Expanding students' involvement in proof problems: Are geometry teachers willing to depart from the norm?[1]

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Abstract

Using a multimedia questionnaire we explore the extent to which secondary mathematics teachers recognize a hypothesized norm of doing proofs in geometry—that the teacher is in charge of providing the 'given' and the 'prove.' We also explore whether teachers who recognize the norm make a negative appraisal of its breach and find that geometry teachers are able to see some departures from the norm in a positive light. This finding suggests that it may be possible to expand geometry students' involvement in proof problems.

Introduction

Recent standards documents have emphasized the importance of engaging students in reasoning and proving (CCSSM, 2010; NCTM, 2000). Historically, proof was limited to the secondary geometry course, but the last decade has seen progress in making room for proof-related tasks in other classes. This can be seen in the existence of professional development materials for elementary and middle school teachers that encourage students to make conjectures and justify them (e.g., Fosnot & Jacob, 2010; Steele, Arbaugh, & Boyle, 2011). Also, researchers have documented proving activities in the elementary and middle grades (e.g., Bieda, 2010; Ellis, 2007; Knuth et al., 2009; Schifter, 2009; Stylianides, 2007). This scholarship shows that efforts have been made to engage students not only in justifying claims, but also in formulating those claims, thus providing for a richer engagement of students in proving.

We consider the question of how much of that richer engagement can be expected in secondary geometry classes. With that in mind, this study investigates the extent to which experienced geometry teachers look favorably on sharing the responsibility for stating the proposition to be proved with students. The study contributes to earlier work on the

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teaching of proof in geometry (Herbst & Brach, 2006; Martin, et al., 2005) that explored the division of labor between teacher and students in regard to doing proofs in geometry. We take advantage of a novel technique: the use of multimedia questionnaires delivered over the internet.

Theoretical Framework

We draw inspiration from sociological and linguistic studies (e.g., Mehan, 1979) that attempt to uncover the norms that regulate human behavior in recurrent situations. A fundamental assumption in this work is that recurring social interaction is facilitated by customary or unmarked ways of acting; we call those norms. Norms describe what experienced participants of a situation default to doing. The psychological notion of script (Schank & Abelson, 1977) and, in particular, the notion of cultural script (e.g., Stigler & Hiebert, 1999), points in the same direction: It identifies sources of regulation of behavior that, unlike the rules of games, are tacit. Norms are also different than rules because even though a norm may privilege some behaviors, they are not inevitable and the consequences of breaching them are not always punitive: Participants may negotiate departures from the norm, departures that may occasion more work but whose consequences can be felicitous. Further, while norms may support an activity by compelling participants to do things, norms may also limit the scope of activity, as is the case with the norm we investigate in this study.

Based on the observation of proof activity in American geometry classrooms, Herbst, Chen, Weiss, and González (2009) identified a number of norms that regulate the work of doing proofs. One of those norms is that the teacher is responsible for stating the proposition to be proved, including the 'givens' and the 'prove,' in a proof problem. While students' share of work includes making statements and reasons that connect the 'givens' to the 'prove,' they rarely participate in shaping what the 'givens' or the 'prove' could be. To the extent that "doing proofs" is a recurrent activity in geometry classrooms, it is natural that norms exist to regulate how teachers and students relate to each other and to the specific subject matter they are learning. Along those lines, for the teacher to provide the 'givens' and the 'prove' may be an appropriate way of scaffolding the work of proving. Yet for students to expand what they know about the subject it is likely that norms will need to evolve as instruction progresses. Teachers might thus negotiate the establishment of norms to support students' learning at one time, and later negotiate how to handle breaches of those norms, also for the sake of enhancing students' learning. This paper explores how teachers who recognize the norm relate to the possibility that students collaborate in fashioning the 'givens' and the 'prove.'

Method

To investigate our research questions, we used multimedia questionnaires consisting of item sets that required participants to refer to or interpret a visual media representation of an episode from a mathematics lesson. Within each item set, participants responded to various questions about the media. We report on responses to four item sets, each of which consisted of individual items that were related to a representation of instruction depicting a lesson in which a teacher was facilitating work on a proof. While this was not told to the viewer, each of these representations showed a teacher breaching the target norm by involving the students in the work of determining the 'givens' and 'prove' for a proof problem. Each story was represented as a sequence of images, where each frame depicted a classroom scene with cartoon characters representing teacher and students and speech bubbles represented their dialogue. In "A proof about a quadrilateral" (item 22002, "quadrilateral"), the teacher presents students with a diagram of a quadrilateral that appears to represent a rhombus and its diagonals, then asks the students to come up with what is to be given and a statement to be proved, thereby breaching the norm. In this story, the teacher accepts givens from the students that are inconsistent with each other, and this leads to students getting frustrated. In "A proof about a parallelogram" (item 22003, "parallelogram"), the teacher helps the class decide on a set of givens that are consistent with each other and asks the students to choose a prove statement, thereby breaching the norm. In "A proof about parallel and intersecting lines" (Item 22005, "Lines"), students work together in pairs to come up with a list of possible givens for a diagram that shows a set of three parallel lines cut by two transversals. Then, through class discussion, the students choose a set of a givens and a statement to prove, thereby breaching the norm. Finally, in "A proof problem for homework" (item 22006, "homework"), at the end of class students are assigned a proof problem based on a diagram that the teacher has just drawn on the board. The teacher asks the students to come up with an appropriate 'given' and 'prove' for the diagram and then complete the proof for homework, thereby breaching the norm. Across this set of items, the breach of the norm in "quadrilateral" (item 22002) was designed to be more egregious than the breaches of the norm in the other stories, in that the teacher in "quadrilateral" breaches the norm with no apparent instructional goal for doing so. We conjectured that this unjustified breach would provoke more negative reactions from the participants and our findings support this conjecture.

After viewing the scenario, each participant was asked the questions listed in Table 1.

Table 1. Questions used in multimedia questionnaire

Question	Format
1. Describe what you saw happening in this scenario	Open response, re-viewing of story allowed
2. How appropriately did the teacher facilitate work on a proof in this scenario?	Open response, re-viewing of story allowed
3. How appropriate was the teacher's facilitation of work on a proof in this scenario?	Likert Scale, 6 choices ranging from 1-Very inappropriate to 6-Very appropriate
4. How typical is it for a teacher to facilitate work on a proof in the way that is shown in this scenario?	Likert Scale, 4 choices ranging from 1-It never or hardly ever happens to 4-It always or almost always happens

We coded open-ended responses to questions 1 and 2 to ascertain (1) whether participants noticed a breach of the norm and (2) if they appraised the breach positively or negatively. Definitions of these these codes are listed in Table 2.

Table 2. Variables used to code responses to questionnaire

Variable	Description	Possible Values
NR(j)	In item set j, did the participant remark upon the norm that the teacher gives the 'given' and the 'prove'?	{0: response contains no reference to norm, 1: response contains reference to norm}
NR	The sum of participants' scores on all the NR(j) codes.	{0 - 4}
BA(j)+	In item set j, did the participant appraise positively the handling of the 'given' and the 'prove'?	{0: response contains no positive appraisal, 1: response contains a positive appraisal}
BA(j)-	In item set j, did the participant appraise negatively the handling of the 'given' and the 'prove'?	{0: response contains no negative appraisal, 1: response contains a negative appraisal}
App_j	In item set j, how appropriate did the participant rate the teachers' actions in the story	[selected by participant] {1 Very Inappropriate 2 Inappropriate 3 Somewhat Inappropriate 4 Somewhat Appropriate 5 Appropriate 6 Very Appropriate}

Three research team members contributed to coding the data and each response was coded by two coders. Reliability in the coding was established though comparison of assigned codes. Disagreements were discussed and resolved. Initial kappa scores can be seen in Table 3. After reconciliation agreement on all codes was 100%. An example of response that was coded as a "1" for "NR(2002)" (that is, a response in which a participant recognized the breach of the norm in story 22002 "quadrilateral") is: "It is very appropriate to give students some ownership of a problem by *making them figure out what they want to prove*" (participant 639, key text in italics). In this response, the participant references that the students were made to "figure out what they want to prove". This was coded as a "yes" for "recognizes breach" because the participant clearly indicates that it was the students, not the teacher, that came up with the statement to be proved.

Table 3. Kappa scores for reliability of coding

	NR(j)	BA(j)+	BA(j)-
22002	0.72	0.51	0.72
22003	0.76	0.28	0.38
22005	0.90	0.57	0.65
22006	0.85	0.48	0.73

Data Sources

A total of fifty secondary mathematics teachers took the multimedia questionnaire. Thirty-three of the participants had completed three or more years teaching high school geometry. Participants were recruited from two states in the American Midwest.

Results

Below we provide evidence aimed at answering the follow research questions:

- Do secondary mathematics teachers recognize a breach of the 'given' and 'prove' norm? Is the proportion of recognizers of this norm higher among experienced geometry teachers than among all secondary mathematics teachers?
- How do recognizers of the norm relate to breaches of the norm? Are they more likely to appraise these breaches negatively?
- When participants do recognize the norm is there a relationship between their appraisal of the handling of the 'given' and 'prove' and their rating for how appropriately the teacher acted in the episode as a whole?

For the "quadrilateral" (22002), "parallelogram" (22003), and "lines" (22005) stories, at least 70% of the participants recognized a breach of the norm (see Table 4). Binomial tests rejected the null hypothesis that recognition and nonrecognition of the norm are equally likely with a *p*-value of 0.01. For the "homework" story (22006), only 14% of the participants recognized the norm in the story, which is also significant. Inspection of this data shows however that all recognizers of the breach in this item were recognizers in other items, which suggests that the item may be a more difficult item.

Table 4. Binomial test for recognition of the norm for all participants

	% of participants who recognize the norm (# who recognized/total =)	Binomial Tests (null hypothesis .5) p value
22002	(37/50 =) 74%	0.000
22003	(35/50 =) 70%	0.002
22005	(36/50 =) 72%	0.001
22006	(7/50 =) 14%	0.000

The subset of experienced geometry teachers (EGT), defined here as those teachers with 3 or more years experience teaching the subject, performed similarly to the whole sample (see Table 5), with at least 70% of these teachers recognizing the norm in item sets 22002, 22003, and 22005. For the 22006 item, set 18% of EGT recognized the norm. All of these rates of recognition are significantly different than 0.5 at the *p*=.01 level.

Table 5. Binomial test for recognition of the norm for experienced geometry teachers

	% of experienced geometry teachers who recognize the norm (# of EGT who recognized/total EGT =)	Binomial Tests (null hypothesis .5) p value
22002	(26/33 =) 79%	0.001
22003	(24/33 =) 73%	0.004
22005	(25/33 =) 76%	0.002
22006	(6/33 =) 18%	0.000

If the provision of the given and prove by the teacher is truly a norm of geometry instruction than we would expect that EGTs would be more likely to recognize a breach of the norm than those without that experience. This conjecture was supported by our analysis. Using the NR variable that aggregates recognition scores across all item sets, we see that (seeTable 6) EGTs were significantly more likely to recognize the breach of the norm than their counterparts.

Table 6. Mann-Whitney test for difference of median NR scores	Mann-Whitney U	Z vale	Asymp. Sig (2-tailed)
Experienced Geometry Teachers v. Non experienced	186.000	-2.005	0.045

The coding scheme also kept track of whether and how participants appraised the breach of the norm. We operationalized appraisal using the linguistic framework by Martin and White (2007) whereby appraisal is realized as expressions of the respondents' affect (e.g., "I liked…"), as judgments of the participants (e.g., "she was rude", "she was welcoming"), or as appreciations of goods or services (e.g., "that diagram was too small"). An example of a positively appraised breach is "it is very appropriate to give students some ownership of a problem by making them figure out what they want to prove" where "appropriate" is a positive appraisal that refers to the teacher's willingness to let students decide on a prove. Similarly, an example of a negatively appraised breach is "I did not think the teacher's facilitation was appropriate […] it teaches students to make assumptions that may not be true". Table 7 reports the number of recognizers that positively or negatively appraised the breach of the norm in each item set. Table 8 reports those proportions for the group of teachers who had experience teaching geometry. The counts suggest that recognition of a breach of a norm does not imply a negative appraisal. The nature of the appraisal seems to be dependent on other considerations that might be specific to the case at hand—while there were more people who appraised positively than negatively the breach of the norm in item sets 22003 and 22004, the relationship was different in 22002.

Table 7. Positive and negative appraisal of breach, given recognition of the norm (all teachers)

	Total recognizers	Positive appraisal of breach	Negative appraisal of breach
22002	37	9	17
22003	35	14	9
22005	36	16	8
22006	7	1	2

Table 8. Positive and negative appraisal of breach, given recognition of the norm (experienced geometry teachers)

	Total recognizers	Positive appraisal of breach	Negative appraisal of breach
22002	33	7	13
22003	24	9	7
22005	25	10	7
22006	6	1	2

To explore the relationship between participants' appraisals of the handling of the 'given' and 'prove' in each story and their rating of appropriateness of the story as a whole we computed the correlation between participants' appraisal (as measured by BA(j)+ and BA(j)–) and the participants' appropriateness rating for the story (as measured by App_j) Table 9 shows the correlations for each item set. Looking only at data from the participants who recognized the breach of the norm in the story, for item sets 22002, 22003, and 22005 there is a significant positive correlation between BA(j)+ and App_j and a significant negative correlation between BA(j) – and App_j. For 22006 there is a significant positive correlation between BA(6)+ and App_6. The negative correlation for 22006 is not significant.

Table 9. Correlations between appraisals on the breach and appropriateness rating

	Correlation between BA(j)+ and App_j	Significance (2-tailed)	Correlation between BA(j)- and App_j	Significance (2-tailed)
22002	.430**	0.008	-.714**	0.000
22003	.461**	0.005	-.721**	0.000
22005	.659**	0.000	-.506**	0.002
22006	.801*	0.031	-0.062	0.895

We interpret these results to mean that participants' ratings of the appropriateness of the teacher's actions in the episodes were importantly related to the appraisal of the teacher's actions regarding the norm. While it is conceivable that participants would base their appropriateness ratings on features of the story that are unrelated to the norm, such as the teacher's relationship with the students or the mathematical content of the problem in the story, the data shows that the teacher's treatment of the 'given' and 'prove' was clearly a factor in participants' rating—regardless of whether appraisals were positive or negative.

Conclusion

The items sets in this multimedia questionnaire seem to be successful in detecting teachers' recognition of a breach of a norm related to 'doing proofs' by geometry teachers. The data also shows that recognition of a breach does not necessarily imply negative appraisal of the breach of the norm. Rather, some breaches of a norm are positively appraised and this appraisal correlates with participants' ratings of the work of the teacher in general. These preliminary results suggest that expanding students' share of labor in the work of proving may be possible in geometry classrooms. While enabling students to formulate the proposition to be proved is not normative, teachers may see reason to depart from the norm. In general, this observation suggests that instructional norms do not need to play a constraining role. Rather they serve as points of departure (and arguably also scaffolds) for classroom mathematical work. This study shows how such exploration could be done with other situational norms.

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