ways in which teachers initiate interaction and how they sustain it. Although developed for helping teachers move towards more standards-based practice, this framework contained moves that could account for actions that are seen in lecture-based lessons in ways that other frameworks did not. An earlier version of this framework had two categories of moves, those that initiate the discussion (initiating moves) and those that invite more participation from students (rejoinder moves). We added three codes to this framework: Statement of Problem—an initiating move—to indicate when a new problem started, and Response Right and Response Wrong—rejoinder moves—to mark moves in which the instructor explicitly states that a student response is correct or incorrect. The Statement of Problem code was important for us because differently from the context in which Stein and colleagues were working, we did not have many open ended tasks that were posed to the students, but the instructors brought many examples and solved many problems on the board that were an important feature of the lessons. Likewise, Response Right and Response Wrong appeared frequently in the data that we agreed was an important feature to capture (see also Nunn, 1996).

We also made two modifications to this framework as we applied it to our data. First, rather than coding at the turn level as Scherrer and Stein did, we coded by groups of clauses that conveyed the meaning proposed by the codes in the framework. We called these clauses or groups of clauses, moves. This allowed the initiating moves to be mutually exclusive, and the rejoinder moves to be 'added on' to an initiating move, thus allowing us to tease out the type of invitations that teachers used to engage students in the conversation. Second, and because we wanted to have a group of codes that were mutually exclusive and would allow us to code all possibilities of dialogue in the classroom (reform oriented or not) we re-classified the code Collect as an initiating move (See Table 2).

---Insert Table 2 around here---

To facilitate discussion of the moves, we further categorized them into the two main voices, Monogloss and Heterogloss. Heterogloss moves were further classified depending on whether the move sought to expand or contract the conversation, or whether it was possible, in context, for the move to be used either way. We classified one code, Provide Information as Monogloss; eight codes, Think Aloud, Literal, Lot, Repeat, Response Right, Response Wrong, Terminal, and Uptake-Literal, as Heterogloss Contract; three codes, Launch, Collect, and Push-Back as Heterogloss Expand; and four codes Re-Direct, Re-Initiate, Connect, and Uptake as Heterogloss moves that could be either contracting or expanding the conversation depending on the context. The Statement of the problem code was not included in this classification because it was not considered germane to the analysis (see ---Insert Figure 1). In ---Insert Table 3 we provide examples of this classification. In some of these examples we include additional text that was used in making the classification; the underlined text was coded as the specific move.

---Insert Figure 1 around here---

---Insert Table 3 around here---

A two-person team including one who participated in the Analysis 1 team coded the Teacher Moves. They coded 30 minutes of a lesson independently using the code descriptions. Agreement on the initiating codes was 78% and 76% on the rejoinder moves. The discrepant cases were discussed allowing us to make our definitions clearer. With the revised system, one person coded the rest of the lessons.

Episode Parsing

We contextualized these two analyses by parsing the transcripts into three mutually exclusive Episodes that occurred in the classroom, namely: introducing New Material, Review, and Other. In New Material episodes teacher and students discuss material that had not been presented previously in the course. In Review episodes teachers and students discuss material covered in a previous class, solutions to past assignments (e.g., homework, quizzes or examinations), or topics or examples that would be covered on a future examination. Episodes classified as Other did not include discussions of mathematical content (e.g., providing dates for upcoming tests, taking an in-class examination, returning graded work). Each transcript was parsed into these three mutually exclusive Episodes. The parsing was done prior to the coding of questions and moves. A team of three researchers, the two authors and a graduate student, parsed the lesson transcripts, with weekly discussions to determine final Episodes.

Results

We present the results of the two main analyses performed, the Mathematical Questions and the Teacher Moves. We then present these results as they unfold over time and relative to the type of Episodes.

Novelty of Mathematical Questions

---Insert Table 4 shows the frequency of mathematical questions and the proportion of Novel questions asked by the teachers over the three lessons observed including the proportion of those questions that were aborted.

---Insert Table 4 around here---

In ---Insert Table 4 we see that instructors asked a large number of mathematical questions in each 85-minute class period (average = 55, min = 38, max = 85). Most of these questions were Routine (70%), which suggests that students were more frequently expected to give answers they already knew rather than answers they did not. In other words, students were not asked to struggle with the content that was being presented very often, a key characteristic of these lectures. This percentage is lower than what Nunn (1996), citing Barnes (1983), reported: 80% of questions that faculty ask in mathematics classes are of the lowest cognitive level (recall of facts, p. 245). In addition, on average 33% of Novel questions were aborted compared to 26% of Routine questions aborted ($\chi^2(1) = 5.94$, $p < .05$). Thus, these instructors were more likely to abort Novel questions than Routine questions and the number of Novel questions that students were engaged with was small (average = 11, min = 4, max = 20).

---Insert Table 5 presents the frequencies and proportions of questions students asked by type over the three lessons observed. Overall, about one third of the questions students asked were coded as Novel, but there is substantial variation in the amount of questions students asked in each class period (mean = 13, min = 4, max = 41), and in the number and proportion of Novel questions asked (mean = 4, min = 2, max = 8). It is also notable that there is variation for the instructor who taught three sections of the same course.

---Insert Table 5 around here---

Teacher Moves

We coded in total 5,094 Teacher Moves across all the lessons. Among the initiating moves, Provide Information, Literal, and No Code accounted for the majority of these moves (46%, 33%, and 13% respectively, see ---Insert Table 6). Thus, consistent with images of lectures, the teachers in these classes delivered the content at hand and asked the students very specific, pointed questions. The moves that were categorized as No Code corresponded to classroom management comments (238, 34%), references to the textbook (202, 28%), administrative announcements (201, 28%), personal stories unrelated to the content (59, 7%), and inaudible text (32, 3%).

---Insert Table 6 around here---

---Insert Figure 2 shows the distribution of these moves in terms of the Heterogloss-Monogloss classification. The figure excludes the No Code moves (n = 658) and the Statement of Problem moves (n =93).

---Insert Figure 2 around here---

In general, the figure shows remarkable similarity across teachers in the absence of Heterogloss-Expand initiating moves (only 4 moves in all by Ed) and limited use of moves that may be considered Heterogloss-Expand depending (Heterogloss-Expand/Contract moves). We see differences, however, across the teachers in terms of the number of moves they used. Three teachers, Ed, Elizabeth, and Elliot, used over 750 moves, whereas Emmett and Evan used less than 500 moves. It is also remarkable that in terms of Monogloss moves (Provide Information), Elliot had almost three times as many as Emmett or Evan, and almost 1.5 times as many as Ed or Elizabeth indicating that the length of each Monogloss move is shorter in Elliot's class. Proportionally there is consistency across teachers in their use of Heterogloss-Contract moves (around 40%). In general, this classification suggests that about half of the Teacher Moves are Monogloss; while teachers do use Heterogloss moves, these tend to be contracting, thus closing the conversation rather than expanding it.

The rejoinder moves were assigned concurrently with initiating moves but could be assigned independently or concurrently with other rejoinder moves. Rejoinder moves were used less frequently than initiating moves—only 17% of all the moves the teachers used. Among the rejoinder moves, Repeat, Uptake, and Response Right were the most frequently assigned (respectively, 40%, 20%, and 17% of the total, see ---Insert Table 7).

---Insert Table 7 around here---

---Insert Figure 3 represents the distribution of these moves in terms of the Heterogloss-Monogloss classification (see ---Insert Figure 1) over the three lessons observed per instructor.

---Insert Figure 3 around here---

We see some variation across teachers in their use of rejoinder moves. Proportionally, Emmett 4, Emmett 5 and Evan use Heterogloss-Contract moves about 60% of the time, whereas for the other instructors this proportion is nearly 70%. Again, there is variation in the number of rejoinder moves: three teachers, Ed, Elizabeth, and Elliot, used these moves over 150 times across the three lessons observed, whereas Emmett and Evan, used these types of moves less than 100 times. Again, this analysis corroborates that in general, even with Heterogloss moves, which seek students' engagement in the conversation, these instructors tended to use contracting rather than expanding moves.

Episodes

Regarding the Episodes, on average 70% of the time was devoted to presenting New Material (min = 40%, max = 88%), 24% was devoted to Review (min = 2%, max = 53%), and 6% (min = 0%, max = 17%) was devoted to Other activities (assessment, discussions before or after class).

Representing the Novelty of Mathematical Questions and Teacher Moves Analyses Simultaneously

The previous analyses portray similar enactments of lessons, with some variation across teachers regarding the ways in which the teachers used questions and moves in their lessons, but do not provide an idea of how these questions and moves are deployed over time. To get a view of this process we mapped the two codings over the duration of the lesson and accounted for the different class Episodes. These maps show a very detailed, yet complicated view of these lessons.

For representation purposes we used different colors to differentiate the Novelty of Mathematical Questions and the types of Teacher Moves: green represents Novel questions, moves that are Heterogloss-Expand (Collect, Launch, and Pushback), and those moves that could be either Heterogloss-Expand or Contract (Connect, Re-Direct, Re-Initiate, and Uptake); brown represents Heterogloss Contract moves (Think Aloud, Literal, Lot, Repeat, Response Right, Response Wrong, Terminal, and Uptake-Literal); and red represents Routine questions and Monogloss moves (Provide Information). We used the same coloring scheme for student questions and black dots to represent when students otherwise interacted—either in responding to a teacher question or providing other utterances such as volunteering information. To better represent the Teacher Moves we divided them into two categories: (a) those best represented as an instance in time (Collect, Launch, Pushback, Connect, Re-Direct, Re-Initiate, Uptake, Literal, Lot, Repeat, Response Right, Response Wrong, Terminal, and Uptake-Literal), and (b) those best represented as an interval of time (Provide Information and Think Aloud). We used dots and Xs to make it easier to visualize the data. The representations also include the type of activities that were done in the lesson, mainly presenting New Material (yellow) and doing homework or exam Review (blue).

To make evident the affordances, specifically the differences that can be captured with these representations, we present 20 minutes of a lesson from Elizabeth, Elliott, Emmett, and Evan (---Insert Figure 4). These four segments were taken from episodes in which New Material was being discussed. Each segment was chosen because it illustrates possible differences between classrooms as well as the complexity of interaction within a classroom. The representation shows that the relationship between the types of questions asked, teacher moves, and student participation is not a simple one. In addition, adding the time component within and across lessons provides a richer picture of the dynamics of the classroom interaction.

In the 20-minute segment of Elizabeth's lesson we can see that she uses both Novel and Routine questions at about the same frequency; the Teacher Moves are mostly Heterogloss-Contract; and the intervals of providing information are frequent, but relatively short. The students are quite engaged in this class both in asking and responding to questions or volunteering information. In the 20-minute segment of Elliot's lesson students are also quite engaged and frequently ask questions even though he asks questions much less frequently and uses less Heterogloss moves than Elizabeth. The 20-minute segments from Emmett's and Evan's lessons show cases in which the students are not very active. In both of these segments the amount of time and length of each instance of Providing Information is greater than Elizabeth's and Elliot's and the use of Heterogloss moves is infrequent and mostly contracting. This segment of Emmett's class has no teacher questions and Evan's segment includes quite a few Novel questions but very few Routine questions. Evan's representation illustrates a phenomenon that we observed frequently—instructors asking Novel questions that were not answered, either because the teacher rephrased the question to make it simpler for the student or because the teacher himself answered it.

---Insert Figure 4 around here---

In the Appendix we present lessons from our five instructors, Ed, Elizabeth, Elliot, Emmett 4, and Evan that illustrate the different ways in which they interacted with the students. The representations are similar in that they show an infrequent use of Novel questions and of Heterogloss-Expand moves. The full-lesson representation reveals other interesting features, such as the differences in student participation and the Novel questions teachers asked between Monogloss moves. When the three representations of the three consecutive lessons are put side by side, we notice an interesting consistency of the diagrams, suggesting, perhaps unsurprisingly, that the individual teachers follow a similar pattern as they teach their lessons day in and day out (See ---Insert Figure 5). This consistency is useful in characterizing what teachers do with their lessons, it suggests that teachers have *signatures*, particular ways of organizing instruction that are recognizable and predictable.

---Insert Figure 5 around here---

Discussion

While the two analyses emphasize the complexity of teaching mathematics and the importance of both mathematical questions and teaching moves, combining them gives a richer picture of classroom interactions. These two analyses separately show regular patterns in the classroom interaction in these trigonometry classes. The analysis of the mathematical questions shows a consistent pattern; the level of mathematics students are expected to engage with is low—the questions are mostly Routine and teachers interrupt the thinking process by aborting a sizable proportion of questions posed. The analysis of the Teacher Moves reveals how instructors use various additional ways to engage students beyond asking questions, although this is also limited. Teachers for the most part Provide Information and rarely use Heterogloss-Expand moves, such as Collecting and Push-Back.

By representing the two analyses together we see that even with the commonalities of interaction seen in the figures, there are visible differences in how the questions and moves unfold over time. While some instructors use moves that limit the amount of student participation, others use moves that seek it. We see with these combined representations, however, that in these lessons that would have been rated 'traditional' or 'lecture', instructors provide many opportunities for student engagement with the material and that the way in which these instructors use these devices varies.

Before discussing the affordances and challenges of using this methodology to understand the classroom interaction, we give some interpretations of why we see these behaviors.

Interpreting the Findings

Part of the reason for instructors offering mostly Routine questions at the same time that they provide opportunities for student engagement might be rooted in their belief that students who attend community college have experienced failure in previous mathematics courses, which have left them with low self-confidence in their ability to do mathematics. Thus instructors, in an effort to help them gain confidence, will ask questions that they think students can answer without risking failure. As a result, the questions they ask tend to be of a routine nature, thus creating a situation in which students are rarely exposed to or expected to answer difficult questions, thus constraining opportunities for the students (Mesa, 2010, 2012). Another possibility is that teachers fear that students might not be able to take on Novel questions on their own or without the appropriate scaffolding. This was partially confirmed in a later study with Elliot (not reported here), in which we suggested that he engage students in a group activity that involved solving a problem that they had not seen before (graphing inverse tangent, after seeing how inverse sine was graphed using values in the unit circle). During the planning sessions, Elliot disagreed with the idea of asking students to do something he had not modeled for them before (doing at least a few of the key values for tangent), and although he agreed to give the task to the students, during the lesson he stepped into the solution process providing the information he felt was necessary for a successful completion of the task, effectively reducing the complexity of the activity for the students. His concerns for the well being of the students and his doubts that they would be able to generate the graph greatly determined how the task unfolded.

Part of the reason why teachers do not use Novel questions and Heterogloss-Expand moves (Uptake, Push-Back, Collect) more frequently might be that teachers do not know how to use them in the classroom or may not be aware of their benefits for student learning. In general, community college instructors have little time and opportunity to participate in faculty development programs and, when available, these programs may not be geared specifically towards mathematics or towards managing classroom discourse (Sowder, 2007). While some colleges have programs for faculty, their teaching load (five courses per term) may impede their participation (Grubb & Associates, 1999). While one can argue that teachers' lack of knowledge about the role of language in opening or closing mathematical discussions can explain some of these results, it is also possible that the instructors' perception of the nature of trigonometry may suggest that Monogloss and Heterogloss-Contract moves are more appropriate. Trigonometry is perceived as a course with an extensive amount of information that students need to be exposed to and gain competency in using. Instructors indicate that the most efficient way to make this content available to students is by providing that information directly, illustrating how to solve the problems, and providing a template for students to repeat the process. Given the limited amount of time in a semester to accomplish this task, instructors may see engaging in explorations (e.g., Launches and Uptakes with Novel questions) as a threat to their responsibility to make sure that students have seen all the content of the course.

Affordances and Challenges in Using this Methodology

The analyses of the novelty of mathematical questions and Teacher moves both individually and together show the importance of attending to these two aspects when describing classroom interaction. While this is not necessarily new, representing both the Novelty of Mathematical Questions and Teacher Moves over time reveals various patterns across teachers that otherwise would go unnoticed. Only after we represented the results of these analyses together could we corroborate the differences that we experienced while observing these classes. Thus these representations allow for a better description of the nature of the classroom interaction that occurred in these classes.

As we have indicated earlier, one advantage of combining these two ways of looking at the classroom data is that it provides richer descriptions for how teaching actions unfold in the classroom, more prominently in cases in which other analyses would simply classify the teachers' work as 'traditional.' Our methodology seeks to understand from the perspective of the local system how instruction happens and the value that teachers give to this form of interaction. Assuming that lecturing or providing information is uniformly "bad" is problematic because we fail to see what does exist in this context that promotes student confidence and learning of the material. We argue that there is a role for researchers in making visible how instructors manage instruction and in finding ways to represent and describe their work that captures the complexity of what they do. This analysis allows us to observe important differences in how interaction in the classroom is organized, differences that would not have been possible to describe with one of the analyses alone. Another important benefit of our analyses is that they allow us to see how lessons unfold over time, to understand the back and forth between teachers and students in ways that might not be apparent by using transcripts or one of the analyses alone.

A major challenge of this methodology lies in managing the level of detail that it seeks to achieve. What is the appropriate unit of analysis that one needs to attend to? In our case we have several units of analysis, the questions, the utterances, and time. Coordinating these into a single representation was a challenge that can be appreciated when looking at the full lessons shown in the Appendix. Whereas seeing these images side to side show different patterns or signatures in these teachers' lessons (especially when their three lessons are put together), the amount of information presented can be overwhelming. We have spent a great deal of energy in looking for ways to convey the richness of the analyses and at the same time seeking to synthesize the patterns that we observe. No representation can be good enough to approach reality, but the ones that we proposed gives us a good idea of how lessons evolved over time and how teachers managed interaction in the classroom.

Future Research

Whereas these analyses and representations are revealing, they are also limited in several ways. Attempting to characterize what teachers do from these three lessons alone could be misleading. However, we believe that the analyses capture important characteristics of lectures that current frameworks don't examine. These analyses can be easily replicated and can produce reliable information about the activities in the classroom.

One major task that lies ahead is developing a system that would allow a comparison, numerical or qualitative, that can account for the differences observed in these lessons. The representations we use provide an account for these differences by contrasting how questions and moves unfold over time. Another task that lies ahead is proposing ways to measure the differences that are observed in these representations, that would better capture the differences over the five teachers and seven sections and take into account all the lessons observed.

We could then use such measures to answer questions such as, are Heterogloss-Expand moves more likely to be used in tandem with Novel or with Routine questions? Are there particular patterns in the use of Heterogloss-Expand, Heterogloss-Contract, or Monogloss moves and questions? Are there differences between use of these moves and use of Novel questions when teachers conduct a Review or when they present New Material? Making these representations highlights that time is an important aspect of the analysis, how these activities unfold over time can shed light on the complexity of teachers' work.

An important feature of lectures is the preponderance of Providing Information which is the core of such a teaching methodology. Further research is needed in order to determine the conditions under which this mode of instruction, augmented with the Heterogloss moves that we see and with the Novel questions posed, is beneficial to students. Does this mode of instruction combined with students' practice and individual work produce sustained learning and good outcomes in the course? This seems to be a strongly held belief of community college mathematics instructors, yet we have not been able to establish whether this is the case or not.

Repeating these analyses with other courses (e.g., basic algebra, calculus) would allow us to see the extent to which these findings depend on the course content and to explore the role of teachers' perceptions of their students' abilities to do mathematics on these findings. It would be important for example to determine whether instructors teaching courses beyond trigonometry would ask more Novel questions and use more engaging moves with their students, under the assumption that the content is more interesting or that their students are more capable. Contrasting these results with courses for honors students would also be useful in testing the hypothesis that teachers' perceptions of their students determine the extent to which they ask more Novel questions or use more engaging moves with the students. Such information can be used to design programs for faculty development that increases instructors' knowledge of ways to use Novel questions and interactional moves that engage students with mathematics in ways that preserve its depth and complexity.

References

Anderson, L. W., Krathwohl, D. R., Airasian, P. W., Cruikshank, K. A., Mayer, R. E., Pintrich, P. R., . . . Wittrock, M. C. (Eds.). (2001). *A taxonomy for learning, teaching, and assessing*. New York, NY: Longman.

Bakhtin, M. M. (1981). *The dialogic imagination*. Austin, TX: University of Texas Press.

Barnes, C. P. (1983). Questioning in the college classroom. In C. L. Ellner & C. P. Barnes (Eds.), *Studies of college teaching* (pp. 61-81). Lexington, MA: D.C. Heath.

Bass, R. (1999). The scholarship of teaching: What's the problem? *Inventio: Creative thinking about learning and teaching, 1*, Available at: http://www.doiiit.gmu.edu/archives/feb98/rbass.htm.

Blair, R. (Ed.). (2006). *Beyond crossroads: Implementing mathematics standards in the first two years of college*. Memphis, TN: American Mathematical Association of Two Year Colleges.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Sciences, 10*(1/2), 113-163.

Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25*, 119-142.

Coppin, C. A., Mahavier, W. T., May, E. L., & Parker, G. E. (2009). *The Moore Method: A pathway to learner-centered instruction*. Washington, DC: The Mathematical Association of America.

Dancy, M. H., & Henderson, C. (2009). Pedagogical practices of physics faculty. *AIP Conference Proceedings, 1179*(1), 121-124.

Dowd, A., Bensimon, E. M., Gabbard, G., Singleton, S., Macias, E., Dee, J. R., . . . Giles, D. (2006). *Transfer access to elite colleges and universities in the United States: Threading the needle of the American dream*. Jack Kent Cooke Foundation, Lumina Foundation, Nellie Mae Education Foundation.

Doyle, W. (1984). Academic tasks in classrooms. *Curriculum Inquiry, 14*, 129-149.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist, 23*, 167-180.

Fennema, E., & Sherman, J. (1986). *Fennema-Sherman Mathematics Attitudes Scales: Instruments designed to measure attitudes towards the learning of mathematics by females and males*. Madison, Wisconsin: Wisconsin Center for Education Research.

Frederick, P. J. (1987). Student involvement: Active learning in large classes. In M. Weimer (Ed.), *Teaching large classes well* (pp. 45-56). San Francisco: Jossey-Bass.

Grubb, N. W., & Associates. (1999). *Honored but invisible: An inside look at teaching in community colleges*. New York: Routledge.

Johnson, D. W., Johnson, R. T., & Smith, K. A. (1998). Cooperative learning returns to college: What evidence is there that it works? *Change, July/August*, 27-35.

Lutzer, D. J., Rodi, S. B., Kirkman, E. E., & Maxwell, J. W. (2007). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States: Fall 2005 CBMS Survey*. Washington, DC: American Mathematical Society.

Males, L. M., Otten, S., & Herbel-Eisenmann, B. (in press). Challenges of critical colleagueship: Examining and reflecting on study group interactions. *Journal of Mathematics Teacher Education*. doi: DOI:10.1007/s10857-010-9156-6

Martin, J. R., & White, P. R. R. (2005). *The language of evaluation: Appraisal in English*. New York: Palgrave-Macmillan.

Mesa, V. (2010). Student participation in mathematics lessons taught by seven successful community college instructors. *Adults Learning Mathematics, 5*, 64-88.

Mesa, V. (2011). Similarities and differences in classroom interaction between remedial and college mathematics classrooms in a community college. *Journal of Excellence in College Teaching, 22*(4), 21-56.

Mesa, V. (2012). Achievement goal orientation of community college mathematics students and the misalignment of instructors' perceptions. *Community College Review, 40*(1), 49-77.

Mesa, V., Celis, S., & Lande, E. (2013). *Teaching approaches of community college mathematics faculty: Do they relate to classroom practices?* Manuscript submitted for publication.

Mesa, V., & Chang, P. (2010). The language of engagement in two highly interactive undergraduate mathematics classrooms. *Linguistics and Education, 21*(2), 83-100.

Moschkovich, J. (1996). Learning math in two languages. In L. Puig & A. Gutiérrez (Eds.), *Proceedings of the 20th Conference of the International Group of the Psychology of Mathematics Education* (Vol. 4, pp. 27-34). Valencia, Spain: Universidad de Valencia.

Nathan, M. J., & Kim, S. (2009). Regulation of teacher elicitations in the mathematics classroom. *Cognition and Instruction, 27*(2), 91-120.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

Nunn, C. (1996). Discussion in the college classroom: Triangulating observational and survey results. *The Journal of Higher Education, 67*(3), 243-266.

Nystrand, M., Wu, L. L., Gamoran, A., Zeiser, S., & Long, D. A. (2003). Questions in time: Investigating the structure and dynamics of unfolding classroom discourse. *Discourse Processes, 35*(2), 135-198.

Pollio, H. R. (1989). Any questions now? Knoxville, TN: University of Tennessee.

Prince, M. (2004). Does active learning work? A review of the research. *Journal of Engineering Education, 93*(3), 223-231.

Rodi, S. B. (2007). Snapshot of mathematics programs at two-year colleges in the U.S. (Available at http://www.amatyc.org). Austin, TX: Austin Community College.

Scherrer, J., & Stein, M. K. (2012). Effects of a coding intervention on what teachers learn to notice during whole-group discussion. *Journal of Mathematics Teacher Education*. doi: 10.1007/s10857-012-9207-2

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Secada, W. G. (1995). Social and critical dimensions for equity in mathematics. In W. G. Secada, E. Fennema & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 146-164). New York: Cambridge University Press.

Silver, E. A., & Stein, M. K. (1996). The QUASAR project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education 30*, 476-521.
Smith, H., & Higgins, S. (2006). Opening classroom interaction: The importance of feedback. *Cambridge Journal of Education, 34*, 485-502.
Soter, A. O., Wilkinson, I. A., Murphy, P. K., Rudge, L., Reninger, K., & Edwards, M. (2008). What the discourse tells us: Talk and indicators of high-level comprehension. *International Journal of Educational Research, 47*, 372-391.
Sowder, J. (2007). The mathematical education and development of teachers. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157-224).
Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*, 455-488.
Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.
Truxaw, M. P., & DeFranco, T. C. (2008). Mapping mathematics classroom discourse and its implications for models of teaching. *Journal for Research in Mathematics Education, 39*(5), 489-525.
Vygotsky, L. S. (1986). *Thought and language* (A. Kozulin, Trans.). Cambridge, Mass: M.I.T. Press.
Wells, G., & Arauz, R. M. (2006). Dialogue in the classroom. *The Journal of the Learning Sciences, 15*(3), 379-428.
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*, 458-477.
Yackel, E., Cobb, P., Wood, T., Wheatley, G., & Merkel, G. (1990). The importance of social interaction in children's construction of mathematical knowledge. In T. Cooney (Ed.), *Teaching and learning mathematics in the 1990s*. Reston, VA: National Council of Teachers of Mathematics.

TABLES AND FIGURES

Table 1: Instructor Characteristics

Instructor	Academic Background	Years of college teaching experience	Status
Ed	Mathematics, BS, MS	3	Part-time
Elizabeth	Mathematics, BS, MA	7	Full -time
Elliot	Economics, BS	6	Part-time
Emmett[a]	Physics, PhD	16	Full-time
Evan	Physics, BS; Mathematics, BS; Mathematics, MA	8	Part-time

Note. a. Emmett taught three sections of the course.

Table 2: Framework for Analyzing Teachers Moves (Adapted from Scherrer & Stein, 2012).

Initiating Moves (mutually exclusive assignment)

1. <u>Collect: Teacher seeks to gather more responses to a question from more students.</u>
2. Launch: Teacher asks an open-ended question meant to invite student thinking.
3. Literal: Teacher asks a question seeking retrieval of factual information. The teacher often is looking for a specific answer
4. Provide Information: Teacher gives information (answer or method) related to the instructional task at hand. Teacher reviews or reveals relevant information from prior work.
5. Re-Direct: Teacher asks a question that invites student thinking in a different direction from a preceding question. The initial question was never fully answered.
6. Re-Initiate: Teacher asks a question that repeats the same or slightly reworded question.
7. *Statement of Problem: Teacher poses the problem to be worked on.*
8. Think Aloud: Teacher talks about how she or he is thinking about a passage or a problem.

Rejoinder Moves (assigned individually or in addition to initiating moves)

9. Connect: Teacher asks a question or makes a statement so the students make an explicit connection.
10. (parking-)Lot: Teacher acknowledges student responses and states that the class will deal with the comment later.
11. Push-Back: The teacher challenges a student response in order to encourage students to rethink or defend their responses.
12. Repeat: Teacher echoes a student response.
13. *Response Right: Teacher tells the student that his or her response or contribution is correct.*
14. *Response Wrong: Teacher tells the student that his or her response or contribution is incorrect.*
15. Terminal: An utterance that discontinues a student's response and often implicitly or explicitly evaluates students' responses
16. Uptake-Literal: Teacher asks a question for retrieving factual information building on a student response.
17. Uptake: Teacher uses a student response to extend, deepen, clarify, or elaborate the discussion.

Other

18. No Code: A move cannot be categorized using one of the codes above. These included classroom management comments, administrative announcements, references to the textbook, and personal stories unrelated to the content.

Note: Italicized text corresponds to codes added to the original framework proposed by Scherrer and Stein. Underlined text corresponds to changes of codes across categories.

Table 3: Examples of Teacher Moves from the corpus.

Monogloss	
Provide Information	Today we're gonna wrap up section 3.1 on graphing [six basic] trig functions, remember we got through sine, cosine and tangent [on Tuesday] and we're going wrap up doing the other three which actually follow uh, pretty easily from the graphs of sine, cosine, and tangent.
Heterogloss Contract	
Literal	T: so in this example, $y = x^2$, what's the range?
Lot	T: Well that's, yeah, and that actually will not give rise to 'cannot define,' let's hold off on that for a second. Where are the places where we would be dividing by zero, for cotangent?
Repeat	M: Arbitrarily small. T: Arbitrarily small. Does everyone see that?
Response Right:	M: And you kind of do this for anything with a negative because if you add a full period it shouldn't change anything, right? T: Correct. So anytime you get a negative radian answer you know that it's supposed to be positive, you start adding periods, however many periods you need.
Response Wrong	T: But where would the 120 be? What letter would be 120? M: [B] T: Not B. Sorry?
Terminal	T: Then can I write the function now? M: Um, no. T: Heck yeah, we got it, right? So we can now say y is 30 sin pi over seven T. Ok. Cool
Think Aloud	T: Ok, I want to draw the graph for this function, so x is between 0 and 2pi. The way I'm going to do it, so let's see. Just base this of off what I know about this regular sine. Let's draw the graph of, say, sine x. (Pause 10 seconds) This is, well, $y = \sin x$
Uptake-Literal	T: what effect, what would this pi over two affect? M: Period. T: Period, right? It would affect period. And, we know something about period, don't we? M: It's fourteen seconds.
Heterogloss Expand	
Collect	T: I'll get what? M: Sine squared. T: (writes on board) What else? M: Minus 2 sine cosine.

Launch	T: We found the *A* and *B* and we wrote this. So now let's talk about well what's that mean. What's does that mean? This thirty. Well what are we answering first?
Push-Back	T: What's our *y*-axis? M: Uhh. M2: [it is] feet. T: Oh, really? Hmm, why is this feet? Does that help or no? Ok. M: Why would it be the *y*-axis if *x* can measure the amplitude?
Heterogloss Contract or Expand	
Connect	**Contract**: T: .0125. So it's 1/100 of a second roughly. That's the time it takes for one wave, that's the time it takes to complete one wave. This is the lowest frequency of a male speech. Now the highest for a male is 240 and the lowest for a female is 140. The range they're giving you from 80 to 240, that's the range of male speech. 140 to 500 is the range of female speech. Compact disc is from 0 to 22,050. It makes sense because it has all kinds of sounds coming from a compact disc. Piano is from 28 to 4,186. And human hearing is from 20 to 20,000 hertz. Ok. Any questions? There are more applications that have to do with electricity, we're running out of time so we'll get to these applications next time. I do have your quizzes. **Expand**: T: So, I'm going to back up over here, I just want to try and draw some parallels. What do we know about this?
Re-Direct	**Contract:** T: And how do we find *C* from that number? What do we do to it? If sine *C* equals this number here, what would *C* be? (pause 6 sec) Let me calculate that number. We need 5 divided by 7.6. 5 divided by 7.6 is .65789. **Expand:** T: What's the unit of the phase shift? (pause 5 sec) First of all what is a phase?
Re-initiate	**Contract**: T: But where would the 120 be? What letter would be 120? **Expand**: T: For this particular frequency, how do we find the time? What do we do to frequency to find the time?

Uptake	**Contract**: M: It's going to shrink T: it shrinks by a factor of M: One-third. T: Yeah. **Expand**: Ok. Any suggestions on that? M2: (laughs) The rate. T: We could look at a rate! That's a good idea!

Table 4: Average frequency per 85-minute class period of mathematical questions; frequency and percent of Novel questions, Novel and Routine questions aborted, and Novel questions not aborted.

		Mathematical Questions					
	N	Novel Questions		Aborted Questions		Novel Questions Not Aborted	
Instructor		n	%[a]	%[b] Novel	%[c] Routine	n	%[a]
Ed	85	24	29%	18%	17%	20	23%
Elizabeth	73	25	35%	30%	24%	18	24%
Elliot	52	10	19%	24%	7%	7	14%
Emmett 1	55	14	25%	37%	38%	9	16%
Emmett 4	39	11	29%	38%	40%	7	18%
Emmett 5	38	19	50%	34%	23%	13	33%
Evan	46	12	26%	66%	43%	4	9%
Average	55	16	30%	33%	26%	11	20%

Notes: a. Percent calculated out of all mathematical questions asked in the lessons. b. Percent calculated out of all Novel questions asked in the lessons. c. Percent calculated out of all Routine questions asked in the lessons.

Table 5: Average frequency and percent of mathematical questions and Novel questions students asked per 85-minute class period.

	Student Mathematical Questions		
	N[a]	Novel[a]	%[b]
Ed	14	7	54%
Elizabeth	8	2	25%
Elliot	41	8	20%
Emmett 1	5	3	57%
Emmett 4	12	6	49%
Emmett 5	9	2	27%
Evan	4	2	45%
Average	13	4	32%

Note: a. The number of questions is the average over the three lessons rounded to the nearest integer. b. The percentage of Novel student questions is calculated before rounding

Table 6: Frequency of Initiating Teacher Moves by Instructor Across All Observed Lessons.

	Initiating Moves								
	COL	LAU	LIT	NC	PIn	ReD	ReI	SoP	ThA
Ed (n=913)	1	3	278	137	415	5	25	12	37
Elizabeth (n=958)	0	0	318	155	384	4	33	11	53
Elliot (n=1158)	0	0	340	155	612	1	10	18	22
Emmett 1 (n=528)	0	0	202	50	232	2	20	19	3
Emmett 4 (n=523)	0	0	190	48	252	0	14	19	0
Emmett 5 (n=526)	0	0	200	42	239	5	29	8	3
Evan (n=488)	0	0	168	71	213	2	16	6	12
Total (N=5,094)	1	3	1696	658	2347	19	147	93	130
Percent	0%	0%	33%	13%	46%	0%	3%	2%	3%

Table 7: Frequency of Rejoinder Teacher Moves by Instructor Across All Observed Lessons.

	Rejoinder Moves								
	CON	Lot	PBK	REP	RR	RW	TRM	U	UL
Ed (n=165)	19	2	2	64	44	5	6	20	3
Elizabeth (n=184)	15	3	5	74	27	3	18	34	5
Elliot (n=220)	5	1	4	85	47	0	26	49	3
Emmett 1 (n=75)	3	0	0	40	6	0	7	18	1
Emmett 4 (n=64)	10	0	0	29	2	0	4	18	1
Emmett 5 (n=82)	11	0	2	32	10	2	4	20	1
Evan (n=67)	12	1	1	22	12	3	5	11	0
Total (N=857)	75	7	14	346	148	13	70	170	14
%[a]	9%	1%	2%	40%	17%	2%	8%	20%	2%
%[b]	2%	0%	0%	8%	3%	0%	2%	4%	0%

Notes: CON: Connections. PBK: Push-Back. REP: Repeat. RR: Response Right. RW: Response Wrong. TRM: Terminal. U: Uptake. UL: Uptake Literal. a. Percent taken out of the rejoinder moves only. b. Percent taken out of all moves, excluding No Codes.

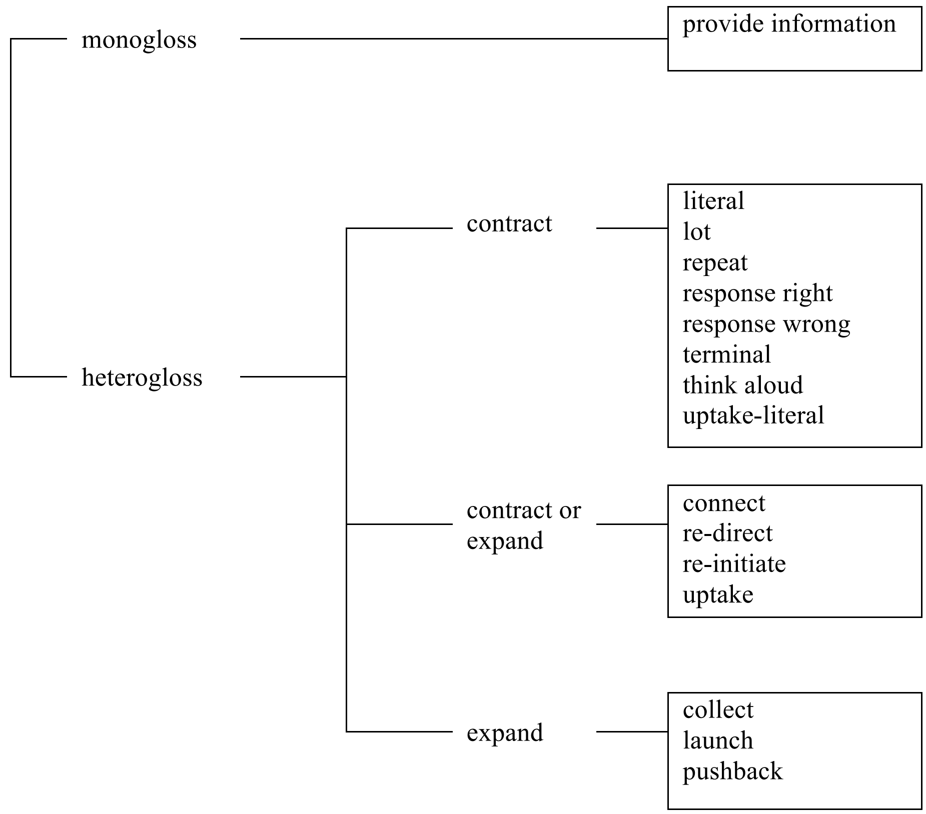


Figure 1: Classification of Teacher Moves according to Monogloss-Heterogloss voices.

Notes: COL: Collect. LAU: Launch. LIT: Literal. NC: No Code. PIn: Provide Information. ReD: Re-Direct. ReI: Re-Initiate. SoP: Statement of problem. ThA: Think Aloud.

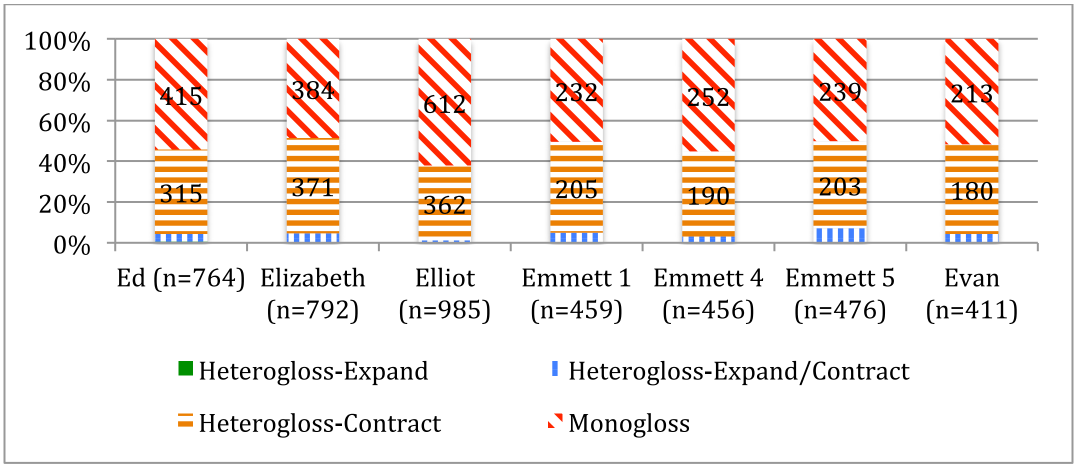


Figure 2: Average distribution of Heterogloss versus Monogloss initiating moves by instructor. (N = 4,436, excludes No Codes, n = 658 and Statement of Problem moves, n = 93).

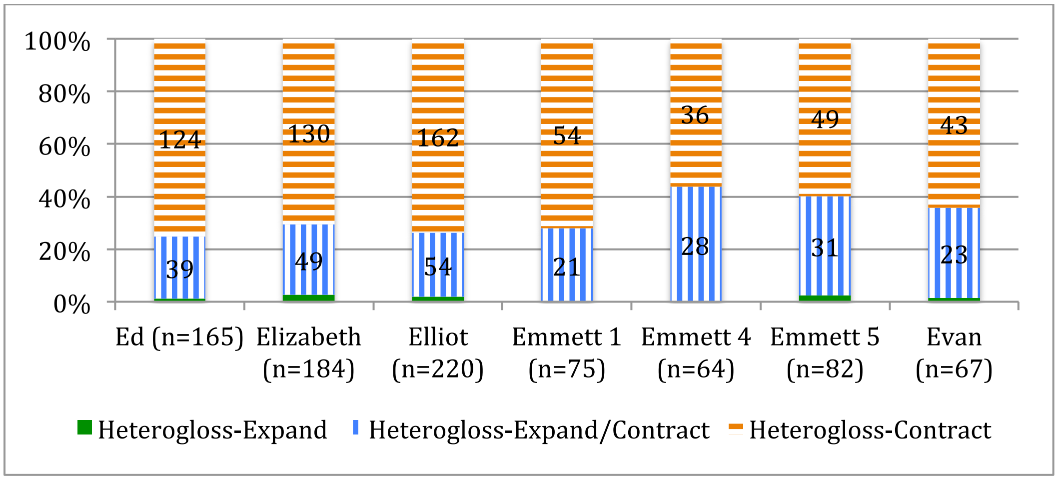
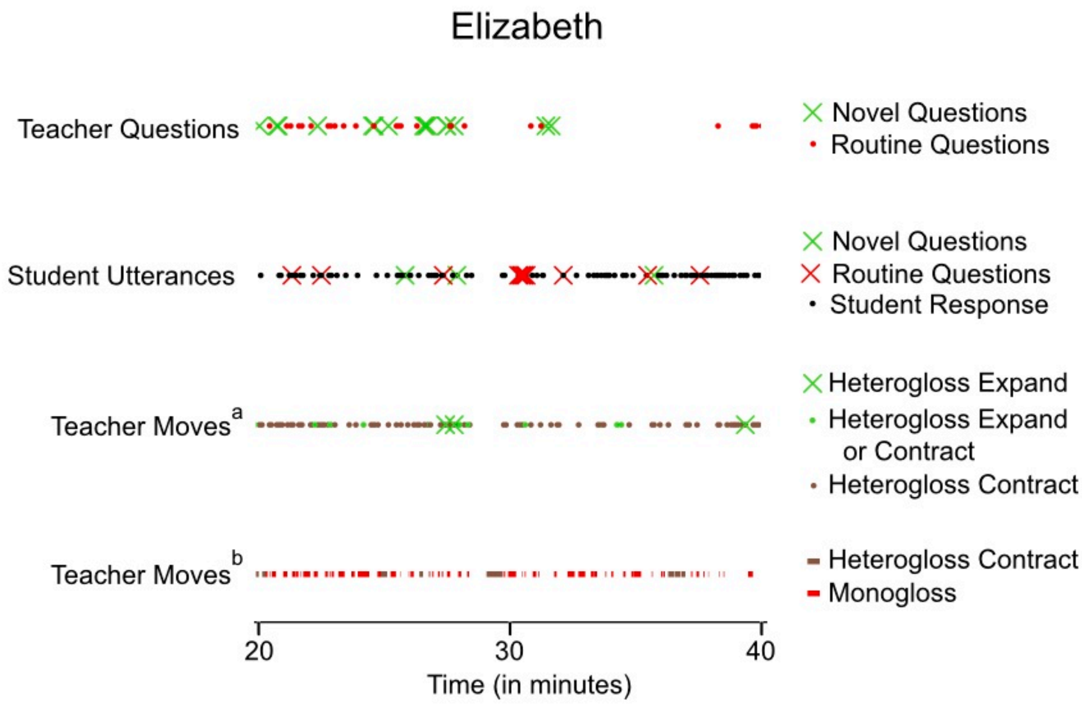
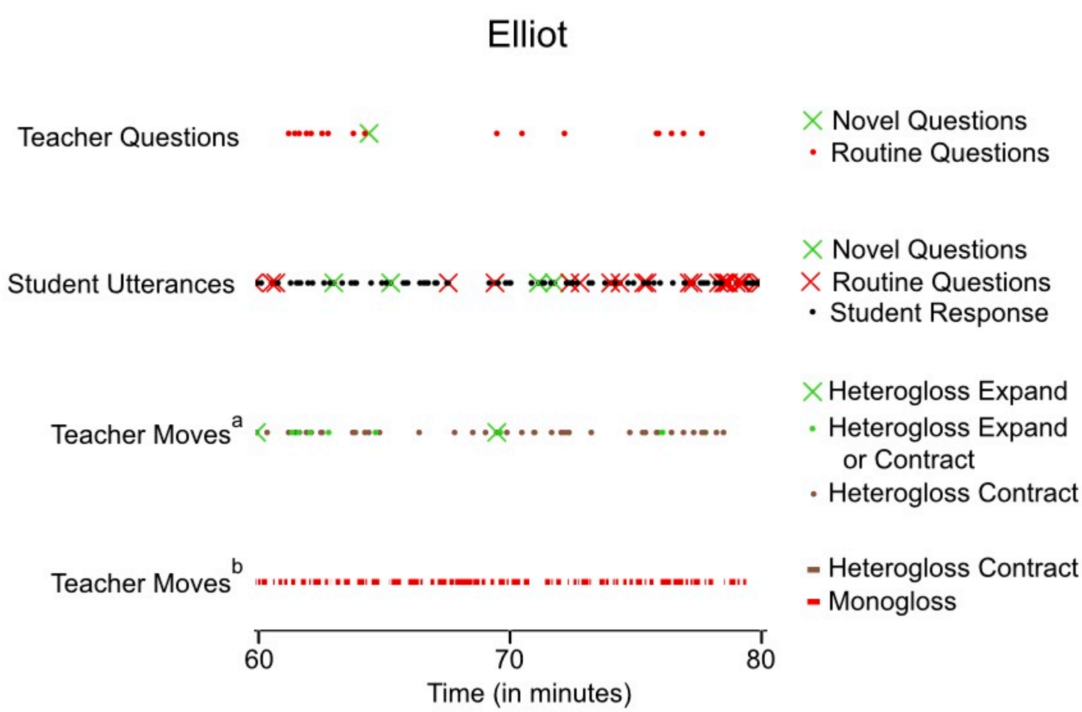
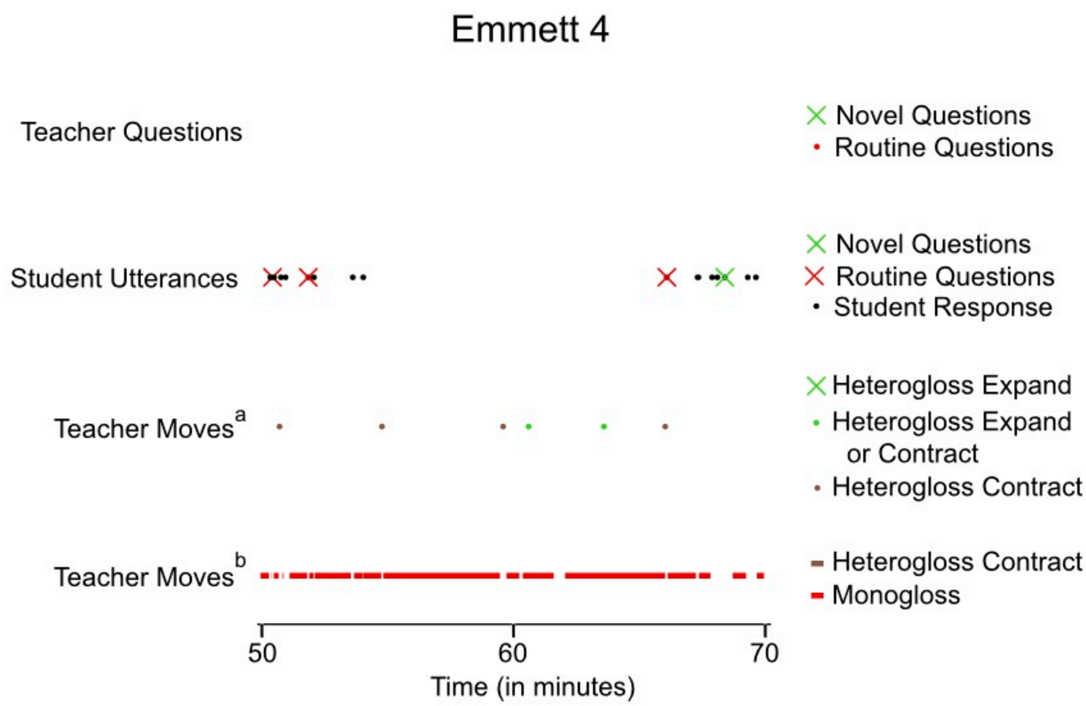


Figure 3: Distribution of Heterogloss versus Monogloss rejoinder moves by instructor (N = 857).


Elizabeth
Teacher Questions
Novel Questions
Routine Questions
Student Utterances
Novel Questions
Routine Questions
Student Response
Teacher Moves[a]
Heterogloss Expand
Heterogloss Expand or Contract
Heterogloss Contract
Teacher Moves[b]
Heterogloss Contract
Monogloss
20
30
40
Time (in minutes)


Elliot
Teacher Questions
Novel Questions
Routine Questions
Student Utterances
Novel Questions
Routine Questions
Student Response
Teacher Moves[a]
Heterogloss Expand
Heterogloss Expand or Contract
Heterogloss Contract
Teacher Moves[b]
Heterogloss Contract
Monogloss
60
70
80
Time (in minutes)



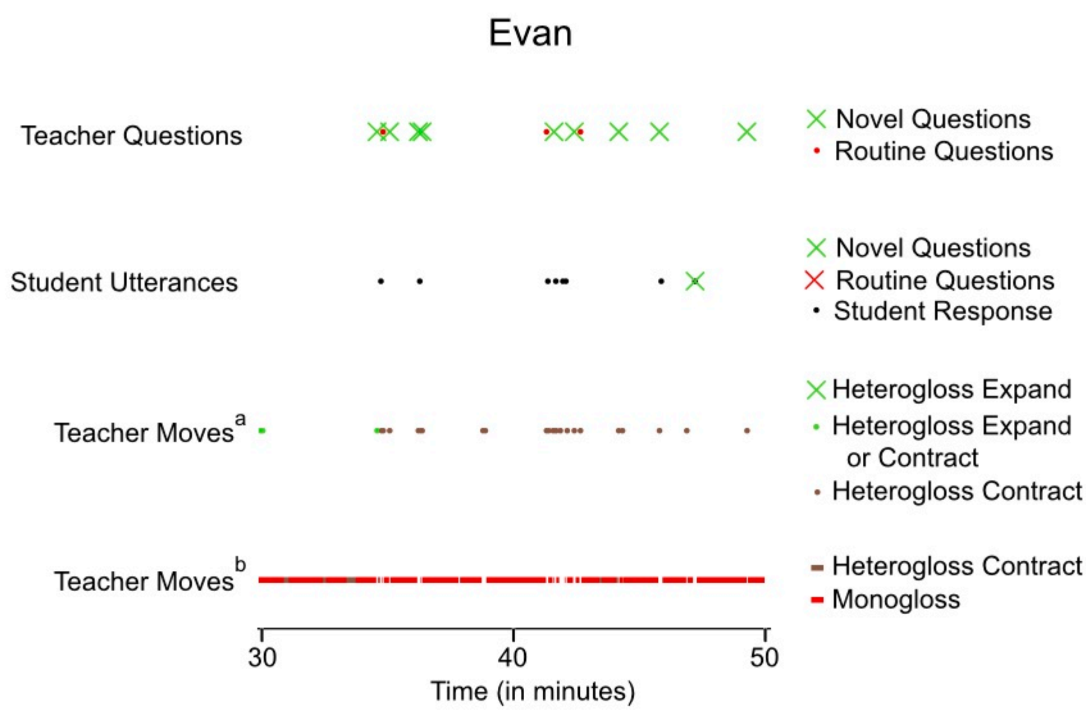


Figure 4: Representations of the Novelty of Teacher Questions, student utterances, and Teacher Moves, as they unfolded over a 20-minute segment of New Material, for four instructors

(a) Elizabeth's first lesson, (b) Elliot's second lesson, (c) Emmett 4's first lesson, and (d) Evan's first lesson. Notes: a. Includes the following moves: Collect, Launch, Pushback, Connect, Re-Direct, Re-Initiate, Uptake, Literal, Lot, Repeat, Response Right, Response Wrong, Terminal, and Uptake-Literal. b. Includes the following moves: Provide Information and Think Aloud.

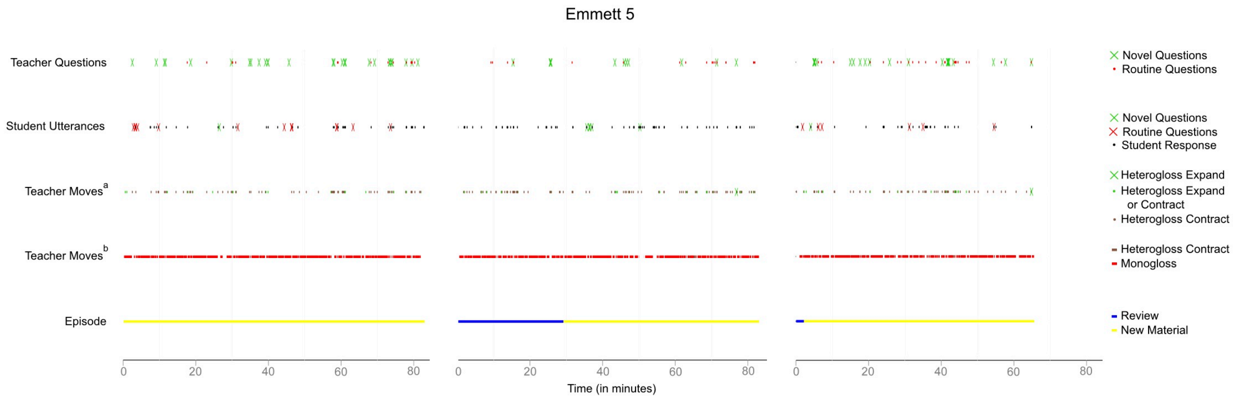
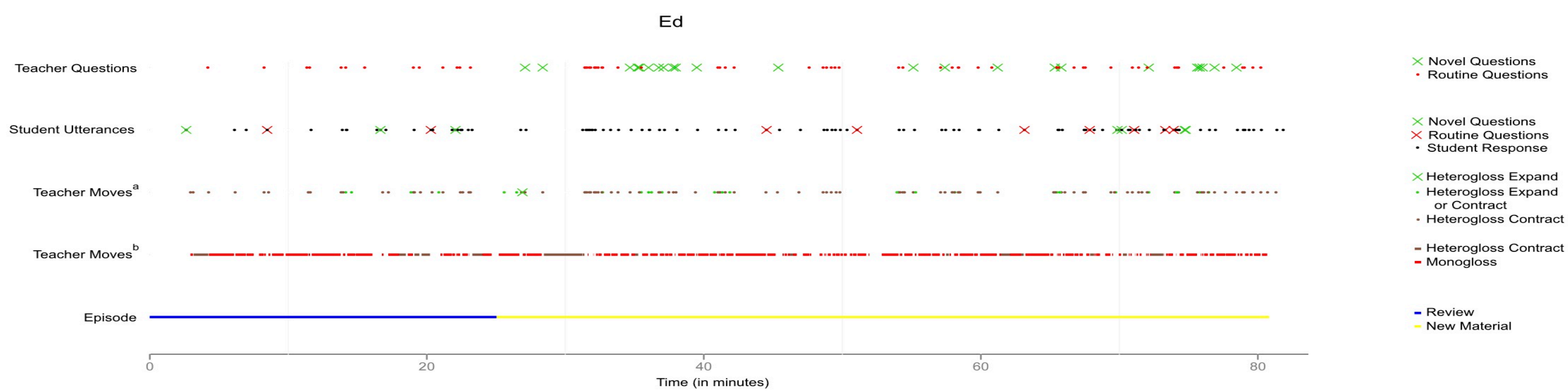
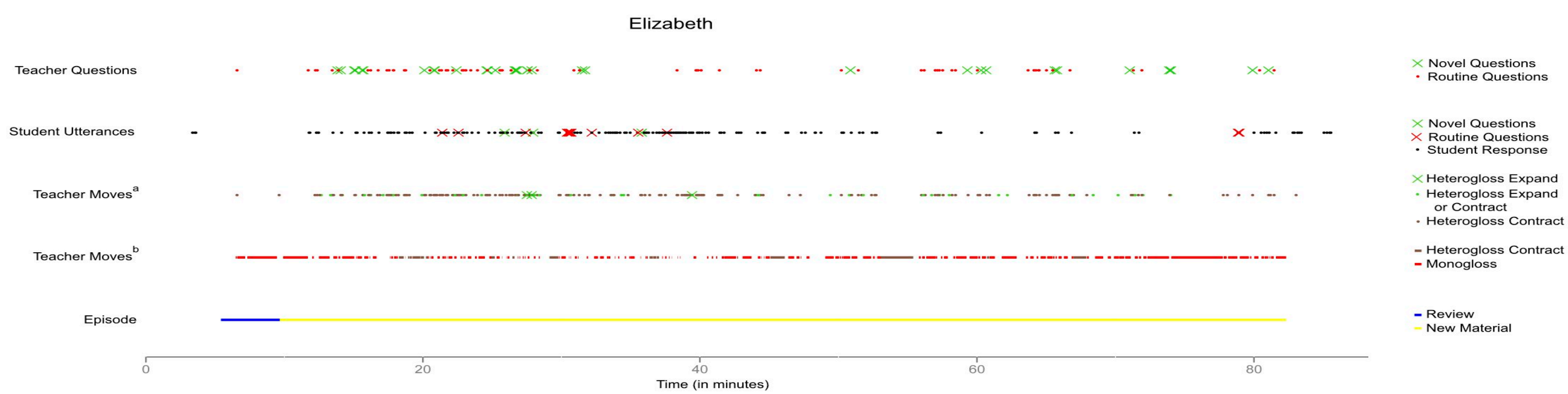


Figure 5: Representation of Emmett 5's three consecutive lessons by the Novelty of Teacher Questions, student utterances, Teacher Moves, and Episodes as they unfolded over time. Notes: a. Includes the following moves: Collect, Launch, Pushback, Connect, Re-Direct, Re-Initiate, Uptake, Literal, Lot, Repeat, Response Right, Response Wrong, Terminal, and Uptake-Literal. b. Includes the following moves: Provide Information and Think Aloud.

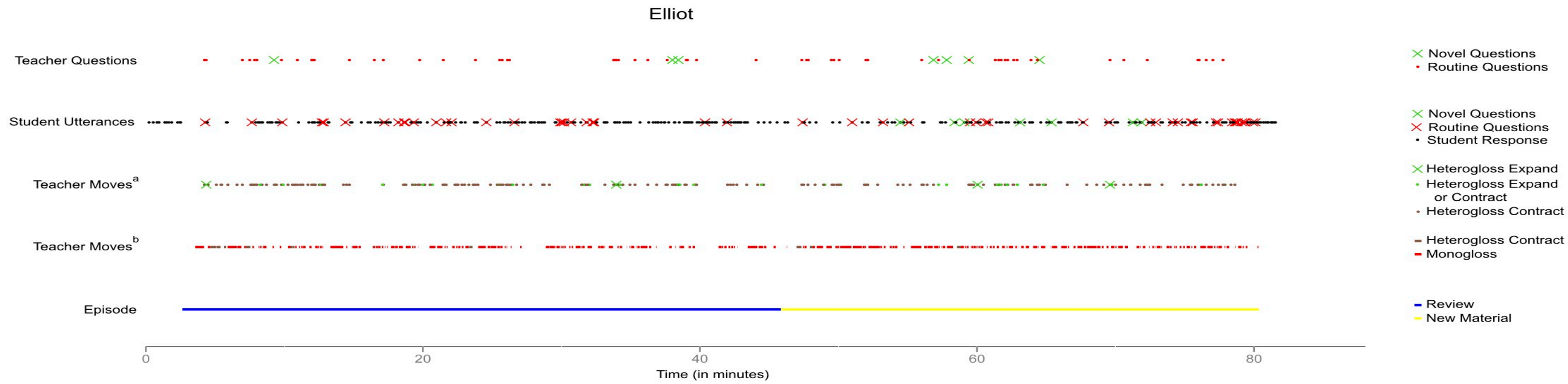
Appendix


Ed
Teacher Questions
Student Utterances
Teacher Moves^a
Teacher Moves^b
Episode
0
20
40
60
80
Time (in minutes)
Novel Questions
Routine Questions
Novel Questions
Routine Questions
Student Response
Heterogloss Expand
Heterogloss Expand or Contract
Heterogloss Contract
Heterogloss Contract
Monogloss
Review
New Material

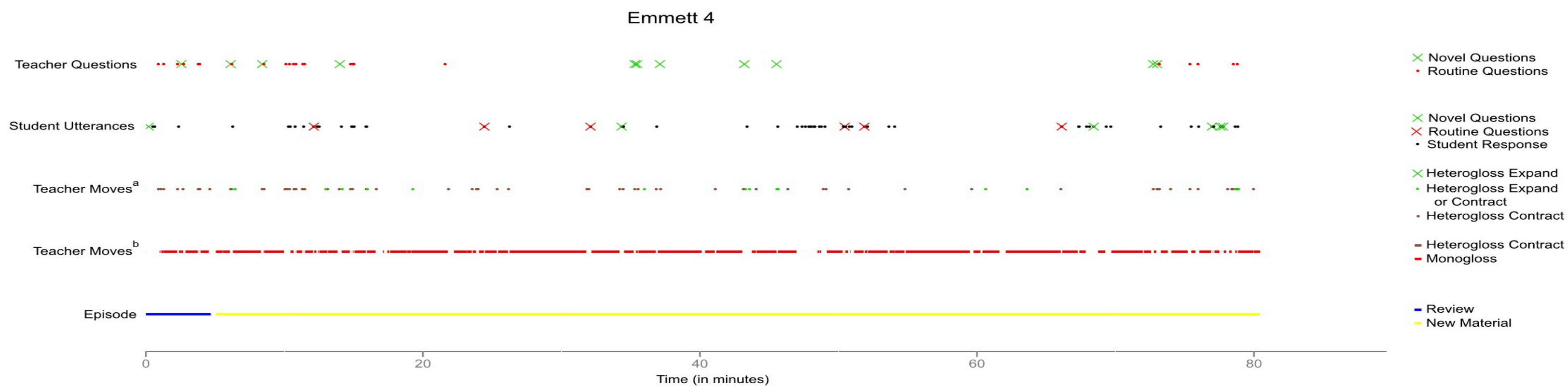
(a)


Elizabeth
Teacher Questions
Student Utterances
Teacher Moves^a
Teacher Moves^b
Episode
0
20
40
60
80
Time (in minutes)
Novel Questions
Routine Questions
Novel Questions
Routine Questions
Student Response
Heterogloss Expand
Heterogloss Expand or Contract
Heterogloss Contract
Heterogloss Contract
Monogloss
Review
New Material

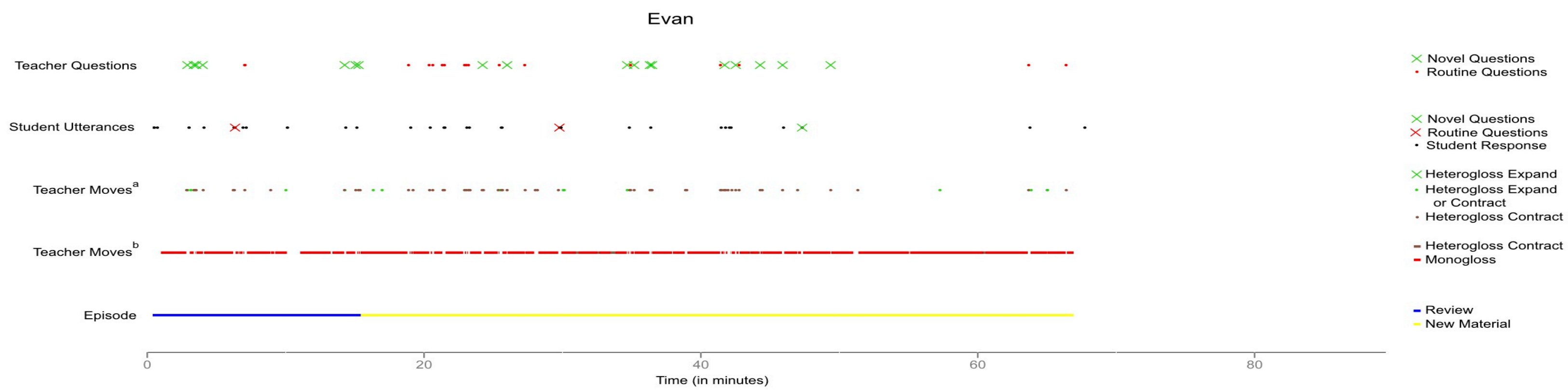
(b)


Elliot
Teacher Questions
Student Utterances
Teacher Moves[a]
Teacher Moves[b]
Episode
Novel Questions
Routine Questions
Novel Questions
Routine Questions
Student Response
Heterogloss Expand
Heterogloss Expand or Contract
Heterogloss Contract
Heterogloss Contract
Monogloss
Review
New Material
0
20
40
60
80
Time (in minutes)

(c)


Emmett 4
Teacher Questions
Student Utterances
Teacher Moves[a]
Teacher Moves[b]
Episode
Novel Questions
Routine Questions
Novel Questions
Routine Questions
Student Response
Heterogloss Expand
Heterogloss Expand or Contract
Heterogloss Contract
Heterogloss Contract
Monogloss
Review
New Material
0
20
40
60
80
Time (in minutes)

(d)



(e)

Representations of the Novelty of Teacher Questions, student utterances, Teacher Moves, and Episodes as they unfolded over time, for the five instructors (a) Ed's second lesson, (b) Elizabeth's first lesson, (c) Elliot's second lesson, (d) Emmett 4's first lesson, and (e) Evan's first lesson. Note: Notes: a. Includes the following moves: Collect, Launch, Pushback, Connect, Re-Direct, Re-Initiate, Uptake, Literal, Lot, Repeat, Response Right, Response Wrong, Terminal, and Uptake-Literal. b. Includes the following moves: Provide Information and Think Aloud. The lessons are 85-minutes long.

Note to Appendix:

In Emmett’s lesson the marks for teacher questions and student utterances are denser than in Elizabeth’s and Elliot’s lessons. In Elliot’s lesson, student utterances are well distributed over the whole lesson, whereas in Elizabeth’s lesson, there are some stretches of time where only Elizabeth is speaking. Ed, Elliot, and Elizabeth use Heterogloss-Expand moves more frequently throughout the whole lesson; Ed and Elizabeth use more Novel questions and engage the students more throughout the lesson than the other instructors, although in their case the student participation is less than in Elliot’s case. Finally, Elizabeth devotes little time to Reviewing, compared to the other instructors, who devoted about half of the time in their lesson to such activity.