Dispersion of Rayleigh Waves Across
the Atlantic Ocean Basin

by

Orhan Baykal

November 1946

59

M.S. Feb. '47

## Introduction

The earthquake discussed in this study occured on November 25, 1941, at $18^h 03^m 53^{sec}$ Universal Time. The location of the epicenter some 600 miles east of the Portuguese coast, was favorable for a study of the structure of the Atlantic Ocean basin.

Through the data secured from the seismograms the following factors have been studied; dispersion of Rayleigh waves along oceanic and continental paths; effects of refraction of surface waves at continental boundaries; and the Atlantic crustal structure as determined from the dispersion of Rayleigh waves.

## Theory of Rayleigh Waves

In 1885, in the Proceedings of the London Mathematical Society the third Lord Rayleigh published a study about the "behaviour of waves upon the plane free surface of an infinite homogeneous isotropic elastic solid, their character being such that a disturbance is confined to a superficial region of thickness comparable with the wave length." These type of waves have since been recognized on seismograms and are referred as "Rayleigh" or "R" waves.

They are a combination of dilatational and shearing distortion. Their amplitudes die out exponentially with depth. In an elastic solid with Poisson's ratio equal to $\frac{1}{4}$, x in the direction of propagation, z vertically downward, Lord Rayleigh found:

$$u = A(e^{-rz} - .5773\, e^{-sz}) \sin k(x-ct)$$

$$w = A(.8475\, e^{-rz} - 1.4679\, e^{-sz}) \cos k(x-ct)$$

$$c = .9194\beta \qquad s = .3933\, k \qquad r = .8475\, k$$

where c is the velocity of "R" waves and $\beta$ of shear waves, $k = \frac{2\pi}{\lambda}$, A is a constant, u and w are respectively the components of the horizontal and vertical displacement.

The combined values of u and w give an elliptical orbit. At the surface, for $z = 0$, the motion is retrograde and the ratio of horizontal to vertical displacement is 0.68. As the depth increases the horizontal amplitude decreases. For $z = .19\lambda$, there is no horizontal component and the particle describes a vertical path perpendicular to the direction of propagation. For z greater than $.19\lambda$, the motion is reversed and for $z = .46\lambda$, the greatest horizontal displacement is obtained in this reversed direction. At greater depth the limiting value of the ratio of horizontal to vertical is 0.39.

As observed on the seismogram, the velocities, periods and amplitudes of the R waves do not fully agree with the theory as developed by Lord Rayleigh. In his theoretical study of the waves the velocity is independent of the wave-length, on the seismogram however the waves show dispersion, i. e. ., dependence of velocity to wave-length. He further assumed a semi-infinite medium whereas in most geologic cases there is at least one surface layer with different density and elastic properties overlying a semi-infinite solid. The case for such a superficial layer has been worked out by Stoneley (1), Lee (2), Sezawa (3), Jeffreys (4), and others.

Stoneley, following the procedure used by Love for an incompressible solid, gives the general equation for the wave-velocity as

$$\xi \eta' - \xi' \eta = 0$$

where

$$\xi = b\left(X \cosh rT + \frac{r'}{r} Y \sinh rT\right) - 2\frac{s}{k}\left(\frac{r'}{k} W \sinh sT + \frac{k}{s} Z \cosh sT\right)$$

$$\xi' = b\left(\frac{s'}{k} W \cosh rT + \frac{k}{r} Z \sinh rT\right) - 2\frac{s}{k}\left(X \sinh sT + \frac{s'}{s} Y \cosh sT\right)$$

$$\eta = b\left(\frac{r'}{k} W \cosh sT + \frac{k}{s} Z \sinh sT\right) - 2\frac{r}{k}\left(X \sinh rT + \frac{r'}{r} Y \cosh rT\right)$$

$$\eta' = b\left(X \cosh sT + \frac{s'}{s} Y \sinh sT\right) - 2\frac{r}{k}\left(\frac{s'}{k} W \sinh rT + \frac{k}{r} Z \cosh rT\right)$$

and

$$X = \frac{\mu' c^2}{\mu \beta'^2} - 2\left(\frac{\mu'}{\mu} - 1\right)$$

$$Y = 2\left(\frac{\mu'}{\mu} - 1\right) + \frac{c^2}{\beta^2}$$

$$Z = \frac{\mu' c^2}{\mu \beta'^2} - 2\left(\frac{\mu'}{\mu} - 1\right) - \frac{c^2}{\beta^2}$$

$$W = 2\left(\frac{\mu'}{\mu} - 1\right)$$

$$(r/k)^2 = 1 - c^2/3\beta^2 ; \qquad (s/k)^2 = 1 - c^2/\beta^2 ; \qquad (r'/k)^2 = 1 - c/3\beta'^2 ;$$

$$(s'/k)^2 = 1 - c^2/\beta'^2 ; \qquad b = 2 - c^2/\beta^2 ;$$

$\mu$ and $\mu'$ are the rigidity, T is the thickness of the upper layer, and the accents are used to denote the underlying material.

Jeffreys (5), has reworked Stoneley's equation and gives from the boundary conditions:

$$-(2 - c^2/\beta^2)\frac{\kappa}{r} A \sinh rT + (2 - c^2/\beta^2) B \cosh rT - 2\frac{s}{\kappa} C \sinh sT + 2\frac{s}{\kappa} D \cosh sT = 0$$

$$2A \cosh rT - 2B \sinh rT + (2 - c^2/\beta^2) C \cosh sT - (2 - c^2/\beta^2) D \sinh sT = 0$$

$$A + C = E + F$$

$$\frac{\kappa}{r}B + \frac{s}{\kappa}D = -\frac{\kappa}{r'}E - \frac{s'}{\kappa}F$$

$$\mu\left[(2 - c^2/\beta^2)\frac{\kappa}{r} B + 2\frac{s}{\kappa} D\right] = \mu'\left[(2 - c^2/\beta'^2)\frac{\kappa}{r'} E + 2\frac{s'}{\kappa} F\right]$$

$$\mu\left[2A + (2 - c^2/\beta^2) C\right] = \mu'\left[2E + (2 - c/\beta'^2) F\right]$$

where A, B, C, D, E, and F are constants. Either set of equations requires laborious computations and the solution is by trial and error.

Using the principle of stationary periods, Jeffreys (6), gives an approximate solution for the wave-velocity equation;

$$\frac{c^2}{\beta^2}\left[1.2409 + (\frac{\rho'}{\rho} - 1)M\right] = (\frac{\mu'}{\mu} - 1)L + 0.8453\left[1.2409 + (\frac{\mu'}{\mu} - 1)M\right]$$

where L and M are evaluated from the values for the homogeneous case.

For small values of kT ( less than .2) Lee's (7), approximate formula may be used; where

$$kT = \frac{1 - 1.0877\ c/\beta'}{c^2/\beta^2 - c^2/\beta'^2} \times 3.70\frac{\mu'}{\mu}$$

For the comparison with the data secured on the earthquake of November 25, three dispersion curves were used.

The first one was taken from Jeffreys. His curve is for the case of a crustal layer where $\beta = 3.3$ and $\beta' = 4.5$. The second curve was computed by Sezawa. This fits roughly the case of an upper layer having $\beta = 3.6$ and the lower medium being as previously stated. Sezawa gives no tables and the curves available are too small to permit accurate reading. Consequently the values are subject to inaccuracies.

The third curve was computed by the writer taking $\rho'/\rho = 7/6$ and $\mu'/\mu = 10/7$, so that $\beta = 4.0$ and $\beta' = 4.5$ km./sec. For the computation of this curve both Stoneley's and Jeffreys equations were used. For $kT < .20$, the calculations were based on Lee's formula. Table 1 gives the approximate value of $c/\beta$ obtained by Jeffreys stationary period method, table 2 is from the formal solution. The values from tables 1 and 2 are shown graphically in figure 1.

## TABLE 1

Approximate Computation

$\rho'/\rho = 7/6 \quad \mu'/\mu = 10/7$

| kT | c/β |
|---|---|
| 0.0 | 1.01739 |
| 0.1 | 1.01455 |
| 0.2 | 1.01297 |
| 0.3 | 1.01219 |
| 0.4 | 1.01190 |
| 0.6 | 1.01194 |
| 1.0 | 1.01074 |
| 1.5 | 1.00395 |
| 2.0 | 0.99225 |
| 2.5 | 0.97881 |
| 3.0 | 0.96570 |
| 4.0 | 0.94513 |
| 5.0 | 0.93264 |
| 7.0 | 0.92245 |
| ∞ | 0.91940 |

## TABLE 2

Theoretical Dispersion

$\rho'/\rho = 7/6 \quad \mu'/\mu = 10/7$

| c/β | kT | βT/P | C/β |
|---|---|---|---|
| 1.0174 | 0.000 | ∞ | 1.01740 |
| 1.0170 | 0.015 | 411.87627 | ------- |
| 1.0150 | 0.065 | 95.23542 | ------- |
| 1.0140 | 0.090 | 68.84920 | 1.01061 |
| 1.0120 | 0.200 | 31.04337 | 1.01012 |
| 1.0100 | 0.964 | 6.45327 | 1.00422 |
| 1.0000 | 1.585 | 3.96415 | 0.97099 |
| 0.9900 | 1.945 | 3.26314 | 0.92738 |
| 0.9800 | 2.214 | 2.89584 | 0.88702 |
| 0.9700 | 2.445 | 2.60141 | 0.86293 |
| 0.9600 | 2.710 | 2.41512 | 0.85865 |
| 0.9400 | 3.500 | 1.90978 | 0.89135 |
| 0.9300 | 4.687 | 1.44146 | 0.90235 |
| 0.9250 | 5.804 | 1.17033 | 0.90585 |
| 0.9210 | 7.450 | 0.91572 | 0.91131 |
| 0.9194 | ∞ | 0.00000 | 0.91940 |

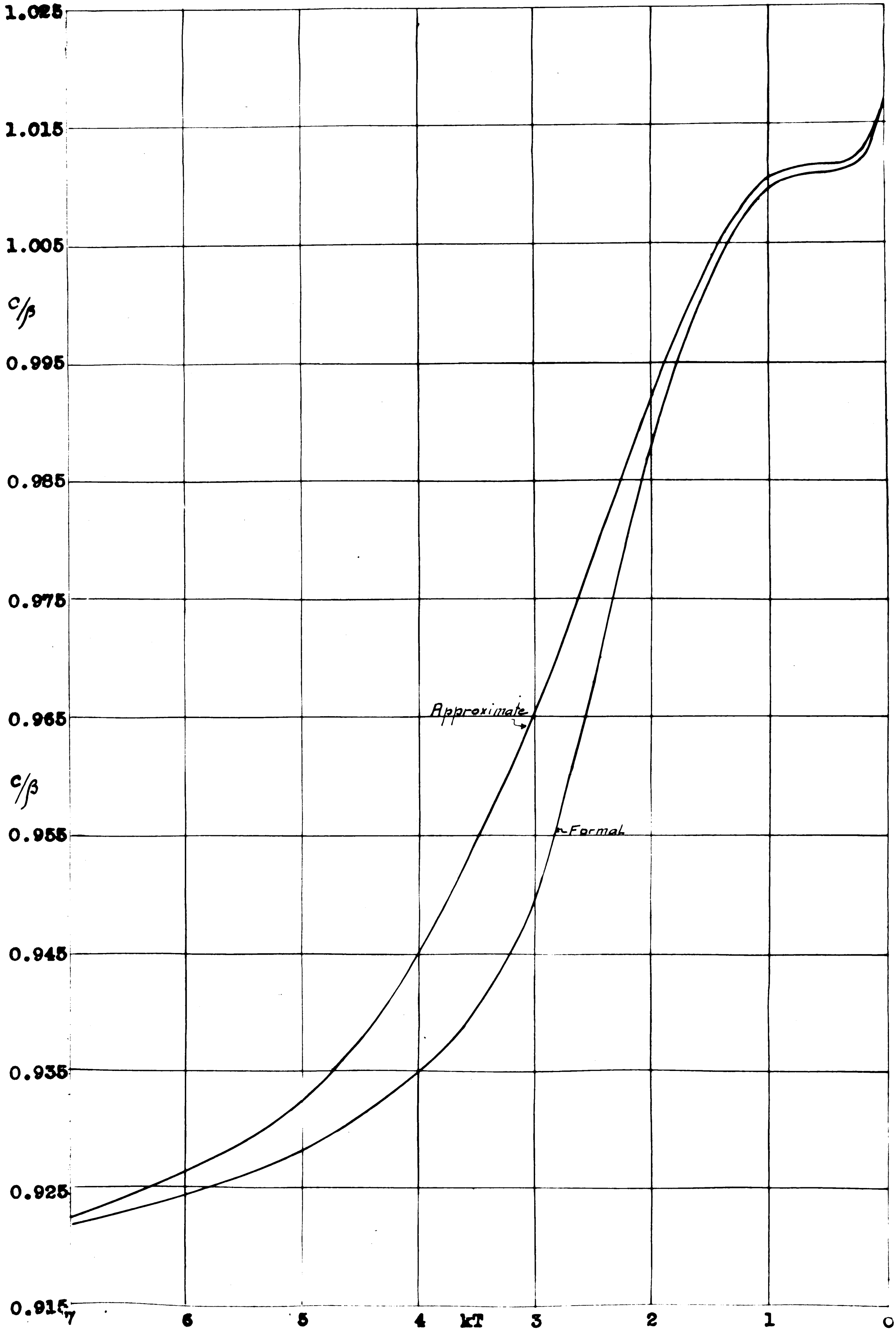


Fig. 1. Wave-Velocity as a Function of kT

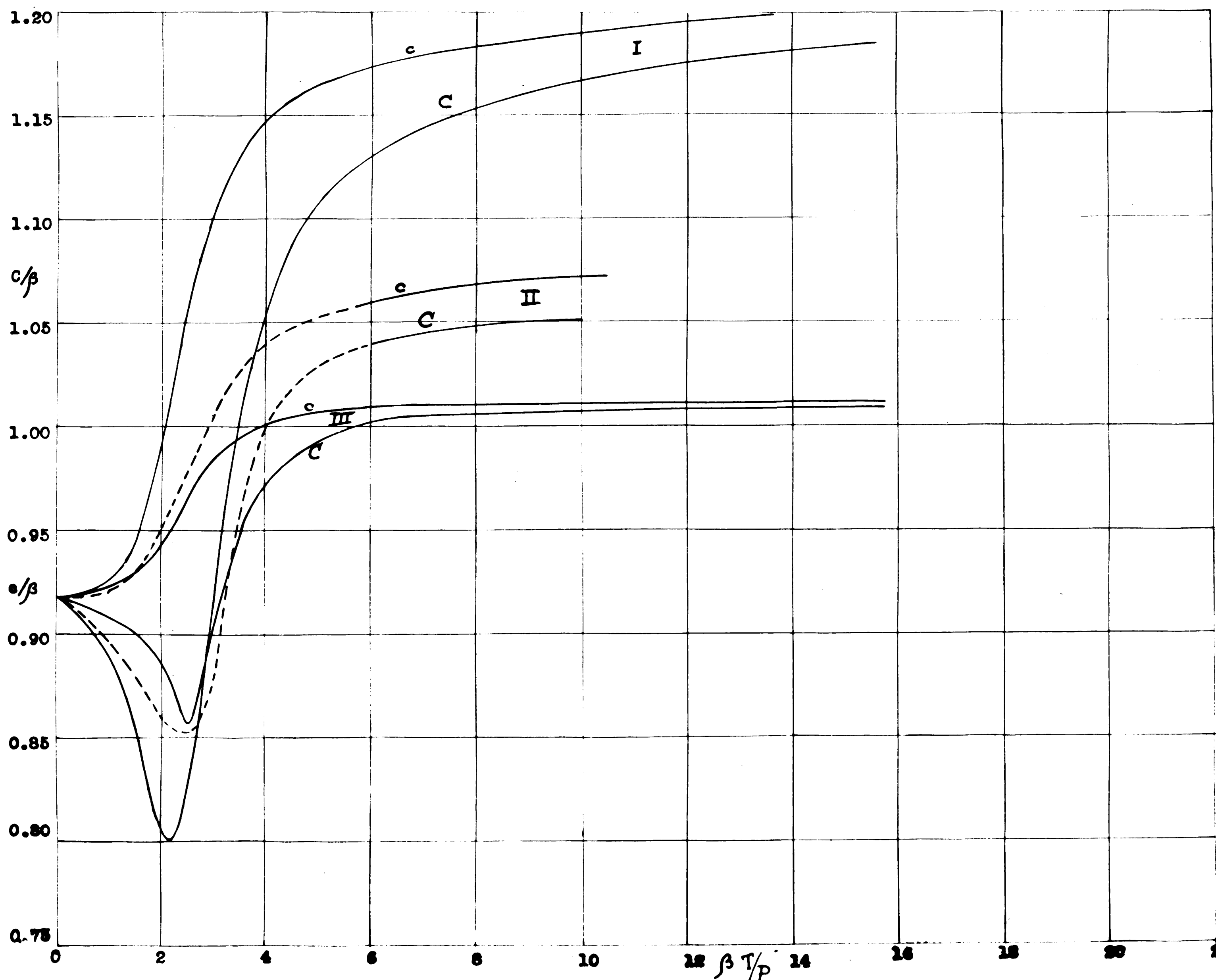


Fig. 2. Dispersion Curves

The group-velocity was calculated from wave-velocity using the well known relation

$$\frac{C}{\beta} = \frac{c}{\beta} + kT \frac{d\, c/\beta}{d\, kT}$$

where C is the group-velocity. The derivative, $d\, c/\beta \, / dkT$, was obtained graphically. The results are also given in table 2. In fig. 2, the dispersion curves for the three cases are plotted.

## Data

The earthquake from which the data were obtained occured Nov. 25, 1941. The location of the epicenter and the time of occurence was provided by Dr. J. T. Wilson.

Location $\varphi = 37°00'$ Time of occurence $0 = 18^h : 03^m ; 53^{sec.}$ U.T.
$\lambda = 19°00'$

The location of the epicenter was such that surface waves were received at a number of stations after having travelled a large portion of their paths across the Atlantic basin. Vertical component data from six stations were available - Weston, Fordham, St. Louis, Mt. Wilson and Huancayo. Most of the seismograms show a well developed group of Rayleigh waves. The seismograms of St. Louis and Weston are more irregular than those at the other stations.

In measuring the periods an arbitrary zero line was drawn (8). The times of crossing the zero line were carefully measured. The time of each crossing point was taken as the time of arrival for the following full period. This method offers the advantage of simplicity and elimination of any possible error in the reading of any single period.

By the beats of the waves and the amplitudes, the beginning of the Rayleigh waves were noted carefully. The readings were taken within the groups where there was no visible interference by any other type of waves. For the Weston record it was possible to compare the vertical and horizontal components of the disturbance. The resulting motion was a retrograde ellipse, having the same characteristics expected for a surface wave of Rayleigh type.

The seismograms of Weston, St. Louis and Mt. Wilson are reproduced in fig. 3. The group of Rayleigh waves for Which period readings were taken are marked AA.

The instruments used at different stations were for:

Berkeley - Small Wiechert Vertical
Weston - Short Period Benioff Vertical
Fordham - Short Period Benioff Vertical
Huancayo - Long Period Benioff Vertical
Mt. Wilson - Short Period Benioff Vertical
St. Louis - Sprengretten Short Period Vertical

The periods and the corresponding "mixed" group-velocities for the different stations are given in table 3. The velocities for each period were obtained by dividing the distance from epicenter to station by the difference between the time of arrival and the adopted time of occurence. This is in accordance with the theory of disper-

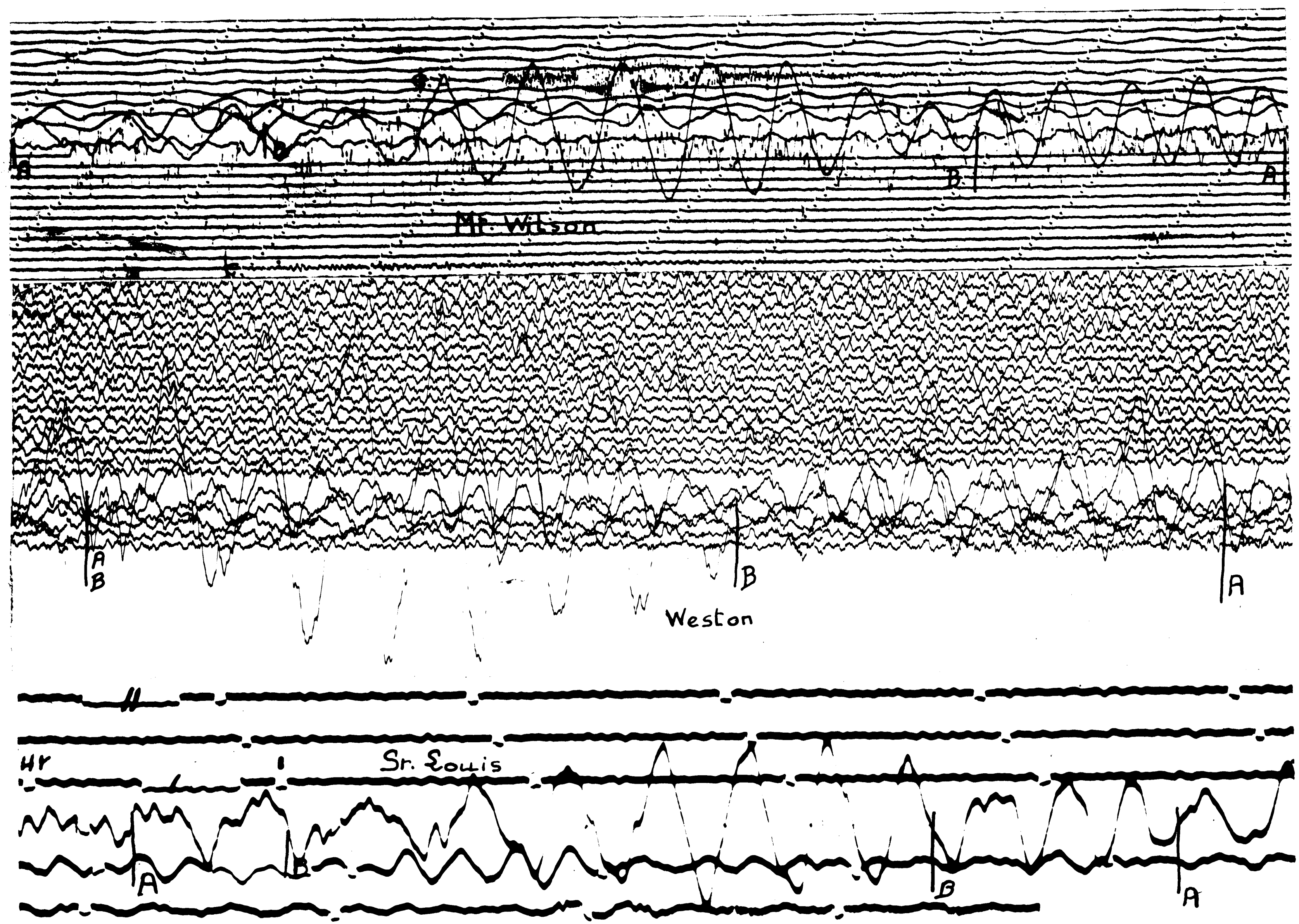


Fig. 3 - Seismograms

TABLE 3

"OBSERVED" DISPERSION DATA

| Berkeley | | Fordham | | Huancayo | | Mt. Wilson | | St. Louis | | Weston | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P | V | P | V | P | V | P | V | P | V | P | V |
| 22 | 3.36 | 23 | 3.68 | 25 | 3.61 | 24 | 3.41 | 22 | 3.67 | 26 | 3.73 |
| 21 | 3.34 | 22 | 3.66 | 25 | 3.58 | 25 | 3.40 | 23 | 3.65 | 23 | 3.69 |
| 22 | 3.32 | 21 | 3.63 | 23 | 3.56 | 24 | 3.38 | 26 | 3.62 | 22 | 3.65 |
| 21 | 3.31 | 20 | 3.60 | 20 | 3.54 | 24 | 3.36 | 28 | 3.59 | 21 | 3.62 |
| 21 | 3.29 | 21 | 3.57 | 22 | 3.52 | 24 | 3.34 | 27 | 3.57 | 22 | 3.58 |
| 21 | 3.27 | 21 | 3.54 | 24 | 3.50 | 26 | 3.33 | 22 | 3.55 | 21 | 3.55 |
| 20 | 3.26 | 20 | 3.52 | 22 | 3.48 | 24 | 3.32 | 23 | 3.52 | 20 | 3.52 |
| 19 | 3.25 | 19 | 3.49 | 21 | 3.47 | 22 | 3.30 | 22 | 3.50 | 20 | 3.49 |
| 19 | 3.24 | 19 | 3.47 | 22 | 3.45 | 22 | 3.29 | 21 | 3.47 | 19 | 3.47 |
| 18 | 3.23 | 19 | 3.44 | 22 | 3.43 | 21 | 3.27 | 21 | 3.44 | 18 | 3.44 |
| 18 | 3.22 | 19 | 3.41 | 22 | 3.42 | 21 | 3.26 | 21 | 3.42 | 20 | 3.42 |
| 18 | 3.21 | 18 | 3.39 | 20 | 3.41 | 21 | 3.25 | 21 | 3.41 | 20 | 3.40 |
| 16 | 3.20 | 18 | 3.37 | 19 | 3.39 | 20 | 3.23 | 21 | 3.39 | 20 | 3.37 |
| 14 | 3.19 | 18 | 3.35 | 20 | 3.37 | 21 | 3.22 | 19 | 3.37 | 19 | 3.34 |
| 15 | 3.18 | 18 | 3.33 | 19 | 3.36 | 20 | 3.21 | 17 | 3.36 | 19 | 3.32 |
| 16 | 3.17 | 18 | 3.31 | 19 | 3.35 | 18 | 3.19 | 19 | 3.35 | 20 | 3.30 |
| 16 | 3.16 | 19 | 3.29 | 20 | 3.34 | 18 | 3.18 | 20 | 3.33 | 18 | 3.28 |
| 16 | 3.15 | 20 | 3.26 | 20 | 3.33 | 18 | 3.17 | 21 | 3.30 | 21 | 3.25 |
| 15 | 3.14 | 18 | 3.24 | 18 | 3.31 | 16 | 3.16 | 20 | 3.28 | 20 | 3.23 |
| 16 | 3.13 | 17 | 3.22 | 15 | 3.29 | 15 | 3.15 | 17 | 3.26 | 17 | 3.20 |
| 17 | 3.12 | 17 | 3.20 | 17 | 3.28 | 14 | 3.14 | 18 | 3.25 | 18 | 3.18 |
| 17 | 3.11 | 17 | 3.18 | 19 | 3.37 | 14 | 3.14 | 14 | 3.23 | 18 | 3.15 |
| 17 | 3.10 | 17 | 3.16 | 20 | 3.26 | 16 | 3.13 | 14 | 3.22 | 16 | 3.13 |
| 17 | 3.09 | 18 | 3.14 | 20 | 3.24 | 16 | 3.12 | | | 15 | 3.11 |
| 15 | 3.09 | 17 | 3.12 | 18 | 3.23 | 14 | 3.11 | | | 16 | 3.09 |
| 13 | 3.08 | 16 | 3.11 | | | 17 | 3.10 | | | 17 | 3.07 |
| 13 | 3.07 | 16 | 3.09 | | | 15 | 3.09 | | | 17 | 3.05 |
| 13 | 3.06 | 17 | 3.07 | | | 15 | 3.08 | | | 16 | 3.04 |
| 14 | 3.05 | 17 | 3.05 | | | | | | | 16 | 3.03 |
| 15 | 3.04 | 16 | 3.04 | | | | | | | 15 | 3.01 |
| 16 | 3.03 | 17 | 3.03 | | | | | | | 14 | 3.00 |
| | | 16 | 3.01 | | | | | | | | |
| | | 15 | 3.00 | | | | | | | | |
| | | 16 | 2.98 | | | | | | | | |
| | | 16 | 2.96 | | | | | | | | |
| | | 17 | 2.95 | | | | | | | | |

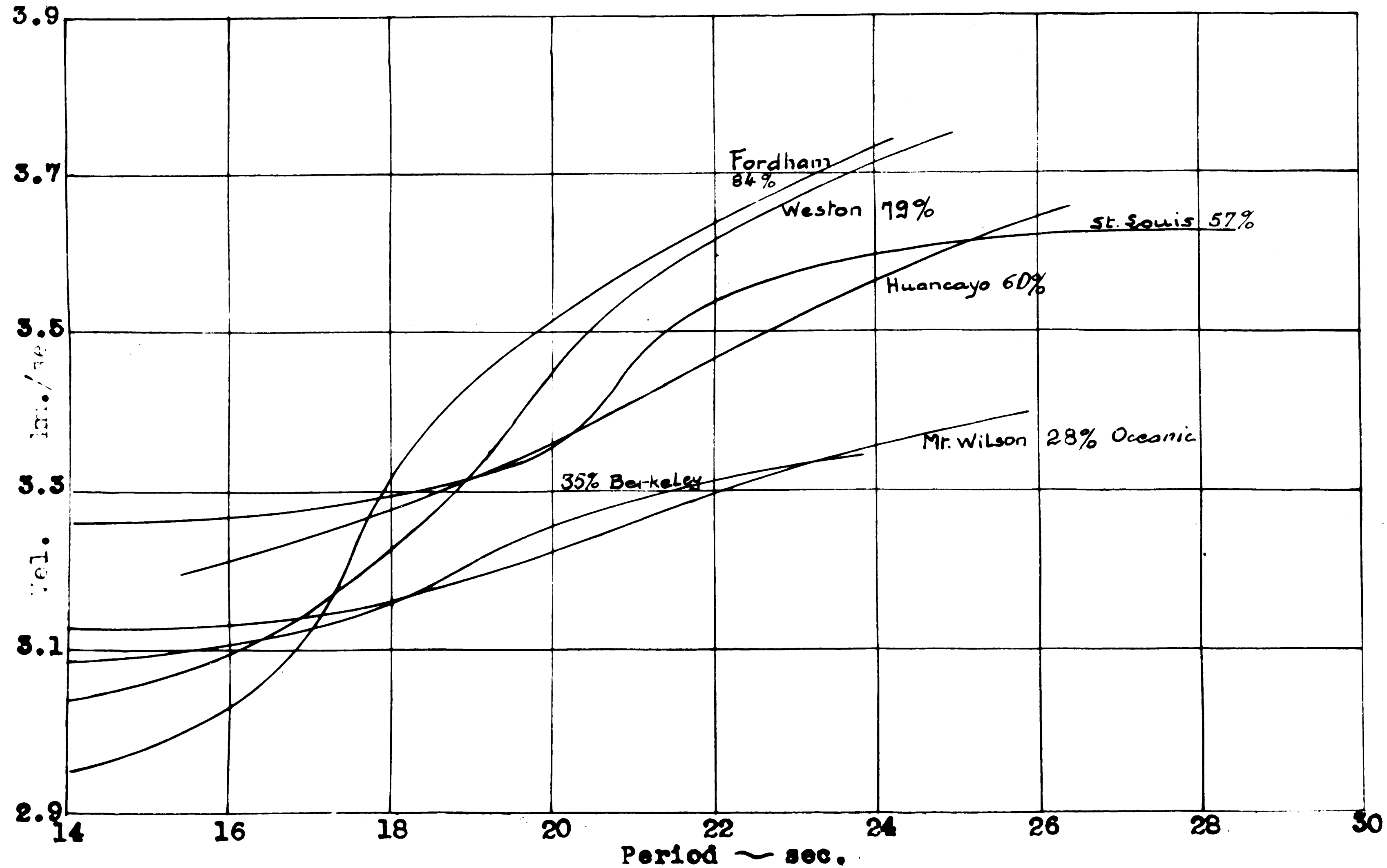


Fig. 4. "Observed" Dispersion Curves

sion (9), where considering a zone of disturbance, the waves spreading from this zone are recorded at distant stations as a train of waves of gradually changing periods; each period travelling with the appropriate group-velocity and arriving at a time corresponding to that group-velocity.

The dispersion curves for each station, obtained from table 3, are given in figure 4.

## The Effect of Refraction

In accordence with the ordinary theory of wave propagation, an incident wave at the interface of two mediums of different densities and elastic constants will be reflected and refracted. Surface waves crossing the continental shelf are subject to such refraction in a horizontal plane due to the variation in thickness and composition of the rocks forming the continental and oceanic basements. At the continental edge a certain amount of energy is lost, but it is assumed that a large portion of this energy is converted into refracted waves of the same type as the incident wave. As a result of refraction, the path followed by the wave will not be the geodesic track, as it is assumed to be for an unrefracted wave.

The angle of incidence and refration follows Snell's law

$$c_1 / c_2 = \sin \theta_1 / \sin \theta_2$$

where $c_2$ and $c_1$ are respectively the continental and oceanic wave-velocities; $\theta_1$ angle of incidence; $\theta_2$ angle of refraction. The waves are refracted according to wave-velocities, whereas they travel with the group-velocities (10). The time of transit of the waves across the interface is considered to be stationary.

Stoneley (11) has studied the effect of refraction at the continental margin, but no examples involving actual data are given.

The continental shelf was assumed to be at the 5,000 ft. subsea contour. The coast line was taken as a straight line in the region of the crossing. The geodesic path from station to epicenter was then located. In fig. 5, the geodesic path for the six stations and the angle of this path with respect to the continental edge for four stations, Weston, St. Louis, Mt. Wilson and Huancayo, are given. Five or six points were computed on each side of the coastal line to locate the wave path which was then plotted on a large scale map and the angle at the crossing read directly. The distance from epicenter to the crossing was computed and the percentage of oceanic to continental path was thus obtained.

Two sets of equation are involved in computing the effects of refraction: The time equation, $t = r_1 / C_1 + r_2 / C_2$, where $r_1$ and $r_2$ represents the oceanic and continental paths of the refracted wave, $C_1$ and $C_2$ the group-velocities; and the previously stated Snell's law. The subscripts 1 and 2 refer respectively to ocean and continent.

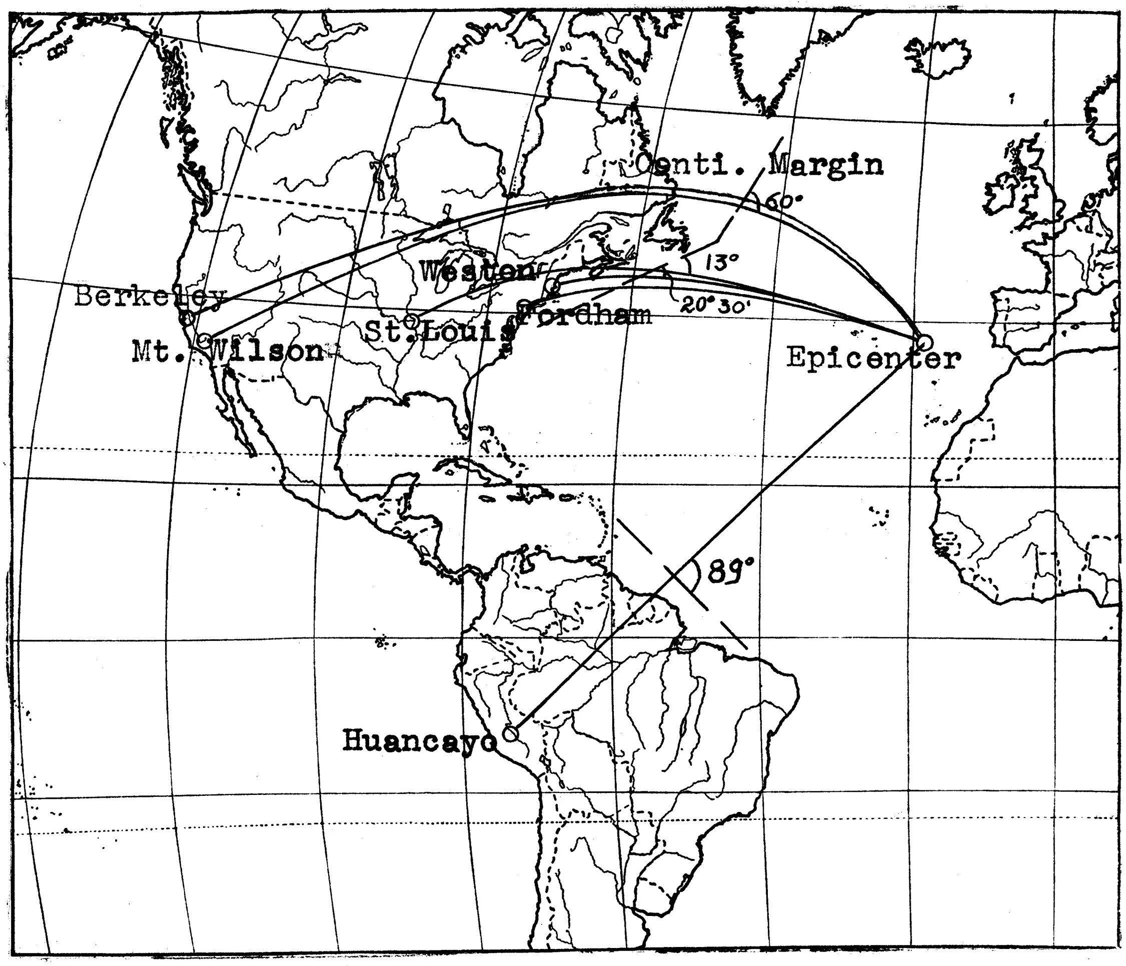


Fig. 5. Geodesic Paths to the Stations

To compute the effect of refraction the values of $c_1$ $C_1$ and $c_2$ $C_2$, were taken respectively from curves 1 and 111 (fig. 2) and the values for T were assumed. For the continent $T_2 = 20$ km., for the ocean $T_1 = 15$ km. Results are given in tab. 4.

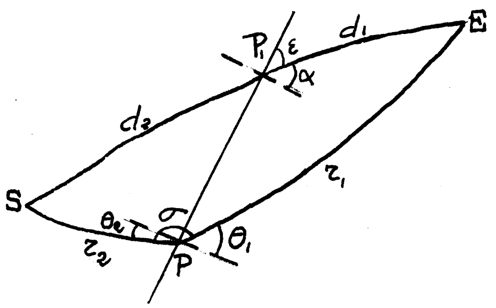


Fig. 6.

Considering fig. 6, the spherical triangle SPE gives; $\sigma = 180 - (\theta_1 - \theta_2)$, and

$$\cos D = \cos r \cos r - \sin r \sin r \cos(\theta_1 - \theta_2) \qquad (1)$$

In triangle $EP_1P$ and $SP_1P$:

$$\sin r_1 = \sin d_1 \cos\alpha / \cos\theta_1 \qquad (2)$$

$$\sin r_2 = \sin d_2 \cos\alpha / \cos\theta_2 \qquad (3)$$

In solving the equations, first a value of $\theta_1$ is assumed and the corresponding value of $\theta_2$ computed from Snell's equation. From equation (2) and (3) the values of $r_1$ and $r_2$ are found and substituted in (1). The solution is by trial and error. Since the Value of D is known, the process is repeated untill equation (1) is solved. Those values

TABLE 4

| P | $C/\beta$ | $C/\beta$ | $C/\beta$ | $C/\beta$ | $c_2/c_1$ |
|---|---|---|---|---|---|
| 25 | 4.040 | 4.024 | 3.802 | 3.515 | 0.9409 |
| 20 | 4.032 | 3.984 | 3.713 | 3.218 | 0.9207 |
| 15 | 4.004 | 3.888 | 3.468 | 2.706 | 0.8662 |
| 10 | 3.896 | 3.468 | 3.185 | 2.772 | 0.8173 |
| 5 | 3.696 | 3.624 | 3.049 | 2.987 | 0.8254 |
| 0 | 3.678 | 3.678 | 3.034 | 3.034 | 0.8249 |

TABLE 5

Effect Of Refraction

| P | St. Louis | | | Weston | | | Mt. Wilson | | |
|---|---|---|---|---|---|---|---|---|---|
| | 56.72% oceanic | | $\varepsilon = 13^\circ$ | 79.40% oceanic | | $\varepsilon = 20^\circ 30'$ | 27.80% oceanic | | $\varepsilon = 60^\circ$ |
| P | V | Vav. | % | V | Vav. | % | V | Vav. | % |
| 25 | 3.89 | 3.80 | 2.3 | 3.93 | 3.91 | 0.5 | 3.65 | 3.64 | 0.3 |
| 20 | 3.81 | 3.65 | 4.2 | 3.85 | 3.80 | 1.3 | 3.40 | 3.40 | 0.0 |
| 15 | 3.60 | 3.38 | 6.1 | 3.69 | 3.57 | 3.3 | 2.97 | 2.96 | 0.3 |
| 10 | 3.37 | 3.17 | 5.9 | 3.37 | 3.30 | 2.1 | 2.95 | 2.94 | 0.3 |
| 5 | 3.50 | 3.35 | 4.3 | 3.52 | 3.47 | 1.4 | 3.15 | 3.14 | 0.3 |
| 0 | 3.56 | 3.40 | 4.5 | 3.57 | 3.52 | 1.4 | 3.20 | 3.19 | 0.3 |

of $r_1$ and $r_2$ that solve equation (1) are then used in the time equation to obtain t.

The computation was carried for three stations: Weston, St.-Louis and Mt. Wilson. For these stations, the problem was also solved considering the earth flat. Between the two considerations, the divergence in time and velocity was found to be less than 1 per cent.

According to the previously mentioned theory of dispersion, the velocity for the corresponding period, for a value of t, was obtained by dividing the total distance by t. The results are given in table 5; where V is the velocity obtained by taking the refraction into account, Vav. is the velocity of the waves, with same period, but assumed to be fallowing the geodesic path.

From a study of table 5 and fig.5, the effect of refraction is seen to depend on two factors.

1- The ratio of oceanic to continental path

2- The angle at which the path crosses the continental edge.

For the first factor it is fairly obvious that if the continental portion of the total path is negligible, the effect of refraction will likewise be negligible. Approximately 10% continental path seems to be necessary before the refraction shows appreciable effect.

The second factor is actually the more important. The angles of crossing of Weston, St. Louis and Mt. Wilson are respectively 20°30′, 13° and 60°, For Mt. Wilson, where the angle is 60°, the effect is negligible. For such angle the refraction is expected to be small, since the angle of incidence is approching the normal. This result agrees with Stoneley's (12). In his case, the angle of incidence was taken 55°49′, the resulting effect found to be less than 1%. But for Weston and St. Louis, the effect of refraction is as high as 6%, which is a fairly serious divergence. For both paths the angle of crossing is small and the amount of refraction relatively large. Thus the effects of refraction are particularly important when the angle of crossing is small and the continental path an appreciable portion of the total path.

Of couse, the values of the "mixed" group-velocities depend on the assumptions of $c_1 C_1$, $c_2 C_2$, $T_1$ and $T_2$. A reasonable change in these values, would not alter the result appreciably.

## Dispersion Curves and Structures

In observing fig.4, it can be seen that for periods of 20 sec. or over, the dispersion curves for the different stations present the same characteristics. They fall in logical order with respect to period, "mixed" velocity and oceanic path percentage. Since $C_1 > C_2$, the station with a higher oceanic path will have a higher "mixed" group-velocities. But for periods less than 20 sec., the velocities for Fordham and Weston are seen to fall below those for St. Louis.

The numbers of period received at any station should be approximately proportional to the epicentral distance. However, examination of table 4 shows that even more periods were read at some of the nearer stations than at those more distant. It is possible that periods beyond the Rayleigh wave group have been read.

As an example the Mt. Wilson seismogram and the corresponding table and graph will be considered. The first period of the Rayleigh waves was considered to start at $10^h$ 15' 34". The waves started with a period of 24". Howewer, at 46' 33", the period is found to be 26". From 46' 33" to 49'07", the periods are decreasing. At 49' 07", the period is 14". From this point, although the appearance suggest Rayleigh waves, the period increases and then decreases. Therefore, starting from the first wave, the periods passes by a maximum, decreases, goes through a minimum, increases and decreases again. The obvious conclusion to such behaviour is the interference of some other types of waves with the Rayleigh waves. The amplitudes were not measured, but in appearance they yield the same conclusion.

The revision of the number of full periods considered to be "R" waves was based on the arguments cited above. The subsequently drawn curves are shown in fig. 7. Here the dispersion curves do not cross and the number of periods considered to be R waves at each station are approximately proportional to their epicentral distances. Except for some expected minor irregularities the periods have a steadily decreasing values. In fig. 3, AA represents the former group, BB the revised group.

From the dispersion curves of fig.7, the intrinsic value of $C_1$ and $C_2$ was obtained graphically. This graphical process is shown in fig. 8. The straight lines for each period are drawn on the assumption that for a given period the group-velocity is a linera function of the oceanic or contbinental path. The value of $C_1$ and $C_2$ thus obtained are compiled in table 6.

Using the refraction data, values of $C_1$ and $C_2$ were also computed algebrically for several station pairs. Four sets of stations were used: Mt. Wilson-St. Louis, Mt. Wilson-Weston, St. Louis-Weston, and Mt. Wilson-Huancayo. The "observed" velocities were taken from fig. 4. The value of $C_1$ and $C_2$ obtained for each group, for a given period, were averaged. For periods of 20 sec. or larger, the values of $C_1$ and $C_2$ for a corresponding period are found to be in close agreement with the figures given in table 6. For periods less than 20 sec., large discrepancies were found, particularly when the data from Weston were used. This of course can now be explained easily since the curves of fig. 4, had to be redrawn thus changing the values of the "observed" velocities. The change were not pronounced for periods larger than 20 sec., but was appreciable in case of Weston and Fordham for P less than 20 sec.

## Oceanic Basin Structure

For comparison with the data three theoretical curves were used. They are dispersion curves 1, 11, 111 of fig. 2.

From the study of "P" waves, for the lowermost layer $\beta'$ is found to be between 4.4 and 4.55 km/sec. In curve 111, $\beta$ was taken as 4.0 and $\beta'$ 4.5 km/sec. Such combination of $\beta$ and $\beta'$ is considered to fit closely the oceanic structural combination with basic material resting on ultrabasic material. Since $C_1$ pertains to oceanic group-

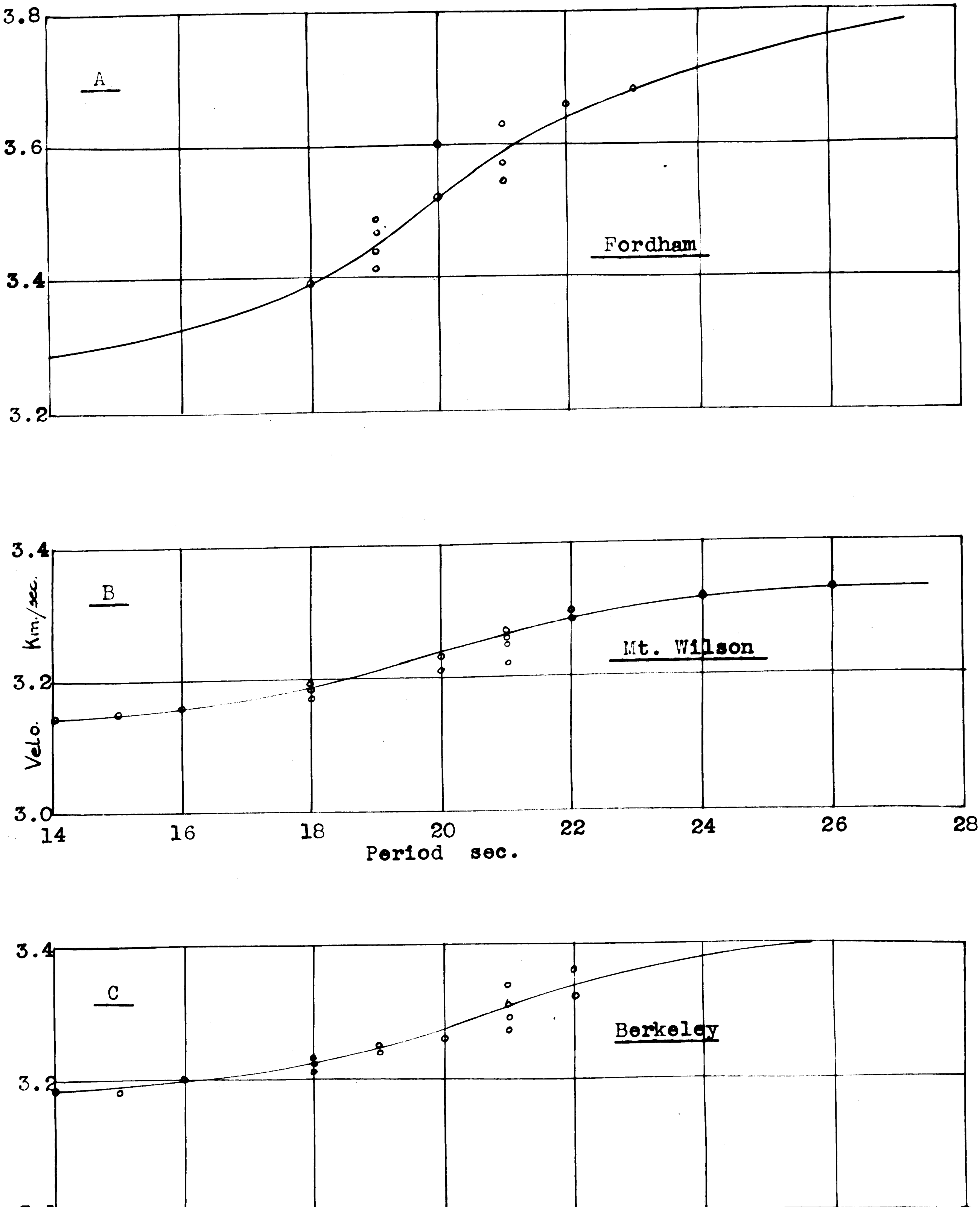


Fig. 7. "Observed" Dispersion Curves

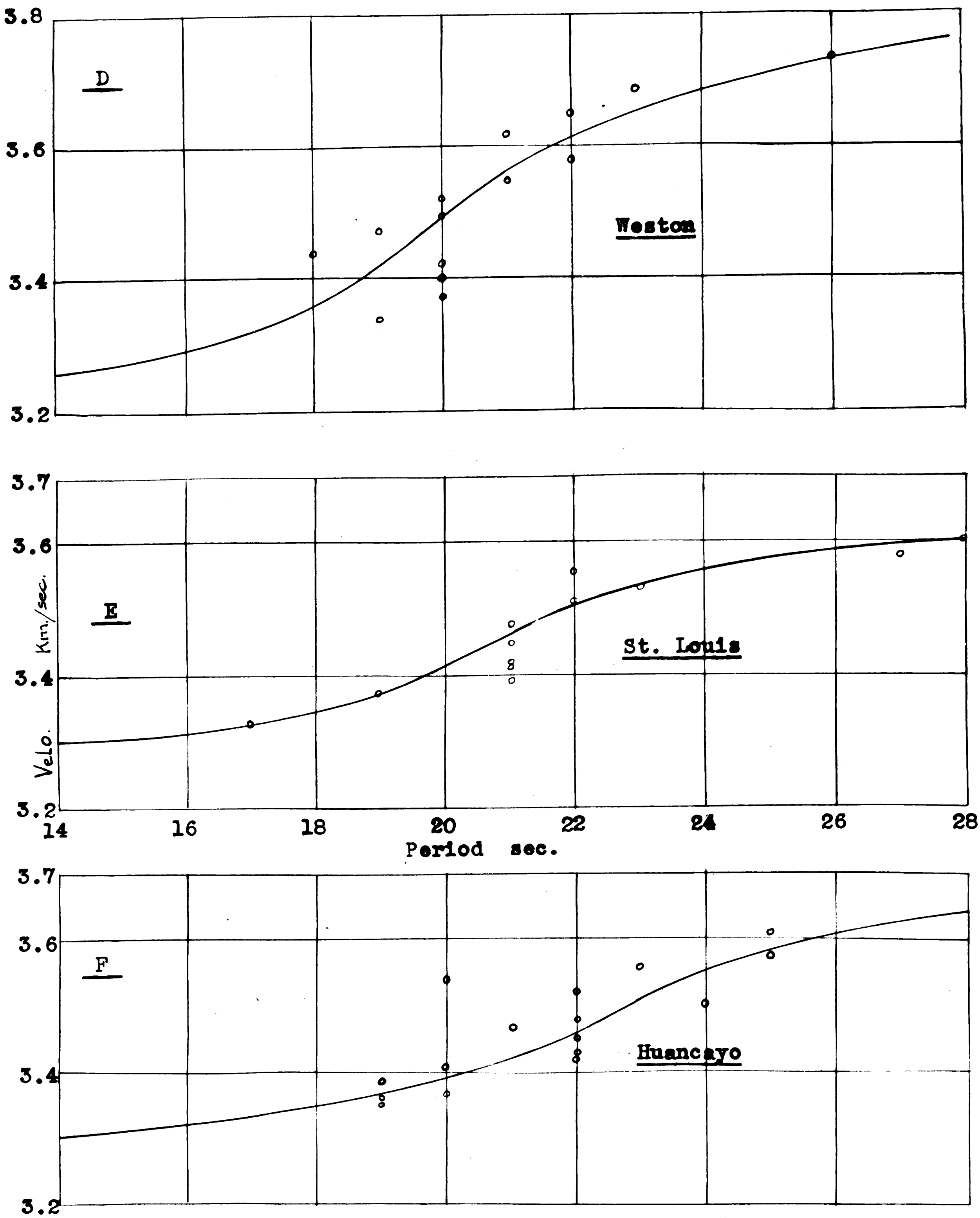


Fig. 7. (Continued)

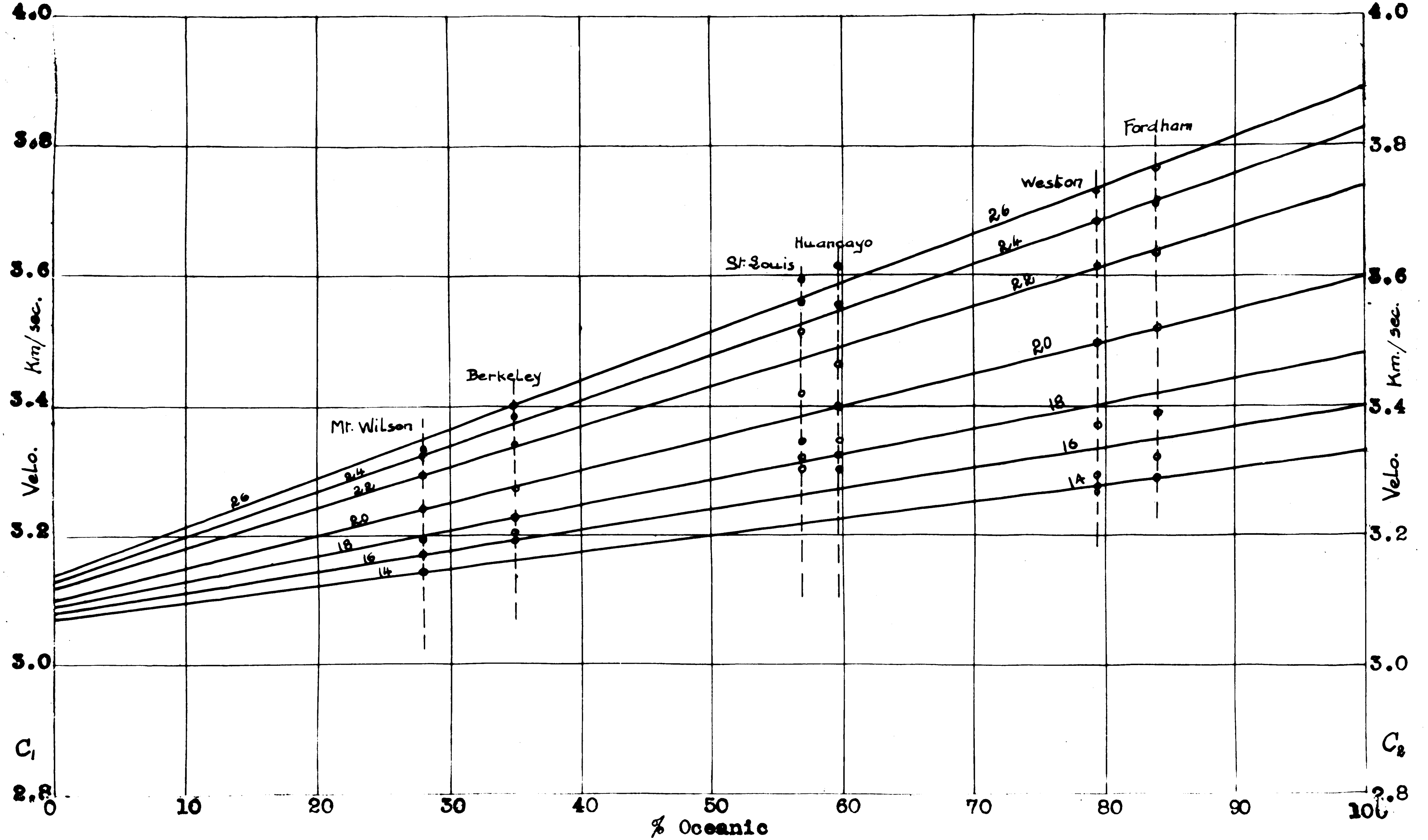


Fig. 8. Graphical Solution of $C_1$ and $C_2$

TABLE 6

"Observed" Goup-Velocities

| P | $C_1$ | $C_2$ |
|---|---|---|
| 14 | 3.33 | 3.07 |
| 16 | 3.40 | 3.08 |
| 18 | 3.48 | 3.09 |
| 20 | 3.60 | 3.10 |
| 22 | 3.74 | 3.12 |
| 24 | 3.83 | 3.13 |
| 26 | 3.89 | 3.14 |

TABLE 7

Theoretical Group-Velocities

| P | $\beta$=4.00 T=26 km. $C_1$ | $\beta$=4.06 T = 29 km. $C_1$ | $\beta$=4.11 T = 30 km. $C_1$ |
|---|---|---|---|
| 10 | 3.61 | ---- | 3.73 |
| 12 | 3.58 | ---- | 3.70 |
| 14 | 3.53 | 3.61 | 3.67 |
| 16 | 3.43 | 3.55 | 3.62 |
| 18 | 3.48 | 3.49 | 3.53 |
| 20 | 3.62 | 3.56 | 3.58 |
| 22 | 3.74 | 3.67 | 3.70 |
| 24 | 3.84 | 3.80 | 3.85 |
| 26 | 3.89 | 3.89 | 3.93 |
| 28 | 3.92 | ---- | 3.98 |
| 30 | 3.95 | ---- | 4.02 |

velocity, curve 111, was used to match the oceanic data given in table 6.

Computations based on curve 111, were carried out for $\beta = 4.00$, 4.06 and 4.11 km/sec. The corresponding thickness is found by trial and error. The result are given in table 7. For $\beta = 4.0$ km/sec. and $T = 26$ km., the values found for $C_1$ agrees very closely to those given in table 6. In fig. 9, this agreement is shown graphically.

For periods over 18 sec., the fit is very good. For periods less than 18 sec., the "observed" velocities fall below the theoretical velocities. Since for shorter periods the waves are influenced more by material near the surface, the discrepancy suggests a very thin layer on top.

Consequently, the matching of the theoretical dispersion curve with the data gives a layer thickness in the order of 26 km., with a probable thin layer of lower velocity material above. Gutenberg (13), finds a 25 km. thick layer with $\beta = 3.3$ km/sec. Under the present assumption of $\mu'/\mu = 10/7$ and $\rho'/\rho = 7/6$, the thickness of the surface structure is thought to be about 26 to 30 km.

## Continental Structure

Data secured from local earthquake (14) gives for the continental structure $\beta = 4.5$ km/sec. for the lowermost layer, $\beta = 3.6$ km/sec. for the intermediate layer, $\beta = 3.3$ km/sec. for the upper one. The sedimentary layer has here been omitted.

For periods greater than 20 sec., it was assumed that the waves are influenced mainly by the lower section where $\beta' = 4.5$ and $\beta = 3.6$ km/sec. This fits the case of a two layer problem, where the bottom layer is "Dunitic" and the upper "Basaltic". Sezawa's curve, curve 11 in fig. 2, computed for $\beta' = 4.5$ and $\beta = 3.6$ km/sec. was used in the matching of the continental data.

A value of T was found as previously by trial and error where the theoretical and observed $C_2$ has been made to agree. The results are given in table 8. For $T = 31$ km. and for periods more than 18", there is a close agreement. The fit is shown graphically in fig. 9. For periods less than 18 sec., the observed velocities fall below the theoretical velocities. In this case, it is obvious that the "granitic" layer causes the drop in velocity for the shorter periods. For periods of about 18 sec., the effect of the upper layer with

Table 8 - for $T = 31$ km.

| P | $C_2$ |
|---|---|
| 14 | 3.12 |
| 18 | 3.08 |
| 20 | 3.09 |
| 22 | 3.10 |
| 24 | 3.13 |
| 26 | 3.14 |

$\beta = 3.3$ km/sec. becomes apparent.

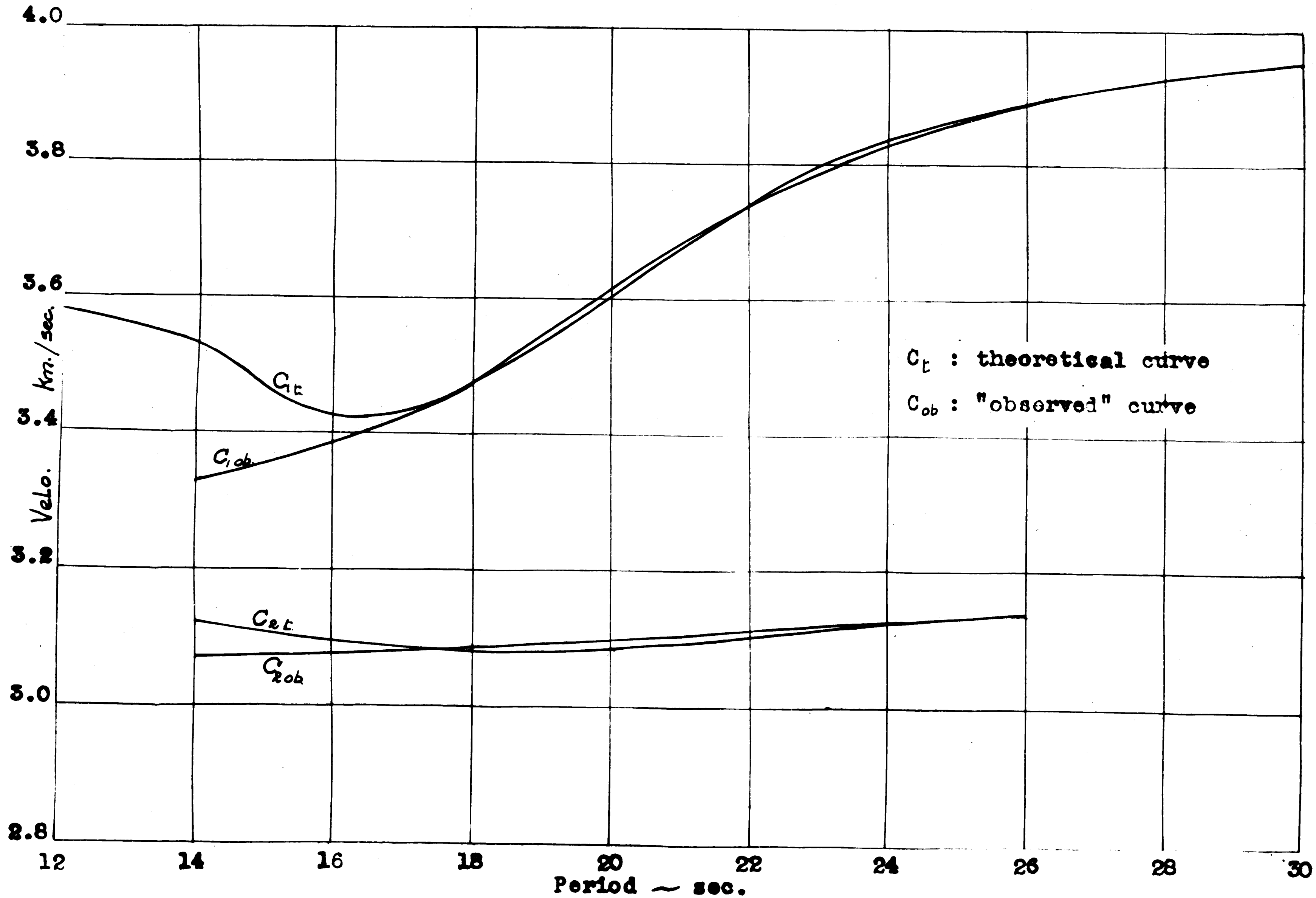


Fig. 9. Theoretical and "Observed" Dispersion Curves

The thickness of the "granitic" layer has not been computed. In order to form a true picture of this structure, observation of shorter periods is necessary. The smallest period obtained from the records was only 14 sec. Moreover, the theoretical curves have been computed on the basis of two layers, for periods of about 18 sec., the problem actually involves three layers.

Wilson (15), from the study of the dispersion of Love waves, finds about 30 to 40 km. to be the thickness of the continental surface structure. Taking into account the 31 km. found in this study for the "basaltic" layer this leaves a thickness of 9 to 10 km. for the "granitic" layer, which is a logical value for that section. Gutenberg (16) gives a total thickness of about 45 km. for Eurasia.

## Summary

Three theoretical curves were drawn to match the Rayleigh wave dispersion data obtained from an earthquake that occured in the Atlantic Ocean on November 25, 1941.

The effects of refraction on velocity and path were discussed. The effect of refraction is found to depend on the angle at which the path crosses the continental margin and on the ratio of oceanic to continental path. It is found that when the angle of crossing is small and the continental path is a significant part of the total track, the effect of refraction should be taken into account.

The matching of the data with theoretical curves yielded the following conclusions as to probable crustal structure. Under the Atlantic Ocean there is probably a layer of 26 km. thick basic material overlying ultrabasic rocks, thin layer of lower velocity material probably lies on top. The thickness of the whole section is indicated to be about 30 km. For the continent, the thickness of the intermediate layer is found to be of the order of 30 km. Considering the total thickness of about 40 km. found by Wilson for the continent, the thickness of the "granitic" layer is believed to be about 10 km.

The writer wishes to express his sincere appreciation and thanks to Dr. J. T. Wilson, of the University of Michigan, for the inestimable advice he has given and under whose direction this study was made.

## References


1- R. Stoneley, "A Rayleigh Wave Problem," Leeds Philo. and Litt. SO., Scientific section Vol.1, part Vl,1928, pg.219.

2- A. W. Lee, "The Effect of Geological Structure upon Microseismic Disturbance," Roy. Astron. So., Geophys. Suppl., Vol. 3, 1932, pg. 90.

3- K. Sezawa, "Rayleigh and Love waves transmitted through the Pacific Ocean and the Continents," Tokyo Imp. Uni., Earthquake Resear, Insti., Vol. 13, part 2, June 1935, pg.245.

4- H. Jeffreys, "The surface waves of Earthquake," Roy. Astron. So., Geophys. Suppl., Vol.3, 1935, pg. 253-261.

5- Op. Cit. 4

6- Op. Cit. 4

7- Op. Cit. 2

8- J. T. Wilson," The Love waves of the South Atlantic Earthquake of August 28, 1933," Bull. Seismo. So. of Ame., Vol. 30, No . 3, July 1940.

9- T. H. Havelock, Camb. Tract.,No. 17.

10- R. Stoneley, "Dispersion of Seismic Waves," Roy. Astron. So., Geophys. Suppl., Vol. 1,1925, pg. 280
H. Jeffreys, "Surface waves of Earthquakes," Roy. Astron. So., Geophys. Suppl., Vol 1,1925, pg. 282.

11- R..Stoneley, "Earthquakes waves over the surface of the Earth" Roy. Astron. So., Geophys. Suppl., Vol. 3, 1935, pg. 262

12- Op. Cit. 11

13- B. Gutenberg, "Dispersion und Extinktion von seismishen Oberflächenwellen und der Aufbau der obersten Erdschichten," Physical. Zeitschr., 25:377-381 (1924)

14- "Handbook of Physical Constants," Geolo. So. of Ameri. No. 36, Jan. 31, 1942, sect. 7, pg. 93-106

15- Op. Cit. 8

16- Op. Cit. 13