Essays on Investment Dynamics under Market Imperfections

by

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Dedicated with love to my parents.

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CHAPTER I

Do Intangible Assets Aggravate Financial Market Imperfections?

1.1 Introduction

External financing is crucial for the creation of new firms and the expansion of existing ones.[1] For this reason, financial frictions have important consequences for firm dynamics. One major obstacle to external financing is the enforceability of a financial contract. When a firm has the ability to repudiate a contract, optimal contractual arrangements may set endogenous borrowing limits based on how much the firm can credibly repay.

Financial frictions, however, do not affect all firms equally. The financing of investment in intangible assets, such as research and development, employee training, and expenditures on marketing and strategy consultants, is more susceptible to financial frictions than investment in physical assets because of the inalienable and firm-specific nature of intangible assets. This paper presents evidence that the sensitivity of firm size, growth, and market value on financial frictions is determined by the firm's intangible assets. My theoretical model offers detailed predictions on the dynamics of firms with heterogeneous technologies and assets, for which I find robust empirical support from data on U.S. public firms.

[1] See, for example, Meyer and Kuh (1957), Fazzari et al. (1988), and Stein (2003).

Empirically, financial frictions associated with intangible assets have short-run and long-run consequences on firm dynamics, which I find nontrivial in magnitude. I construct new measures of firm-level intangible and physical assets using accounting information on U.S. public firms from Compustat data. I find that the share of intangible assets in total assets explains 44% of the variance in the size of physical assets of entrant firms, 19% of the variance in the size of their total assets, 7% of the variance in sales, 10% of the variance in employment, and 17% of the variance in firm growth. The predictive power of the share of intangible assets on firm size remains strong for firms up to 40 years old; its predictive power on firm growth remains strong for firms up to 10 years old. In addition to size and growth variations, the market value per unit of assets is higher among entrant firms with a higher share of intangible assets. As firms age, the share of intangible assets eventually ceases to matter: Firm size, growth and the market value per unit of assets more or less converge among older firms. These patterns are illustrated in Figure 1.2.[2] It is worth noting that while the market value per unit of assets falls and converges to a constant level among firms up to 30 years old (see the lower left panel of Figure 1.2), the market value per unit of physical assets (i.e., the conventional Tobin's q) does not converge: Firms with a higher share of intangible assets on average have permanently higher Tobin's q even among mature firms (see the lower right panel of Figure 1.2).

This paper provides a simple yet comprehensive model that explains these salient features of the data. I incorporate heterogeneous firms and dynamic investment decisions on physical and intangible assets into a model of limited contract enforcement based on Albuquerque and Hopenhayn (2004) and Cooley et al. (2004). Three

[2]The sample of firms is from the annual Standard and Poor's Compustat industrial file and the Center for Research in Security Prices (CRSP) monthly stock file. See Section 1.3 for detailed descriptions of the data, variable measurement and results.

assumptions underpin the model: (i) Young firms have insufficient internal funds and are partly financed by external debt. (ii) Financial frictions arise because debt contracts have limited enforceability and a repudiation-free contract sets an endogenous borrowing limit. (iii) Physical and intangible assets affect the borrowing limit asymmetrically because intangible assets have higher residual value for the firm if the debt contract is repudiated.

The inalienable and firm-specific features of intangible assets motivate the last two assumptions. Although a creditor may seize physical assets when the firm repudiates the contract, intangible assets tend to be inalienable from the owner of the firm, whether they are managerial skills developed by running the firm or patents in the name of the owner. Intangible assets are often specific to the firm, having large value inside the firm but little value to creditors. This feature of intangible assets also motivates the use of state-contingent debt contract, which distinguishes my model from models with collateral constraints.[3] Although collateralized debt is the typical form of financing for certain firms, for firms that are intensive in intangible assets it is usually not available because of the specificity of their assets. Outside investors have to rely on direct contractual incentives.

My model offers an explanation for Figure 1.2 as follows. Young firms are unable to internally finance start-up costs and large investment expenditure on long-lived assets. An external financing contract sets an endogenous borrowing limit based on how much the firm can credibly repay. Young firms are constrained to be small and assets earn a high rate of return, which is reflected in a high market value per unit of assets. As firms accumulate more assets and pay off debt gradually, financial constraints are relaxed over time. In the model, heterogeneous financial

[3] See, for example, Kiyotaki and Moore (1997).

frictions arise because firms have technologies that differ in the intensity of intangible assets. As a result, the share of intangible assets strongly correlates with the size and growth dynamics of young firms, but it eventually ceases to matter as firms become financially unconstrained.

In the model, financial frictions lead to three sources of inefficiencies: First, they discourage the creation of firms whose technology relies heavily on intangible assets. Second, firms, particularly those that are intensive in intangible assets, are constrained to be inefficiently small for a prolonged period of time. Third, financial frictions distort investment away from intangible assets and leads to misallocation between physical and intangible assets.

Quantitatively, I establish a new strategy to identify structural parameters from firm-level panel data, using the theoretical relationship between financial frictions and dynamic investment paths of heterogeneous firms, which allows me to quantitatively evaluate financial frictions associated with intangible and physical assets. This is not possible with previous models with homogeneous firms and assets because they cannot separate the effect of limited contract enforcement from that of financial leverage.

My model also informs empirical studies of firm growth and investment, in particular the so-called investment regression. The model predicts that among young firms, in addition to Tobin's q and cash flow, the share of intangible assets will correlate with firm growth. Empirically, I verify that the share of intangible assets is a significant variable conditional on Tobin's q and cash flow. This result contrasts with predictions of the q theory of investment in which Tobin's q is a sufficient statistic for investment.[4] It also suggests that empirical investment models using cash flow

[4]See, among others, Summers (1981), Hayashi (1982), Hayashi and Inoue (1991).

as a proxy of financial frictions should be augmented to include the share of intangible assets.[5] Whereas the variation in cash flow mostly captures the availability of funds due to profit shocks, the variation in the share of intangible assets captures the ability to borrow.

This paper relates in a broad sense to the literature on firm dynamics. Empirically, a large number of studies find that firm growth is dependent on firm age and size.[6] Various studies provide explanations to these empirical regularities, such as learning or persistent shocks to the production technology.[7] These models without financial frictions are successful in explaining unconditional growth characteristics, but they are unable to explain the dependence on age and size simultaneously.[8] A separate but related literature relates growth to the average market value of capital (i.e., Tobin's q). These models, conventionally called the q theory of investment models, can explain investment rates and firm growth in terms of adjustment costs. But they are insufficient to match all of the facts in Figure 1.2.[9] This paper's empirical findings shed light on an important yet lesser-known aspect of firm dynamics. I show that intangible assets, when interacting with financial frictions, can generate substantial heterogeneity among entrants, as well as convergence inertia among young firms. Moreover, the share of intangible assets is an important factor in firm dynamics, in addition to age and size.

My work also contributes to the literature on firm dynamics and financial frictions – empirically, theoretically and quantitatively – by studying the direct implications of intangible assets on firm dynamics in the presence of financial friction.[10] I in-

[5]See, among others, Fazzari et al. (1988) for evidence on the predictive power of cash flow on investment rate.

[6]See, among others, Evans (1987) and Hall (1987) for earlier results and Haltiwanger et al. (2010) for more recent findings.

[7]Jovanovic (1982) has a model with learning and Hopenhayn (1992) has a model with persistent shocks to the production technology.

[8]See Cooley and Quadrini (2001) for a detailed discussion.

[9]See the discussion on model implications for more details.

[10]See, among others, Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), Quadrini (2004), and

corporate the dynamic investment decisions of heterogeneous firms in a problem of financial contracting. This extension to the model offers an intuitive yet comprehensive explanation to the short-run and long-run dependence of firm dynamics on intangible assets. Modeling the accumulation of physical and intangible assets also allows me to explain the age and cross-sectional patterns of Tobin's q, which has eluded prior studies that abstract from the dynamic investment decisions of firms.[11]

This paper contributes to the developing literature on intangible assets.[12] I construct new measures of intangible assets by accumulating firms' selling, general, and administrative (SG&A) expense as reported in the Compustat data. SG&A expense is an appropriate proxy of intangible investment because it includes most of the expenditures firms use to accumulate intangible assets, such as employee training, marketing, research and development, and payments to strategy consultants.[13] I document evidence that resources allocated to SG&A expense have long-run impacts on firm performance and predict the size and market value of the firm.

The paper is organized as follows. Section 2 describes the model and derives its testable implications. Section 3 describes the data, documents a systematic relationship between intangible assets and firm dynamics, and compares empirical facts with the model's predictions. Section 4 studies the quantitative properties of the model. Section 5 concludes.

Arellano et al. (Forthcoming).

[11]For example, Albuquerque and Hopenhayn (2004) abstract from investment and assume a constant book value of assets. Their model implies that Tobin's q rises with age, which is counterfactual. See Section 4 for detailed discussions.

[12]See, among others, Atkeson and Kehoe (2005), McGrattan and Prescott (2000), and Laitner and Stolyarov (2003).

[13]See Lev (2001) and Lev and Radhakrishnan (2005).

1.2 A model of intangible assets and financial frictions

I incorporate heterogeneous firms and dynamic investment decisions on physical and intangible assets into a model of dynamic financial contracting based on Albuquerque and Hopenhayn (2004).

1.2.1 Model set up

Time is discrete and the time horizon is infinite. A firm is born with zero assets and an investment project. The project requires a sunk setup cost I_0 and generates flow profit $F(k, a, z)$, where k is physical assets, a is intangible assets, and z is an idiosyncratic shock. I make the following assumptions about the profit function and the shock.

Assumption 1: The function $F : \mathcal{K} \times \mathcal{A} \times \mathcal{Z} \rightarrow \mathcal{R}_{+}$ is strictly increasing, strictly concave, and twice continuously differentiable in k and a, and increasing in z. It satisfies $F(k, 0, z) = F(0, a, z) = 0$ and has decreasing returns to scale, that is, $\forall \iota > 1, f(\iota k, \iota a) < \iota f(k, a)$.

Assumption 2: The stochastic process of z has bounded and finite support $\mathcal{Z} = [z_1, z_2, ..., z_N]$, $N \geq 2$. z follows a common stationary and monotone (increasing) first-order Markov transition function.

Decreasing returns to scale implies an optimal firm size. This property may result from limited managerial technology, as in Lucas (1978).

The firm can seek debt financing from outside investors, or debtholders. I define debt by its claim structure and control rights. In exchange for funds, the firm promises contingent payments and control rights. If the firm repudiates the contract, the debtholders can exercise their control rights and take over the firm. The debtholders and the firm have the same discount rate $r > 0$.

The timing of actions and associated payoffs is as follows. At time zero, initial productivity z_0 is realized and observed publicly. The firm signs a long-term contract. A contract c specifies funds for the setup cost I_0, delivered from the debtholders, and a triplet of functions $c : \{k_t, a_t, d_t\}_{t=1}^{\infty}$, which determines funds for assets k_t and a_t delivered at the end of period $t-1$, a dividend payment d_t to the firm, and a debt payment to the debtholders delivered at the end of period t.[14] Even though the dividend payment d_t is made at the end of the period t, it is contracted in period t. This timing assumption simplifies the analysis considerably.[15] The triplet $\{k_t, a_t, d_t\}_{t=1}^{\infty}$ implicitly defines a sequence of payments to the debtholders, who receive all cash flow net of dividends. The funds and payments in the contract are contingent on the history of shocks. This specification is consistent with empirical debt contracts with covenants restricting firms' investment, dividend and financing policies.[16]

Although debtholders behave competitively and have full commitment to the contract, the firm has limited liability and can choose to repudiate the contract. At the end of each period, the firm can divert $D(k_t, a_t)$ from the funds for k_t and a_t. I refer to $D(k_t, a_t)$ as the repudiation value and will discuss its specification later. In case of a repudiation, the debtholders take control. Because debtholders cannot run a firm, perhaps because of the lack of skills, they commit no further funds and liquidate the business. If the firm does not repudiate the contract, it survives to the next period with probability $1-\varphi$. At the beginning of the next period, conditional on survival,

[14] It is important for the purpose of this model that physical and intangible assets are separately specified in a contract. One natural extension is to assume that intangible assets are not contractible perhaps due to non-veriafiability of the value of intangible assets. The resulting constraints would be more restrictive.

[15] Following this assumption, the Bellman equation (1.11) does not depend on d_t implicitly. Even though d_t is determined before shock z_t is realized, the structure of the problem (see Proposition I.4) ensures that the optimal d_t is never greater than the firm's revenue for any $z_t > 0$ so the firm can always pay the contracted dividend if it does not default.

[16] A contingent debt repayment schedule can be thought about as a fixed payment schedule without prepayment penalties. Specifically, the contract can specify a minimum fixed payment (possibly zero) and allow for increased payment at the borrower's discretion.

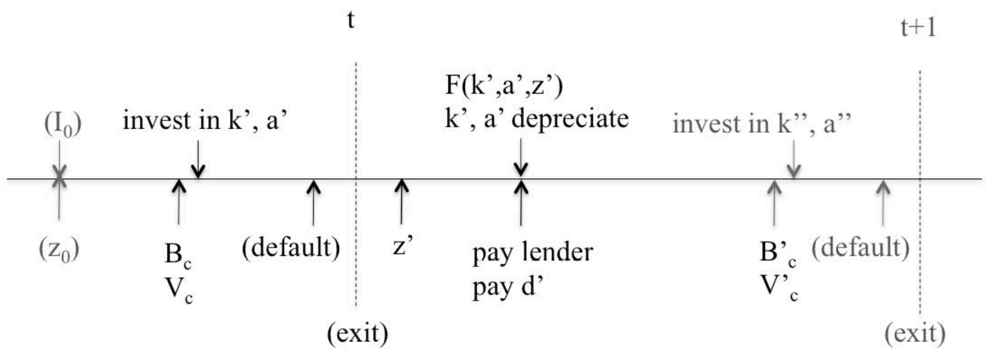

Figure 1.1: Timing

z_t is realized and production takes place. Payments to the firm and debtholders are made as specified in the contract. The timing of events is illustrated in Figure 1.1.

Let B_c denote the contract value to the debtholders from some contract c and let V_c denote the contract value to the firm. The value to debtholder satisfies the following recursion:[17]

$$B_c = -R_k k' - R_a a' + \beta \mathbb{E}_{z_i'|z} \left[F\left(k', a', z_i'\right) - d' + B_c' \right], \tag{1.1}$$

where the discount rate is $\beta = \frac{1-\varphi}{1+r}$ (recall the firm survives to the next period with probability $1-\varphi$). The flow payoff to the debtholders consists of the expected present value of debt repayment $\beta \mathbb{E}_{z_i'|z} \left[F\left(k', a', z_i'\right) - d' \right]$, net of the cost of investment $R_k k' + R_a a'$. The cost of investment, $R_j \equiv 1 - \beta\left(1 - \delta_j\right) - \frac{\varphi}{1+r}$, $j = \{k, a\}$, can be interpreted as follows: For every unit of assets, the firm makes a dollar of investment at the end of the current period, which is a cost; conditional on survival, the assets' value in the next period after production and depreciation is $1 - \delta_j$; with probability φ, the firm exits and the liquidation value is one.

The value to the firm can also be written in a recursive form:

$$V_c = \beta \left(d' + \mathbb{E}_{z_i'|z} V_c' \right). \tag{1.2}$$

[17] I shall use letters without subscripts to denote current period values and with a prime to denote next period's value. $\mathbb{E}_{z_i'|z}$ denotes the conditional expectation of z' given z.

Let $h_t = \{z_0, z_2, ..., z_t\}$ denote the history of shocks up to date t and let $\mathcal{C}(h_{t-1})$ denote the set of contracts contingent on history h_{t-1}. A contract is *feasible* if the dividend payment is nonnegative

$$d' \geq 0, \tag{1.3}$$

for any history h_{t-1} and $(k, a, d) = \mathcal{C}(h_{t-1})$. This condition says that the firm is financially constrained and has no additional source of funds.[18]

For a contract to be *enforceable*, the firm should not have an incentive to repudiate. In case of a repudiation, the firm diverts η_k, a fraction of the value of physical assets, and η_a, a fraction of the value of intangible assets. The repudiation value to the firm is

$$D(k', a') = \eta_k k' + \eta_a a'.$$

Diversion generates private benefits to the firm, which may result from various limitations of the enforceability of the contract, for example, the inalienability of assets from the owner of the firm, costly verification of wrongdoing, or imperfectly defined property rights. The severity of limited enforceability is captured by η_k and η_a. When η_k and η_a are large, the value of repudiating a contract is high. I assume $0 \leq \eta_k < \eta_a \leq 1$, so the repudiation value to the firm is higher for intangible assets than physical assets. A perfect financial market requires $\eta_k = \eta_a = 0$.

The firm does not have incentive to repudiate if $D(k', a')$ is less than the value of continued production; thus, the enforcement constraint is given by

$$\beta\left(d' + \mathbb{E}_{z_i'|z} V_c'\right) \geq D(k', a'), \tag{1.4}$$

for any history of the shocks.

[18]This assumption can be relaxed by allowing firms to obtain additional funds from equity holders at a cost. However, some sort of constraints on additional funds is crucial. Without it, financial frictions of the long-term contract become irrelevant because the first best could be achieved by having the firm pay a lump sum transfer to debtholders at the beginning of the contract, which would be forfeited if the firm defaults.

The maximum debt that can be credibly repaid from time zero can be derived as follows. Competitive lending implies that the initial debt value is equal to the setup cost $B(v_0, z_0) = I_0$. The contracting problem at time zero is to find a feasible and enforceable contract that gives the highest value to the firm consistent with the debtholders breaking even, so the initial value to the firm is $v_0 = \sup\{v : B(v, z_0) = I_0\}$ and the initial value to debtholders is

$$B(v_0, z_0) = \sup_{c \in \mathcal{C}(z_0)} \{B_c(z_0) \,|\, V_c(z_0) = v_0\} \tag{1.5}$$

An optimal contract cannot be Pareto-dominated after any realization of the shocks. Otherwise, it would be possible to make a Pareto improvement by replacing the part of the contract that was dominated and relaxing all previous enforcement constraints. It follows that the set of optimal contracts can be characterized by a Pareto frontier defined for any history h_{t-1} and for any feasible value to the firm v that maximizes the debt value of the contract.[19]

$$B(v, z) = \sup_{c \in \mathcal{C}(h_{t-1})} \{B_c(h_{t-1}) \,|\, V_c(h_{t-1}) = v\}. \tag{1.6}$$

Following (1.5) and (1.6), $B(.)$ depends on the history h_{t-1} only through z_{t-1}, because the constraint set is stationary and the information set is identical in these two equations. It follows that the value to the debtholders on the Pareto frontier can be written in a recursive form:

$$B(v, z) = \max_{\substack{k', a', d' \\ v_i', i=(1,\ldots,N)}} \left\{ \begin{array}{c} -R_k k' - R_a a' \\ +\beta \mathbb{E}_{z_i'|z} \left[F(k', a', z_i') - d' + B(v_i', z_i')\right] \end{array} \right\}, \tag{1.7}$$

subject to

$$d' \geq 0, \tag{1.8}$$

[19] V is feasible in the sense that there exists $c \in \mathcal{C}(h_{t-1})$ that yields this value.

$$\beta\left(d' + \mathbb{E}_{z_i'|z} v_i'\right) \geq D\left(k', a'\right), \tag{1.9}$$

$$\beta\left(d' + \mathbb{E}_{z_i'|z} v_i'\right) = v. \tag{1.10}$$

The first two constraints ensure the feasibility and enforceability of the contract. The last constraint ensures the dynamic consistency: The current value to the firm v is equal to the discounted expected value of dividend payment plus the continuation value.[20]

In this problem, the current value to the firm, v, can be viewed as a state variable. Given values v and z, the optimal contract specifies funds for assets k' and a' which takes place at the end of the current period and a dividend payment d' which takes place after production in the next period. The contract also specifies a continuation value v_i' contingent on the realization of the shock $\forall z_i' \in \{z_1, ..., z_N\}$. The law of motion for the state variable v is simple. The future value of v equals the continuation value currently promised by the contract depending on the realization of the shock.

So far the problem is characterized by maximizing the debt value $B(v, z)$ given v. Let $W(v, z) \equiv B(v, z) + v$ denote the total surplus of the contract. It is easy to see that the optimal contract also maximizes $W(v, z)$ given v. Using (3.18), the enforcement constraint (1.9) simplifies to

$$v \geq D\left(k', a'\right).$$

Using (1.8), (3.18) simplifies to

$$v \geq \beta \mathbb{E}_{z_i'|z} v_i'.$$

[20] Assuming debtholders' full commitment to the contract simplifies the problem considerably. Without it, the debtholders' participation constraint would have to be imposed for every period: $B(v_{t-1}, z_{t-1}) \geq 0$ and the problem could not be transformed into a standard dynamic programming problem. In principle, $B(v_{t-1}, z_{t-1}) \geq 0$ is satisfied if the shock z never gets too low. To ensure that this assumption does not affect the solution of the problem, I check in the numerical solution that $B(v_{t-1}, z_{t-1}) \geq 0$ for all parameterizations used in the paper.

An equivalent dynamic programming problem to (1.7) is given by[21]

$$W(v,z) = \max_{\substack{k',a' \\ v_i', i=(1,\dots,N)}} \left\{ \begin{array}{c} -R_k k' - R_a a' \\ +\beta \mathbb{E}_{z_i'|z} \left[F(k',a',z') + W(v_i', z_i') \right] \end{array} \right\}, \quad (1.11)$$

subject to

$$v \geq D(k',a'). \quad (1.12)$$

$$v \geq \beta \mathbb{E}_{z_i'|z} v_i'. \quad (1.13)$$

where $R_j \equiv 1 - \beta(1-\delta_k) - \frac{\varphi}{1+r}$, $j = \{k, a\}$ and $D(k',a') = \eta_k k' + \eta_a a'$. The continuation value v_i' must be supported by a feasible continuation contract. Any nonnegative continuation value is feasible because it can be obtained by giving the firm a transfer equal to v_i' and committing to zero asset in the future. Any $v_i' < 0$ is infeasible since it is inconsistent with the nonnegative dividend constraint. $v_i' \geq 0$ is the domain restriction indicated in problem (1.11). Standard dynamic programming results show that there is a unique solution to the function $W(.)$.

1.2.2 Properties of an optimal contract

Contract value and the efficient frontier

In problem (1.11), current value v sets a limit on assets k' and a' by constraint (1.12). v sets a limit on the continuation value v_i' by constraint (1.13). As a result, the choice of k' and a' can be solved in a static maximization problem separate from the dynamic choice of v_i'. The static problem is to maximize the indirect profit function

$$\pi^*(v,z) = \max_{k',a'} \left\{ -R_k k' - R_k a' + \beta \mathbb{E}_{z_i'|z} F(k',a',z_i') \right\}, \quad (1.14)$$

[21] The alternative formulation eliminates dividend payment from the problem and simplifies analysis.

subject to (1.12). The solution to this problem is simple: A value v exists such that constraint (1.12) does not bind. Let

$$(k^*(z), a^*(z)) = \arg\max_{k',a'} \left\{-R_k k' - R_a a' + \beta \mathbb{E}_{z_i'|z} F(k', a', z_i')\right\} \tag{1.15}$$

be the unconstrained level of assets and define $V^*(z) = D(k^*(z), a^*(z))$. $V^*(z)$ is the smallest value for the firm that is compatible with unconstrained expected profit maximization. If $v < V^*(z)$, the unconstrained assets level cannot be achieved and k' and a' are picked so that $v = D(k', a')$. If $v \geq V^*(z)$, the policy is identical to one obtained in an unconstrained problem; however, this does not mean that the firm has grown out of constraint indefinitely. Stronger conditions are necessary to ensure that the enforcement constraint will not bind in the future. To see this, let $V^n(z)$ denote the minimal value needed so that the enforcement constraint does not bind for at least $n \geq 1$ periods. Let $V^0(z) = 0$ and define

$$V^n(z) = \max\left\{V^*(z), \beta \mathbb{E}_{z_i'|z} V^{n-1}(z_i')\right\}. \tag{1.16}$$

Assume that for every $z \in \mathcal{Z}$, a constant $M < 0$ exists such that $V^*(z) \leq M$. Since $V^n(z)$ is an increasing sequence, this assumption guarantees that the sequence is uniformly bounded and has a limit. Denote this limit by $\tilde{V}(z) \equiv \lim_{n\to\infty} V^n(z)$. Applying Lebesgue's dominated convergence theorem to (1.16) implies that $\tilde{V}(z)$ satisfies the following dynamic programming equation:

$$\tilde{V}(z) = \max\left\{V^*(z), \beta \mathbb{E}_{z_i'|z} \tilde{V}(z_i')\right\}.$$

Blackwell's sufficient conditions can be verified so the solution $\tilde{V}(z)$ is unique. $\tilde{V}(z)$ defines the minimum value for the firm that is consistent with unconstrained profit maximization. Once $\tilde{V}(z)$ is reached, financial constraint no long matters. I henceforth call $\tilde{V}(z)$ *the efficient frontier* to capture the result that once it is reached, the total surplus of the contract cannot be improved by manipulating the debt level. This is formally stated in Lemma I.1.

Lemma I.1. *(i)* $W(v,z)$ *is weakly increasing in* V; *(ii) Let* $\tilde{W}(z)$ *be the solution to*

$$\tilde{W}(z) = \mathbb{E}_{z_i'|z}\left[\pi^*(z) + \beta\tilde{W}(z_i')\right].$$

For all $v \geq \tilde{V}(z)$, $W(v,z)$ *is constant and equal to* $\tilde{W}(z)$; *(iii) For all* $v < \tilde{V}(z), W(v,z) < \tilde{W}(z)$.

Proof. See Appendix. □

The next two lemmas discuss concavity and monotonicity of the total surplus function W.

Lemma I.2. $W(v,s)$ *is strictly concave in* v *when* $v < \tilde{V}(z)$.

Proof. See Appendix. □

Lemma I.3. *If* $v < \tilde{V}(z)$, $W(v,z)$ *is strictly increasing in* v.

Proof. See Appendix. □

A direct implication of the strict monotonicity of $W(.)$ in v for all $v < \tilde{V}(z)$ is that the optimal policy requires no dividends be distributed before firm's value reaches the efficient frontier and all earnings be allocated as debt repayment. The intuition is simple. The firm's input is constrained by its current value by the enforcement constraint (1.12). Postponing dividend payment increases the continuation firm value and relaxes future constraints. Formally,

Proposition I.4. *If* $v < \tilde{V}(z)$, *the optimal policy requires no dividends be distributed.*

Market value and the book value of assets

Before the firm reaches the efficient frontier, the market value of the firm reflects constraint status and growth potential. Let $\mathbb{W}_t$ denote the market value, which is given by the sum of the expected discounted stream of earnings net of investment expenditure on physical and intangible assets i_k and i_a, plus liquidation value upon exit. Investment expenditure satisfies the following assets accumulation equations:

$$i_{k,t} = k_{t+1} - (1 - \delta_k)\, k_t, \tag{1.17}$$

and

$$i_{a,t} = a_{t+1} - (1 - \delta_a)\, a_t, \tag{1.18}$$

It is convenient to express $\mathbb{W}_t$ through (1.17) and (1.18) so it does not depend on investment explicitly:

(1.19)
$$\mathbb{W}_t = \mathbb{E}_t \sum_{s=1}^{\infty} \beta^{s-1} \begin{pmatrix} \frac{\varphi}{1+r} k_{t+s} + \frac{\varphi}{1+r} a_{t+s} + \beta F\left(k_{t+s}, a_{t+s}, z_{t+s}\right) \\ -\beta\left[k_{t+s+1} - (1-\delta_k)\, k_{t+s}\right] - \beta\left[a_{t+s+1} - (1-\delta_a)\, a_{t+s}\right] \end{pmatrix}.$$

The market value of the firm per unit of assets, denoted by q^{adj},[22] is defined by

$$q_t^{adj} \equiv \frac{\mathbb{W}_t}{I_0 + k_t + a_t}. \tag{1.20}$$

Tobin's q, conventionally defined as the ratio of the market value of the firm to the book value of physical assets, denoted by q_t^{Tobin}, is defined by

$$q_t^{Tobin} \equiv \frac{\mathbb{W}_t}{I_0 + k_t}. \tag{1.21}$$

The relation between q^{adj} and W is shown in next lemma.

[22] I use q^{adj} for adjusted q to capture the notion that this ratio is analogous to the convention Tobin's q measure but adjusts the book value to reflect both physical and intangible assets.

Lemma I.5. *The market value of a firm captures the expected surplus value of its financial contract and the book value of assets. In particular, the market value per unit of assets is given by*

$$q_t^{adj} = \frac{W_t - I_0}{I_0 + k_{t+1} + a_{t+1}} + 1,$$

Proof. See Appendix. □

This equality has a nice interpretation. The first term represents the surplus value (adjusted by I_0 and normalized by the book value of assets) resulting from the contract with limited enforcement. It depends on the severity of financial frictions and the firm's constrained status through W_t. The second term represents the book value, or the replacement costs, of assets.

1.2.3 Model implications on heterogeneous firms

Heterogeneous technology

This paper studies heterogeneity in firms' investment in physical and intangible assets. There are different ways to introduce heterogeneity. One way is to introduce heterogeneous productivity of physical assets and intangible assets. An alternative way is to introduce heterogeneous costs of investment. For example, the costs of investment in k relative to a is firm-specific: $R_{k,i}/R_{a,i} = C_i$ for some constant C_i. In the absence of financial constraint, both formulations give similar predictions for the optimal level of assets. Since their implications are similar, I henceforth focus on the first formulation. I assume that the profit function is Cobb-Douglas $F(k,a,\gamma) = e^z \left(k^\gamma a^{1-\gamma}\right)^\theta$, with firm-specific production parameter γ drawn from a common distribution. With this parameterization, it is easy to check that the share of physical assets is increasing in γ among unconstrained firms.[23]

[23]For unconstrained firms, γ and τ satisfy the following relationship: $\frac{1-\beta(1-\delta_k)}{1-\beta(1-\delta_a)} = \left(\frac{1}{\tau} - 1\right)\frac{\gamma}{1-\gamma}$.

Intangible assets and firm dynamics: unconditional correlation

I now provide analytical results regarding the share of intangible assets and firm dynamics, under two simplifying assumptions: (i) Firms have identical and deterministic shock z; (ii) The repudiation value of physical assets is zero, so $D(k', a') = \eta_a a'$. Under these assumptions, firm i's problem is a special case of (1.11). All previous results go through, except that the value and policy functions will depend on the firm specific production technology parameter γ. The efficient frontier also depends on γ. To see this, note that with deterministic z, the efficient frontier equals the smallest V that is compatible with unconstrained expected profit maximization. In particular, let $V^*(\gamma) = D(k^*(\gamma), a^*(\gamma)) = \eta_a a^*(\gamma)$. Following Proposition I.4, $d = 0$ before the efficient frontier is reached; thus, $v = \beta V'$, implying the firm value is growing. If the enforcement constraint does not bind in the current period, it will never bind in the future; therefore, $\tilde{V}(\gamma) = V^*(\gamma)$. The enforcement constraint becomes $\tilde{V}(\gamma) = \eta_a a^*(\gamma)$. It is easy to check that $a^*(\gamma)$ is decreasing in γ so $\tilde{V}(\gamma)$ is decreasing in γ. In other words, a firm with a production technology that is more intensive in intangible assets has a higher efficient frontier.

The next result discusses the monotonicity of entrant size in the share of intangible assets.

Proposition I.6. *Entrant firms with a higher share of intangible assets have lower value to the firm (v_0) and smaller size of physical and intangible assets (k_1 and a_1).*

Proof. See Appendix. □

This result has a simple intuition. Firms with a higher share of intangible assets face more restrictive constraints; consequently, their initial size is smaller.

Growth in intangible assets is also related to the share of intangible assets. Following $v = \eta_a a'$ and zero dividend, $\eta_a a' = v = \beta v' = \beta \eta_a a''$, so intangible assets grow at rate $1/\beta - 1$ before the firm reaches the efficient frontier. The growth rate is zero afterwards.[24] Given the initial size premium and the negative relationship between γ and $\tilde{V}(\gamma)$, firms with low γ (i.e., a technology more intensive in intangible assets) take a longer time to mature.

Proposition I.7. *Among young age categories, firms with a higher share of intangible assets on average have higher growth rate in firm value (v) and intangible assets (a). All firms have the same growth rate eventually.*

This result is a reflection of financial constraint being relaxed with age. Among young age categories, firms with lower γ are more likely to be constrained. The share of intangible assets predicts the constrained status and in turn predicts firm growth and size. As age increases, more firms become unconstrained and the correlation between the share of intangible assets and growth diminishes. With deterministic z, the convergence in growth rate is exact since all firms have zero growth eventually. With stochastic z, the growth rate of an unconstrained firm depends on the realization of the shock. Conditional on z, growth converges in expectation. The convergence in size is more subtle. If $\delta_k = \delta_a$, then the cost of investment in k and a are identical. It is easy to check from the optimality conditions that unconstrained firms' (k^*, a^*) is symmetric in γ, that is, $k^*(\gamma) + a^*(\gamma) = k^*(1-\gamma) + a^*(1-\gamma)$. Because of the nonlinearity of $F(k, a, z)$ in k and a, total assets size $k^*(\gamma) + a^*(\gamma)$ is not independent of γ. In general, when the decreasing return to scale parameter θ is small, the variance in total assets is small. If $\delta_k \neq \delta_a$, then in general unconstrained firms' optimal (k^*, a^*) is not independent of γ. This observation suggests that only

[24]The growth rate of k is more subtle. Since the share of physical assets falls as the firm ages until it reaches the efficient frontier (as shown at the end of this section), k grows at a slower rate than a. The relationship between growth in k and γ cannot be derived analytically. I shall defer this discuss until Section 1.4.4.

when $\delta_k = \delta_a$ and θ is small does the model predict size convergence.

Intangible assets and firm dynamics: conditional correlation

The model also has implications for the conditional correlation: the share of intangible assets is correlated with firm growth conditional on Tobin's q and cash flow. I shall discuss the intuitions in a simplified economy with zero repudiation value in physical assets $\eta_k = 0$. Since Tobin's q is decreasing in v and increasing in z. An observed Tobin's q could mean a high v combined with a low z, or a low v combined with a high z. In contrast with the standard q theory of investment. Tobin's q is not a sufficient statistic for the growth of an unconstrained firm, neither is cash flow.[25]

The predictive power of γ results mainly from its correlation with the constrained status of a firm. Consider a sample of constrained firms with identical Tobin's q but different γ. Following Lemma I.5, the long-run Tobin's q is higher for firms with lower γ. For a given Tobin's q, firms with lower γ are more likely to be constrained and therefore have higher growth. This implies that:

Corollary I.8. *Among young age categories, firms with a lower share of intangible assets on average have higher growth rate, conditional on Tobin's q and cash flow.*

This analysis can be extended to the market value per unit of assets q^{adj}. In a region where $v < \tilde{V}(\gamma)$, q^{adj} is not a sufficient statistic for firm size and growth. Once v reaches the efficient frontier, it is not relevant for firm size. As a result, W is a sufficient statistic and so is q^{adj}. The same cannot be said about Tobin's q because Tobin's q also depends on the firm-specific technology γ even after the efficient frontier is reached. In the model, unconstraint firms have zero growth on

[25] In this simplified economy, cash flow is given by $F_i(k', a', z') + (1-\delta_k)k' + (1-\delta_a)a' - k'' - a''$. An observed cash flow could mean a high V_i combined with a low z, or a low V_i combined with a high z.

average but Tobin's q is permanently higher for firms whose technology is more intensive in intangible assets.

Intangible assets and firm dynamics: age pattern

The model also implies an age pattern in the share of intangible assets.

Proposition I.9. *Before a firm reaches the optimal size, its share of intangible assets in total assets increases over time.*

Proof. See Appendix. □

Intuitively, a firm invests relatively more intensively in k' when it is constrained because the enforcement constrained is more restrictive in intangible assets. In the model, the age variation in the share of intangible assets reflects financial frictions of a firm being relaxed over time whereas the cross-sectional variation reflects the severity of financial frictions across firms due to heterogeneous production technologies.[26]

Deriving analytical expressions for firm growth and size in the case of stochastic z and general repudiation values is difficult due to the lack of analytical solutions for decisions rules. Nevertheless, all the intuitions from the simplified economy carry over to the more general case. Quantitative results of the full model will be presented in Section 1.4.

1.3 Empirical findings on intangible assets and firm dynamics

I use data on U.S. public firms to derive key empirical results, which are consistent with the implications of my model.

[26]Quantitatively, the age variation in the share of intangible share is small in the model and in the data.

1.3.1 Data

The sample of firms is from the annual Standard and Poor's Compustat industrial file and the Center for Research in Security Prices (CRSP) monthly stock file. The sample period is 1979 to 2009. I select the sample by first deleting any firm-year observations for which the book value of total assets (item AT), physical assets (i.e., gross property, plant, and equipment, item PPEGT), employment (item EMP), or sales (item SALE) is either zero or negative. Following the literature, I remove firm-year observations if large discontinuities in total assets or assets stock exist because these discontinuities are likely to be caused by mergers and acquisitions. Firms with primary standard industrial classifications between 4900 and 4999 or between 6000 and 6999 are omitted because the theory is unlikely to be applicable to regulated or financial firms. I deflate all series by the Consumer Price Indexes (CPI) from the Bureau of Labor Statistics.

Measuring assets

Compustat reports the book value of a firm's gross property, plant, and equipment (item PPEGT); however, firms may have an incentive to overreport because of favorable tax treatments on depreciation. To alleviate this problem, I impute physical assets from physical investment expenditure because investment is harder to misreport and potentially more reliable. I calculate implied depreciation rate for every firm using

$$\delta_{k,i} = \mathbb{E}\left[\frac{i_{it-1}}{\hat{k}_{it-1}} - \frac{\hat{k}_{it} - \hat{k}_{it-1}}{\hat{k}_{it}}\right], \tag{1.22}$$

where $\hat{k}_{it}$ is the reported book value of physical assets (item PPEGT) and i_{it} is investment expenditure (item CAPX) minus the sale of property, plant, and equipment (item SPPE). I eliminate firms with $\delta_{k,i} < 0$ or $\delta_{k,i} > 1$. For the rest of the sample, I impute physical assets with a perpetual inventory equation and set the initial stock to the first observation of physical assets (item PPEGT) for each firm.

$$\begin{aligned} k_{it} &= (1-\delta_{k,i})\, k_{it-1} + i_{it-1}, \\ \text{with } k_{i0} &= \hat{k}_{i0}. \end{aligned}$$

One way to check the consistency between the data on investment flow and the stock of physical assets is to examine the implied depreciation rate in (1.22). I group the sample of firms with $0 \leq \delta_{k,i} \leq 1$ into 17 industries using the Fama and French (1997) classification and calculate the mean of $\delta_{k,i}$ by industry. As reported in Table 1.1, the depreciation rate ranges from 0.056 for automobile industry to 0.085 for the oil industry. This range is consistent with the literature.

In contrast to property, plant, and equipment, some assets are not physical by nature. Common examples of intangible assets are brand name, managerial skills and firm-specific technologies. The challenge of measuring intangible assets is well recognized. Under the U.S. generally accepted accounting principles (GAAP), internally generated intangible assets are not reported. Estimations on firm-level intangible assets are scarce.[27] To bridge this gap, I extend the approach in Corrado et al. (2006), which imputes aggregate intangible assets using investment data, to the firm level. Intangible investment is measured by expenditure on Selling, General, and Administrative (SG&A). The GAAP defines the Selling, General, and Administrative expense as all commercial expenses of operation, such as expenses not directly related to production, incurred in the regular course of business pertaining to the securing of operating income. Companies usually explicitly discuss the level and changes of SG&A in the managerial discussion and analysis (MD&A) section of their

[27] One exception is Eisfeldt and Papanikolaou (Forthcoming), whose similarly impute organization capital for U.S. public firms. They find that firms with more organization capital have higher stock returns than their industry peers, suggesting that these firms are more risky.

10-K financial reports. In general, SG&A includes most of the expenditure related to business intangible outlays and knowledge input, such as advertising and marketing expense, (company-sponsored) research and development, employee training, payment to systems and strategy consultants, and the cost of setting up and maintaining internet-based supply and distribution channels. As a consequence, SG&A expenditure usually has a long-term impact on firm performance because it supports activities that improve employee incentive, operation efficiency, and customer loyalty (Lev (2001) and Lev and Radhakrishnan (2005)). I construct the stock of firm i's intangible assets a_{it} using a perpetual inventory equation

$$a_{it} = (1 - \delta_a)\, a_{it-1} + SGA_{it-1}, \tag{1.23}$$

where SGA_{it} is the SG&A expenditure (Compustat annual item XSGA). Two additional elements are needed to implement (1.23): initial intangible assets a_{i0} and depreciation rate δ_a. I set the initial stock of intangible assets to the first reported value of intangible assets (item INTAN). The model provides some guidance on the choice of δ_a. It suggests that if physical assets and intangible assets have different depreciation rates, market value per unit of assets of mature firms does not converge; neither does firm size. As a benchmark, I set identical depreciation rate for physical and intangible assets $\delta_a = \delta_k = \delta$. I choose $\delta = 0.08$, which is within the range of depreciation rates estimated from the data on physical assets. I also experiment with higher values for δ_a.[28]

Using imputed physical and intangible assets, the share of physical assets in total assets is defined as $\tau_{it} \equiv k_{it}/\left(k_{it} + a_{it}\right)$, so the share of intangible assets is $1 - \tau_{it}$.

[28] To examine how the choice of δ_a and δ_k affects the empirical results, I first check that by setting $\delta_a = \delta_k = 0.08$, the data shows convergence in the market value per unit of assets and firms size among unconstrained firms with different asset composition. I also experiment with alternative empirical measures of intangible assets by setting different values for δ_a up to 0.35. I find that most of the empirical results in the paper remains qualitatively the same. This is not surprising because δ_a mostly affect the size variation in asset composition. The qualitative relationship between asset composition and firm dynamics should not be affected.

Growth in physical assets is defined as $g_{k,it} = (k_{it+1} - k_{it}) / k_{it}$. Growth rates of other variables are defined similarly. Age is defined as the number of years (plus one) since the year of the company's initial public offering (IPO). The year of IPO is approximated with the earliest of (a) the year in which the firm appears on CRSP; (b) the year in which the firm is included in Compustat; and (c) the year for which a valid link is found between CRSP and Compustat (based on item LINKDT). I describe the measurement of other variables in Section 1.6.2 in the Appendix.

1.3.2 Empirical findings and tests of model implications

The characteristics of physical and intangible assets

The first important observation is that the share of intangible assets shows substantial cross-sectional variations. Figure 1.3 plots the distribution and kernel density estimation for $\tau = k/(k+a)$ for entrant firms (i.e., age 1). τ ranges from below 5% to above 95%. A substantial, and nearly constant, fraction of the sample spread across every bin of the histogram between 10% and 90%. A relatively larger fraction of the sample has τ between 15% and 25%. To examine the dynamics of τ, I classify firms into three classes according to their initial share of intangible assets. The sample of firms is restricted to those with observation at age 1. The class of firms with a high share of intangible assets includes firms whose initial τ is less than or equal to 0.4; the class with a low share of intangible, above 0.7; and the class with a medium share of intangible assets, between 0.4 and 0.7. As shown in the upper left panel of Figure 1.2, the age variation in τ is small in all classes. More importantly, the initial heterogeneity in the share of intangible assets persists up to 30 years.[29]

[29] I also regress asset composition on age using fixed effects estimations with year fixed effects and firm fixed effects. The coefficient on age is close to zero (0.0004), which again suggests that the age variation in asset composition is very small.

The unconditional relationship between intangible assets and firm dynamics

This subsection presents the key empirical findings of the paper. The goal is to document a systematic relationship between intangible assets and firm dynamics. To help illustrate key findings before proceeding to a more formal analysis, Figure 1.2 plots the mean of several variables of interest for different classes of intangible shares. The upper right panel of Figure 1.2 plots firm size (i.e., the natural logarithm of total assets) for firms with a high, medium, and low share of intangible assets. Several observations emerge. First, young firms are smaller on average. This general pattern holds for firms in all classes. Second, a substantial size premium exists: Among entrants, firms with a low share of intangible assets are on average 6 times as large as firms with a high share of intangible assets. The size premium diminishes over time: Among older firms, firm size more or less converges. The first observation implies an inverse age and size relationship that is consistent with the literature. The second and third observations are the key: Substantial heterogeneity exists in the size of young firms, and this heterogeneity is related to the share of intangible assets in total assets. The size difference diminishes over time and exhibits convergence inertia.

The middle panels of Figure 1.2 plot growth in total assets and growth in physical assets respectively. Similar patterns emerge. First, young firms grow fast on average. Growth rate drops substantially as age increases. Second, an initial growth rate premium exists. Among entrants, firms with a high share of intangible assets grow faster on average. Third, the growth rate premium diminishes over time. The average growth rate of different classes converges among old firms.

The lower left panel of Figure 1.2 plots the market value per unit of assets. Among young age categories, firms with a higher share of intangible assets on average have higher market value per unit of assets; however this value converges as age increases.

The long run level Tobin's q is different from 1, as shown in the lower right panel of Figure 1.2, which plots the market value per unit of physical assets. This is in contrast with the lower left panel of Figure 1.2, where the market value per unit of total assets converges among old firms. The finding that the market value per unit of physical assets is significantly higher among firms with large shares of intangible assets suggests that the market value captures the value of intangible assets, so modeling intangible assets is essential in order to explain the heterogeneity in the market value of old firms.

Figure 1.2 previews primary findings on firm dynamics. The rest of this subsection describes the tests of the following hypotheses:

Hypothesis 1 (the unconditional relationship between intangible assets and firm dynamics): Among young age categories, firms with a higher share of intangible assets on average have higher growth rate, smaller size, and higher market value per unit of assets.

Hypothesis 2 (the diminishing effect of intangible assets): The sensitivity of firm growth and size with respect to the share of intangible assets diminishes among old age categories.

Hypotheses 1 and 2 follow directly from Proposition I.7. Among young cohorts, the share of intangible assets predicts the probability of being financially constrained. Because the constrained status of a firm is reflected in it growth rate, firm size and market value per unit of assets, the share of intangible assets also predicts these aspects of firm dynamics. The predictive power diminishes as the firm ages and becomes constrained.

To test these hypotheses, I first estimate the following regression model using the sample of entrant firms.

$$\text{size}_{ijt} = \alpha_0 + \alpha_1 \tau_{ijt} + \mu_j + \theta_t + \varepsilon_{ijt}, \tag{1.24}$$

where i, t and j denote firm, year, and industry respectively. I use different measures for size_{it}: the natural logarithm of (i) physical assets, (ii) total assets, (iii) sale, and (iv) employment. τ_{ijt} is the share of physical assets in total assets. Because entrant size varies by industry, I control for detailed industry fixed effects.[30] I also control for year fixed effects to abstract from cyclical consideration. The results under different measures of firm size are reported in Panels A to D in Table 2.19. τ returns positive and significant coefficients for all measures of firm size, suggesting that the share of physical assets in total assets positively correlates to entrant size. The model explains a significant 51% and 31% of the variance in the size of physical assets and total assets. It also explains 20% and 15% of the variance in entrants' sale and employment. τ alone has strong predictive power. Measured by partial R^2, τ explains 0.44 of the variance in the size of physical assets; 0.19, in the size of total assets; 0.07, in sale; and 0.10, in employment.

I examine the relationship between entrant growth and the composition of physical and intangible assets using the following model:

$$\text{growth}_{ijt} = \alpha_0 + \alpha_1 \tau_{ijt} + \mu_j + \theta_t + \varepsilon_{ijt}.$$

All independent variables are specified the same as model (1.24). I use two measures for the dependent variable: growth in physical assets and growth in total assets. The results are reported in Panels E and F of Table 2.19. τ returns positive and significant coefficients for either measure of firm growth. Similar to the size regression in (1.24),

[30]Industry fixed effects are based on the 17-industry classification following Fama and French (1997).

the model explains a sizable fraction of the variance in entrant growth. Measured by partial R^2, τ explains 0.04 of the variance in the growth in physical assets; 0.15, in the growth in total assets.

To examine the relationship between the composition of physical and intangible assets and firm dynamics by firm age, I pool all firm-year observations and estimate the following regression model:

$$y_{its} = \alpha_0 + \alpha_{1s} Age_s + \alpha_{2s} \tau_{its} \times Age_s + \mu_i + \theta_t + \varepsilon_{its},$$

where y is the variable of interest: growth, size, or the market value per unit of assets. Age_s is a set of dummy variables, which take a value of 1 if age equals s and 0 otherwise. τ_{its} is the share of physical assets in total assets. To investigate the short-run and long-run effects of τ, I run the regression, using two measures: current τ or initial τ. Current τ is the share of physical assets in total assets measured in the current year; initial τ is measured upon entry and restricted to the sample of firms with observation at age 1. I control for firm fixed effects to address potential heterogeneity across firms. I also control for year fixed effects to abstract from cyclical consideration. Given this specification, the estimated coefficients for α_{2s} represent the marginal effect of τ on the left-hand-side variable for different age groups.

I report estimation results in Table 1.3. Given the detailed age classes and alternative specifications, a large number of coefficients are reported. I find it easier to discuss the results with figures that illustrate the pattern of estimated coefficients. Figure 1.4 plots the coefficient returned for τ corresponding to each age category with a dotted line and the 95% confidence intervals with dashed lines. The upper left panel displays regression results with growth in total assets as the left-hand-side variable; the upper right panel displays results with growth in physical assets as the left-hand-side variable; the lower left and lower right panel display results with size and the market value per unit of assets as left-hand-side variable respectively. Starting with the upper panels, the plotted curves show a strong inverse relationship between growth and the share of physical assets for young firms. For example, among entrants, firms that are 10 percentage points lower in the share of physical assets on average have a growth rate in total assets about 7 percentage points higher. The effect diminishes as age increases. The growth rate premium for firms with a low share of physical assets remains significant until age 15, but it is close to zero among older age categories. In the lower left panel, the plotted curve shows a strong positive relationship between total assets size and the share of physical assets. Among entrants, firms that are 10 percentage points higher in the share of physical assets are on average 35% larger than their industry peers. The effect declines more or less monotonically as age increases. The size premium remains significant until age 45. Finally, the lower right panel illustrates the relationship between the composition of physical and intangible assets the market value per unit of assets. The plotted curve shows a similar pattern as the upper panels. Firms with higher shares of physical assets have significantly lower market value per unit of assets and this effect diminishes around age 10.

It is worth pointing out that the long run level Tobin's q is different from 1, as shown in the lower right panel of Figure 1.2, which plots the market value per unit of physical assets. Although this value falls among firms in all three classes, it remains permanently higher for firms with a low share of physical assets. This is in contrast with the lower left panel of Figure 1.2, where the market value per unit of total assets converges among old firms. This result may not seem surprising given previous discussions on the market value per unit of total assets, because in that case the denominator is smaller for firms with a low share of physical assets. Nevertheless, this result is important in its own right. The finding that the market value per unit of physical assets is significantly higher among firms with large shares of intangible assets suggests that the market value captures the value of intangible assets. This finding suggests that including intangible assets in the model is essential in order to explain the heterogeneity in the market value of old firms.

The conditional relationship between intangible assets and firm dynamics

Hypothesis 3 (the conditional relationship between intangible assets and firm dynamics): Among young age categories, firms with a lower share of intangible assets on average have higher growth conditional on Tobin's q and cash flow.

This hypothesis states a conditional relationship between growth and the share of intangible assets following Corollary I.8. With heterogeneity in the share of intangible assets, Tobin's q and cash flow are not sufficient statistics for firm growth. Under the null hypothesis, the share of physical assets correlates positively with growth conditional on Tobin's q and cash flow. This effect is stronger among younger firms because the correlation between their share of intangible assets and the constrained status is stronger. To test this hypothesis, I sort firms into three classes by age: young (aged 0 to 8), middle-aged (age 9 to 24) and old (aged 25 and older). After firms are sorted, estimations are carried out separately for firms across different classes.[31] The empirical model is

$$\text{growth}_{it} = \alpha_0 + \alpha_1 q_{it} + \alpha_2 CF_{it} + \alpha_3 \tau_{it-1} + \mu_i + \theta_t + \varepsilon_{it}, \tag{1.25}$$

where growth_{it} is the growth in total assets or the growth in physical assets, q_{it} is the market value per unit of physical assets (i.e., Tobin's q) or the market value per

[31] Prior empirical studies on investment and financial frictions also sort firms into different classes based on *a priori* criteria associated with the financing constraints firms face (e.g. Fazzari et al. (1988), Erickson and Whited (2000), Almeida et al. (2004), Hennessy and Whited (2007)).

unit of assets (i.e., q^{adj}), and τ_{it} is the share of physical assets in total assets. I use two measures for the cash flow term CF_{it}. The first is the standard cash flow rate in the literature, defined as the sum of income before extraordinary items (item IB) and depreciation (item DP) divided by physical assets (item PPEGT). One potential problem about this measure is that SG&A expenditure is treated as an operating expenditure and deducted from the firm's income. As a result, cash flow not only captures profitability of the firm but intangible investment as well. For example, a firm with large intangible investment will have smaller cash flow compared to another firm with otherwise the same income but smaller intangible investment. To address this concern, I constructed an adjusted cash flow measure to capture the firm's income before intangible investment. In particular, I adjust the standard cash flow measure by adding the tax-deductible fraction of intangible investment. Adjusted cash flow is income before extraordinary items (item IB) plus depreciation (item DP) plus $(1-\Gamma)$ multiplied by SG&A (item XSGA), where Γ is the firm's marginal tax rate. For simplicity, I set marginal tax rate to be 0.32, which is consistent with the range of values suggested by the literature. Adjusted cash flow rate is defined as the ratio of adjusted cash flow to physical assets. Finally, μ_i and θ_t are firm and year fixed effects respectively.

Table 1.4 presents estimation results with growth in physical assets as the dependent variable. Coefficients associated with each of the three age classes are reported in separate rows labeled Young, Middle-aged and Old. Consistent with the hypothesis, large and significant coefficients are returned for τ. The predictive power of τ conditional on Tobin's q and cash flow arises from its correlation with the constrained status τ in the young class under all specifications. Under some specifications, the τ coefficients remain positive and significant even among old firms. Two interpretations are possible. First, a fraction of the old firms are still constrained. These are firms with a relatively lower share of physical assets and higher growth. This interpretation is consistent with the model. Second, firms that are more intensive in intangible assets have growth opportunities that are unrelated to financial constraints. This could be the case, for example, if the accumulation of firm-specific knowledge leads to higher productivity. This effect may not be entirely captured by Tobin's q or cash flow resulting from measurement errors.

More importantly, large differentials exist between the coefficients of τ across age classes. As expected, the coefficients for middle-aged and old classes are progressively smaller. As age increases and more firms become financially unconstrained, the sensitivity of growth with respect to τ diminishes. The model also explains a greater proportion of the variance in firm growth for the young class relative to the middle-aged and old class. For example, using market value per unit of assets, adjusted cash flow, and τ as independent variables, the R^2 are 0.20, 0.06, and 0.05 for the young, middle-aged, and old class as shown in Panel D.

Estimation results with growth in total assets as the dependent variable are reported in Table 1.5. A similar pattern emerges as in Table 1.4. Detailed discussions of these results are omitted from the paper for space considerations.

Next, I examine the robustness of previous results with respect to changes in estimation techniques and specifications. The error structure of (1.25) presents some problems that might affect the econometric results. To the extent the stock market is excessively volatile, market value may not reflect fundamental value of the firm. The presence of measurement errors may also bias the results. To address these problems, Tobin's q, q^{adj}, cash flow and adjusted cash flow are instrumented with up to three lags of the associated variables. The model is estimated using the two-step dynamic panel GMM estimator proposed by Arellano and Bond (1991). I use pooled firm-year observations and restrict the sample of firms to those with at least three consecutive years, which is the minimum number of years required, given the lag structure of the regression model and the instrumental-variables approach. Estimation results with growth in physical assets and growth in total assets are reported in Table 1.6 and Table 1.7 respectively. Similar to previous results, τ attracts positive and statistically significant coefficients under all specifications. The coefficients for τ in the young class are higher than those returned for the middle-aged and old classes.

Robustness

Robustness to other measures of assets Previous results are robust to alternative measures of assets. I measure physical assets by their reported book value (item PPEGT) and use the ratio of reported physical and total assets (i.e., $\hat{\tau} \equiv$PPEGT/AT) as an indication of the asset tangibility. Because the reported total assets include current assets (e.g., inventory and cash) but not internally generated intangible assets, it is a noisy measure of the firm's total productive assets. Nevertheless, if the market (e.g., equity traders and lenders) uses this information and the share of intangible assets indeed affects investment and lending decisions, it is likely that $\hat{\tau}$ is correlated with firm dynamics. I perform the same regression analyses described in Section 1.3.2 using $\hat{\tau}$ in place of τ. Estimation results show that the coefficients for $\hat{\tau}$ are in the same direction as those for τ but the coefficients on $\hat{\tau}$ are generally smaller. Moreover, the R^2 from models using $\hat{\tau}$ are generally smaller.

Robustness to other explanations One important theoretical explanation of the size and growth dynamics is the selection theory: Some firms are more likely to exit perhaps because of adverse shocks and tend to have higher growth and higher market value conditional on survival. If the selection effect is higher among firms with a higher share of intangible assets, it may confound the effect of financial frictions. Fortunately, this is not the case. I estimate a binomial model and find that firms with a higher share of intangible assets on average are less likely to exit.[32]

1.3.3 Discussion

This subsection discusses several empirical findings and their implications for the model.

Long-run Tobin's q

As shown in the lower right panel of Figure 1.2, the market value per unit of physical assets (i.e., the conventional Tobin's q) does not converge among firms. Firms with a lower share of physical assets have permanently higher market value per unit of physical assets even among mature firms. This contrasts with the predictions of standard q theory of investment (e.g., Hayashi (1982) and Summers (1981)), which suggests Tobin's q converges in the long run. My model offers an explanation to this with intangible assets and heterogeneous production technologies. The market value–the numerator of Tobin's q–reflects future profits generated by both physical and intangible assets; the denominator measures only physical assets. Firms that are more intensive in intangible assets have a larger fraction of market value in excess of the book value of physical assets. Their Tobin's q is permanently higher.

Extensions to the model

The data show a strong age pattern in both the market value per unit of assets q^{adj} and Tobin's q. Conditional on the share of intangible assets, both values fall among young firms. They converge and remain roughly constant for firms up to 45

[32]Exit is defined when a firm drops out of the sample. In the Compustat data, this may mean a firm stops operation, a firm exits the stock market, or it is merged or acquired by another firm.

years old. Beyond this age, q^{adj} and Tobin's q seem to rise among older firms from 50 to 80 years old. The model can match the falling and converging age pattern well but it does not lead to rising q^{adj} and Tobin's q among older firms. I do not view this as a limitation of the model. For simplicity, the model does not distinguish a firm from a production technology. But it can easily be extended by allowing a firm to expand (i.e., by acquiring new production units) or to switch (i.e., by changing their main line of production) its production technology. In real life, older firms acquire younger firms all the time allowing them to update their technology or to expand their market. If an acquired young firm's technology is operated under the name of an old firm, then the assets and growth potentials of the young firm are reflected in the market value of the old firm. Simple extension to the model accommodates this. For example, with some probability an old firm can acquire a young firm every period. When acquisition occurs, the assets and income of the young firm are merged into the old firm. Following the basic mechanics of the model, the parent firm will have high q^{adj} and high Tobin's q even though it appears to be old. As age increases and more old firms acquire young firms, the average q^{adj} and Tobin's q rise as well.

The data also show that a sizable fraction of the firm-year observations have q^{adj} or Tobin's q below one even though on average q^{adj} and Tobin's q are above 1. This observation suggests that disinvestment cost may be large for these firms; otherwise, they will reduce the size of their assets or liquidate. Introducing costly disinvestment to the current model is possible but will complicate the model in a number of ways. (i) The surplus value W will be an outcome of discrete choices. If the firm does not disinvest, W simply equals the sum of flow profit and the discounted future value; otherwise, any disinvestment cost will be subtracted from W. (ii) The size of current assets $\{k, a\}$ will affect the disinvestment decision and will be state variables of the

Bellman equation. (iii) The possibility of disinvestment may introduce non-convexity to the problem.

1.4 Quantitative implications of the model

This section studies quantitatively how a firm's share of intangible assets affects firm dynamics through the financial channel.

1.4.1 Calibration and discretization

The period in the economy is one year. The interest rate is set to $r = 0.04$. The probability of exit is set to $\varphi = 0.1$, which implies a discount rate $\beta = 0.865$. The production function is specified as $F(k, a, z) = Ze^{z}\left(k^{\gamma}a^{1-\gamma}\right)^{\theta}$.[33] This production function abstracts from labor input; nevertheless, $F(.)$ can map to a production function with labor with an appropriate choice of θ. For example, I can write the production function as $G(k, a, l, z) = e^{z}\left(k^{\gamma}a^{1-\gamma}\right)^{\theta} l^{\epsilon-\theta}$ and set $\epsilon = 0.95$ so $G(.)$ has a small degree of decreasing return. This parameterization makes the production function not too different from a standard constant return to scale function and is consistent with the literature (e.g. Gomes (2001)). Once ϵ is determined, θ is set so the labor share is equal to 0.7. This implies $\theta = 0.25$.[34] The firm-specific production parameter γ is set matches the distribution of physical and intangible assets in the data.[35] I calibrate $\delta_k = 0.08$ and $\delta_a = 0.08$ to match the values in the empirical section (1.23). I assume that the stochastic process for incumbent productivity follows $z' = \rho z + \varepsilon'$, where ε is assumed to follow a (truncated) normal

[33]The parameter Z is introduced to scale the average level of assets.

[34]Recall from the model's analytical results that the size convergence of firms with different γ requires θ to be small. To check that $\theta = 0.25$ is consistent with this implication, I simulate a sample of firm using this θ value and fixed all other parameters at their benchmark calibration value. I find that the variation of unconstrained total assets size with respect to γ is small: The standard deviation is 0.11 and the average size of the medium class is 7% lower than that of the low class.

[35]I assume that γ is drawn from a cumulative distribution function $H(\gamma)$, with support $[0.1, 0.9]$. $H(\gamma)$ is set so that the model's distribution of physical and intangible assets among unconstrained firms matches the distribution of physical and intangible assets among old firms in the data. I discretize the support of γ into 5 points.

distribution with mean zero, standard deviation σ and finite support $[-2\sigma, 2\sigma]$. I discretize the stochastic process by two points following Tauchen (1991) and pick $\rho = 0.40$ and $\sigma = 0.05$ to match the mean and variance of investment rate among old firms.[36] Finally, I discretize the state space of v, v', k' and a' into 90-point uniform grids.

1.4.2 Estimation strategy

I solve the model numerically using value function iteration on a multigrid scheme. Given the realization of firm-specific production technology and optimal decision rules, the model generates a simulated panel of firms. In particular, I simulate a model economy with 10,000 firms over 200 periods. Using simulated method of moments (SMM), I jointly calibrate parameters $\{\eta_k, \eta_a, I_0\}$. The SMM approach minimizes the distance between the key moments of the simulated data and those of the empirical data. Let ϑ be the vector of structural parameters, $m(\vartheta)$ be the moments of simulated data, and $\hat{m}$ be the moments of the empirical data. The SMM estimator is defined as

$$\hat{\vartheta} = \arg\min_{\vartheta} L\left(\hat{\vartheta}\right) = \arg\min_{\vartheta} \left(m(\vartheta) - \hat{m}\right)' \left(m(\vartheta) - \hat{m}\right).$$

As I shall discuss in the next subsection, the identification of structural parameters results from certain key moments, in particular, the size and growth rate premium associated with intangible asset. I estimate the following regression for the simulated data and the empirical data:

$$y_i = \alpha_0 + \alpha_1 \tau_i + \varepsilon_i, \tag{1.26}$$

where i indexes the firm, y_i is firm size (measured as the natural logarithm of total assets or physical assets) or growth (measured as the growth in total assets or growth

[36] Specifically, I set $z_1 = -2\sigma$, $z_2 = 2\sigma$ and the transitional probability from z_1 to z_1 is $p_{11} = \Phi(2\rho) - \Phi(-4 + 2\rho)$. Entrant firm's productivity is drawn from a discrete uniform distribution over $[z_1, z_2]$.

in physical assets), and τ is the share of physical assets.[37] The target moments of SMM are regression coefficients, α_1, in the four specifications of regression (1.26). Because my target moments are parameters of an auxiliary regression model, this procedure is also called indirect inference in SMM.

1.4.3 Identification

The model's implications on the dynamics of heterogeneous firms are essential for the identification of structural parameters. It is worth noting that the set up cost I_0 and repudiation values η_k and η_a have similar implication and may not be identified separately in a model with representative firms;[38] however, observations from heterogeneous firms can separately identify I_0, η_k, and η_a.

To see how identification results from the size premium associated with heterogeneous firms, suppose entrant firm sizes are observed for three classes of firms with different γ denoted by $\{L, M, H\}$. Identical setup cost and the enforcement constraint imply

$$B^L\left(\eta_k k^L + \eta_a a^L; \eta_k, \eta_a\right) = B^M\left(\eta_k k^M + \eta_a a^M; \eta_k, \eta_a\right) = B^H\left(\eta_k k^H + \eta_a a^H; \eta_k, \eta_a\right)$$

This equation identifies η_k and η_a using observed size differentials associated with γ. Once η_k and η_a are identified, the setup cost I_0 is the level of entrant debt $B^L(.) = B^M(.) = B^H(.) = I_0$.

To illustrate how identification results from properties of the model, I plot the value and policy functions of the model using parameter values in Table 1.8 except $\eta_k = 0.3$ and $\eta_a = 0.95$. The upper left panel of Figure 1.5 plots total surplus value W as a function of v for firms with a low current z, for firms with a high, medium

[37]To control for unobserved heterogeneity, the regression model for the empirical sample also includes industry fixed effects and year fixed effects.

[38]I_0 affects the initial size of entrant firms, which in turn affects the age of firms when they become unconstrained. η_k and η_a affect the entrant size and time to mature in similar manners as I_0.

and low share of intangible assets (i.e., $\gamma = 0.05, 0.45$, and 0.85) respectively.[39] W is strictly increasing and strictly concave in v until v reaches the threshold value: the efficient frontier $\tilde{V}(\gamma)$. For any given value of v smaller than the threshold value, the rate of change in W with respect to v is larger for firms with higher γ.

The upper middle panel of Figure 1.5 plots B as a function of v. The model predicts that as v increases, B decreases more than one-to-one with respect to v as a result of postponing dividend payment until $v = \tilde{V}(\gamma)$. In the graph, the highest level of B, which I shall call $\bar{B}(\gamma)$, is the maximum debt value that can be achieved under the optimal contract. If $\bar{B}(\gamma) \geq I_0$, then the optimal contract picks $B_0(\gamma) = I_0$; otherwise no contract can be arranged that satisfies debtholders' initial incentive constraint $B_0(\gamma) \geq I_0$. The graph shows that $\bar{B}(\gamma)$ is increasing in γ, suggesting that firms with a high share of physical capital can sustain a larger initial debt. For any I_0 that satisfies $\bar{B}(\gamma) \geq I_0$, the initial value to shareholders satisfies $B_0(v_0) = I_0$.[40] Also evident from the graph is the monotonicity of $v_0(\gamma)$ in γ: firms with a higher share of physical assets have higher v_0.[41] The upper right panel of Figure 1.5 plots the continuation value as a function of v for firms with a low current z. For all three values of γ, $V'(\gamma)$ increases at a constant rate until v reaches a threshold value $\tilde{V}(\gamma)$. This threshold value is higher for firms with a higher share of intangible assets, reflecting their more restrictive borrowing constraints. In a region where $v \geq \tilde{V}(\gamma)$, v is not relevant for the total surplus. Many payment schedules are possible. Here, I illustrate the case in which the firm pays interests and keeps the value to debtholders constant.[42]

[39] For more compact notation, I shall suppress the argument z in the rest of this subsection.

[40] For any $I_0 \leq \bar{B}(\gamma)$, the I_0 line crosses $B(v, \gamma)$ twice as a result of the concavity of $B(v, \gamma)$ in γ. The optimal contract corresponds to the one with higher v because v satisfies $v = \sup\{v : B(v_0, z_0) \geq I_0\}$.

[41] Strictly speaking, this results holds when I_0 is high. When I_0 is sufficiently low, $v_0(\gamma)$ is the same for all γ. This the case when the initial debt is so low that the financial constraint does not bind for an entrant firm.

[42] To see this, recall that $W(v, \gamma)$ is constant in v in a region where $v \geq \tilde{V}(\gamma)$. Constant B then implies constant v.

To illustrate the effect of a decrease in financial frictions, the middle panels of Figure 1.5 plot value function and policy rules with a different parameterization: $\eta_k = 0.05$ and $\eta_a = 0.95$. Compared to the previous parameterization, financial frictions are lower for all firms but firms with a low share of intangible assets are most affected. These firms also reach the efficient frontier much sooner and have a higher $\bar{B}(\gamma)$. Finally, to illustrate the effect of a decreased spread between the repudiation value of intangible assets and physical assets, I set $\eta_k = 0.5$ and $\eta_a = 0.75$ and plot the results in the lower panels of Figure 1.5. As expected, this change reduces the differences between firms. For example, the efficient frontier $\tilde{V}(\gamma)$ are closer between firms with different shares of intangible assets. The variation in entrant value v_0 is also smaller.

1.4.4 Quantitative results

Table 1.8 summarizes the parameter values. Overall, the model is successful in matching the targeted moments of the data. In Table 1.9, I report the coefficients returned for τ for the empirical sample and the simulated sample for regression (1.26). Consistent with the data, the model generates a positive and significant relation between firm size and τ, and a negative and significant relation between firm growth and τ. The magnitudes of the coefficients are also similar. For example, in the model, the coefficient for τ is 4.85 for the regression on total assets size and -0.14 for the regression on total assets growth. In the data, the coefficients are 3.27 and -0.68 respectively.

Given parameter values estimated by SMM, I perform an external validity check by examining the model's ability to match untargeted moments, including growth rate and market value per unit of assets by firm age. I classify firms from the simulated economy into three classes according to their initial share of physical assets and compute their moments separately. The cutoff points for the three classes are consistent with those in the empirical exercise.[43] I then compare the mean statistics by age from the model with data. The upper left panel of Figure 1.6 plots the average growth rate in physical assets by age for firms with a high, medium, and low share of intangible assets. Corresponding statistics from the data are plotted in the upper right panel. The model can successfully replicate the age pattern and the growth rate differentials of the data. Firm growth has a strong age pattern. Conditional on the τ, young firms grow faster. Firm growth slows down over time and eventually falls to zero among older firms. This age pattern is strongest in the class with a low share of physical assets: their initial growth is the highest. Heterogeneity among firms with different share of intangible assets also exists with respect to the time to mature. The high class reaches unconstrained status sooner than the medium or low class. The average age when a firm becomes unconstrained is between 7 to 11 years, which is also consistent with the data. Quantitatively, the model tends to underpredict the growth rate. In particular, the model predicts that firms stop growing once they reach the efficient frontier. In the data, old firms continue to grow at a slow rate. One possible interpretation of the slow growth of old firms is technological progress. If technological progress increases the marginal return to assets, unconstrained firms will continue to grow. Adding a technology trend to the model can potentially eliminate the discrepancy between the model prediction and the data.

The lower left panel of Figure 1.6 plots the mean of market value per unit of assets q^{adj} by age for firms with a high, medium, and low share of intangible assets. An age pattern emerges. In all three classes, q^{adj} is high initially but falls as age

[43] Specifically, the low group consists of firms whose share of physical assets is less or equal to 0.4, the high group consists of firms whose share of physical assets is greater than 0.7 and the medium group for all other firms.

increases. The model predicts that after the firm reaches the efficient frontier, the expected market value per unit of assets is constant, which is consistent with the data as plotted in the lower right panel. The model can also capture the market value differentials between firms with different shares of intangible assets at young ages. Consistent with the data, the model predicts that the age pattern is strongest in the class with the lowest share of physical assets. Quantitatively, the model fits the data well for the majorities of the age categories. The model underpredicts q^{adj} for entrant firms which have on average very high q^{adj} in the data. The predicted value of q^{adj} among unconstrained firms is slightly higher than the data.

The trajectory of the market value per unit of assets has a simple intuition. Before the firm reaches the efficient frontier, assets are constrained to be smaller; however, the market value is affected less because it is a forward-looking variable that captures the expected discounted value of all future net profits. When the discount rate is sufficiently small, unconstrained profits in the future dominates and the market value is less affected by the constrained status of the firm. In my model, the market value per unit of physical assets, Tobin's q, also falls among unconstrained firms following the same argument. Note that my model's implication on the age pattern differs from that of Albuquerque and Hopenhayn (2004). In their model, assets are short-lived, (i.e., they depreciate 100% after production). Tobin's q is measured as the ratio of the market value to the setup cost I_0. As W rises with age, Tobin's q rises with age as well.[44]

1.5 Conclusion

I study investment decisions on heterogeneous assets and firm dynamics in the presence of financial frictions. My paper contributes to the developing literature on

[44]Their model does not have intangible assets so Tobin's q is the same as q^{adj}.

intangible assets and financial frictions empirically, theoretically, and quantitatively. I document robust empirical evidence of the long-run consequence of financial frictions particularly related to intangible assets. Theoretically, I incorporate dynamic investment decisions on heterogeneous assets into a model of limited contract enforcement and establish the theoretical relationship between financial frictions and firm dynamic, which leads to a new identification strategy to quantify structural parameters of financial frictions. Moreover, I integrate firm-level accounting information into the measurement of intangible assets. This methodology can be applied to other dataset to enhance our understanding of intangible assets. I leave for future researchers to study financial frictions and firm dynamics under aggregate fluctuations. For instance, the framework I propose in this paper can prove useful in analyzing the cyclical cross-sectional firm dynamics and studying the long-run impact of severe financial frictions during recession. This framework can also be extended to cross-country studies of intangible assets and financial development.

Industry	Mean δ_{it}
Food	0.061
Mines	0.075
Oil	0.085
Clothes	0.070
Durables	0.074
Chemicals	0.074
Consumption goods	0.075
Construction	0.060
Steel	0.057
Fabricated products	0.060
Machinery	0.073
Automobile	0.056
Transportation	0.064
Retail	0.067
Other	0.079

Table 1.1: Imputed Physical Depreciation Rate by Industry

The table reports the mean of imputed depreciation rate by industry. Firms are sorted according to the 17-industry classification following Fama and French (1997).

Dependent variable	Independent variables τ	Industry fixed effects	Year fixed effects	R^2	Partial R^2 for τ	N
			Panel A			
Size of physical assets	5.6372^{**} (0.1148)	Yes	Yes	0.5124	0.4354	2,692
			Panel B			
Size of total assets	3.2686^{**} (0.1125)	Yes	Yes	0.3140	0.1937	2,692
			Panel C			
Sale	2.4694^{**} (0.1447)	Yes	Yes	0.2050	0.0710	2,692
			Panel D			
Employment	2.4960^{**} (0.1328)	Yes	Yes	0.1508	0.0968	2,692
			Panel E			
Growth in physical assets	-0.3665^{**} (0.0390)	Yes	Yes	0.0513	0.0357	2,630
			Panel F			
Growth in total assets	-0.6798^{**} (0.0347)	Yes	Yes	0.1662	0.1543	2,630

Table 1.2: The Relationship Between Asset Composition and Entrant Size

The table reports results from fixed effects estimations. Physical assets size and total assets size are measured in natural logarithm. All specifications include an intercept, industry fixed effects, and year fixed effects. Only the coefficients for τ are reported. The standard errors (in parentheses) are clustered at the firm level. **,* indicate statistical significance at the 1 and 5 percent (two-tail) test levels, respectively.

Independent variable	Dependent variable					
	Growth		Size		Market value to the book value of assets	
	Model 1 Current τ	Model 2 Initial τ	Model 3 Current τ	Model 4 Initial τ	Model 5 Current τ	Model 6 Initial τ
$\tau \times$ *Age* 1	-0.7030^{**}	-0.6948^{**}	3.0692^{**}	3.3206^{**}	-3.7916^{**}	-3.6034^{**}
	(0.0138)	(0.0166)	(0.1122)	(0.1085)	(0.0936)	(0.1181)
$\tau \times$ *Age* 2	-0.3803^{**}	-0.3278^{**}	2.4952^{**}	2.8545^{**}	-1.3378^{**}	-0.8672^{**}
	(0.0121)	(0.0181)	(0.0948)	(0.1327)	(0.0792)	(0.1212)
$\tau \times$ *Age* 3	-0.2131^{**}	-0.1593^{**}	2.3590^{**}	2.6464^{**}	-0.6100^{**}	-0.2658^{*}
	(0.0131)	(0.0201)	(0.0987)	(0.1327)	(0.0824)	(0.1318)
$\tau \times$ *Age* 4	-0.1021^{**}	-0.0865^{**}	2.1193^{**}	2.4679^{**}	-0.2658^{**}	-0.0682
	(0.0140)	(0.0216)	(0.1062)	(0.1327)	(0.0886)	(0.1444)
$\tau \times$ *Age* 5	-0.0469^{**}	-0.0428	2.0293^{**}	2.3982^{**}	-0.0977	0.1002
	(0.0151)	(0.0237)	(0.1144)	(0.1435)	(0.9550)	(0.1562)
$\tau \times$ *Age* 6	-0.0356^{*}	-0.0196	2.0437^{**}	2.5292^{**}	-0.0652	0.0772
	(0.0162)	(0.0258)	(0.1231)	(0.1561)	(0.1028)	(0.1699)
$\tau \times$ *Age* 7	-0.0356^{*}	-0.0067	1.9240^{**}	2.4781^{**}	-0.1736	-0.1202
	(0.0170)	(0.0274)	(0.1324)	(0.1709)	(0.1105)	(0.1860)
$\tau \times$ *Age* 8	-0.0897^{**}	$= 0.0184$	1.8811	2.5451^{**}	-0.0921	-0.0352
	(0.0153)	(0.0289)	(0.1195)	(0.1808)	(0.0997)	(0.1968)
$\tau \times$ *Age* 9	-0.0576^{**}	-0.0049	1.8681^{**}	2.4830^{**}	-0.0221	0.0516
	(0.0160)	(0.0311)	(0.1254)	(0.1925)	(0.1047)	(0.2095)
$\tau \times$ *Age* 10	-0.0310^{*}	0.0139	2.0180^{**}	2.5163^{**}	0.0600	0.2709
	(0.0171)	(0.0343)	(0.1305)	(0.2048)	(0.1089)	(0.2229)
$\tau \times$ *Age* 15	-0.0252	0.0183	2.2791^{**}	2.1065^{**}	0.0129	0.3870
	(0.0213)	(0.0520)	(0.1658)	(0.3161)	(0.1384)	(0.3440)
$\tau \times$ *Age* 20	-0.0471	0.0396	1.7142	1.3112^{**}	-0.1913	0.4277
	(0.0247)	(0.0844)	(0.1955)	(0.5028)	(0.1632)	(0.5472)
$\tau \times$ *Age* 25	-0.0048	0.0954	1.7373^{**}	0.5719	-0.0945	1.4758
	(0.0324)	(0.1416)	(0.2251)	(0.8427)	(0.1879)	(0.9171)
$\tau \times$ *Age* 30	-0.0168	-0.0007	1.5991^{**}	0.3456	-0.2896	0.7408
	(0.0345)	(0.3312)	(0.2746)	(1.8682)	(0.2292)	(2.0332)
$\tau \times$ *Age* 35	0.0215		1.7318^{**}		-0.1594	
	(0.0380)		(0.3056)		(0.2551)	
$\tau \times$ *Age* 40	-0.0044		1.5815^{**}		-0.1631	
	(0.0552)		(0.4393)		(0.3667)	
$\tau \times$ *Age* 45	-0.0704		1.1508^{**}		-0.4101	
	(0.0697)		(0.5263)		(0.4394)	
$\tau \times$ *Age* 50	-0.0202		1.2944		-0.6898	
	(0.0916)		(0.7306)		(0.6099)	
R^2	0.4148	0.5117	0.4026	0.3236	0.2293	0.2630
N	60,274	18,312	67,024	21,000	67,024	21,000

Table 1.3: The Relationship Between Firm Dynamics and Asset Composition by Age

The table reports results from fixed effects estimations. Physical assets size and total assets size are measured in natural logarithm. All specifications include an intercept, year fixed effects, and firm fixed effects. Only the coefficients for the interaction term $\tau_i \times Age_s$ are reported. The standard errors (in parentheses) are clustered at the firm level. **,* indicate statistical significance at the 1 and 5 percent (two-tail) test levels, respectively.

Dependent variable Growth in physical assets	Independent variables q^{Tobin}	q^{adj}	Cash flow	Adjusted cash flow	τ	R^2	N
			Panel A				
Young	0.0076**		0.0000		0.1190**	0.1218	18,356
	(0.0003)		(0.0000)		(0.0081)		
Middle-aged	0.0013**		0.0000		0.0307**	0.0059	24,046
	(0.0003)		(0.0004)		(0.0043)		
Old	0.0001		0.0002**		0.0215**	0.0064	12,945
	(0.0002)		(0.0000)		(0.0040)		
			Panel B				
Young	0.0076**			0.0000	0.1190**	0.1218	18,356
	(0.0003)			(0.0000)	(0.0079)		
Middle-aged	0.0013**			0.0002	0.0308**	0.0068	24,046
	(0.0003)			(0.0001)	(0.0004)		
Old	0.0001			0.00008**	0.0218**	0.0063	12,945
	(0.0002)			(0.0000)	(0.0040)		
			Panel C				
Young		0.0576**	0.0000		0.0320**	0.2069	18,356
		(0.0013)	(0.0000)		(0.0074)		
Middle-aged		0.0323**	0.0001		0.0207**	0.0582	24,046
		(0.0017)	(0.0004)		(0.0041)		
Old		0.0229**	0.0002**		0.0271**	0.0545	12,945
		(0.0017)	(0.0000)		(0.0038)		
			Panel D				
Young		0.0576**		0.0000**	0.0320**	0.2069	18,356
		(0.0013)		(0.0000)	(0.0074)		
Middle-aged		0.0323**		0.0002*	0.0209**	0.0592	24,046
		(0.0017)		(0.0001)	(0.0041)		
Old		0.0229**		0.0000**	0.0273	0.0541	12,945
		(0.0017)		(0.0000)	(0.0038)		

Table 1.4: Conditional Relationship Between Asset Composition and Growth in Physical Assets: OLS Fixed-Effect Estimation

The table reports results from fixed effects estimations. All specifications include an intercept, year fixed effects, and industry fixed effects. The standard errors (in parentheses) are clustered at the firm level. Young firms are age 1 to 8 firms; middle age firms are age 9 to 20 firms; old firms are age 21 and above. **,* indicate statistical significance at the 1 and 5 percent (two-tail) test levels, respectively.

Dependent variable Growth in total assets	q^{Tobin}	q^{adj}	Cash flow	Adjusted cash flow	τ	R^2	N
				Panel A			
Young	0.0100**		0.0000		0.0338**	0.2092	18,356
	(0.0002)		(0.0000)		(0.0078)		
Middle-aged	0.0038**		0.0001**		0.0189**	0.0802	24,046
	(0.0002)		(0.0005)		(0.0039)		
Old	0.0022**		0.0002**		0.0150**	0.0452	12,945
	(0.0002)		(0.0000)		(0.0039)		
				Panel B			
Young	0.0100**			0.0000	0.0339**	0.2092	18,356
	(0.0002)			(0.0000)	(0.0078)		
Middle-aged	0.0038**			0.0004*	0.0192**	0.0834	24,046
	(0.0002)			(0.0001)	(0.0038)		
Old	0.0022**			0.0001**	0.0154**	0.0460	12,945
	(0.0002)			(0.0000)	(0.0039)		
				Panel C			
Young		0.0730**	0.0000		0.0825**	0.3227	18,356
		(0.0012)	(0.0000)		(0.0070)		
Middle-aged		0.0446**	0.0002**		0.0154**	0.1591	24,046
		(0.0015)	(0.0004)		(0.0035)		
Old		0.0319**	0.0002**		0.0043	0.1136	12,945
		(0.0015)	(0.0015)		(0.0036)		
				Panel D			
Young		0.0730**		0.0000**	0.0824**	0.3225	18,356
		(0.0012)		(0.0000)	(0.0070)		
Middle-aged		0.0446**		0.0004**	0.0150**	0.1625	24,046
		(0.0015)		(0.0001)	(0.0035)		
Old		0.0319**		0.0001**	0.0045	0.1145	12,945
		(0.0015)		(0.0000)	(0.0036)		

Table 1.5: Conditional Relationship Between Asset Composition And Growth in Total Assets: OLS Estimation

The table reports results from fixed effects estimations. All specifications include an intercept, year fixed effects, and industry fixed effects. The standard errors (in parentheses) are clustered at the firm level. Young firms are age 1 to 8 firms; middle age firms are age 9 to 20 firms; old firms are age 21 and above. **,* indicate statistical significance at the 1 and 5 percent (two-tail) test levels, respectively.

Dependent variable Growth in physical assets	Independent variables Intercept	q^{Tobin}	q^{adj}	Cash flow	Adjusted cash flow	τ
			Panel A			
Young	-0.0519^{**}	0.0064^{**}		0.0001^{**}		0.3008^{**}
	(0.0002)	(0.0002)		(0.0000)		(0.0088)
Middle-aged	-0.0143^{**}	0.0011^{**}		-0.0002^{**}		0.1686^{**}
	(0.0002)	(0.0001)		(0.0000)		(0.0041)
Old	-0.0040^{**}	-0.0004^{**}		0.0002^{**}		0.0589^{**}
	(0.0000)	(0.0000)		(0.0000)		(0.0004)
			Panel B			
Young	-0.0575^{**}	0.0061^{**}			0.00002^{**}	0.3494^{**}
	(0.0001)	(0.0002)			(0.0000)	(0.0058)
Middle-aged	-0.0131^{**}	0.0008^{**}			0.0004^{**}	0.1707^{**}
	(0.0001)	(0.0001)			(0.0000)	(0.0023)
Old	-0.0037^{**}	-0.0002^{**}			0.0001^{**}	0.0515^{**}
	(0.0000)	(0.0000)			(0.0000)	(0.0003)
			Panel C			
Young	-0.04743^{**}		0.0439^{**}	0.0002^{**}		0.1768^{**}
	(0.0001)		(0.0010)	(0.0000)		(0.0027)
Middle-aged	-0.0109^{**}		0.0228^{**}	-0.0001^{**}		0.0715^{**}
	(0.0002)		(0.0007)	(0.0000)		(0.0036)
Old	-0.0035^{**}		0.0144^{**}	0.0003^{**}		0.0129^{**}
	(0.0000)		(0.0000)	(0.0000)		(0.0003)
			Panel D			
Young	-0.0506^{**}		0.0439^{**}		0.00003^{**}	0.1970^{**}
	(0.0001)		(0.0006)		(0.0000)	(0.0013)
Middle-aged	-0.0106^{**}		0.0225^{**}		0.0009^{**}	0.0698^{**}
	(0.0001)		(0.0004)		(0.0000)	(0.0024)
Old	-0.0033^{**}		0.0151^{**}		0.0002^{**}	0.0099^{**}
	(0.0000)		(0.0000)		(0.0000)	(0.0000)

Table 1.6: Conditional Relationship Between Growth and Asset Composition: GMM Estimation

The table reports results from the two-step dynamic panel GMM estimator (Arellano and Bond (1991)). q^{Tobin}, q^{adj}, cash flow and adjusted cash flow are instrumented with three lags of the associated variables. Young firms are age 1 to 8 firms; middle age firms are age 9 to 20 firms; old firms are age 21 and above. $^{**},^{*}$ indicate statistical significance at the 1 and 5 percent (two-tail) test levels, respectively.

Dependent variable Growth in total assets	Intercept	q^{Tobin}	q^{adj}	Cash flow	Adjusted cash flow	τ
			Panel A			
Young	-0.0678^{**}	0.0122^{**}		0.0001^{**}		0.4987^{**}
	(0.0002)	(0.0001)		(0.0000)		(0.0094)
Middle-aged	-0.0195^{**}	0.0026^{**}		-0.0002^{**}		0.2445^{**}
	(0.0002)	(0.0001)		(0.0000)		(0.0029)
Old	-0.0071^{**}	0.0013^{**}		0.0003^{**}		0.0904^{**}
	(0.0000)	(0.0000)		(0.0000)		(0.0004)
			Panel B			
Young	-0.0735^{**}	0.0118^{**}			0.0000^{**}	0.5601^{**}
	(0.0001)	(0.0002)			(0.0000)	(0.0039)
Middle-aged	-0.0184^{**}	0.0025^{**}			0.0008^{**}	0.2428^{**}
	(0.0001)	(0.0000)			(0.0000)	(0.0016)
Old	-0.0073^{**}	0.0013^{**}			0.0001^{**}	0.0934^{**}
	(0.0000)	(0.0000)			(0.0000)	(0.0004)
			Panel C			
Young	-0.0614^{**}		0.0770^{**}	0.0002^{**}		0.3120^{**}
	(0.0002)		(0.0009)	(0.0000)		(0.0059)
Middle-aged	-0.0156^{**}		0.0165^{**}	-0.0001^{*}		0.1812^{**}
	(0.0002)		(0.0006)	(0.0000)		(0.0028)
Old	-0.0055^{**}		0.0104^{**}	0.0004^{**}		0.0664^{**}
	(0.0000)		(0.0000)	(0.0000)		(0.0005)
			Panel D			
Young	-0.0652^{**}		0.0732^{**}		0.00003^{**}	0.3484^{**}
	(0.0001)		(0.0008)		(0.0000)	(0.0019)
Middle-aged	-0.0150^{**}		0.0174^{**}		0.0013^{**}	0.1754^{**}
	(0.0001)		(0.0003)		(0.0000)	(0.0018)
Old	-0.0054^{**}		0.0100^{**}		0.0002^{**}	0.0678^{**}
	(0.0000)		(0.0000)		(0.0000)	(0.0004)

Table 1.7: Conditional Relationship Between Growth and Asset Composition: GMM Estimation (continued)

The table reports results from the two-step dynamic panel GMM estimator (Arellano and Bond (1991)). q^{Tobin}, q^{adj}, cash flow and adjusted cash flow are instrumented with three lags of the associated variables. Young firms are age 1 to 8 firms; middle age firms are age 9 to 20 firms; old firms are age 21 and above. **,* indicate statistical significance at the 1 and 5 percent (two-tail) test levels, respectively.

Calibrated parameters		
Interest rate	r	0.04
Exit rate	φ	0.10
Return to scale	θ	0.25
Depreciation rate (physical)	δ_k	0.08
Depreciation rate (intangible)	δ_a	0.08
Shock persistence	ρ	0.40
Stochastic shock variance	σ	0.05
Estimated parameters		
Repudiation value (physical)	η_k	0.52
Repudiation value (intangible)	η_a	0.87
Setup cost*	I_0	0.56

Table 1.8: Parameter Values

*The setup investment is normalized by the unconstrained size of total assets of a firm with median γ

Independent variable		τ	R^2
Size of total assets	Data	3.2686^{**}	0.3140
		(0.1125)	
	Model	3.8724^{**}	0.2436
		(0.0824)	
Size of physical assets	Data	5.6372^{**}	0.5124
		(0.1148)	
	Model	5.8580^{**}	0.4023
		(0.0927)	
Growth in total assets	Data	-0.6798^{**}	0.1662
		(0.0347)	
	Model	-0.2112^{**}	0.2190
		(0.0022)	
Growth in physical assets	Data	-0.3665^{**}	0.0513
		(0.0390)	
	Model	-0.2679^{**}	0.1023
		(0.0345)	

Table 1.9: Model Quantitative Results

The table reports results from ordinary least squares (fixed effects) estimation. All specifications include an intercept. Specifications using the empirical sample include year fixed effects and industry fixed effects. The standard errors (in parentheses) are clustered at the firm level. **,* indicate statistical significance at the 1 and 5 percent (two-tail) test levels, respectively.

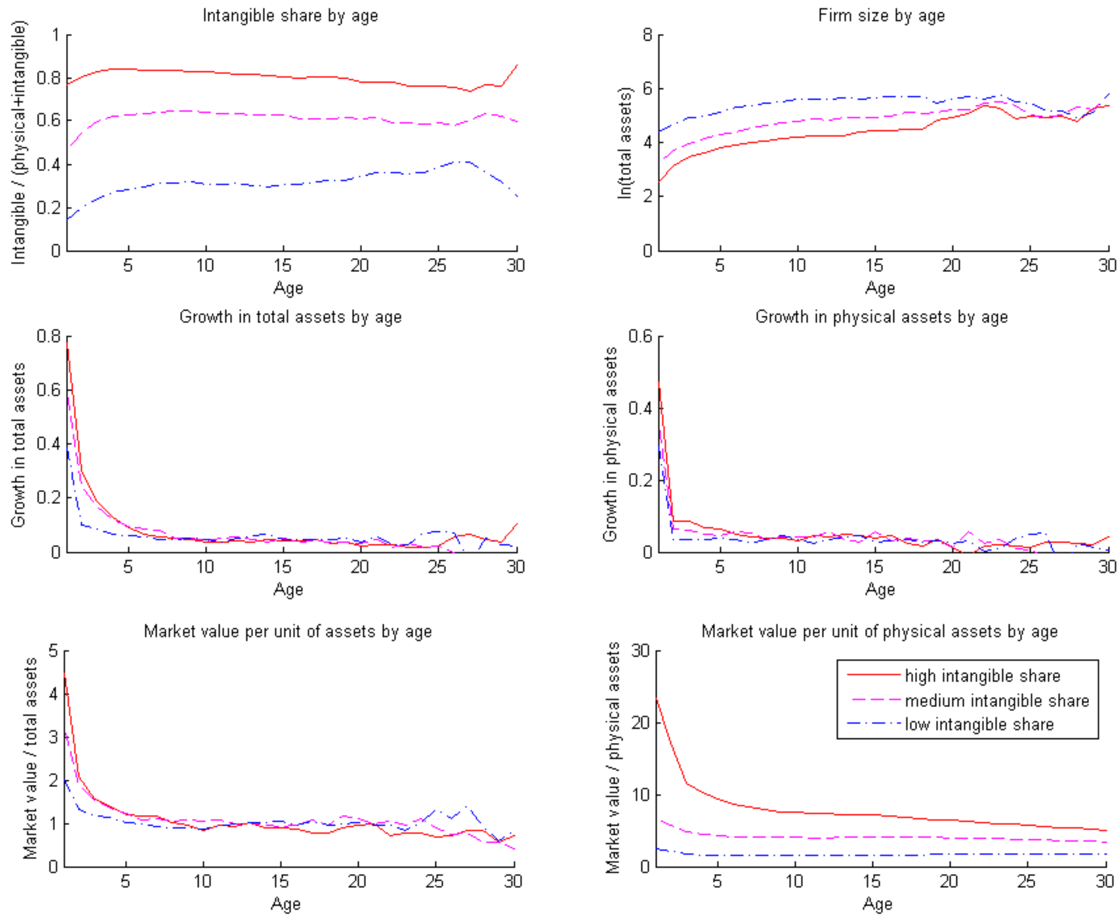

Figure 1.2: Intangible Assets and Firm Dynamics

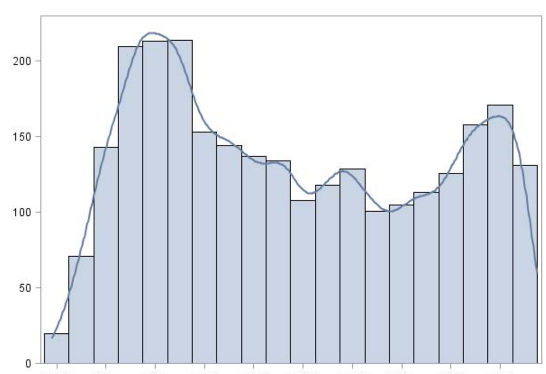

Figure 1.3: Distribution of Initial Physical Share

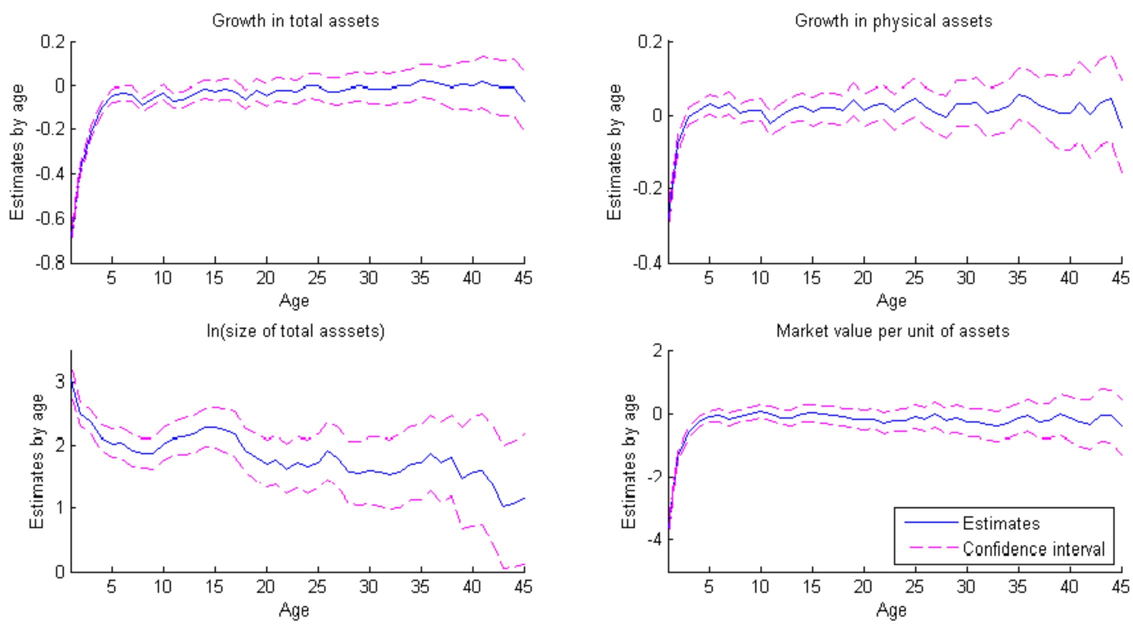

Figure 1.4: The Relationship Between Asset Composition and Firm Dynamics

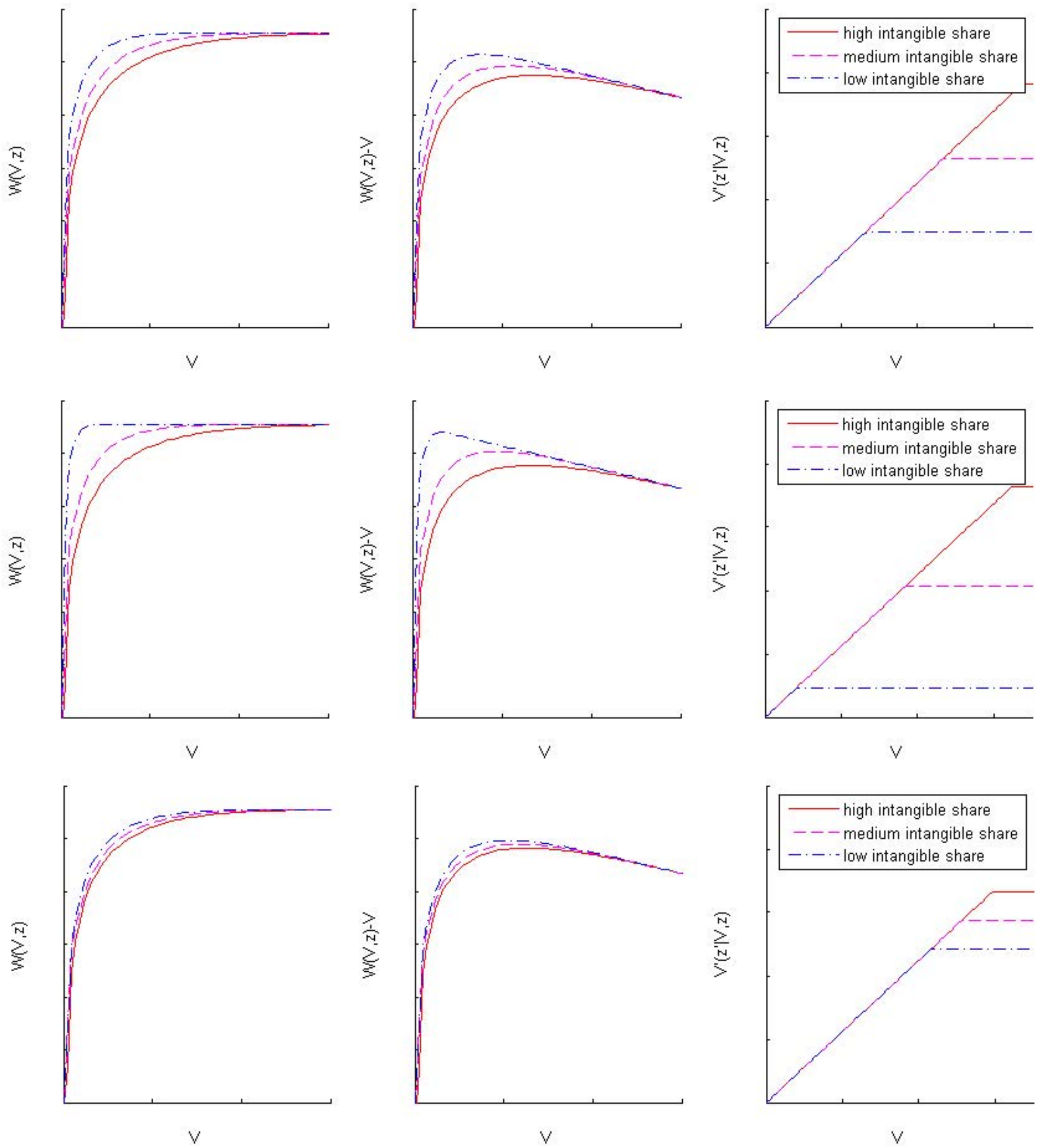

Figure 1.5: Identification

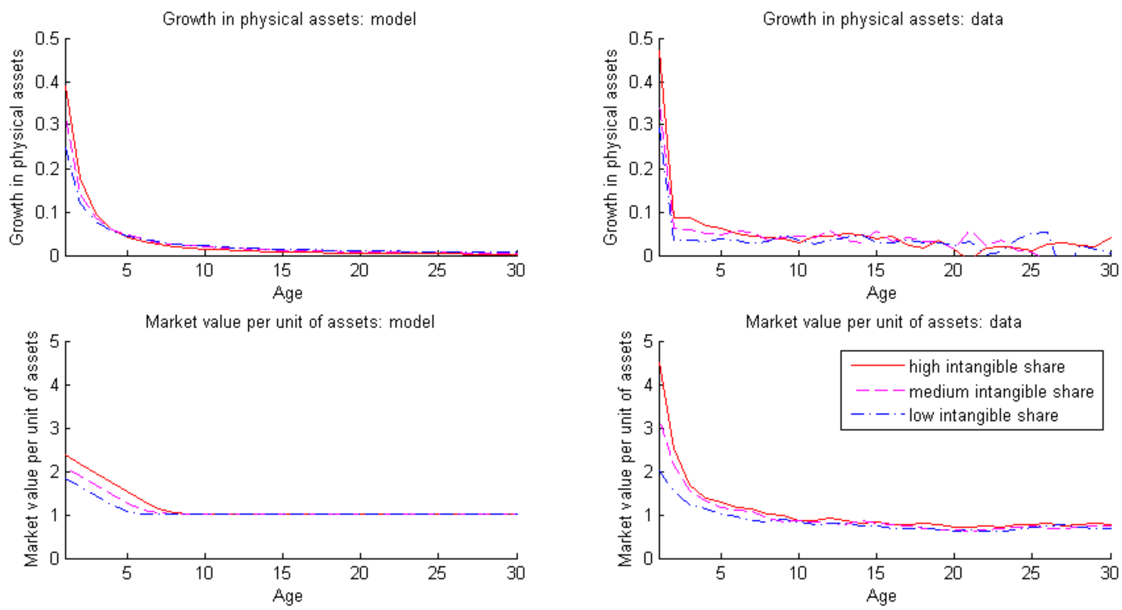

Figure 1.6: Model Quantitative Results

1.6 Appendix

1.6.1 Proofs

Proof of Lemma I.1

Proof. (i) $\pi(v,z)$ is weakly increasing in v following the monotonicity of the constraint set. The weak monotonicity of $W(v,z)$ in v follows immediately from the monotonicity of $\pi(v,z)$ in v applying standard dynamic programming arguments. (ii) Suppose for all $v \geq \tilde{V}(z)$, $W(v,z_i') = \tilde{W}(z_i')$ and let $v_i' = \tilde{V}(z_i')$. (1.11) implies that

$$\begin{aligned} W(v,z) &= \mathbb{E}_{z_i'|z}\left[\pi^*(z) + \beta W\left(\tilde{V}(z_i'), z_i'\right)\right] \\ &= \mathbb{E}_{z_i'|z}\left[\pi^*(z) + \beta \tilde{W}(z_i')\right] \\ &= \tilde{W}(z). \end{aligned}$$

(iii) If $v < \tilde{V}(z)$, then $v < V^n(z)$ for some n. I shall proceed with induction. For $n=1$, $v < V^*(z)$ implies $-R_k k' - R_a a' + \beta \mathbb{E}_{z'|z} F(k', a', z') < \pi^*(z)$. Abusing notation, let v_i' be the optimal continuation value given (v,z), then

$$\begin{aligned} W(v,z) &< \mathbb{E}_{z_i'|z}\left[\pi^*(z) + \beta W(v_i', z_i')\right] \\ &\leq \mathbb{E}_{z'|z}\left[\pi^*(z) + \beta \tilde{W}(z_i')\right] \\ &= \tilde{W}(z). \end{aligned}$$

To proceed, suppose $v < V^{n-1}(z)$ implies $W(v,z) < \tilde{W}(z)$ for all $z \in \mathcal{Z}$.For $v < V^n(z)$ to hold, one need $v < V^*(z)$ or $v < \beta \mathbb{E}_{z_i'|z} V^{n-1}(z_i')$. The proof for the former case follows the same argument as in $n=1$. For the latter, the continuation policy v_i' is such that $V_i' < V^{n-1}(z')$ for some $z \in \mathcal{Z}$ by induction, it follows that $v = \beta \mathbb{E}_{z_i'|z} v_i' < \beta \mathbb{E}_{z_i'|z} V^{n-1}(z_i')$.[45] □

[45] The proof of (ii) and (iii) follows from Albuquerque and Hopenhayn (2004).

Proof of Lemma I.2

Proof. This proof is similar to the proof of Lemma 3 in Appendix C in Albuquerque and Hopenhayn (2004). First, the indirect profit function $\pi(v, z)$ is strictly concave in v following strict concavity of the production function $F(.)$ and convexity of the constraint set. It follows that $W(v, z)$ is concave by Theorem 9.8 in Stokey et al. (1989). If $v < \tilde{V}(z)$, then an n exists such that $v < V^n(z)$. I shall proceed with induction on n to show that this implies strictly concavity of $W(v, z)$ in the neighborhood of V. For $n = 1$, $v < V^n(z)$ and strictly concavity of $W(v, z)$ in v follows strict concavity of $\pi(v, z)$. Now suppose the result hold for all z and $v < V^{n-1}(z)$ and $V^u(z) \leq v < V^n(z)$. The optimal continuation value $v'_i < V^{n-1}(z'_i)$ on a subset of s with positive measure given s. Strict concavity follows by inductions. □

Proof of Lemma I.3

Proof. Suppose some $v_2 < v_1 < \tilde{V}(z)$ exists such that $W(v_2, z) = W(v_1, z)$, then concavity of $W(v, z)$ implies $W(v_2, z) = W\left(\tilde{V}(z), z\right)$, which contradicts Lemma I.1. □

Proof of Lemma I.5

Proof. Substituting (1.19) to (1.20) gives the left-hand side. Expanding and comparing terms with (1.11) establishes the equality. □

The following proofs are based on a Cobb-Douglas production technology $F(k, a, \gamma) = e^z (k^\gamma a^{1-\gamma})^\theta$ with deterministic z and assumes $\eta_k = 0$.

Proof of Proposition I.6

Proof. Suppose some $\gamma_2 < \gamma_1$ and v exist such that $v < \tilde{V}(\gamma_1) < \tilde{V}(\gamma_2)$. The indirect profit function is given by

$$\pi(v, \gamma) = \max_{k', a'} \{-R_k k' - R_k a' + \beta F(k', a', \gamma)\},$$

subject to $v \geq \eta_a a'$. Equality of the enforcement constraint implies that $a'(v, \gamma_1) = a'(v, \gamma_2) = \frac{V}{\eta_a}$. It is easy to check that the optimality conditions for k' (i.e., $R_k = \beta \frac{\partial F(k', a', \gamma)}{\partial k'}$) implies that $k'(v, \gamma)$ is increasing in γ and the indirect profit function can be simplified as $\pi(v, \gamma) = \beta(1 - \theta\gamma) F(k'(v, \gamma), a'(v, \gamma), \gamma) - R_a a'(v, \gamma)$; so $\pi(v, \gamma)$ is increasing in γ given v. Applying standard dynamic programming arguments, $W(., \gamma)$ is increasing in γ following the monotonicity of $\pi(., \gamma)$ in γ. Since $B(v, \gamma) = W(v, \gamma) - v$, $B(., \gamma)$ is also increasing in v.

By Lemma I.2 and I.3, $W(v, \gamma)$ is strictly increasing and strictly concave in v in a region where $v < \tilde{V}(\gamma)$. It follows that $B(v, \gamma)$ is strictly concave in v as well. Now consider the problem of entrants. Suppose I_0 is such that the debtholders' initial participation constrained is satisfies (i.e., $B_0(v_0, \gamma) \geq I_0$) for some $v_0 \geq 0$ so the initial contract is signed. The optimal initial equity value $v_0(\gamma)$ is in a region where $B(v, \gamma)$ is decreasing in v. To see this, suppose otherwise. By the strictly concavity of $B(v, \gamma)$ in v, a $\hat{V}_0$ exists such that $\hat{v}_0 > v_0$ and $B(\hat{v}, \gamma) \geq I_0$, which contradicts $v_0 = \sup\{v : B(v_0, \gamma) \geq I_0\}$. It follows that v maps one-to-one $B(v, \gamma)$, for all possible values of V that satisfy the optimal contract. Now define the transformation $v = B^{-1}(B(v, \gamma), \gamma)$, where $B^{-1}(., \gamma)$ is the inverse function of $B(., \gamma)$. Total differentiation of the transformation function gives

$$0 = \frac{\partial B^{-1}(B, \gamma)}{\partial B} \frac{\partial B(v, \gamma)}{\partial \gamma} + \frac{\partial B^{-1}(B, \gamma)}{\partial \gamma}.$$

$\frac{\partial B^{-1}(B(v,\gamma),\gamma)}{\partial B} < 0$ follows from the argument that $B(v, \gamma)$ is decreasing in v and $\frac{\partial B(v,\gamma)}{\partial \gamma} > 0$ as shown previously. It follows that the value to the firm can be expressed as a function of the debt value $V(B, \gamma)$ and $\frac{\partial V(B,\gamma)}{\partial \gamma} = \frac{\partial B^{-1}(B,\gamma)}{\partial \gamma} > 0$, in particular $\frac{\partial V(I_0,\gamma)}{\partial \gamma} > 0$, which says that given identical initial debt value (i.e. I_0), the initial equity value is increasing in γ; Following $V_0 = \eta_a a_1$ and the monotonicity of k'/a' in γ, entrant asset size is increasing in γ. □

Proof of Proposition I.9

Proof. Let $\lambda \geq 0$ denote the Lagrange multiplier for the enforcement constraint. The optimality conditions with respect to k and a imply $\frac{k}{a} = \frac{\gamma}{1-\gamma} \frac{R_a+\lambda\eta_a}{R_k+\lambda\eta_k}$. Conditional on γ, the size of physical assets relative to intangible assets is increasing in λ if $\eta_a > \eta_k$ and $R_a = R_k$.[46] Recall from Lemma I.2 and Lemma I.3 that the total surplus value W is strictly concave and strictly increasing in v. It follows that as the firm ages, the shadow value λ decreases until the firm reaches the efficient frontier and λ reaches 0. As a result, a firm's share of physical assets is higher than its unconstrained level and fall monotonically until the firm becomes unconstrained. □

1.6.2 Variable measurement

Total debt is long-term debt (item DLTT) plus short-term debt (item DLC). Market value is defined as the sum of total debt, the market value of common equity (CRSP December market capitalization), and the book value of preferred stock (item PSTKRV), minus cash and short-term investments (item CHE) and inventory (item INVT). Tobin's q is market value divided by physical assets. Cash flow is the sum of income before extraordinary items (item IB) and depreciation (item DP). Cash flow rate is cash flow divided by physical assets. To alleviate impacts of outliers, I

[46] $R_a = R_k$ holds when $\delta_a = \delta_k$.

windsorize growth and financial measures (including Tobin's q and market value to total assets) at 2% and 98%.

Compustat reports both stock and flow variables at the end of year t. However, the model requires stock variables subscripted t to be measured at the beginning of year t and flow variables subscripted t to be measured over the course of year t. I take any year t stock variable, for example k_t, from the year $t-1$ balance sheet and any year t flow variable from the year t income or cash flow statement.

CHAPTER II

Embodied Technological Progress and Investment in Vintage Capital

2.1 Introduction

The recent revolution in information technology provides abundant evidence on technological progress that improves the quality of capital: more powerful computers, faster telecommunication equipment, and robotization of assembly lines. The introduction of more efficient capital goods represents an important source of growth from *embodied* technology, which is distinct from the more traditional Hicks-neutral, or *disembodied*, technology; however, embodied technology is hard to disentangle from disembodied technology because of well-known difficulties in productivity measurement. Many recent researchers have used changes in the price of investment goods relative to consumption goods to measure embodied technological progress;[1] however, this approach is controversial because investment price indices may not properly reflect quality improvements in capital.[2]

In this paper, I propose a new vintage capital model with costly capital reversibility to infer the rate of embodied progress from observed capital service life and investment allocation. The model explores the implications of embodied progress on investment demand for capital of different vintages. The intuition is simple. Because

[1]See, for example, Greenwood et al. (1997), Cummins and Violante (2002), and Fisher (2006).
[2]See, for example, Pakes (2003).

the frontier of embodied technology can be adopted only by capital of the latest vintage, a faster rate of embodied progress makes new vintage more advantageous and reduces demand for old vintages. When capital reversibility is costly, the decision to retire old vintages will also depend on the resale value of retired capital. The endogenous exit decision determines the economic service life of capital.

Investment allocation also depends on the substitutability of capital goods. When capital of different vintages are perfect substitutes, all investment goes to the latest vintage; when they are imperfect substitutes, for example, when consumers have a taste for old-fashioned goods, the model features a nontrivial allocation of new investment across capital of different vintages. The basic mechanisms are as follows: Old-fashioned goods, for instance, hand paintings and handcrafted furniture, are produced with vintage capital (brushes and hand tools as opposed to digital printers and assembly lines). As aggregate technology advances, aggregate demand growth provides an incentive to expand the production of old-fashioned goods. Producers face incentives in the opposite direction as well: Modern sectors drive up labor costs and induce old-fashion firms to exit. As a result, technological progress, capital reversibility, and the elasticity of substitution jointly determine capital service life and investment allocation.

The model implications are consistent with three important patterns in the data. (i) Old and new vintages coexists in production for a prolonged period of time. For example, the Volkswagen Beetle was produced in Puebla, Mexico, for 36 years beginning in 1967. The same engine was used in all Beetle models from 1974 to 1993. (ii) Capital has finite service life: Old equipment is retired and production lines are upgraded at regular intervals. (iii) Investment in old capital may occur. For example, after World War II, many steam ships were produced in Germany when motor ships–ships of a newer vintage–were also being produced.

Existing models that can be conveniently used to measure embodied progress cannot interpret these data. These models belong to one of three main categories. In the first category, as in Solow (1960), the production characteristics of machines are fixed once they are built and cannot be changed *ex post*. Because different vintages are perfect substitutes, all investment flows to the latest vintage where the efficiency of investment is highest. Disinvestment in old capital is prohibited, so service life is infinite.[3] In the second category, as in Greenwood et al. (1997), even though capital of different vintages can participate in the same production process, the elasticity of substitution is infinite; consequently, all investment is, again, in the latest vintage. Old capital is never retired so service life is also infinite. Finally, in putty-clay models as in Johansen (1959), capital service life is finite because the old labor-intensive machines are retired as wage rises with embodied growth; however, firms never invest in old vintages.

My model implies a mapping between the rate of embodied progress, capital service life, capital resale value, and aggregate markup in the steady-state equilibrium. I estimate an aggregate capital service life from U.S. postwar data and infer the rate of embodied progress. Using the growth accounting framework implied by the model, I infer the contribution of embodied and disembodied technology to aggregate productivity growth. My results suggest that embodied progress contributes to approximately 62% of the growth of U.S. labor productivity.

My paper contributes to the literature of productivity measure both theoretically and empirically. Theoretically, I show how costly reversibility and imperfect substitution can be embedded in a vintage capital model and how investment in vintage

[3] The assumption for no disinvestment is essential with embodied progress. Without it, firms would like to disinvest in old capital, exchanging it for the new.

capital can be analyzed. Empirically, I use a new approach to estimate the rate of embodied progress that does not rely on the use of investment price indices. These estimates complement results from prior studies.

The paper is organized as follows. Section 2 presents a baseline vintage capital model with costly capital reversibility. Section 3 presents an extended model with imperfect substitution between vintages that allows for continued investment in vintage capital. Empirical strategy and results are discussed. Section 4 concludes.

2.2 Baseline: A vintage capital model with perfect substitution

2.2.1 Elements

Suppose all firms behave competitively and have identical production functions with constant returns in capital and labor. Firms have access to two types of technology: embodied technology Z, and disembodied technology A. The frontier of embodied technology can only be applied to capital of the latest vintage because all capital embodies the technology it is used with. The frontier of disembodied technology can be applied by all vintages. Let X_{vt} be the date t stock of vintage v capital that embodies technology Z_v and let L_{vt} denote the amount of labor that works with technology Z_v. The output produced with technology Z_v is

$$Y_{vt} = A_t X_{vt}^{\alpha} L_{vt}^{1-\alpha}. \tag{2.1}$$

Firms can produce with all technologies so far. Total output is

$$Y_t = \int_{-\infty}^{t} Y_{vt} dv.$$

Let output Y_t be the economy's numéraire. Output is homogeneously divisible into consumption and investment. The cost of investment is unity. Investment has the following properties. First, embodied progress improves the efficiency of investment.

In particular, every unit of investment delivers Z_v units of vintage v capital. In other words, the cost of investment in vintage v is $1/Z_v$. Second, even though existing capital cannot be used with a new technology, it can be recycled at a cost. Recycled capital can recovered $\eta \in (0, 1)$ fraction of its replacement cost. Costly reversibility may arise because of lemon problem in the resale market as in Akerlof (1970), or as a result of the specificity of the capital. The capital law of motion is

$$\dot{X}_{vt} = \begin{cases} Z_v I_{vt} - \delta X_{vt}, \text{ if } I_{vt} \geq 0, \\ \frac{Z_v I_{vt}}{\eta} - \delta X_{vt}, \text{ if } I_{vt} < 0, \end{cases}$$

$\forall t \geq v$, where X_{vt} is measured in the number of efficient units.[4]

As a preview of results to follow, consider the model's implications for a single change in Z. Suppose for $t < t_1$, the embodied technology is Z_0, and for $t \geq t_1$, the technology frontier improves to $Z_1 > Z_0$. After date t_1, optimal investment in the new vintage ensures that the (shadow) value of investment equals its cost: $V_1 = 1/Z_1$. In order to have full employment of labor, the value of investment in the old vintage must drop at date t_1. As I shall show later, the value of old capital depends on embodied progress: $V_0 = 1/Z_1$. Because the cost of investment in the old vintage remains $1/Z_0$, any investment in old vintage entails an immediate capital loss of $1/Z_0 - 1/Z_1$. As a result, firms only invest in the new vintage.

What about existing old vintage? If the resale price is less than its value, $\eta/Z_0 < V_0$, firms will choose to keep the old vintage; otherwise it is optimal to retire and recycle it.

This simple example demonstrates how capital service life is jointly determined by embodied progress and investment reversibility. If embodied progress and capital reversibility are both high, old vintage will be retired at date t_1; otherwise it will

[4]For simplicity, I assume initial capital can be built instantaneously. Firms can also liquidate existing capital stock instantaneously.

continue to operate.

To extend this reasoning to continuous technological progress, I shall proceed with a formal definition of equilibrium. Let embodied and disembodied technology grow at rate g^e and g^d respectively:

$$\dot{Z}_t = g^e Z_t,$$

$$\dot{A}_t = g^d A_t.$$

Let W_t be the wage rate, R_t be the rental rate on new capital. I assume that the economy saves a constant fraction σ of the final output.

The next proposition establishes the definition and existence of the equilibrium and characterizes its properties.

Proposition II.1. *(Elements and properties of an equilibrim) A steady state equilibrium is a sequence of functions of time for factor prices* $\{W_t, R_t, V_{vt}\}, \forall v \leq t$ *and quantities* $\{K_{vt}, I_{vt}, L_{vt}\}$ *such that*

1. Firms maximize profits:

$$(X_{vt}, L_{vt}) = \arg\max_{(x,l)} \left(A_t x^a l^{1-\alpha} - R_t x - W_t l\right),$$

$\forall v \leq t.$

2. The (shadow) value of investment in capital embodying the frontier technology equals its cost:

$$V_{tt} = \frac{1}{Z_t}. \tag{2.2}$$

3. The (shadow) value of a vintage v falls continuously at a rate equal to embodied progress:

$$V_{vt} = \frac{1}{Z_t}, \forall v \leq t. \tag{2.3}$$

4. Old capital is retired at age S when the value of investment equals the resale value:

$$\frac{\eta}{Z_{t-S}} = \frac{1}{Z_t}. \tag{2.4}$$

5. Firms only invest in the latest vintage and operate old vintage for a finite period of time S:

$$X_{vt} = \begin{cases} \sigma Y_t + \frac{\eta X_{(t-S)t}}{Z_{t-S}}, & t = v. \\ X_{vv} e^{-\delta(t-v)}, & t \in (v, v+S)\,. \\ 0, & t \geq v + S. \end{cases} \tag{2.5}$$

6. (Market clearing) Labor and goods markets clear:[5]

$$\int_{-\infty}^{t} L_{vt} dv = L_t.$$

7. (Aggregation) Let $X_t \equiv \int_{t-S}^{t} X_{vt} dv$ denote aggregate capital stock. Aggregate price and output can be expressed as

$$R_t = \alpha \frac{Y_t}{X_t},$$

$$W_t = (1-\alpha) \frac{Y_t}{L_t},$$

$$Y_t = A_t X_t^{\alpha} L_t^{1-\alpha}. \tag{2.6}$$

X_t follows the law of motion

$$\dot{X}_t = \sigma Y_t Z_t - \delta X_t. \tag{2.7}$$

Proof. See Appendix. □

[5]Goods market clearing implies investment supply equals investment demand. Because in equilibrium, firms only invest in the latest vintage, the goods market clearing condition is characterized by the capital law of motion for the latest vintage, given by the first line of equation 2.5.

2.2.2 Investment allocation and capital service life

Investment dynamics characterized by Proposition II.1 imply that capital service life is jointly determined by embodied progress and capital reversibility. Intuitively, a faster rate of embodied progress discourages positive investment in old vintages. Keeping old vintages is costly because embodied progress leads to capital obsolescence; retiring them is also costly because of imperfect reversibility. This trade-off endogenously determines capital service life. Costly reversibility is important to explain investment behavior. Without it, firms would like to disinvest, exchanging their old capital for the new. Capital service life would be zero. In a conventional vintage capital model, disinvestment is prohibited and service life is infinite. Following the relation given by 2.4, data on capital service life S and resale value η informs about the rate of embodied progress g^e.

2.2.3 Growth accounting

The aggregation property of the model makes it suitable for growth accounting. Aggregate growth in total factor productivity (TFP) arises from disembodied growth, embodied growth, and labor growth. For simplicity, I have assumed, up to now, that only one type of capital (i.e., equipment) exists. But it is easy to allow embodied progress to improve the efficiency of equipment but not structures, as it is commonly assumed in the literature.[6] Proposition II.2 shows that aggregate growth then depends on the relative share of equipment and structures.

Proposition II.2. *(Growth accounting) Aggregate output growth g follows*

$$g = \frac{g^d + \alpha^E g^e}{1-\alpha} + n, \tag{2.8}$$

[6]See, for example, Cummins and Violante (2002) and Greenwood et al. (1997).

where α^E is the equipment share, g^d and g^e are growth in disembodied and embodied technology respectively, and n is labor growth.

Proof. See Appendix. □

2.2.4 Calibration

I set parameter values using U.S. data for the postwar period. The model is parsimonious in that only two parameters are needed to infer the rate of embodied progress: capital service life and the resale value of retired capital. First, I use data from the U.S. Bureau of Economic Analysis (BEA) to infer the aggregate capital service life. The BEA publishes service life and the value of fixed assets by detailed asset classes.[7] Using these data, I estimate the value-weighted average service life of all equipment to be 18 years. Second, I refer to previous empirical estimates for the resale value of retired capital. For example, Ramey and Shapiro (2001) collect data from plant closing from three large aerospace companies and estimate resale value of capital at various ages. Their estimates imply an equivalent annual depreciation of 0.04 for 20 year old equipment. Hulten et al. (1989) use data from the Machine Dealers National Association from 1954 to 1983 to estimate economic depreciation rates for machine tools. According to Oliner (1996), their estimates imply an annual depreciation rate of 0.05 for a 12-year-old milling machine. Oliner (1996) uses the same data and estimates the average depreciation rate for all machine tools to be 0.035. Based on these estimates, I set the annual depreciation rate to be 0.035. [8] The implied the resale value in then $\eta = 0.527$ based on geometric depreciation during 18 years of service life.

[7] Source: http://www.bea.gov/scb/account_articles/national/0597niw/maintext.htm. See Table A for capital service life and Table 3 for the value of fixed private capital.

[8] I choose the number at the lower end of the depreciation rate implied by the cited studies. This is because the estimated of those studies are based on a sample of retired capital and liquidated capital resulting from plant closing. In generally resale value of retired capital is higher than liquidated capital, perhaps because plant closing leads to deeper discount; therefore, the implied depreciation rate should be smaller.

Parameter	Calibration	Data
g	0.025	GDP growth
n	0.005	employment growth
$1-\alpha$	0.68	labor share
α^E	0.19	equipment share
η	0.527	resale price of capital
S	18.0	capital service life

Parameter	Estimate	Interpretation
g^e	0.0356	embodied growth
g^d	0.0068	disembodied growth
g^e share	0.6253	share of embodied growth
g^d share	0.3747	share of disembodied growth

Table 2.1: Calibration: The Model with Perfect Substitution

Once the rate of embodied progress is obtained, the rate of disembodied progress can be calculated as the residual from aggregate TFP measure following (2.8). This identity also allows me to break down the contribution of embodied and disembodied progress on labor productivity: The share of embodied growth is $\frac{g^e}{(1-\alpha)(g-n)}$ and the share of disembodied progress is $\frac{\alpha^E g^d}{(1-\alpha)(g-n)}$. Three more parameters are need for this exercise. Following conventions in the literature, I set the long-run growth rate of GDP $g = 0.025$, labor growth $n = 0.005$. I choose $\alpha = 0.32$ to match a labor share of 68% and $\alpha^E = 0.19$ so that equipment represents 60% of the aggregate capital.

The calibrated values and results are summarized in Table 2.1. Embodied technology grows at 0.0356 annually, it accounts for 62.5% of the growth in labor productivity in the post-war period. Disembodied technology accounts for the rest, 37.5%.

2.3 Extension: A vintage capital model with imperfect substitutes

The baseline model shows how the demand for vintage capital is affected by embodied progress and how the rate of embodied progress can be inferred from data on capital service life and resale value. For parsimony, the model assumes that different vintages are perfect substitutes, so investment in old vintage never occurs. To use information on investment allocation on different vintages, the model can be extended to introduce imperfect substitution. This section presents an extension where different vintages are imperfect substitutes because consumers have taste for old-fashioned goods. The demand for vintage capital, driven by aggregate demand and factor prices, introduces a nontrivial investment allocation between capital vintages. The incorporation of other realistic features such as exogenous exits and expanding capital variety also leads to more comprehensive quantitative results.

2.3.1 Elements

The intermediate-goods sector consists of a continuum of monopolistic firms indexed by i and the time of entry v. Each firm owns a capital variety and produces a differentiated intermediate good, using labor and variety-specific capital X_{ivt}:

$$Y_{ivt} = A_t X_{ivr}^{\alpha} L_{ivt}^{1-\alpha}, \alpha \in (0,1), \tag{2.9}$$

where A_t is the time t disembodied technology that grows at constant rate g^d

$$\dot{A}_t = g^d A_t. \tag{2.10}$$

As before, X_{vt} embodies technology Z_v so the cost of investment is $1/Z_v$. Reversing capital is costly and only recovered $\eta \in (0,1)$ fraction of its replacement cost can be recovered from recycled capital. The capital law of motion is

$$\dot{X}_{ivt} = \begin{cases} Z_v I_{ivt} - \delta X_{ivt}, \text{ if } I_{vt} \geq 0. \\ \frac{Z_v I_{vt}}{\eta} - \delta X_{ivt}, \text{ if } I_{vt} < 0. \end{cases} \tag{2.11}$$

The final-goods sector produces final output with a constant elasticity of substitution (CES) technology from differentiated intermediate goods:

$$Y_t = \left(\int_0^{Q_t} Y_{ivt}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1, \tag{2.12}$$

where ε is the elasticity of substitution between intermediate goods and Q_t denotes the number of varieties at time t. New varieties arrive at exogenous rate $\varphi > 0$. At exogenous rate $\lambda > 0$, an existing variety becomes obsolete. Think of it as a new variety being introduced with perfect substitutability to an old one. The capital used to produce the intermediate good becomes useless and the monopolistic firm exits. Given variety entrant and exit, Q_t follows the law of motion

$$\dot{Q}_t = \varphi Q_t - \lambda Q_t,$$

with Q_0 given. The economy can have either expanding variety (if $\varphi > \lambda$) or constant variety (if $\varphi = \lambda$).[9] For simplicity, I assume that the number of varieties of the same vintage also grows at rate $\varphi - \lambda$.

Many equivalent institutional arrangements can support the production structure as described. For example, blueprints of production design come from innovations outside production sectors. Members of households discover blueprints exogenously through luck or inspiration and appropriate them with patents. A firm buys one and only one patent, becomes a monopolistic firm, produces an intermediate good variety, and sells it to the final-goods sector. The assumption that patent owners are the sole producers is only a convenience. They extract the same monopoly profits whether they produce the goods by themselves or not.

Let P_{ivt} be the price of intermediate good i. Profit maximization in the final-goods sector gives the demand function for intermediate good i

[9] For simplicity, I also assume that the number of varieties of the same vintage also grows at rate $\varphi - \lambda$.

$$Y_{ivt} = Y_t \left(\frac{P_{ivt}}{P_t} \right)^{-\varepsilon}, \tag{2.13}$$

and the price index for final output

$$P_t = \left(\int_0^{Q_t} P_{ivt}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} = 1.$$

Let final output be the numéraire and normalize the price index to 1. With this normalization, Y_t also equals value of final output. In what follows, where no confusion occurs, I shall suppress the index i.

Household. The economy has a representative household. The household supplies labor inelastically and saves a constant fraction σ of its income.

2.3.2 Steady-state equilibrium

Optimal investment decision

The economy has no aggregate uncertainty so the interest rate is constant r. Equilibrium properties on the production-side can be analyzed as follows. *A continuing firm* with capital X_{vt} is valued at $V(X_{vt})$, which equals the expected sum of the present value of operating profit before exit and the capital resale value upon exit. The expectation is taken with respect to exit time τ.[10]:

$$\begin{aligned} & V(X_{vt}) \\ = & \max_{\{L_{vs}, I_{vs}, P_{vs}\}} \int_t^{\infty} \lambda e^{-\lambda(\tau-t)} \left[\int_t^{\tau} e^{-r(s-t)} \left(P_{vs} Y_{vs} - I_{vs} - W_s L_{vs} \right) ds + e^{-r(\tau-t)} \frac{\eta X_{v\tau}}{Z_v} \right] d\tau \end{aligned}$$

subject to

$$\dot{X}_{vs} = \begin{cases} Z_v I_{vs} - \delta X_{vs}, \forall s \geq v, \text{ if } I_{vs} \geq 0, \\ \frac{Z_v I_{vs}}{\eta} - \delta X_{vs}, \forall s \geq v_i, \text{ if } I_{vs} < 0, \end{cases} \tag{2.14}$$

[10] Under the assumption that obsolescence occurs with Poisson rate λ, survival time follows an exponential distribution with mean $1/\lambda$.

$$P_{vs} = \left(\frac{Y_s}{Y_{vs}}\right)^{\frac{1}{\varepsilon}}, \tag{2.15}$$

$$Y_{vs} = A_s {X_{vs}}^{\alpha} {L_{vs}}^{1-\alpha}, \tag{2.16}$$

$$X_{vt} \text{ is given}, \tag{2.17}$$

where the term in the square bracket is the value of capital upon exit at time τ.

In the maximization problem (3.18), a firm faces uncertainty on exit time. Applying Fubini's Theorem to the double integral, (3.18) simplifies to

$$V(X_{vt}) = \max_{\{P_{vs}, I_{vs}, L_{vs}\}} \int_t^{\infty} e^{-(\lambda+r)(\tau-t)} \left(P_{v\tau} Y_{v\tau} - I_{v\tau} - W_\tau L_{v\tau} + \lambda \frac{\eta X_{v\tau}}{Z_v} \right) d\tau,$$

subject to (2.14) to (2.17). This simplification follows from the assumption of a Poisson exit rate. It implies that the original problem is equivalent to one with discount rate $\lambda + r$ and no uncertainty on exit. Thus, the firm's investment policy is equivalent to that in a perfect foresight equilibrium.

In equilibrium, a firm's investment policy depends on its current capital stock and aggregate conditions. In particular, firms make positive investment in a capital vintage if the marginal value is greater than the marginal cost; they make negative investment if the marginal value is less than the marginal cost. Because monopolistic firms face decreasing demand, the marginal value of capital depends on the size of the capital stock and macroeconomic conditions. Because of embodied technology and costly reversibility, the marginal cost of capital is vintage specific and depends on whether the firm makes positive or negative investment. These results are formally summarized in the following lemma.

Lemma II.3. *Let $MV(X_{v\tau})$ denote the marginal value of capital $X_{v\tau}$. Let MC_v^+ denote the marginal cost for positive investment and MC_v^-, for negative investment.*

The firm makes positive investment if $MV(X_{v\tau}) < MC_v^+$; it makes negative investment (i.e. disinvestment) if $MV(X_{v\tau}) > MC_v^-$. If η is strictly less than 1, then a region exists such that $MC_v^+ < MV(X_{v\tau}) < MC_v^-$ where the firm neither invest nor disinvest. $MV(X_{v\tau})$, MC_v^+ and MC_v^- are given by

$$MV(X_{v\tau}) = \left(\frac{X_{v\tau}}{Y_\tau}\right)^{\frac{1}{\alpha(1-\varepsilon)-1}} \left(\frac{W_\tau}{1-\alpha}\right)^{-\frac{(1-\alpha)(1-\varepsilon)}{\alpha(1-\varepsilon)-1}} Z_\tau^{-\frac{\varepsilon-1}{\alpha(1-\varepsilon)-1}} \alpha \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\alpha(1-\varepsilon)-1}},$$

$$MC_v^+ = \frac{r+\delta+(1-\eta)\lambda}{Z_v},$$

$$MC_v^- = \frac{[r+\delta+(1-\eta)\lambda]\,\eta}{Z_v}.$$

Proof. See appendix. □

An entrant firm chooses the optimal initial capital by maximizing the value of capital net of its production cost. For simplicity, I assume that entrant capital is built instantaneously.[11] Let X_{vt}^0 be the optimal entrant capital.

$$X_{vt}^0 = \arg\max_{X_{vt}} \left[V(X_{vt}) - \frac{X_{vt}}{Z_v}\right] \tag{2.18}$$

With perfect foresight and exponential discounting, the entrant firm makes investment so that initial capital is on the optimal path; in other words, the optimal initial capital is such that its marginal value equals to its marginal cost

$$V'\left(X_{vt}^0\right) = \frac{r+\delta+(1-\eta)\lambda}{Z_v}, \tag{2.19}$$

where V' denotes the first derivative. This is the user cost of capital adjusted for exit hazard, $(1-\eta)\lambda$, and the cost of investment $1/Z_v$.

Proof. See appendix. □

[11] This assumption is innocuous. Allowing for gradual accumulation of entrant capital stock will complicate the problem without affecting qualitative implications of the model.

A few remarks are in order. The user cost of capital reflects the compensation for forgone interest r/Z_v and depreciation Δ/Z_v, where the depreciation rate $\Delta \equiv \delta + (1-\eta)\lambda$ captures the rate of wear and tear δ and obsolescence $(1-\eta)\lambda$. The obsolescence term reflects capital obsoletes at rate λ, in which case exiting capital loses $1-\eta$ fraction of its book value. Depreciation rate in standard neoclassical models does not have the obsolescence term because capital does not obsolete ($\lambda = 0$) and fully recovers its book value ($\eta = 1$). Standard neoclassical models do not have the production cost term because they assume unit cost of capital, that is, $Z_v = 1, \forall v$.

To complete the description of a firm's investment problem, I also allow for voluntary liquidation. Liquidation is optimal if the expected value of capital is less than its liquidation value.

$$V(X_{vt}) < \frac{\eta X_{v\tau}}{Z_v}$$

In equilibrium, firms will not exercise the liquidation option because $V(X_{vt}) > \frac{X_{vt}}{Z_v}$ always holds as a result of monopoly profits.

Aggregation

Aggregate investment I_t at time t is given by the sum of investment from entrant and continuing firms.

$$I_t = \varphi A_t \frac{X_{itt}}{Z_t} + \int_0^{Q_t} I_{ivt} di, \tag{2.20}$$

where φA_t is the measure of entrant firms and X_{itt}/Z_t is the investment of an entrant firm, $I_{ivt}, v \neq t$, is the investment of continuing firm i.

Let K_t be the time t aggregate book value of capital.

$$K_t \equiv \int_0^{Q_t} \frac{X_{ivt}}{Z_v} di$$

Capital exits at Poisson rate λ and is liquidated for η_v fraction of its book value. Let E_t be the time t aggregate value of reinvested capital; then E_t equals the value of capital in exiting firms and disinvestment in continuing firms.

$$E_t = \eta\lambda \int_0^{Q_t} \frac{X_{ivt}}{Z_v} di + \int_0^{Q_t} \frac{\eta X_{ivt}}{Z_v} \mathbb{I}\left(\dot{X}_{ivt} < 0\right) di. \tag{2.21}$$

where $\mathbb{I}_{ivt}$ is an indication function that takes a value of 1 if X_{ivt} is disinvested at time t and 0 otherwise.

Equilibrium properties

I now have all the elements for the definition of general equilibrium.

Definition II.4. An equilibrium is a time path for prices $\{P_{ivt}, W_t, r_t\}$, $\forall v \leq t, \forall t \geq 0$ and quantities $\{Y_t, Y_{ivt}, X_{ivt}, I_{ivt}, L_{ivt}, C_t\}$, $\forall v \leq t, \forall t \geq 0$ satisfying

1. Profit maximization in the final-goods sector, with the final good producer taking P_{ivt} as given;

2. Profit maximization in the intermediate-goods sector, with intermediate good producers choosing the time path of P_{ivt} and I_{ivt}, subject to the capital law of motion (2.11) and inverse demand function (2.13), taking X_{ivt} and W_t as given;

3. Full employment of labor

$$\int_0^{Q_t} L_{ivt} di = L_t;$$

4. Investment market clearing. Aggregate investment expenditure equals the sum of household saving and the value of disinvested capital.

$$I_t = \sigma Y_t + E_t. \tag{2.22}$$

Proposition II.5. *1. The aggregate production function is*

$$Y_t = A_t \left(\mathcal{Z}_t K_t\right)^{\alpha} L_t^{1-\alpha}, \tag{2.23}$$

where

$$\mathcal{Z}_t \equiv \left(\int_0^{Q_t} Z_{iv}^{\alpha(\varepsilon-1)} di \right)^{\frac{1}{\alpha(\varepsilon-1)}} \tag{2.24}$$

is an index for aggregate capital quality. Z_{iv} *denotes the technology embodied in variety* i *of vintage* v*.*

$$K_t \equiv \int_0^{Q_t} \frac{X_{ivt}}{Z_v} di;$$

2. Aggregate capital law of motion is

$$\dot{K}_t = \sigma Y_t - \Delta K_t, \tag{2.25}$$

where $\Delta \equiv \delta + (1-\eta)\lambda$ *is the depreciation rate.*

Proof. See Appendix. □

Proposition II.5 says that the economy can be characterized by a simple aggregate production function and a capital law of motion. Equations (2.23) and (2.25) are mathematically identical to the familiar Solow (1956) model but have different measures of TFP and capital input. (2.23) measures TFP with $A_t \mathcal{Z}_t{}^{\alpha}$. This measure incorporates neutral technology and capital quality, the latter incorporating embodied technology and capital variety. The corresponding measure of capital input is the aggregate book value of capital. The capital law of motion can be used to match the National Income and Product Accounts (NIPA) data, which uses a perpetual inventory equation with a constant depreciation rate to construct the aggregate capital stock. (2.25) implies that the depreciation rate of NIPA corresponds to $\Delta \equiv \delta + (1-\eta)\lambda$.

The following proposition characterizes the aggregate growth rate.

Proposition II.6. *(Growth accounting) Aggregate output growth* g *follows*

$$g = \frac{g^d + \alpha^E g^e}{1-\alpha} + \frac{\varphi - \lambda}{(1-\alpha)(\varepsilon - 1)} + n \tag{2.26}$$

Proof. See Appendix. □

The aggregate production function (2.23) shows that labor productivity growth comes from disembodied growth, embodied growth and expanding variety. When variety creation occurs more rapidly than obsolescence, that is, $\varphi > \lambda$, the expanding variety effect is positive and decreasing in the elasticity of substitution ε. Interestingly, when $\varphi = \lambda$, (2.26) is the same as the growth accounting identity (2.8) in the baseline model with perfect substitution.

Investment in vintage capital In the model, investment demand depends on technological progress and the degree of capital reversibility. To see this, consider an entrant firm at time t. Optimal entrant investment equates the marginal value of X_{tt} to its marginal cost: $MR(X_{tt}) = MC_t^+$. Optimal future investment in vintage t should also equate the marginal value to its marginal cost. Following Lemma II.3, the marginal value depends on aggregate demand and factor prices; however, the marginal cost of investment is constant at MC_t^+ or MC_t^-. Firms invest in a region where the marginal value is greater than MC^+ and disinvest in a region where marginal value is less than MC^-. Firms do not invest nor disinvest when the marginal value is in the interval (MC^-, MC^+). The investment decision is characterized by two threshold conditions: (C1) the condition under which the firm is indifferent of disinvesting in existing vintages and (C2) the condition under which the firm is indifferent of investing.

Condition (C1) can be derived as follows. Entrant capital is given by Lemma II.3:

$$X_{tt} = Y_t A_t^{\varepsilon-1} \left(\frac{W_t}{1-\alpha}\right)^{(1-\alpha)(1-\varepsilon)} \left(\frac{MC_t^+}{\alpha}\right)^{\alpha(1-\varepsilon)-1} \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}.$$

Let dt be an infinitesimal time interval. With no investment, a unit of capital is reduced by δdt. The firm finds disinvestment optimal if the marginal value of the undepreciated capital falls sufficiently fast that it is under the marginal cost for disinvestment. In other words, the disinvestment threshold is

$$X_{v,t+dt} = Y_{t+dt} A_{t+dt}^{\varepsilon-1} \left(\frac{W_{t+dt}}{1-\alpha}\right)^{(1-\alpha)(1-\varepsilon)} \left(\frac{MC_v^-}{\alpha}\right)^{\alpha(1-\varepsilon)-1} \left(\frac{\varepsilon}{\varepsilon-1}\right)^{-\varepsilon}.$$

Let $dt \to 0$. Recall that by definition $MC_v^- = \eta MC_v^+$, and in the steady state, Y, A, and W grow at rate g, g^d, and $g-n$ respectively. The two equations above can be rewritten as

$$-\delta = g + (\varepsilon-1)\, g^d - (1-\alpha)(\varepsilon-1)(g-n) + [\alpha(\varepsilon-1)+1](1-\eta).$$

If the left-hand side is less than the right-hand side, disinvestment occurs, in which case $1-\eta$ can be interpreted as instantaneous depreciation (or capital loss). No disinvestment at time $t+dt$ does not necessary mean that disinvestment will never occur. In particular, when capital loss due to resale is front-loaded–young capital loses a larger proportion of its value in resale–the cost of disinvestment will be falling. The retirement of capital at age S is optimal when the following equality holds:

$$-\delta = g + (\varepsilon-1)\, g^d - (1-\alpha)(\varepsilon-1)(g-n) + [\alpha(\varepsilon-1)+1](1-\eta(S)), \tag{2.27}$$

where $1-\eta(S)$ is the equivalent annual loss resulting from capital resale.[12]

Combining (2.26) and (2.27) gives a *service life condition* relating embodied growth g^e and capital service life S:

$$g^e = \frac{g+\delta-(\varphi-\lambda)+[\alpha(\varepsilon-1)+1](1-\eta(S))}{(\varepsilon-1)\,\alpha}. \tag{2.28}$$

This is one of the main results on the model. It implies that capital service life is long if embodied progress g^e is low, reversibility is costly, and the elasticity of substitution

[12] For example, if capital of age S sells for a fraction η of its value, then $\eta(S)$ satisfies $\eta(S)^S = \eta$.

ε is low. These parameters can be linked to output growth g and variety growth $\varphi-\lambda$, which are observed in the steady state.

In the model, firms may also invest in old capital. Condition (C2) can be derived in exactly the same manner as condition (C1). Investment is optimal up to a point where the marginal value of capital equals its marginal cost, so

$$-\delta < g + (\varepsilon - 1) g^d - (1-\alpha)(\varepsilon - 1)(g - n).$$

It follows that the *investment condition* is given by

$$g^e < \frac{g + \delta - (\varphi - \lambda) + [\alpha(\varepsilon - 1) + 1]}{(\varepsilon - 1)\alpha}. \tag{2.29}$$

It is easy to see that (2.29) can be viewed as a special case of the *service life condition* (2.27) by setting $\eta = 0$. This observation has a natural interpretation. When investment is perfectly reversible (i.e. $\eta = 1$), the investment decision is smooth, so the thresholds for investment and disinvestment collapse to a single point.

This analysis also suggests that the condition that firms invest in old capital imposes upper bounds on the rate of embodied progress.

2.3.3 Calibration

Parameter values

I use the same set of parameter values described in Section 2.2.4. Additionally, the extended model introduces the elasticity of substitution ε and the degree of expanding variety. The specification of the model leads to a straightforward way to calibrate ε. The CES aggregate in (2.12) implies that the monopoly markup is given by $\varepsilon/(\varepsilon - 1)$. I choose $\varepsilon = 16$ to match aggregate profits of 7%.[13] I consider two calibrated values for $\varphi - \lambda$. First, I consider the case of expanding variety, which is consistent with evidence on the growing variety of products. For example,

[13]For estimates of aggregate profits, see, for example, Domowitz et al. (1988) and Cummins and Violante (2002).

Parameter	Calibration	Data
g	0.025	GDP growth
n	0.005	employment growth
$1-\alpha$	0.68	labor share
α^E	0.21	equipment share
$1-\eta(T)$	0.04	annual depreciation
S	18.0	capital service life
ε	16	elasticity of substitution
δ	0.01	rate of wear and tear

Table 2.2: Calibration: The Model with Imperfect Substitution

Jovanovic and Rousseau (2005) estimate that the annual growth rate of patents is 1.9% and trademark is 3.9% in US between 1971 and 2006. The number of business establishment grew at 2.4% between 1977 and 2005. [14] I use $\varphi - \lambda = 0.025$, which is in the range of values suggested by the data. Second, I consider the simpler case of constant variety: $\varphi - \lambda = 0$, which is a commonly assumed in prior papers of productivity measurement. Finally, I choose the rate of wear and tear $\delta = 0.01$. The parameter values are summarized in Table 2.2.

Numerical results

I present estimates for the constant variety case and the expanding variety case. In each case, I first infer the rate of embodied progress g^e using the *service life condition* (2.28). I interpret the result as a point estimate of the rate of embodied progress that is consistent with the aggregate capital service life.

Case 1: Expanding variety. Table 2.3 presents estimation results with $\varphi - \lambda = 0.025$. Embodied progress is 4.44% and disembodied progress is 0.35%. The contribution of labor productivity growth from embodied progress ranges is 62%. The contribution by disembodied progress and expanding variety is 26% and 12% respec-

[14] To obtain the growth rate of business establishment, I calculate the average birth rate of business establishment at 13.6 percent and the average exit at 11.2 percent from the Business Dynamics Statistics (BDS). The data period is from 1977 to 2005.

Parameter	Calibration	Interpretation
g^e	0.0444	embodied progress
g^d	0.0035	disembodied progress
$\varphi - \lambda$	0.025	expanding variety
$S(embodied)$	0.6200	contribution of embodied progress
$S(disembodied)$	0.2575	contribution of embodied progress
$S(variety)$	0.1225	contribution of expanding variety

Table 2.3: Estimation Results: Expanding Variety

Parameter	Calibration	Interpretation
g^e	0.0496	embodied progress
g^d	0.0042	disembodied progress
$\varphi - \lambda$	0	expanding variety
$S(embodied)$	0.6927	contribution of embodied progress
$S(disembodied)$	0.3073	contribution of disembodied progress
$S(variety)$	0.00	contribution of expanding variety

Table 2.4: Estimation Results: Constant Variety

tively.

Case 2: Constant variety. Table 2.4 presents estimation results with $\varphi - \lambda = 0$. The values for embodied and embodied progress is the same as in the expanding variety case but their contributions differ from before. Embodied growth is 5.0% and the contribution of labor productivity growth from embodied progress ranges is 69%. The remaining growth is explained by disembodied progress.

2.3.4 Implications for the use of investment price indices

A number of recent studies include the use of investment price indices to estimate embodied progress (e.g., Greenwood et al. (1997), Cummins and Violante (2002), Fisher (2006)). This approach is practically very attractive because investment price indices can be used to estimate the time series of embodied progress on the aggregate and industry level; however, criticism to this approach has also arisen (e.g., Pakes (2003)). To date, no broad agreement has emerged in the literature on whether and how investment price indices bias the estimation of embodied technology. Two distinct features of my model encourage its use as an independent check of prior estimates. My theoretical model yields a new mapping between parameters of technology and data independent of investment price indices. Additionally, my theoretical model nests prior models for productivity measurement, in particular the Greenwood et al. (1997) model, as special cases. As a result, investment price indices can also used in my model, which can be compared to my benchmark results independent of investment price.

The estimation based on investment price indices is obtained as follows. I calibrate the embodied growth g^e using changes in an investment price index P_t^I but keep the other calibration equations as before.

Define time t investment price index P_t^I as the ratio of investment expenditure to units of newly produced capital:

$$P_t^I \equiv \frac{\varphi A_t I_{it}(t) + \int_0^{A_t} I_{it}(v_i)di}{\varphi A_t X_{it}(t) + \int_0^{A_t} X_{it}(v_i)di}. \tag{2.30}$$

The growth rate of P_t^I equals the negative of the growth rate of Z_t in the steady-state equilibrium:

$$\frac{\dot{P}_t^I}{P_t^I} = -g^e.$$

I calibrate $g^e = 0.04$ to match the rate of decline of the quality-adjusted equipment price index at an average annual rate of 2.6%. [15] This implies that the growth rate of embodied technology is 0.04 and the fraction of labor productivity growth explained by embodied growth is 56%. These estimates are similar to that of Greenwood et al. (1997). Their calibrated model using a quality-adjusted equipment price index (Gordon (1990)) suggests that the fraction of labor productivity growth explained

[15] The annual rate of 2.6% is based on Jason G. Cummins and Giovanni L. Violante (2002), who extrapolate Gordon's (1990) quality-adjusted equipment price index to 2000 and calculate the annual growth rate of the index from 1954 to 2000, using a weight average of equipment price and structure price.

by embodied technology is 58% from 1954 to 1990. This result is reassuring because my theoretical model does not contradict earlier models. This is expected because the Greenwood et al. (1997) model is a special case of mine with perfect substitution of capital ($\varepsilon = \infty$) and perfect reversibility ($\eta = 1$) and no exits. Using the same data to match aggregate capital quality should yield identical results.

These estimates are very similar to those reported in Table 2.3 and 2.4 (for the model with imperfect substitution) and Table 2.1 (for the model with perfect substitution). It suggests that the implications of investment price data are consistent with those of the data on capital service life and resale value.

2.4 Conclusion

Existing models that can be conveniently used for productivity measurement cannot explain one or more features of observed investment behaviors: (i) Old and new vintages coexists in production; (ii) capital service life is finite; (iii) investment in old capital may occur. These models also rely on investment price indices to estimate embodied progress–a controversial approach. This paper proposes a vintage capital model that incorporates costly capital reversibility and the endogenous exit of old capital vintages. The implications of the model are not only consistent with these investment behaviors but also offer a new strategy to infer embodied progress using data on capital service life and investment allocation. The quantitative results suggest that embodied progress has been an important source of growth, accounting for approximately 62% of aggregate growth in labor productivity in the US during the postwar period.

2.5 Appendix

2.5.1 Proofs

Proof of Proposition II.1 Take any technology $v > t - S$ and any date $t \geq v$,

$$\frac{R_{vt}}{R_{tt}} = \frac{V_{vt}}{V_{tt}}.$$

The first-order conditions for profit maximization are

$$\alpha \frac{Y_{tt}}{X_{tt}} = R_{tt} \tag{2.31}$$

$$\alpha \frac{Y_{vt}}{X_{vt}} = R_{vt} = \frac{V_{vt}}{V_{tt}} R_{tt} \tag{2.32}$$

$$(1-\alpha)\frac{Y_{vt}}{L_{vt}} = (1-\alpha)\frac{Y_{tt}}{L_{tt}} = W_t. \tag{2.33}$$

The production function (2.1) and first-order condition (2.33) imply

$$1 = \frac{Y_{tt}/L_{tt}}{Y_{vt}/L_{vt}} = \frac{\left(\frac{X_{tt}}{L_{tt}}\right)^{\alpha}}{\left(\frac{X_{vt}}{L_{vt}}\right)^{\alpha}}. \tag{2.34}$$

Dividing (2.32) by (2.31), and using (2.33) and (2.34),

$$\frac{V_{vt}}{V_{tt}} = \frac{Y_{vt}/X_{vt}}{Y_{tt}/X_{tt}} = \frac{Y_{vt}/L_{vt}}{Y_{tt}/L_{tt}} \frac{X_{tt}/L_{tt}}{X_{vt}/L_{vt}} = 1.$$

From (2.2) and the above equation,

$$V_{vt} = V_{tt} = \frac{1}{Z_t} < \frac{1}{Z_v} \tag{2.35}$$

This immediately implies

$$I_{vt} = 0, \forall v < t,$$

and

$$\eta = V_{v,v+S} Z_v = \frac{Z_v}{Z_{v+S}}.$$

Since $0 < \eta < 1$, the service life S is finite and constant and satisfies

$$\eta = e^{-g^e S}.$$

From the goods market clearing condition,

$$I_{vt} = \begin{cases} \sigma Y_t + \eta \frac{X_{t-S,t}}{Z_{t-S}}, & v = t. \\ 0, & v < t. \end{cases}$$

Integrating investment with respect to time gives (2.5).

To derive aggregate output, note (2.34) implies

$$\frac{X_{vt}}{L_{vt}} = \frac{X_{tt}}{L_{tt}}.$$

Therefore,

$$\int_{t-S}^{t} \frac{X_{vt}}{L_t} dv = \int_{t-S}^{t} \frac{X_{vt}}{L_{vt}} \frac{L_{vt}}{L_t} dv = \int_{t-S}^{t} \frac{X_{tt}}{L_{tt}} \frac{L_{vt}}{L_t} dv = \frac{X_{tt}}{L_{tt}}$$

Using (2.3), (2.1), and the above expression,

$$\begin{aligned} Y_t &= \int_{t-S}^{t} Y_{vt} dv = \int_{t-S}^{t} A_t X_{vt}^{\alpha} L_{vt}^{1-\alpha} dv \\ &= A_t \int_{t-S}^{t} \left(\frac{X_{vt}}{L_{vt}} \right)^{\alpha} L_{vt} dv = A_t \left(\frac{X_{tt}}{L_{tt}} \right)^{\alpha} L_t \\ &= A_t \left(\int_{t-S}^{t} \frac{X_{vt}}{L_t} dv \right)^{\alpha} L_t = A_t \left(\int_{t-S}^{t} X_{vt} dv \right)^{\alpha} L_t^{1-\alpha} \end{aligned}$$

From (2.32) and let $R_t \equiv R_{vt} = R_{tt}$,

$$\alpha \frac{Y_{vt}}{X_{vt}} = R_t,$$

so

$$\alpha \int_{t-S}^{t} Y_{vt} dv = \int_{t-S}^{t} R_{tt} X_{vt} dv = R_{tt} X_t$$

and

$$\alpha Y_t = R_t X_t.$$

From (2.33),

$$(1-\alpha)\,Y_t = (1-\alpha)\int_{t-S}^{t} Y_{vt}dv = W_t\int_{t-S}^{t} L_{vt}dv = W_tL_t.$$

The capital law of motion is$Z_vI_{vt} - \delta X_{vt}$

$$\begin{aligned}
\dot{X}_t &= X_{tt} - X_{t-S,t} + \int_{t-S}^{t}\dot{X}_{vt}dv \\
&= \left(\sigma Y_t + \eta\frac{X_{t-S,t}}{Z_{t-S}}\right)Z_t - X_{t-S,t} - \delta X_t \\
&= \sigma Y_tZ_t - \delta X_t.
\end{aligned}$$

Proof of Proposition II.2 Goods market clearing implies that aggregate investment and output grow at the same rate g. Assume $X_t = \left(X_t^E\right)^{\alpha^E/\alpha}\left(X_t^S\right)^{\alpha^S/\alpha}$, where α^E is the equipment share and α^S is the structure share. Following the law of motion (2.7), X grows at rate $g+\alpha^Eg^e$. The form of aggregate output (2.6) implies that $g = g^d + \alpha\left(g+\alpha^Eg^e\right) + (1-\alpha)\,n$. Rearranging this identity gives the result.

Proof of Lemma II.3 Applying Fubini's Theorem to the double integral, (3.18) simplifies to

$$V(X_{it}) == \max_{\{L_{vs},I_{vs},P_{vs}\}}\int_t^{\infty} e^{-(\lambda+r)(\tau-t)}\left(P_{v\tau}Y_{v\tau} - I_{v\tau} - W_\tau L_{v\tau} + \lambda\eta\frac{X_{v\tau}}{Z_v}\right)d\tau,$$

subject to (2.14) to (2.17). For easy of exposition, rewrite (2.14) as

$$\dot{X}_{v\tau} = \frac{Z_v}{\eta_x}I_{v\tau} - \delta X_{v\tau}, \forall\tau\geq v,$$

and let $\eta_x = 1$ if $I_{v\tau}\geq 0$ and $\eta_x\in[0,1]$ if $I_{v\tau}<0$.

Substitute $Y_{v\tau} = A_\tau X_{v\tau}^{\alpha}L_{i\tau}^{1-\alpha}$. The present value Hamiltonian is given by

$$\begin{aligned}
H \;=\;& e^{-\lambda(\tau-t)-\overline{r}(t,\tau)}\left[P_{v\tau}A_\tau X_{v\tau}^{\alpha}L_{i\tau}^{1-\alpha} - I_{v\tau} - W_\tau L_{v\tau} + \eta_v\lambda\frac{X_{v\tau}}{Z_v}\right] \\
&+e^{-\lambda(\tau-t)-\overline{r}(t,\tau)}\iota_{v\tau}\left[Y_\tau^{\frac{1}{\varepsilon}}\left(A_\tau X_{v\tau}^{\alpha}L_{i\tau}^{1-\alpha}\right)^{-\frac{1}{\varepsilon}} - P_{v\tau}\right] \\
&+\mu_{v\tau}\left[\frac{Z_v}{\eta_x}I_{v\tau} - \delta X_{v\tau}\right]
\end{aligned}$$

F.O.C.

$$\begin{aligned}
\text{wrt } I_{v\tau} \quad &: \quad e^{-\lambda(\tau-t)-\overline{r}(t,\tau)} = \mu_{v\tau}\frac{Z_v}{\eta_x} \\
&\Leftrightarrow \quad -\lambda - r_\tau = \frac{\dot{\mu}_{v\tau}}{\mu_{v\tau}}, \\
\text{wrt } X_{v\tau} \quad &: \quad e^{-\lambda(\tau-t)-\overline{r}(t,\tau)}\left[\alpha P_{v\tau}A_\tau\left(\frac{X_{v\tau}}{L_{v\tau}}\right)^{\alpha-1} + \frac{\eta\lambda}{Z_v} - \iota_{v\tau}\frac{\alpha P_{v\tau}}{\varepsilon X_{v\tau}}\right] \\
-\mu_{v\tau}\delta \quad &= \quad -\dot{\mu}_{v\tau},
\end{aligned}$$

$$\begin{aligned}
\text{wrt } P_{v\tau} \quad &: \quad Y_{v\tau} - \iota_{v\tau} = 0, \\
\text{wrt } L_{v\tau} \quad &: \quad (1-\alpha)P_{v\tau}A_\tau\left(\frac{X_{v\tau}}{L_{v\tau}}\right)^{\alpha} - W_\tau - \iota_{v\tau}\frac{(1-\alpha)P_{v\tau}}{\varepsilon L_{v\tau}} = 0.
\end{aligned}$$

By (2.36) and (2.36),

$$\frac{Z_v}{\eta_x}\left(\alpha P_{v\tau}A_\tau\left(\frac{X_{v\tau}}{L_{v\tau}}\right)^{\alpha-1} + \frac{\eta\lambda}{q(v_v)} - \frac{\alpha}{\varepsilon}P_{v\tau}A_\tau\left(\frac{X_{v\tau}}{L_{v\tau}}\right)^{\alpha-1}\right) - \delta = -\frac{\dot{\mu}_{v\tau}}{\mu_{v\tau}},$$

$$\frac{\alpha(\varepsilon-1)}{\varepsilon}\frac{q(v_v)}{\eta_x}P_{v\tau}A_\tau\left(\frac{X_{v\tau}}{L_{v\tau}}\right)^{\alpha-1} = r_\tau + \delta + (1-\eta_v)\lambda. \tag{2.36}$$

By (2.36),

$$W_\tau = \frac{(1-\alpha)(\varepsilon-1)}{\varepsilon}P_{v\tau}A_\tau\left(\frac{X_{v\tau}}{L_{v\tau}}\right)^{\alpha}. \tag{2.37}$$

Combine (2.36) and (2.37)

$$P_{v\tau} = \frac{1}{Z_\tau}\left(\frac{W_\tau}{1-\alpha}\right)^{(1-\alpha)}\left(\frac{R_{v\tau}}{\alpha}\right)^{\alpha}\frac{\varepsilon}{\varepsilon-1}, \tag{2.38}$$

where $R_{v\tau} \equiv \frac{[r_\tau+\delta+(1-\eta_v)\lambda]\eta}{Z_v}$.

Substituting (2.38) into (2.13) and collecting terms gives

(2.39)
$$\underbrace{\left(\frac{X_{v\tau}}{Y_\tau}\right)^{\frac{1}{\alpha(1-\varepsilon)-1}}\left(\frac{W_\tau}{1-\alpha}\right)^{-\frac{(1-\alpha)(1-\varepsilon)}{\alpha(1-\varepsilon)-1}}A_\tau^{-\frac{\varepsilon-1}{\alpha(1-\varepsilon)-1}}\alpha\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{\varepsilon}{\alpha(1-\varepsilon)-1}}}_{MV(X_{v\tau})} = \underbrace{\frac{[r_\tau+\delta+(1-\eta)\lambda]\,\eta_x}{Z_v}}_{MC}$$

Equation (2.39) summarizes the optimal (dis)investment decision. The left-hand-side is the time τ marginal value of one unit of capital X_v and the right-hand-side is the marginal cost. The marginal cost equals $MC^+ \equiv \frac{[r_\tau+\delta+(1-\eta_v)\lambda]}{q(v_v)}$ for positive investment and equals $MC^- \equiv \frac{[r_\tau+\delta+(1-\eta_v)\lambda]\eta}{Z_v}$ for negative investment. Call a firm that satisfies (2.39) with $\eta_x = 1$ to be *on the optimal path.* The firm makes positive investment if $MV\left(X_{v\tau}\right) < MC^+$; it makes negative investment (i.e. disinvest) if $MV\left(X_{v\tau}\right) > MC^-$. If η_x is strictly less than 1, (i.e. $MC^- < MC^+$), then there is a region such that $MC^+ < MV\left(X_{v\tau}\right) < MC^-$ where the firm neither invest nor disinvest.

Proof of (2.19) I shall show that an entrant firm's capital is given by equation (2.39) evaluated at $\tau = t$. The first order conditions of an entrant firm's maximization problem (2.18) gives $\frac{d}{dX_{vv}}V(X^0_{vv}) = \dfrac{1}{Z_v}$.

Evaluate the continuing firm i's expected value function (3.18) at entry time v and let $X^*_{vv} = \arg\max\, V(X_{vv})$ denote the optimal capital of the continuing firm given X_{vv}. The co-state variable μ_{vv} equals the shadow value of X^*_{vv} at time v, that is,

$$\frac{d}{dX_{vv}}V(X^*_{vv}) = \mu_{vv}.$$

Evaluating (2.36) at time $\tau = v$ gives

$$\begin{aligned} e^{-\lambda(v-v)-r(v-v)} &= \mu_{vv} Z_v \\ \Leftrightarrow \mu_{vv} &= \frac{1}{Z_v}. \end{aligned}$$

So

$$\frac{d}{dX_{vv}}V(X^*_{vv}) = \mu_{vv} = \frac{d}{dX_{vv}}V(X^0_{vv}),$$

this says that the entrant firm chooses initial capital to be on the optimal path.

Proof of Proposition II.5. By (2.36) and (2.37),

$$R_{vt}X_{vt} = \frac{\alpha(\varepsilon-1)}{\varepsilon}P_{vt}Y_{vt}.$$

Thus,

$$\int_0^{Q_t} R_{ivt}X_{ivt}di = \int_0^{Q_t} \frac{\alpha(\varepsilon-1)}{\varepsilon}P_{ivt}Y_{ivt}di = \frac{\alpha(\varepsilon-1)}{\varepsilon}Y_t. \tag{2.40}$$

Therefore,

$$\frac{R_{ivt}X_{ivt}}{\int_0^{Q_t} R_{ivt}X_{ivt}di} = \frac{P_{ivt}Y_{ivt}}{Y_t} = \left(\frac{Y_{ivt}}{Y_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}.$$

Similarly,

$$\frac{L_{ivt}}{L_t} = \left(\frac{Y_{ivt}}{Y_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}.$$

Using $K_t \equiv \int_0^{Q_t} \frac{X_{ivt}}{Z_v}di$,

$$\begin{aligned}\int_0^{Q_t} R_{ivt}X_{ivt}di &= \int_0^{Q_t} \left[r_t+\delta+(1-\eta)\lambda\right]\frac{X_{ivt}}{Z_v}di \\ &= \left[r_t+\delta+(1-\eta)\lambda\right]K_t.\end{aligned}$$

So

$$X_{ivt} = Z_vK_t\left(\frac{Y_{ivt}}{Y_t}\right)^{\frac{\varepsilon-1}{\varepsilon}}$$

Substituting into (2.9),

$$Y_{ivt} = A_tX_{vt}^{\alpha}L_{vt}^{1-\alpha\varepsilon} = A_t^{\varepsilon}Z_{it}^{\alpha\varepsilon}K_t^{\alpha\varepsilon}L_t^{(1-\alpha)\varepsilon}Y_t^{1-\varepsilon}. \tag{2.41}$$

Substituting into (2.12),

$$Y_t = \left(\int_0^{Q_t} {Y_{ivt}}^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}} = A_t\left(\int_0^{Q_t} Z_{iv}^{\alpha(\varepsilon-1)}di\right)^{\frac{1}{\varepsilon-1}}K_t^{\alpha}L_t^{1-\alpha}. \tag{2.42}$$

Proof of Proposition II.6. Substituting $Q_v = Q_t e^{-(\varphi-\lambda)(t-v)}$ and $Z_v = Z_t e^{-g^e(t-v)}$ into (2.24) gives

$$\mathcal{Z}_t = \left(\frac{\varphi Q_t}{\varphi + g^e\alpha(\varepsilon - 1)}\right)^{\frac{1}{\alpha(\varepsilon-1)}} Z_t.$$

Substituting this into this (2.42),

$$Y_t = Z_t \left(\frac{\varphi Q_t}{\varphi + g^e\alpha(\varepsilon - 1)}\right)^{\frac{1}{\varepsilon-1}} Z_t^{\alpha} K_t^{\alpha} L_t^{1-\alpha}.$$

CHAPTER III

The Tax-Adjusted Q Model of Corporate Investment After Accounting for Intangible Assets: Theory and Evidence from the US in 1998-2006

By Yongjia (Sophia) Chen and Estelle P. Dauchy [1]

3.1 Introduction

This paper sheds new light on research related to the effectiveness of investment tax incentives. We question the accuracy of previous research that analyzes the impact of tax changes on corporate investment. We show that most of this empirical research suffers from misspecification of the model because it ignores intangible assets in corporate investment decisions.

We derive a tax-adjusted q model that includes intangible assets and re-estimate the impact of tax incentive on corporate physical investment using a comprehensive database of industrial assets.

Although the increasing importance of intangible assets is well recognized (Corrado et al. (2009), Dauchy (2013), Fullerton and Lyon (1988), Nakamura (2001)), no research, to our knowledge has considered how they affect the effectiveness of investment tax incentives. Neoclassical theory of investment suggests that firms choose to

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invest until the after-tax cost of assets is equal to tax-adjusted marginal value (i.e., the marginal q). Under constant returns to scale, physical investment is determined by the marginal value of physical assets adjusted for the tax treatment on physical assets, or tax-adjusted q. Although the marginal q of physical assets is usually not observed, our theory suggests that, when tax changes are temporary, it can be approximated from the observed market value per unit of assets (i.e., the average q), tax depreciation allowances, and the share of physical assets. Several episodes of temporary changes in tax depreciation allowances in the early 2000s–known as "bonus depreciation"–provide an opportunity to implement this empirical strategy.[2]

Our theory also suggests that conventional empirical models used to study the effectiveness of investment tax incentives are misspecified because they use the average q as a proxy for the marginal q of physical assets. In the presence of intangible assets, the two are not equivalent. In particular, the average q depends on the marginal q of physical assets, the marginal q of intangible assets, the value of tax depreciation allowances, and the share of physical and intangible assets. The misspecification in existing studies potentially leads to biased estimates of the investment response to changes in both the q term and the tax term. Consider a simple example. A biotech company invests in depreciable equipment assets, which benefit from tax depreciation allowances, and skilled scientists who experiment on a new drug. The firm's average q – the firm's market value divided by the value of equipment and the intangible value of scientists' skills– is a weighted average of the value of equipment and scientists' knowledge. The firm makes investment decisions by comparing the

[2] It is also recognized that firm-specific intangible assets is important in explaining large variations in average firm value, see Gleason and Klock (2006), Megna and Mueller (1991). Corrado et al (2005, 2006) evaluate that from the 1950's decade to the 2000-03 period, the ratio of intangible assets to NIPA tangible assets incleased almost threefold from .54 to 1.36. Their measure of the annual average value of intangible assets (from which we base our own measure of intangible assets at the industry level) in 2000-03 is 1.23 trillion, which is very close to what Nakamura (2000) finds with stock market data.

costs and the benefits of the marginal unit of investment. A change in tax depreciation allowances affects investment in equipment because it affects the after-tax marginal cost of investment. The extent to which investment responds to the tax change depends on the marginal value of equipment in the firm. If this marginal value is not observed by the economist and substituted by an inaccurate proxy, then the estimated investment response will be biased. In the methodology section, we argue that the direction of the bias is unclear, and therefore is an empirical question.

We argue that ignoring intangible assets in previous estimates of the effectiveness of investment tax incentives potentially significantly biases the results because research shows that intangible assets currently represent half of total investment assets (Corrado et al. (2005), Nakamura (2001)), which is also true for intangible assets held by corporations only (Dauchy (2013)).

Our model suggests a new empirical approach to evaluate both the direction and the size of the bias in conventional models. We use a comprehensive panel dataset with investment, assets, tax treatments, and market value. Specifically, we combine firm-level data on physical investment and market value obtained from Compustat, with an industry-level comprehensive dataset that accounts for both physical and intangible assets and which is developed in Dauchy (2013). The resulting dataset, which covers companies and assets from 1998 to 2006, enables us to estimate a tax-adjusted q model of investment in the conventional way (i.e., without intangible assets) and in a correctly specified way with intangible assets. A comparison of these results provides the size and direction of the bias in conventional estimates.

Our results suggest that the bias of the q term and tax terms estimated by conventional models over all corporations is small, provided that the tax terms are properly measured. Nevertheless, we find variation across firms. In order to correctly estimate the impact of tax incentives on investment, controlling for intangibles is more important for large firms than for small firms. This is primary because intangible asssets on average represent a larger fraction of total assets in large firms. We also find that our estimates of the conventional and the correctly specified impact of the equipment and structure tax terms on investment over 8 years are larger than prior studies that use data covering a longer sample period, and that this is the case even if the overall variation is very similar. We note that our sample period only consists of temporary tax incentives–dubbed 'bonus depreciation' (which describes periods when depreciation allowances of short-lived assets were temporarily accelerated), while longer sample periods also cover permanent tax changes.[3] This shows that investment responses to temporary and permanent tax changes are likely to highly depend on idiosyncratic of firms that use them, economic environment, or tax incentive characteristics. Therefore, our findings provide strong evidence on the effectiveness of temporary tax incentive and on the significance of ignoring intangible assets for large firms.

Previous studies on the impact of tax incentives on investment generally use either of two methods to calculate the tax-induced change in the pretax cost of investment: the tax-adjusted average q (Cummins et al. (1994), Cummins et al. (1996), Edgerton (2010)), or the change in the after tax cost of capital weighted by asset types (Jorgenson (1963), House and Shapiro (2008)). Most of this research finds little impact of tax incentives on investment, suggesting implausibly high adjustment costs (Caballero and Engel (1999)), physical assets heterogeneity (Bontempi et al. (2004)), exceptionally low cash flows or asymmetries in taxable status (Edgerton (2010)), or

[3]See Section IV-A and appendix B for more details about temporary tax incentives from 1998 to 2006. Although our database would allow us to cover more recent episodes of temporary tax incentives, we do not include them for worries that they would interfere with the financial crisis. This is explained in more details in the empirical section.

low take-up rates (Knittel (2007)).[4] Our results suggest that previous estimates of the impact of the 2002-2003 bonus depreciation experience, if already small, potentially underestimate the impact of the equipment tax term.

Section II of this paper provides a quick background of the literature on the tax-adjusted q-theory of investment and the difficulties to implement it empirically. Section III proposes a new tax-adjusted q model with intangible assets and discusses its new implications for the empirical approach. Section IV describes our empirical implementation of the model and the data. Section V presents and explains the results. Section VI concludes.

3.2 Intangible Assets and Models of Investment

Many economists have recognized the failure of empirical research to correctly specify neoclassical models of investment based on the q-theory of investment. In spite of this, empirical research generally uses the neoclassical model because of its mathematical soundness and its intuitive implications. The low explanatory power in empirical models using both panel and time-series has been explained in many ways. Among them, models allowing for adjustment costs seem to only provide a partial answer because, depending on the data and methodology, empirical research has either found implausibly large adjustment costs (Hayashi (1982), Summers (1981)), or small adjustment costs (Hall and Jorgenson (1969), House and Shapiro (2008)). Other models have assumed that q does not perfectly capture investment decisions, and that a correct specification should control for other firm-specific characteristics, such as liquidity constraints (Cummins et al. (1996), Gilchrist and Himmelberg

[4] Most of the existing empirical research is based on neoclassical models of investment developed by Jorgenson (1963), and extended by Hayashi (1982) and Summers (1981). Edgerton (2010) finds that the effectiveness of tax incentives depends on firms' taxable status, but that most of this difference is actually explained by low cash flows. Bloom et al. (2009) find that the effectiveness of tax incentives may be dampened by time-varying uncertainty. For other models of investment with heterogeneous fixed assets, see Wildasin (1984) and Cummins and Dey (1998).

(1995)).

Moreover, Hayashi (1982) explains that the requirements for the measured Tobin's q to appropriately proxy for marginal q do not generally hold.[5] If this is true, measured Tobin's q is subject to measurement error. Using a GMM specification with information on the joint distribution of the observed explanatory variables, Erickson and Whited (2000) find a much larger explanatory power of the q-model, and that additional firm-level characteristics, such as proxies for liquidity constraints, are no longer significant. Nevertheless, this strategy is still limited because instead of fully correcting for the measurement error problem, it uses a parametric model that fixes the form of the measurement error. In particular, further research also using a GMM strategy to correct for the measurement error or the endogeneity of average q has found that cash flows may play a significant role (Hayashi and Inoue (1991), Blundell et al. (1992)).

Most of this empirical research is based on the first order condition of the firm's optimization problem, which states that firms invest as long as the shadow value of capital—or marginal q—is larger than the marginal cost of investment. This provides a simple relationship between the investment rate and Tobin's q. However, other research has also noticed that although the neoclassical model of investment holds when investment is a homogenous good, it no longer holds when firms invest in heterogeneous assets (Wildasin (1984), Cummins and Dey (1998), Bontempi et al. (2004)). Hayashi and Inoue (1991) address both the potential endogeneity of average q and the fact that heterogeneous investments prevent a single linear relationship between the investment rate and q. Endogeneity stems from unobserved technological shocks that are captured by firm-specific variables but not by average q. They

[5]For marginal q to be correctly approximated by average q (or Tobin's q), markets need to be efficient, meaning that there must be constant returns to scale and perfect competition (Hayashi, 1982).

instrument for it based on the lagged value of average q and other explanatory variables. Nevertheless, their model does not completely solve the empirical problem, as measures of cash flow and other firm-specific variables still significantly explain the investment rate. Edgerton (2010) evaluates the importance of tax status and internal finance in estimating a q-model of investment on publicly traded companies up to 2005 and finds that previous finding of a lack of effectiveness of investment tax incentives, such as the 2002-03 bonus depreciation provision (Desai and Goolsbee (2004)) are likely to be due to low cash flows during periods of slow growth or recessions, and tax status asymmetries implied by tax code provisions such as tax losses carry-forward.

3.3 Intangible assets and tax-adjusted q: Theory

Our model extends the neoclassical model of investment (Hayashi (1982), Summers (1981)). It allows for intangible assets to accumulate in the stock of capital and to depreciate in the firm maximization strategy. Therefore, firms' q value explicitly incorporates the value of intangible assets, which are observed by firms but not by economists. Traditional models of investment treat intangible assets as intermediate inputs, implying that they are not included in the production function. Nevertheless, prior research has shown that a large part of q that is not explained by companies' fundamentals can be explained by the value of intangible assets. The model extends the relationship between the investment ratio and tax-adjusted q by including intangible assets in the mix of investments, even if they are fully expensed for tax purposes. The model implies that any change in tax policy that only affects assets that are depreciated for tax purposes, such as a change in the corporate tax rate or in depreciation allowances affect both investment in physical assets and in intangible goods, such as R&D and advertising.

3.3.1 The model

Consider a firm that produces with two types of assets K^m (tangible) and K^u (intangible) with a constant return to scale production technology $F\left(K^m, K^u, X\right)$, where X represents stochastic productivity of the firm. The firm decides to invest in each type of assets I^m and I^u to maximize the expected present value of future income stream:

(3.1)

$$V_t = \max_{\substack{\left\{I^i_{t+s}, K^i_{t+s}\right\}^{\infty}_{s=0}, \\ i=\{m,u\}}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta_{ts} \left[\begin{array}{c} \left(1-\tau_{t+s}\right)\left(F\left(K^m_{t+s}, K^u_{t+s}, X_{t+s}\right) -_{i=\{m,u\}} \Psi\left(I^i_{t+s}, K^i_{t+s}\right)\right) \\ -\sum_{i=\{m,u\}}\left(1-k^i_{t+s}-\tau_{t+s}z^i_{t+s}\right) I^i_{t+s} \end{array} \right] \right\}$$

subject to

$$K^i_{t+s+1} = \left(1-\delta^i\right) K^i_{t+s} + I^i_{t+s} \tag{3.2}$$

for $i = \{m, u\}$, where $\mathbb{E}_t$ is the expectations operator conditional on information available in period t, τ is corporate tax rate, k_i represents investment tax credit for assets i and τz_i is the present value of depreciation allowances on a dollar of investment in assets i, and $\Psi\left(I^i_{t+s}, K^i_{t+s}\right)$ represents investment adjustment cost for assets i. As is standard in the literature, adjustment cost is a quadratic and linear homogeneous function for each type of assets, and is parameterized as $\Psi\left(I^i_{t+s}, K^i_{t+s}\right) = \frac{\psi}{2} K^i_t \left(\frac{I^i_t}{K^i_t}\right)^2$. We do not allow for interrelated adjustment costs. β_{ts} is the (possibly stochastic and time-varying) real discount factor applicable in period t to s-period-ahead payoffs with $\beta_{t0} = 1$ and $\beta_{tj} = \beta_{t1} \cdot \beta_{t+1,1} \cdots \beta_{t+j-1,1}$.

Let q^i_t be the Lagrangian multiplier associated with the law of motion (3.2) of assets $i = \{m, u\}$. The first order conditions with respect to I^i_t and K^i_{t+1} are, respectively,

$$q_t^i = 1 - k_t^i - \tau_t z_t^i + (1 - \tau_t)\, \psi \frac{I_t^i}{K_t^i}, \tag{3.3}$$

and

(3.4)
$$q_t^i = \mathbb{E}_t \left\{ \beta_{t1} \left[(1 - \tau_{t+1}) \left(\frac{\partial F\left(K_{t+1}^m, K_{t+1}^u, X_{t+1}\right)}{\partial K_{t+1}^i} - \frac{\psi}{2} K_{t+1}^i \left(\frac{I_{t+1}^i}{K_{t+1}^i} \right)^2 \right) + \left(1 - \delta^i\right) q_{t+1}^i \right] \right\}.$$

From (3.3), we obtain the following expression for investment rate of each type of assets:

$$\frac{I_t^i}{K_t^i} = \frac{q_t^i}{(1 - \tau_t)\, \psi} - \frac{1 - k_t^i - \tau_t z_t^i}{(1 - \tau_t)\, \psi}. \tag{3.5}$$

This equation suggests that investment rate in assets i, $\frac{I_t^i}{K_t^i}$, depends on its own tax-adjusted shadow value $\frac{q_t^i}{(1-\tau_t)\psi}$ (henceforth the q term) and tax treatment $\frac{1-k_t^i-\tau_t z_t^i}{(1-\tau_t)\psi}$ (henceforth the tax term).

Let P_t denote the end-of-period (i.e. ex-dividend) market value of the firm and q_t be the ratio of P_t to the book value of total assets: $q_t \equiv \frac{P_t}{K_{t+1}^m + K_{t+1}^m}$. Since q_t reflects the average market value of per unit of assets, we henceforth call it average q. In our model average q captures the average value the firm's total assets including tangible and intangible. This is a natural extension of the notion of average q in a model with only tangible assets. Yet it is important to note that with more than one type of assets, average q is different from the marginal value of each type of assets. Formally, we show in the next Proposition that under constant return to scale in the production technology and adjustment costs, average q is a weighted average of the marginal q in physical and intangible assets.

Proposition III.1. *The ratio of the ex-dividend market value to the book value of assets q_t is a weighted average of the book value of tangible and intangible assets.*

$$q_t = q_t^m S_t^m + q_t^u \left(1 - S_t^m\right), \tag{3.6}$$

where $S_t^m \equiv \frac{K_t^m}{K_t^m + K_t^u}$ is the share of tangible assets in total assets.

Proof. See Appendix. □

It is easy to see that out model nests as a special case conventional model with only tangible assets. Specifically, conventional model assumes the share of tangible assets equal to 1. Setting $S_t^m = 1$ in (3.6) gives $q_t = q_t^m$, which says that average q equals the marginal value of tangible assets. The investment rate expression (3.5) is then equivalent to

$$\frac{I_t^m}{K_t^m} = \frac{q_t}{\left(1 - \tau_t\right)\psi} - \frac{1 - k_t^m - \tau_t z_t^m}{\left(1 - \tau_t\right)\psi}, \tag{3.7}$$

which implies that the investment rate in tangible assets can be expressed as a function of average q and the tax treatment on tangible assets. Comparing this to the expression of investment rate in the general case (3.5), we know that conventional models have erroneously used average q in place of q^m. Because this implies measurement error in one of the independent variables when the share of physical assets is not 1, conventional estimates of equation (3.7) are biased. When the measurement error in q term correlates with the tax term, estimates of the sensitivity of physical investment with respect to the tax term is biased even when the tax term was measured properly. As a consequence, a model with both physical and intangible assets is essential to correctly evaluate the investment response to tax changes.

If the q term was strictly exogenous to the model, the measurement error in q implies that conventional models of investment tend to overestimate the effect of the q term. Nevertheless, if the q term was not strictly exogenous, and if the remaining of the model was correctly specified with intangible assets, the direction of bias on the tax term is more subtle and depends on the size and direction of the correlation between the q term and the tax term. We defer detailed discussion of this and other econometric issues to the empirical section of the paper.

3.3.2 Short-run approximations for long-lived assets

Our model suggests that the correct relationship between tangible investment rate, tax-adjusted q and tax is given by

$$\frac{I_t^m}{K_t^m} = \frac{q_t^m}{(1-\tau_t)\,\psi} - \frac{1-k_t^m-\tau_t z_t^m}{(1-\tau_t)\,\psi}. \tag{3.8}$$

One difficulty of evaluating (3.8) is that the marginal value of tangible assets q_t^m is not observed. However, as we shall show some fundamental properties of temporary investment tax incentive and shed light on the relation between q_t^m and observed variables.

Suppose the government credibly announces a temporary change in bonus depreciation allowance, which temporarily increases z_t^m. The exact solution to the impact of this change on tangible investment is complicated because of two reasons. First, the optimality conditions (3.3) and (3.4) imply that investment decisions are both forward-looking and backward-looking. Second, if physical and intangible assets are imperfect substitutes, physical investment depends on the stock of intangible assets. However, for sufficiently temporary tax changes, two short-run approximations simplify the problem considerably and yield an analytical expression for q_t^m. In particular, the backward-looking variables K_t^m, K_t^m and the forwarding-looking variables q_t^m, q_t^u are approximated by their associated steady-state values K^m, K^u, q^m and q^u. Approximating long-lived assets with their steady-state values is standard in many setting. When depreciation rate is low, the stock of assets is much bigger than the flow investment. As a result, K_t^m, K_t^m change only slightly in the short-run. Approximating q_t^m and q_t^u with their steady-state level is less common.[6] The rationale comes from the optimality conditions. Expanding (3.4) gives

$$q_t^i = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta_{ts} \left(1-\delta^i\right)^s \left(1-\tau_{t+s+1}\right) \left[\frac{\partial F\left(K_{t+s+1}^m, K_{t+s+1}^u, X_{t+s+1}\right)}{\partial K_{t+s+1}^i} - \frac{\psi}{2} \left(\frac{I_{t+s+1}^i}{K_{t+s+1}^i} \right)^2 \right] \right\}$$

for $i = \{m, u\}$. Because the tax change is temporary, the system will eventually return to its steady-state, which means that variables in the future remains close to their steady-state level. The approximation error, that is the difference between q_t^i and q^i, comes from the first few terms in the expansion. If both the depreciation rate and the discount rate are small, then future terms will dominate the expression of q_t^i and the approximation error will be small. The interpretation is that the value of long-lived assets is forward-looking and and are mostly influenced by long-run considerations. Therefore, the effect of a temporary tax change only has mild effects.

3.3.3 Investment responses to a temporary tax change

We now derive an analytical expression for tangible investment rate. Because $q_t^i \simeq q^i$ and $K_t^i \simeq K^i$, the share of physical assets can be approximated by $S^m = \frac{K^m}{K^m+K^u}$ and average q can be approximated by its steady-state level following (3.6):

$$q = q^m S^m + q^u \left(1 - S^m\right).$$

This expression has similar interpretation as (3.6). It says that in the steady-state average q is a weighted average of the marginal value of tangible and intangible assets, where the weights are given by the share of tangible and intangible assets respectively.

[6] One exception is House and Shapiro (2008). They analyze a model of temporary investment incentive in a model with physical capital and approximate a normalized shadow value of physical capital using its steady-state level. They also include a numerical example to study the approximation error for different depreciation rate and duration of the tax policy. They find that with an annual depreciation rate of 5 percent and a moderate adjustment cost (corresponding to $\psi = 20$), the approximation error in q for a one year duration of tax change is 0.016.

Following (3.5), the steady state level of investment rate $\frac{I^i}{K^i}$ is related to the marginal value q^i:

$$q^i = 1 - k^i - \tau z^i + (1-\tau)\,\psi\left(\frac{I^i}{K^i}\right) \tag{3.9}$$

for $i = \{m, u\}$, which immediately implies

$$q^u = \eta q^m, \tag{3.10}$$

where $\eta \equiv \frac{1-k^u - z^u + (1-\tau)\psi\left(\frac{I^u}{K^u}\right)}{1-k^m - z^m + (1-\tau)\psi\left(\frac{I^m}{K^m}\right)}$. Combining (3.9) and (3.10) gives an identity to express q^m through q :

$$q^m = \frac{q}{S^m + \eta\,(1 - S^m)}. \tag{3.11}$$

This expression is more than an accounting identity. Importantly, it expresses the unobserved variable q^m through q and S^m. q can be observed—although imperfectly—from companies' financial statements. To construct requires time-series of physical and intangible assets. Constructing η requires the time-series of tax rates and investment rate on physical and intangible assets. It also requires assumption on the unknown parameter ψ. As we shall discuss in more details in Section IV, we adopt an empirical strategy to study the sensitivity of our key findings with respect to different assumptions on this term.

Following (3.11) and (3.5), we have

$$\frac{I^m_t}{K^m_t} = \frac{q}{\left[S^m + \eta\,(1 - S^m)\right](1-\tau_t)\,\psi} - \frac{1 - k^m_t - \tau_t z^m_t}{(1-\tau_t)\,\psi}. \tag{3.12}$$

The problem of mismeasurement in conventional model is once again apparent. (3.12) shows that a correct specification of tax-adjusted q term should be $\frac{q}{[S^m + \eta(1-S^m)](1-\tau_t)}$ which depends on the share of tangible assets S^m and the relative intensive of tangible and intangible investment (through η) in a non-linear way. By contrast, the q term from conventional models is $q/(1-\tau_t)$, which is misspecified when S^m is not equal to 1.

3.4 Methodology and Data

3.4.1 Bonus Depreciation Allowance

Our empirical strategy is derived from the model's implication on investment responses to temporary tax incentives. Several episodes of temporary bonus depreciation allowances during the early 2000s allow us to estimate the model. In an attempt to spur business investment, the Job Creation and Worker Assistance Act, passed on March 11, 2002 created a 30 percent first-year "bonus depreciation" allowance, enabling businesses to write off immediately 30 percent of the cost of eligible capital goods. The provision applied retroactively to certain business property acquired after September 11, 2001 and applied to assets purchased before September 11, 2004, and placed in service before January 1, 2005.[7] However, because the Act was passed in March 2002, investors sitting in 2001 did not make their investment decisions based on the reduced asset cost for that year. On May 28, 2003 the Jobs and Growth Tax Relief Reconciliation Act increased the bonus first-year depreciation allowance for capital put in place after that date to 50 percent and extended to December 31, 2004. Eligible property for this special treatment included property with a recovery period of 20 years or less, water utility property, certain computer software, and qualified leasehold improvements.

Two aspects of the bonus depreciation provision are worth noting. First, among qualifying property, the present value of the provision was, putting aside the possibility of taxable losses (Edgerton (2010)), an increasing function of the depreciable lives of qualified capital assets, because from the second year onward the offsetting

[7] Taxpayers who had already filed their 2001 returns before this new provision was passed could still take advantage of the bonus depreciation provision by filing an amended return.

decreases in depreciation allowances of longer-lived assets occur farther into the future, and thus have a lower present value. Second, because the bonus depreciation provision explicitly expired (although the deadline was later extended), it provides an incentive to move investment forward.[8]

3.4.2 Methodology

To assess how ignoring intangible assets affects conventional estimates of the q model of investment, we first estimate the conventional investment model. Then, we estimate a tax adjusted q-theory model with intangible assets based on the model described in Section III, which provides a new formulae for the effect of the tax and the q terms on the physical investment rate. Comparing the estimated effect of investment tax incentives under the two models enables us to appreciate the direction and size of the bias generated by conventional models.

The specification of a conventional model is:

$$\frac{I^m_{i,t}}{K^m_{i,t-1}} = \alpha \frac{q_{i,t}}{1-\tau_t} + \beta \frac{1-\Gamma^m_{j,t}}{1-\tau_t} + \gamma_k X_{k,i,t} + \varepsilon_i + \varepsilon_t + \varepsilon_{i,t}, \tag{3.13}$$

where $\frac{I^m_{i,t}}{K^m_{i,t-1}}$ is the investment rate in physical (or tangible) assets, $\frac{q_{i,t}}{1-\tau_t}$ is the q term from conventional q-theory models, $\frac{1-\Gamma^m_{j,t}}{1-\tau_t}$ is the tax term from conventional q-theory models including $\Gamma^m_{j,t} = k^m_{j,t} + \tau_t z^m_{j,t}$, which is the present value of investment tax incentives, and $X_{k,i,t}$ controls for firms' idiosyncratic characteristics, including $\frac{CF_{i,t}}{K_{i,t}}$, defined as the ratio of cash flow to physical capital stock, and acts as a proxy for liquidity constraints, and $Lev_{i,t}$ which is the leverage ratio. Although the model implies that the investment rate should only depend on tax-adjusted q, previous literature has shown that with the presence of other constraints (e.g., liquidity constraints,

[8]More details about the policy are provided in the Appendix.

cash constraints), firms with lower cash flow intensity or excessive leverage may have smaller investment rates; therefore we also control for proxies of these constraints.[9] ε_i and ε_t are firm and time fixed effects, and $\varepsilon_{i,t}$ is an i.i.d. error term.[10] Equation (3.13) shows that conventional estimates of the effect of the tax term on the investment rate depend on the measure of the present value of tax incentives, Γ_j^m, which only includes physical assets. Disregarding tax credits, this is: $\Gamma_j^m = \sum_{a=1}^{N^m} \frac{I_{aj}}{I_j^m} z_a$, where $I_j^m = \sum_{a=1}^{N^m} I_{aj}$ is total investment in physical assets of industry j and z_a is the present value of tax depreciation allowances for a dollar of investment in asset a.

However, as shown in Section III, equation (3.13) is incorrect because average q does not take into account the presence of intangible assets. Instead, we should estimate

$$\frac{I_{i,t}^m}{K_{i,t-1}^m} = \alpha' \frac{\tilde{q}_{i,t}}{1-\tau_t} + \beta' \frac{1-\Gamma_{j,t}}{1-\tau_t} + \gamma_k X_{k,i,t} + \varepsilon_i + \varepsilon_t + \varepsilon_{i,t}, \tag{3.14}$$

where $\tilde{q}_{i,t} = \frac{q_{i,t}}{S_{i,t}^m + \eta_{j,t}\left(1 - S_{j,t}^m\right)}$.[11]

For exposition purposes, let's assume that the tax term is independent from the q term and that the measurement error in the q-term is independent across industries. In this case, equation (3.13) implies that conventional estimates of the impact of the q term are captured by α, while the true estimate is $\alpha' = \alpha\left[\bar{S}^m + \bar{\eta}\left(1 - \bar{S}^m\right)\right]$, where $\bar{S}^m$ and $\bar{\eta}$ are the industry averages of S_j^m and η_j. Under the assumption that the measurement error in q is independent across industries and with the tax term,

[9]One might not only expect the impact of liquidity constraints on investment to vary across firms, but also to influence the effectiveness of tax incentives. Edgerton (2010) shows that tax incentives have the smallest effect when they are particularly are most likely to be put in place, such as during downturns or recessions when cash flows are low.

[10]Note that although our database allows us to observe financial variables and firms' physical assets at the company level (subscript i), we only observe the share of physical and intangible assets (and therefore the tax term) at the industry level. Also, although not reflected in equations (3.13) and (3.14), the regressions also include a proxy for leverage, and relevant macroeconomic variables. We also provide regressions that allow for the error term to be heteroskedastic, or for firms' financial variables to be predetermined (i.e. where the q-term or liquidity constraints are not strictly exogenous).

[11]During the period 1998-2006 used in the estimations, no broad-based investment tax credit was available to corporations. Conditional tax credits were available for specific expenditures (e.g., qualified R&D, renewable energy), small corporations (under Internal Revenue Code Sections 179 and 168), or qualified employment. For a list of business tax credit, see:

http://www.irs.gov/businesses/small/article/0,,id=99839,00.html and if it is possible to observe $\bar{S}^m$ and $\bar{\eta}$–as we shall see below–it follows that we could predict the direction of the bias on q. For example, if $\bar{\eta} \geq 1$ (respectively if $\bar{\eta} < 1$), then the conventional estimates of the q term would overestimate (respectively underestimate) the true impact of q. Nevertheless, there is no reason to believe that these assumptions should hold. In particular if the q and tax terms are correlated, then any additional measurement error in the tax term would affect the coefficient on q beyond the simple effect described by the difference between 3b1 and 3b1'. However it is clear that the q and tax terms are related, implying that the biases on conventional estimates of the effect of the q and tax terms are unclear.

3.4.3 Data

We use a combination of firm- and industry-level data. The sample period is 1998 to 2006, which includes several episodes of temporary investment tax incentives as described in Section IV A. The main reason for starting in 1998 is the use of a comprehensive database for corporate intangible assets by industry developed during this period only (Dauchy (2013)) and presented in the Appendix. Other reasons are that investment tax incentives have significantly changed since the turn of the century, with permanent tax incentives giving place to temporary tax policy such as partial expensing. We expect that temporary tax incentives have a different impact on the investment rate than permanent tax changes. Moreover, compared to previous decades, the 1990s and after have experienced a significant shift in the composition of corporations' assets, towards more investment in intangible assets. We choose to end the sample period in 2006 before the most recent recession starts. The main reason is that economists recognize that the recent recession is different from previous business cycles in its causes and duration, and that the recovery has had unusual and unpredictable features.[12]

Firm-specific variables are based on financial statements, obtained from Compustat.[13] We exclude industries that are subject to specific tax treatment, which includes all firms in North American Industry Classification System sectors 52 (Finance and Insurance) and 22 (Utilities). Compustat data items and other variables used in regressions are presented in Table 3.1.

The calculation of the tax part of modified Tobin's q and of stocks and flows of investment by asset, industry, and over time is based on data collected and presented in Dauchy (2013) and presented in the Appendix. Starting from the 1997 BEA's capital flow table, which provides the distribution of investment in equipment, software and structures for 23 two-digit industries and 51 asset types, we isolate corporate from non-corporate investment using the annual BEA's Surveys of Current Businesses. Finally we grow corporate investment in physical assets over time based on BEA investment data over time. Stocks of physical assets are calculated based on the perpetual inventory method (PIM). It is worth noting that our methodology for calculating intangible assets and merging them with physical assets adjusts for potential discrepancies between tax and national accounts databases so that the distribution of investment across industries is consistent with our desire to correctly evaluate tax depreciation allowances at the industry level.[14]

To obtain corporate investment in intangible assets, the author uses a comprehensive methodology based on Corrado et al. (2005) and Corrado et al. (2006)(CHS

[12]According to "The Budget and Economic Outlook: An Update," , Congressional Budget Office, U.S. Congress, August 2011. business investment in equipment, software, and structures less depreciations decreased sharply during the recession, and was at its lowest in more than half a century, in part due to idle industrial capacity and vacant real estate, reducing the need for businesses to invest as they usually would (p. 61).

[13]Following previous research, extreme values of many financial statements variables are winsorized at the 2nt and 98th percentiles (Desai and Goolsbee, 2004, Edgerton, 2010; Dauchy and Martinez, 2008; Moyen, 2004).

[14]See Appendix A and B and Dauchy (2013) for details. The distribution across industries is based on the BEA and the IRS, as described in appendix A1. As shown in the appendix, the methodology used in this paper to distribute investment and stocks of corporations (i.e. companies subject to corporate taxation) across industries closely reflect actual investment given available data, and even if BEA's Survey of Current Businesses covers a different set of companies than IRS's tax returns.

hereafter). However, while CHS's measure of intangible assets is at the aggregate level and for years ending in 2000, we improve it to calculate intangible investment for each industry included in the BEA physical asset tables, and use BEA tables on corporate and non-corporate investment to isolate corporate investment in intangibles. The change in the stock of intangible assets is also calculated based on the PIM.[15]

The variation of tax depreciation allowances by industry and over time is calculated based on the methodology explained and used in previous literature (Edgerton (2010), House and Shapiro (2008)). As described previously from 1998 through 2006, the tax treatment of capital assets has varied many times. Contrary to the previous 30 years, many of the tax incentives for investment have been temporary.[16] From these datasets we calculate the tax part of the q model, as described in the methodology.[17]

Summary statistics are presented in Table 3.2. The investment rate, the q term as well as the equipment and structure terms are very similar to those found in previous papers (Desai and Goolsbee, 2004; Edgerton, 2010). Although not shown in Table 3.2, for the sample of publicly traded companies in Compustat, the ratio of corporate investment in intangible to total assets has slightly decreased from 46 percent in 1998 to 44 percent in 2006.[18] The investment rate has decreased over time, from 0.28 in 1998 to 0.25 in 2006.[19] This is not surprising given the much smaller growth rate of GDP, and is in spite of a reduced long-term interest rate.[20] Also, compared to the

[15]See Appendix A.

[16]See Section IV-A and Appendix B for a detailed description of bonus tax depreciation allowances.

[17]Macroeconomic variables include real growth rate of GDP and the 10-year federal funds yield, both obtained from the 2011 Economic Report of the President (See appendix C).

[18]See Dauchy (2013) for detailed data of intangibles over time. Contrary to the sample of publicly traded companies in Compustat, the ratio of corporate investment in intangible to total assets for the aggregate economy has slightly increased from 1998 to 2006, although the differences are small in magnitudes.

[19]All company-level variables are winsorized by 2 percent at the top and bottom of the distribution (this includes the investment rate, average q, the cash flow ratio and the leverage ratio).

[20]The real rate of GDP growth dropped from 4.4 percent to 2.7 percent from 1998 to 2006 (Economic Report of the President, Council of Economic Advisors, 2012). The yield on 10-year Treasury bonds was also lower in 2006

whole sample of corporations, top firms seem to have smaller investment rates, be more intangible intensive, have higher leverage, more cash flows, and have more cost effective investment—reflected by smaller equipment tax terms.

3.5 Results

3.5.1 Baseline results

Table 3.3 presents baseline regressions of physical investment rate on the tax-adjusted-q term, the equipment tax term (ETT), and the structure tax term (STT). Following the literature, we include the ratio of cash flow to capital and the leverage ratio as proxies for financial constraints.[21] To facilitate comparison, we report estimates of the conventional model with the conventional q term, defined by equation (3.13), on the left panel (columns 1 to 4), and estimates of the intangible adjusted q model with the intangible adjusted q term, defined by equation (3.14), on the right panel (columns 5 to 8).

We report estimates from panel regressions with firm fixed effects and year fixed effects in columns 1 and 5 respectively for the conventional model and for the intangible-adjusted model. Following Eberly et al. (2012), we also estimate a standard q-model of investment augmented with lagged investment as explanatory variable.[22] Fixed effect regressions including lagged investment rate as explanatory variables are shown in columns 2 and 6, respectively for the conventional model and our model with intangible-adjusted q.

than in 1998 (4.97 percent compared to 5,35 percent).
http://www.gpo.gov/fdsys/browse/collection.action?collectionCode=ERP

[21] Cash flow and leverage may be correlated with future profitability. If the q-term was pefrectly ovbserved, it would fully capture the future value of future cash flows, and we should not find any additional explanatory power of cash flow or leverage. However, for various reasons including cross-sectional variation in internal funds, liquidity constraints (Gilchrist and Himmelmerg, 1995), or measurement error in q (Erickson and Whited, 2000, 2010), previous literature finds that including cash flow increases the deterministic power of estimated q-models. See also Cummins et al., 1994) for a detailed argument about the cash flow deterministic power, as well as Fazzari, Hubbard, and Peterson (1988), Gilchist and Himmelberg (1991, 1992), Blundell et al. (1992), and Cummins et al. (1994) for examples of empirical models including lagged values of financial variables.

[22] Eberly et al (2012) find that lagged investment is not only a significant predictor of the current investment rate but also has a larger explanatory power than the q and cash flow terms.

As reported, the coefficients for the ETT and STT terms estimate the responsiveness of investment rate to tax. Using our units of measurement, the coefficients can be interpreted as changes in investment rate (i.e., $\frac{I^m}{K^m}$, in percentage) per 1 percent change in applicable tax (i.e., $\frac{1-k-\tau z}{1-\tau}$).[23] In most regressions shown in Table 3.3, estimates from the conventional model and the intangible-adjusted model are similar. In most cases, the coefficients of the ETT and STT terms are negative and significant, as expected. For example, in specification (1), the coefficient for the ETT term and STT term are -2.843 and -1.124 respectively for the conventional model, and -2.852 and -1.120 respectively for the intangible-adjusted model. This suggests that the tax-adjusted q term is largely uncorrelated with the ETT and STT terms. As a result, using different measures of the q term leaves estimates of the tax terms unaffected. We also find that the coefficient for the ETT term is larger than the STT term, similar to previous studies using this a fixed effect methodology (e.g., Edgerton (2010)). As shown in columns (2) and (6), adding lagged physical investment rate on the right hand side reduces the coefficients of the tax terms. This result is consistent with the lagged investment effect, and provides evidence of a Christiano et al. (2005) type of investment adjustment costs.[24] In the presence of this type of adjustment cost, it is likely that lagged investment is also correlated with the tax term, which explains the reduction in tax coefficients in model (3) and (4).

At this point it is worth spending some time on the fact that the magnitude of the coefficients on the ETT term and STT term are in general larger than those suggested by prior studies with longer sample periods (Desai and Goolsbee (2004),

[23] Our result can also be interpreted in terms of the elasticity of investment rate with respect to tax. Using the GMM estimation result with lagged investment, i.e., model (8) of Table 3.2, the elasticity of investment rate with respect to the ETT and STT terms are -3.1 and -1.0 respectively.

[24] Christiano et al. (2005) proposes a model with an adjustment costs depending on lagged and current investment. Eberly et al. (2012) show that the lagged investment effect can be explained by this type of model. They also find that when lagged investment is included as a regressor, the expanory powers of the q and cash flow terms, though significant, are much smaller in magnitude, and the R-square almost doubles.

Edgerton (2010)); however, our results are comparable to those that focus on the temporary tax changes in the early 2000s (House and Shapiro (2008)) as will be seen in Table 3.4. Using similar specifications as column (1) of the conventional model reported in Table 3.4, Edgerton (2010) reports a coefficient for the ETT term to be -0.842. His coefficient for the STT term is small and insignificant. Desai and Goolsbee (2004) report similar results. The difference between our estimates and those from the latter studies highlights the sensitivity of estimations of q-model of investment to both the time period and the coverage of companies. On the one hand, and in part because our sample covers a shorter time period, we are also able to cover many other companies that may not belong to their sample. On the other hand, and more importantly, the differences between our results and those of Desai and Goolsbee (2004) and Edgerton (2010) mostly likely reflect the temporal nature of tax changes in different time periods. The sample period covered by Desai and Goolsbee (2004) and Edgerton (2010) includes many tax changes, some of which are permanent changes.[25] In contrast, our sample period covers several episodes of bonus depreciation, all of which were not only intended to be temporary but also differently affected various types of assets.[26] As noted in House and Shapiro (2008), on the one hand bonus depreciation was explicitly temporary contrary to all tax changes before that. On the other hand, most of the variation in studies that cover the past 40 years comes from the ITC, which broadly covers equipment assets, while bonus depreciation was concentrated among a narrower range of assets, but also included certain quasi-structures. This specificity of bonus depreciation provided room for substitution not

[25]Edgerton (2010) covers years from 1967 to 2005. During these 40 years, the corporate tax rate changed 4 times (1968, 1970, 1979, and 1987) and depreciation rules changed 3 times (in 1971, 1981, and 1987). The investment tax credit was repealed in 1969, reinstated in 1971, increased in 1975, and repealed in 1986 (For more details on these tax reforms, see Cummins et al., 1994). The only temporary tax incentives included in the sample are bonus depreciation rules passed in 2002 and 2003. Desai and Goolsbee (2004) cover years from 1962 to 2003.

[26]It is worth noting that the standard deviations of the ETT and STT terms in Edgerton (2010) and Desai and Goolsbee (2004) are similar to ours. This is because, compared to the 2000s, tax changes over the long sample period are not as frequent but larger in magnitude.

only across assets, but also across industries. Temporary reductions in the after-tax cost of physical assets provide a strong incentive to invest, especially when assets are sufficiently long-lived. This incentive results mainly from firms' ability to arbitrage the after-tax cost of investment over time (House and Shapiro (2008)). For tax changes that last sufficiently longer, the incentive for intertemporal substitution is weak. In sum, the fact that studies covering several decades include tax reforms aimed at different purposes–and therefore with potentially different incentive effects on companies–makes their estimates likely to bias the impact of specific tax reforms. Moreover, in spite of their longer duration coverage, the variation in their tax terms is of very similar magnitude than ours, mostly because only few large tax changes have only occurred in the past 40 years. For example, although Edgerton (2010) covers over 40 years and observing about 390 firms per year on average (for a total of 14,720 firms) the average value and the standard deviation of the ETT respectively are 1.039 and 0.044 (and respectively 1.277 and 0.109 for the average value and standard deviation of the STT). Table 3.2 shows that in our sample, which covers 9 years and almost four times more companies, or about 1,360 companies per year on average (for a total of 10,889 firms), the average value and standard deviation of the ETT respectively are 1.155 and 0.076 (and respectively 1.484 and 0.053 for the average value and standard deviation of the STT).

Finally and more specific to our study, another drawback of using several recent decades is due to the increasing importance of intangible assets in the past 40 years, as documented in Section 2. For example, because the ratio of intangible assets to NIPA assets has increased almost threefold since 1960 to 2003 (Corrado et al. (2006)), the incentive effect across periods and industries is likely to have changed in non-trivial ways over this longer period.[27]

As noted in Arellano and Bond (1991), many financial data from companies' annual reports are likely not to be strictly exogenous. Even if the model was fully determined, many financial variables are likely to be pre-determined.[28] Moreover, our results shown in columns 2 of Table 3.3, which include lagged values of investment, are not correctly specified because the presence of fixed effect automatically makes the error terms serially correlated. To correct for these issues, we re-estimate the conventional and intangible adjusted models of investment with the consistent dynamic GMM estimator developed by Arellano and Bond (1991). The results for the conventional model are presented in columns 3 and 4 of Table 3.3 (respectively not including and including the lagged investment effect), and in columns 7 and 8 for the intangible-adjusted model. Another advantage of the GMM estimation method is that it addresses the issue of the measurement error in q, or other serially correlated unobserved shocks, which may bias previous estimates of q-models. To avoid further risks of potential serial correlation in the measurement error, we exclude the first two lags from the sets of instruments. Similar to the fixed effect estimations, the coefficient returned for the ETT and STT terms are negative and significant. The estimates of the equipment tax term are again of similar magnitude as the pooled OLS estimates of the conventional model, suggesting that the measurement error in q does not significantly affect estimates of the effect of investment incentives. Also

[27] In addition, studies that explain company-level variation with variation derived at both the company and industry levels—which is the case in our study as well as in Edgerton (2010) or Desai and Goolsbee (2004)—cannot allow investment decisions to be dependent within industries (for example by clustering standard errors at the industry level). One of the main reasons is that the impact of tax incentives is measured at the industry level. To the extent that the importance of intangible assets in investment decisions have significantly changed over time and in different ways across industries (as documented in appendix A), standard errors are likely to be significantly biased when estimated over many recent decades and without industry fixed effects.

[28] In fact previous literature estimating q-models of investment alternatively use the lagged values of certain financial variables (such as cash flows and leverage) instead of their current value, because investment decisions in a given year are likely to depend on most recent measures of cash flows (e.g., based on the end of the closest previous year). For more discussion on the use of lagged financial variables, see for example Eberly et al (2012), Blundell et al. (1992), Cummins et al (1994).

similar to fixed effects results, models using the lagged value of the investment rate return smaller coefficients for the ETT and STT terms than those excluding the lagged investment effect.

Interestingly, the coefficients for the ETT term are generally smaller in the GMM estimations whereas the coefficients for the STT term are larger in the fix effects estimations.

Table 3.3 also shows results of the GMM specification tests. As expected, in all specifications that include the lagged physical investment term (i.e., in columns 4 and 8), the AR(1) test is not rejected and the AR(2) test is rejected, consistent with the hypothesis of absence of second-order serial correlation and significant first-order correlation. In specifications that exclude the lagged physical investment rate (i.e., column 3 and 7), the AR(2) test is also rejected, suggesting invalid instruments. This result is similar to previous findings (e.g., Cummins et al. (1994)).[29] Moreover, also similar to previous GMM estimated of the Q-model, the Hansen-Sargan test of over-identifying restrictions is more likely to be rejected when including lagged values of the investment rate.[30] Our interpretation is that current financial variables (q, cash flow and leverage) are either endogenous (simultaneously determined by unobserved factors that also affect the physical investment rate), or pre-determined. Using the lagged value of the investment rate can address this problem.

In most regressions, the cash flow and leverage terms are significant and have the same magnitude. The cash flow term is generally negative and significant. This may seem contrary to the belief that companies with higher cash flow should be more

[29]See also Blundell et al. (1992). Similar to Bond and Cummins (2001) we limit the issue of including instruments that risk to be correlated with the error term, we exclude both first and second lags of all variables in our instrument set. Although this is unlikely to fully solve the remaining second-order serial correlation, AR(3) tests not shown in Table 3 reject the null of no serial correlation. Previous literature does not propose other ways to solve for the remaining AR(2) effect.

[30]See e.g., Bond and Cummins (2001).

able to invest. However, this result is in line with previous literature.[31]

In Table 3.4, we report results using the total tax term TTT, calculated as the value weighted average of the ETT and STT term. This specification is motivated by most theoretical q-models of investment, where the adjustment costs of structure and equipment are assumed identical. In this case, equation (3.12) suggests that the physical investment rate (inclusive of equipment and structure) depends on the marginal after-tax cost of investment (again inclusive of equipment of structure). The intuition is as follows. When firms make physical investment decision, they equate the marginal cost to the marginal value of physical investment. The tax treatment of physical assets affects the cost of physical investment. If the unit cost of physical investment is 1 without taxation, it is measured by the TTT with taxation. Moreover, estimates of the TTT terms are more directly comparable to previous estimates of tax adjusted of q-models that combine q and the tax term in one single variable, such as House and Shapiro (2008). The estimated coefficients for TTT terms are generally negative and significant, and of similar magnitude as House and Shapiro (2008).[32]

3.5.2 Large firms

To check the sensitivity of our regressions to firm size, we re-estimate the model for large firms. The results for samples restricted to the largest firms by assets are presented in Tables 3.5 and 3.6, respectively for the largest 3500 and the largest 1500 firms. Because investment of these two sets of large firms respectively account for 96

[31] There is no reason to believe that cash flow should always be positively related with investment. For example, there is evidence that both after the 2001 recession and the 2008 recession, firms with large amount of cash flows chose not to invest, mostly because of increased uncertainty about the future. Almeida et al. (2004) show that the sensitivity of investment to cash flow varies across firms in non-trivial ways.

[32] House and Shapiro (2008) discuss their finding that the supply elasticity of investment to the temporary bonus depreciation is much larger compared to previous literature (at least 6 times larger), and use data at the aggregate level rather than at the firm level. However, we could indirectly compare their results with ours through their estimate of the coefficient on the combined tax adjusted q-term, which is between 0.33 and 0.7. Our estimate of the coefficient of the TTT term from Table 4 is of similar magnitude.

and 89 percent of total investment over the period considered, aggregate responses to tax changes are largely determined by them.[33]

Although the results for the top firms are qualitatively similar to those of the full sample, there are important quantitative differences. Although the coefficients for the ETT term remain negative and significant in most specifications, they are always smaller than for the whole sample, and smaller for the largest 1500 companies than for the largest 3500 companies. For example, the coefficient for the ETT term drops from -2.852 for the full sample to -2.124 for top 3500 firms, and to -1.851 for the top 1500 firms. This is in line with previous literature, and suggests that temporary investment tax incentives tend to have a smaller impact on firms that represent the largest share of investment in the economy than on smaller firms, which is not surprising considering that large firms may have other opportunities for intertemporal arbitrage.

A more interesting result is shown by the ratio the coefficients of the ETT in our model to the conventional model: in all regressions, for large companies, the coefficient of ETT is consistently larger for the intangible adjusted model than for the conventional model. This is a very interesting and innovative result because it suggests that, in order to correctly estimate the impact of tax incentives, controlling for intangibles is more important for large firms than for small firms. Our interpretation is that large firms on average hold a larger share of intangible assets than small firms. As a result, ignoring intangible assets in conventional models leads to larger bias.[34]

[33] Total investment and large companies' invetsment is defined as investment of all firms of top firms in Compustat that are kept in our regressions, and every years from 1998 to 2006. As noted in previous literature (Desai and Goolsbee, 2004), Compustat companies also represent a large share of the aggregate economy. Compared to NIPA accounts, our sample of companies covers 68 percent of aggregate investment in non-residential fixed assets. The largest 3500 and 1500 companies respectively cover 66 percent and 60 percent of it over the same period.

[34] This interpretation is also supported by the summary statistics. As shown in 3.2, the mean of our tax-adjusted q term is much larger than the conventional tax-adjusted q term for the sample of large firms. The difference is smaller for the full sample

Finally, compared to conventional regressions, the q-term is also positive but much larger and always significant —for both sets of large companies—and of a magnitude in line to previous literature. As in conventional regressions, adding lagged values of the physical investment rate reduces the coefficient of the ETT term. In some cases, the STT term becomes smaller and insignificant, which is also more in line with the expected effect given that bonus depreciation essentially changes the cost of short-lived assets.

3.5.3 Robustness check

Baseline results discussed previously have emphasized our careful attempt to account for intangible assets in estimating investment responses to tax changes. In particular, we use a proxy for the tax-adjusted q term that accounts for intangible assets. To ensure that our results are robust to this new proxy, we investigate regressions using alternative values of the q term.[35] We find that the responses of the physical investment rate to the tax terms are largely unaffected by these alternative values. This is not surprising given that the q term and the tax term are largely uncorrelated, as we have discussed previously.

As a way to deal with common outliers in firm-level financial data, the tradition from the finance and accounting literature is to winsorize the data.[36] To evaluate the sensitivity of our results to the degree of winsorization, we also subject our findings higher levels of winsorization (at 5 percent and 10 percent). Note that outliers tend to be generally more extreme and frequent for smaller firms than for large firms. Higher levels of winsorization produce results that seem stronger (more significant and closer to previous literature) than lower levels of winsorization (at 2 percent).

[35]In particular, we re-run all models by using $\psi = 0.1$ and $\psi = 10$ in our measure of the intangible adjusted q term defined in equation (3.14). Results are available upon request.

[36]We use a baseline of 2 percent at the top and bottom for all financial variables constructed from compustat and in our regressions (i.e., we winsorize the investment rate rather than investment or the capital stock, etc.).

Results are shown in Table 3.7 and 3.8. The coefficient on ETT is negative and significant, and smaller that in baseline regressions. More importantly, as was the case in the regressions limited to large firms, the ratio of the coefficient on ETT from the intangible-adjusted model to the conventional model is always larger than 1. This again, strengthens the finding that the impact of bonus depreciation is likely to be larger than suggested in previous literature, once we correct for the presence of intangible assets in firms' investment decisions. Finally, we test the sensitivity of the results to the time period including in our sample, by ending the data in 2004 or 2005 instead of 2006. Most of these tests produce no qualitative changes to our empirical findings and are omitted from the article for space consideration.[37]

3.6 Conclusion

The rapid growth of intangible assets in the economy and of their importance in corporate investment affect any model of investment that traditionally exclude corporate spending in intangible assets from the tax part of the cost of capital. There is large evidence that the relative size of investment in intangible to physical assets, where intangible assets are comprehensively measured including scientific and non-scientific R&D, brand equity, and firm-specific human capital such as organization and management skills, is close to one and much larger in certain industries such as business management services, wholesale, and finance. This paper investigates how explicitly accounting for intangible assets in corporate investment and asset stock bundles affects conventional estimates of the q model of investment. For this, we derive the theoretical implications of including intangible assets in the model. We then construct a comprehensive measure of corporate intangible assets by industry from 1998 to 2006. This important step provides proxies for the relative size of

[37]Results of these robustness checks are available upon request.

physical and intangible assets by industries and over time, based on multiple sources of investment data. We develop an empirical methodology to estimate a q model of investment with intangibles and implement it for publicly listed companies in Compustat over this period.

The results show a number of shortfalls from conventional estimates of the q-model. First, we show that ignoring intangible assets biases conventional estimates of the impact of the tax terms in the q model of investment, and that the direction of this bias is not obvious, although in our sample period, the size of the bias appears to be small. Second, our estimates of the investment response to temporary tax changes in the sample period are generally larger than those suggested by prior papers using longer sample periods, providing evidence for strong investment response to temporary tax incentives compared to previous permanent tax incentives. This result is particularly important because recent research on the impact of the bonus depreciation policy in 2002 and 2003 generally finds that it had a small impact on stimulating investment. Third, our results suggest that, in order to correctly estimate the impact of tax incentives, controlling for intangibles is more important for large firms than for small firms.

I_{ijt}/K_{ijt-1} =$\mathrm{CAPEX}_{ijt}/\mathrm{PPEVEB}_{ijt-1}$ =Investment rate

$q_{ijt} = \left[\mathrm{AT}_{ijt} + \mathrm{CSHO}_{ijt}\mathrm{PRCC}_{ijt} - \mathrm{CEQ}_{ijt} - \mathrm{TXDB}_{ijt}\right]/\mathrm{AT}_{ijt}$ =Average q

$q\_adj_{ijt} = q_{ijt} * q\_factor_{jt}$ =Average q from Chen and Dauchy model

$q\_factor_{jt} = 1/\left\{\left[S^m_{jt} + \eta_{jt}\left(1 - S^m_{jt}\right)(1-\tau_t)\varphi\right]\right\}$

$\eta_{jt} \equiv \left[1 - k^u_{jt} - z^u_{jt} + (1-\tau_t)\varphi\left(\frac{I^u_{jt}}{K^u_{jt}}\right)\right] / \left[1 - k^m_{jt} - z^m_{jt} + (1-\tau_t)\varphi\left(\frac{I^m_{jt}}{K^m_{jt}}\right)\right]$

$CF_{ijt}/K_{ijt-1} = \left(\mathrm{IB}_{ijt} + \mathrm{DP}_{ijt}\right)/\mathrm{PPEVEB}_{ijt-1}$ =Ratio of cash flow to capital stock

Lev_{ijt} =$\mathrm{DLTT}_{ijt}/\mathrm{CEQ}_{ijt}$ =Leverage ratio=Value of long term debt to equity

ETT_{jt} and $STT_{jt} = \left(1 - \tau_t z^m_{jt}\right)/(1-\tau_t)$ =Equipment & structure tax terms*

Table 3.1: Variable Definitions

(1) Sources: Compustat and authors' calculations using various sources (see Appendix and Dauchy, 2013). Observations are at the firm level (subscript i) or industry level (subscript j). Compustat variables are listed as item and item #, where ppeveb (or item 187) = Property, plant and equipment (Ending balance, Schedule V); capx= Capital expenditures; at=Total assets; csho=Common shares outstanding; prcc=Annual price at closing; ceq=Total common and ordinary equity; txdb=Deferred taxes (Balance sheet); ib=Income before extraordinary items; dp=Depreciation and amortization; dltt9=Total long-term debt. All final variables constructed from Compustat are further winsorized at 2 percent at the top and bottom. (2) *Equipment and structure tax terms are further defined in the Appendix.

	All (N=55,442)		Top 3500 (N=16,943)		Top 1500 (N=7,897)	
Variable	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev
I_{ijt}/K_{ijt-1}	0.228	0.375	0.176	0.250	0.146	0.191
$q_{ijt}/(1-\tau_t)$	4.755	8.065	3.076	3.210	3.032	3.016
$q_{ijt}\_adj/(1-\tau_t)$	4.027	6.746	2.624	2.698	2.580	2.529
q_factor	0.855	0.071	0.857	0.073	0.855	0.073
η_{it}	3.903	1.043	3.975	1.158	3.992	1.165
CF_{ijt}/K_{ijt-1}	-1.182	5.001	0.186	1.377	0.229	0.844
Lev_{ijt}	0.399	1.286	0.724	1.470	0.779	1.410
S^m_{jt}	0.842	0.079	0.848	0.081	0.846	0.080
ETT_{jt}	1.155	0.076	1.159	0.077	1.156	0.074
STT_{jt}	1.484	0.053	1.483	0.052	1.484	0.051
TTT_{jt}	1.101	0.037	1.103	0.039	1.102	0.038

Table 3.2: Summary Statistics

	Conventional				Chen & Dauchy			
Independent	Pooled OLS, fixed effects		GMM		Pooled OLS, fixed effects		GMM	
Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$q_{ijt}/(1-\tau_t)$	0.001	0.001	-0.002	-0.627***				
	(0.001)	(0.001)	(0.003)	(0.084)				
$q_{ijt}\_adj/(1-\tau_t)$					0.001	0.001	-0.003	-0.003
					(0.001)	(0.001)	(0.004)	(0.004)
ETT_{jt}	-2.843***	-1.443***	-1.157***	-0.627***	-2.852***	-1.458***	-1.141***	-0.613***
	(0.553)	(0.490)	(0.093)	(0.084)	(0.554)	(0.490)	(0.091)	(0.081)
STT_{jt}	-1.124***	-0.505	-2.322***	-1.321***	-1.120***	-0.502	-2.313***	-1.315***
	(0.424)	(0.414)	(0.136)	(0.138)	(0.424)	(0.414)	(0.136)	(0.137)
CF_{ijt}/K_{ijt-1}	-0.031***	-0.028***	-0.023***	-0.022***	-0.031***	-0.028***	-0.025***	-0.023***
	(0.001)	(0.001)	(0.008)	(0.007)	(0.001)	(0.001)	(0.008)	(0.007)
Lev_{jt}	0.004***	0.000	0.007	0.026**	0.004***	0.000	0.005	0.026**
	(0.001)	(0.001)	(0.013)	(0.012)	(0.001)	(0.001)	(0.013)	(0.012)
Lag(I_{ijt}/K_{ijt-1})		0.129***		0.263***		0.129***		0.263***
		(0.010)		(0.048)		(0.010)		(0.048)
Const.	5.114***	2.545***	5.007***	2.825***	5.119***	2.556***	4.975***	2.799***
	(1.064)	(0.979)	(0.311)	(0.310)	(1.064)	(0.978)	(0.310)	(0.305)
Obs.	55,442	46,235	55,442	46,235	55,442	46,235	55,442	46,235
No. fixed effects	10,889	9,718	10,889	9,718	10.889	9,718	10.889	9,718
R^2	0.157	0.158			0.157	0.158		
AR(1)			-16.93	-8.588			-16.93	-8.599
p-val			0	0			0	0
AR(2)			-8.346	-1.681			-8.365	-1.693
p-val			0	0.0927			0	0.0904
Hansen			138.9	120.8			134.1	118.8
p-val			8.80e-09	8.23e-4			3.69e-08	0.00125

Table 3.3: Fixed Effect and GMM Regressions with Equipment and Structure Tax Terms

(1) Include firm and year effects. Standard errors (in brackets) are clustered at the firm level. *** p<0.01, ** p<0.05, * p<0.1.

(2) GMM instruments are the q terms, cash flow, leverage, and lagged investment rate.

	Conventional				Chen & Dauchy			
Independent	Pooled OLS, fixed effects		GMM		Pooled OLS, fixed effects		GMM	
Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$q_{ijt}/(1-\tau_t)$	0.001	0.001	-0.001	-0.002***				
	(0.001)	(0.001)	(0.003)	(0.003)				
$q_{ijt}\_adj/(1-\tau_t)$					0.001	0.001	-0.002	-0.003
					(0.001)	(0.001)	(0.004)	(0.004)
TTT_{jt}	-1.050***	-0.486***	-0.349***	-0.148***	-1.046***	-0.483***	-0.337***	-0.133***
	(0.430)	(0.418)	(0.081)	(0.063)	(0.430)	(0.418)	(0.079)	(0.060)
CF_{ijt}/K_{ijt-1}	-0.031***	-0.028***	-0.021***	-0.021***	-0.031***	-0.028***	-0.022***	-0.021***
	(0.001)	(0.001)	(0.008)	(0.007)	(0.001)	(0.001)	(0.008)	(0.007)
Lev_{jt}	0.004***	0.000	0.012	0.027**	0.004***	0.000	0.011	0.027**
	(0.001)	(0.001)	(0.013)	(0.012)	(0.001)	(0.001)	(0.013)	(0.012)
Lag(I_{ijt}/K_{ijt-1})		0.129***		0.278***		0.129***		0.279***
		(0.010)		(0.048)		(0.010)		(0.047)
Const.	1.304***	0.656***	0.601***	0.281***	1.299***	0.652***	0.588***	0.278***
	(0.478)	(0.463)	(0.101)	(0.080)	(0.478)	(0.464)	(0.097)	(0.076)
Obs.	55,442	46,235	55,442	46,235	55,442	46,235	55,442	46,235
No. fixed effects	10,889	9,718	10,889	9,718	10.889	9,718	10.889	9,718
R^2	0.157	0.158			0.157	0.158		
AR(1)			-16.93	-8.869			-16.93	-8.915
p-val			0	0.653			0.486	0.135
AR(2)			-8.317	-1.507			-8.365	-1.496
p-val			0	0			0	0
Hansen			148.9	125.0			146.2	123.4
p-val			3.80e-10	3.38e-4			8.91e-10	4.79e-4

Table 3.4: Fixed Effect and GMM Regressions with Total Tax Terms

(1) Include firm and year effects. Standard errors (in brackets) are clustered at the firm level. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.
(2) GMM instruments are the q terms, cash flow, leverage, and lagged investment rate.

Independent Variables	Conventional				Chen & Dauchy			
	Pooled OLS, fixed effects		GMM		Pooled OLS, fixed effects		GMM	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$q_{ijt}/(1-\tau_t)$	0.018***	0.016***	-0.029***	-0.020***				
	(0.002)	(0.002)	(0.007)	(0.006)				
$q_{ijt}\_adj/(1-\tau_t)$					0.022***	0.019***	-0.033***	-0.023***
					(0.002)	(0.003)	(0.008)	(0.007)
ETT_{jt}	-2.124***	-0.852*	-0.632***	-0.375***	-2.304***	-1.038**	-0.719***	-0.429***
	(0.567)	(0.471)	(0.089)	(0.081)	(0.566)	(0.470)	(0.086)	(0.084)
STT_{jt}	0.270	-0.834*	-1.251***	-0.793***	-0.279***	-0.834*	-1.290***	-0.813***
	(0.494)	(0.433)	(0.148)	(0.136)	(0.494)	(0.433)	(0.147)	(0.137)
CF_{ijt}/K_{ijt-1}	-0.023***	-0.012**	-0.039***	-0.030*	-0.023***	-0.012**	-0.040***	-0.030*
	(0.005)	(0.006)	(0.009)	(0.018)	(0.005)	(0.006)	(0.009)	(0.017)
Lev_{jt}	0.004***	0.001	-0.006	-0.007	0.004**	0.001	-0.003	-0.005**
	(0.002)	(0.001)	(0.010)	(0.010)	(0.002)	(0.001)	(0.011)	(0.010)
Lag(I_{ijt}/K_{ijt-1})		0.236***		0.184**		0.234***		0.189***
		(0.026)		(0.079)		(0.026)		(0.078)
Const.	2.167***	-0.183***	2.684***	1.695***	2.361***	0.033	2.841***	1.786***
	(1.142)	(0.985)	(0.320)	(0.298)	(1.139)	(0.984)	(0.313)	(0.303)
Obs.	16,943	14,705	16,943	14,705	14,705	14,705	16,943	14,705
No. fixed effects	2,748	2,587	2,748	2,587	2,587	2,587	2,748	2,587
R^2	0.151	0.199			0.200	0.200		
AR(1)			-6.829	-5.985			-6.824	-6.036
p-val			0	2.16e-9			0	1.55e-09
AR(2)			-3.403	-1.447			-3.422	-1.395
p-val			6.67e-4	0.148			6.22e-4	0.163
Hansen			81.91	92.06			85.43	95.43
p-val			0.0170	0.101			8.73e-4	0.0653

Table 3.5: Fixed Effect and GMM Regressions for Large Firms: Top 3500 (N=16,943)

(1) Include firm and year effects. Standard errors (in brackets) are clustered at the firm level. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.
(2) GMM instruments are the q terms, cash flow, leverage, and lagged investment rate.

Independent	Conventional				Chen & Dauchy			
	Pooled OLS, fixed effects		GMM		Pooled OLS, fixed effects		GMM	
Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$q_{ijt}/(1-\tau_t)$	0.012***	0.009***	-0.033***	-0.014***				
	(0.003)	(0.003)	(0.007)	(0.004)				
$q_{ijt}\_adj/(1-\tau_t)$					0.015***	0.010***	-0.035***	0.015***
					(0.004)	(0.003)	(0.008)	(0.006)
ETT_{jt}	-1.851**	-0.738	-0.371***	-0.168**	-1.958**	-0.833**	-0.468***	-0.206**
	(0.773)	(0.614)	(0.112)	(0.078)	(0.777)	(0.614)	(0.119)	(0.083)
STT_{jt}	-0.587	-0.127	-0.735***	-0.374***	-0.582	-0.127	-0.777***	-0.389***
	(0.455)	(0.356)	(0.167)	(0.109)	(0.454)	(0.356)	(0.172)	(0.112)
CF_{ijt}/K_{ijt-1}	-0.005	0.009	-0.007	-0.030	0.005	0.009	-0.008	-0.029
	(0.012)	(0.012)	(0.015)	(0.022)	(0.012)	(0.012)	(0.016)	(0.021)
Lev_{jt}	0.004**	0.002	0.005	-0.004	0.004	0.002	0.006	-0.003
	(0.002)	(0.002)	(0.009)	(0.008)	(0.002)	(0.002)	(0.009)	(0.008)
Lag(I_{ijt}/K_{ijt-1})		0.253***		0.296***		0.252***		0.289***
		(0.044)		(0.038)		(0.044)		(0.037)
Const.	3.108***	-1.109	1.560***	0.797***	3.223**	1.218	1.741***	0.865***
	(1.356)	(1.007)	(0.368)	(0.243)	(1.358)	(1.076)	(0.379)	(0.253)
Obs.	7,897	6,914	7,897	6,914	7,897	6,914	7,897	6,914
No. fixed effects	1,197	1,147	1,197	1,147	1,197	1,147	1,197	1,147
R^2	0.106	0.160			0.109	0.162		
AR(1)			-3.730	-4.762			-3.778	-4.803
p-val			1.91e-4	1.91e-6			1.58e-4	1.56e-06
AR(2)			-2.383	-0.670			-2.398	-0.608
p-val			0.0172	0.503			0.0165	0.543
Hansen			75.53	83.94			81.65	87.67
p-val			0.0508	0.249			0.0178	0.170

Table 3.6: Fixed Effect and GMM Regressions for Large Firms: Top 1500 (N=7,897)

(1) Include firm and year effects. Standard errors (in brackets) are clustered at the firm level. *** p<0.01, ** p<0.05, * p<0.1.
(2) GMM instruments are the q terms, cash flow, leverage, and lagged investment rate.

	Conventional				Chen & Dauchy			
Independent	Pooled OLS, fixed effects		GMM		Pooled OLS, fixed effects		GMM	
Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$q_{ijt}/(1-\tau_t)$	0.006***	0.006***	0.011***	0.012***				
	(0.001)	(0.001)	(0.003)	(0.003)				
$q_{ijt}\_adj/(1-\tau_t)$					0.007***	0.007***	0.013***	0.014***
					(0.001)	(0.001)	(0.004)	(0.004)
ETT_{jt}	-2.159***	-1.056***	-0.600***	-0.276***	-2.225**	-1.132***	-0.608***	-0.309**
	(0.384)	(0.339)	(0.068)	(0.047)	(0.384)	(0.339)	(0.066)	(0.044)
STT_{jt}	-0.343	-0.081	-1.367***	-0.607***	-0.335	-0.077	-1.353***	-0.624***
	(0.263)	(0.248)	(0.094)	(0.070)	(0.263)	(0.248)	(0.095)	(0.068)
CF_{ijt}/K_{ijt-1}	-0.031***	-0.026***	-0.032***	0.013*	-0.030***	-0.026***	-0.036***	0.012
	(0.001)	(0.001)	(0.008)	(0.008)	(0.001)	(0.001)	(0.008)	(0.008)
Lev_{jt}	0.009***	0.003**	0.043***	0.031***	0.009***	0.003**	0.043***	0.030**
	(0.002)	(0.002)	(0.015)	(0.012)	(0.002)	(0.002)	(0.015)	(0.012)
Lag(I_{ijt}/K_{ijt-1})		0.188***		0.491***		0.187***		0.482***
		(0.008)		(0.036)		(0.008)		(0.036)
Const.	3.138***	1.437	2.848***	1.265***	3.201**	1.519**	2.832***	1.329***
	(0.718)	(0.647)	(0.222)	(0.163)	(0.717)	(0.646)	(0.220)	(0.156)
Obs.	55,442	46,235	55,442	46,235	55,442	46,235	55,442	46,235
No. fixed effects	10,889	9,718	10,889	9,718	10.889	9,718	10,889	9,718
R^2	0.133	0.155			0.133	0.156		
AR(1)			-22.73	-15.80			-22.79	-15.75
p-val			0	0			0	0
AR(2)			-10.89	1.275			-10.87	1.176
p-val			0	0.202			0	0.240
Hansen			240.9	127.2			227.4	130.2
p-val			0	2.11e-4			0	1.08e-4

Table 3.7: Fixed Effect and GMM Regressions with Winsorization at 5 Percent

(1) Include firm and year effects. Standard errors (in brackets) are clustered at the firm level. *** p<0.01, ** p<0.05, * p<0.1.
(2) GMM instruments are the q terms, cash flow, leverage, and lagged investment rate.

Independent Variables	Conventional				Chen & Dauchy			
	Pooled OLS, fixed effects		GMM		Pooled OLS, fixed effects		GMM	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$q_{ijt}/(1-\tau_t)$	0.009***	0.009***	0.026***	0.014***				
	(0.001)	(0.001)	(0.003)	(0.002)				
$q_{ijt}\_adj/(1-\tau_t)$					0.012***	0.011***	0.033***	0.017***
					(0.001)	(0.001)	(0.004)	(0.003)
ETT_{jt}	-1.667***	-0.937***	-0.260***	-0.149***	-1.743**	-1.030***	-0.306***	-0.183**
	(0.272)	(0.238)	(0.049)	(0.029)	(0.271)	(0.237)	(0.047)	(0.027)
STT_{jt}	-0.228	-0.031	-0.736***	-0.359***	-0.216	-0.026	-0.738***	-0.376***
	(0.165)	(0.152)	(0.066)	(0.040)	(0.164)	(0.152)	(0.066)	(0.039)
CF_{ijt}/K_{ijt-1}	-0.019***	-0.013***	-0.023***	0.022***	-0.019***	-0.012***	-0.028***	0.020
	(0.001)	(0.001)	(0.009)	(0.007)	(0.001)	(0.001)	(0.009)	(0.007)
Lev_{jt}	0.010***	0.004**	0.034***	0.019*	0.011***	0.005**	0.037***	0.019**
	(0.002)	(0.002)	(0.014)	(0.010)	(0.002)	(0.002)	(0.015)	(0.010)
Lag(I_{ijt}/K_{ijt-1})		0.228***		0.557***		0.227***		0.546***
		(0.007)		(0.030)		(0.007)		(0.030)
Const.	2.382***	1.210***	1.453***	0.729***	2.450**	1.308**	1.498***	0.793***
	(0.493)	(0.437)	(0.158)	(0.095)	(0.491)	(0.435)	(0.155)	(0.091)
Obs.	55,442	46,235	55,442	46,235	55,442	46,235	55,442	46,235
No. fixed effects	10,889	9,718	10,889	9,718	10.889	9,718	10,889	9,718
R^2	0.120	0.154			0.122	0.155		
AR(1)			-27.01	-20.14			-26.98	-19.92
p-val			0	0.0185			0	0.514
AR(2)			-11.90	2.355			-11.71	2.276
p-val			0	0.496			0.970	0.023
Hansen			320.7	140.4			296.9	144.4
p-val			0	1.03e-5			0	3.70e-6

Table 3.8: Fixed Effect and GMM Regressions with Winsorization at 10 Percent

(i) Include firm and year effects. Standard errors (in brackets) are clustered at the firm level. *** p<0.01, ** p<0.05, * p<0.1.
(2) GMM instruments are the q terms, cash flow, leverage, and lagged investment rate.

3.7 Appendix

3.7.1 Methodology for Measuring the Stocks and Flows of Intangible and Physical Assets

Intangible Assets

This paper measures intangible assets in a comprehensive way using the methodology developed by Corrado et al. (2005) and Corrado et al. (2006) (hereafter CHS) for the United States, and increasingly used for other countries since then (Edquist (2011), Giorgio Marrano and Haskel (2007), Marrano et al. (2009), Fukao et al. (2009), Jalava et al. (2007), van Rooijen-Horsten and van den Bergen (2008)). In this paper, we extend this methodology to the corporate level. We measure intangible assets by types of assets from 1998 through 2006. The data collection is, to our knowledge, the most comprehensive to this date for this time period.

Our data cover companies that file form 1120 for tax purposes, and therefore are classified by the Internal Revenue Service as corporations paying the corporate tax. The CHS methodology uses various sources to cover intangible assets, including the Bureau of Economic Analysis (BEA)'s Survey of Current Businesses for intangibles that are accounted in national NIPA accounts as physical assets (e.g., computer software). All sources for non-NIPA intangible assets are presented in Table 3.11 and 3.12.

The NIPA measures of aggregate investment and revenues are based on data collected from either "establishments" or "companies." In the Industry section of A Guide to the NIPA's, the BEA states that:

Establishments are classified into an SIC industry on the basis of their principal product or service, and companies are classified into an SIC industry on the basis of the principal SIC industry of all their establishments. Because large multi-establishment companies typically own establishments that are classified in different

SIC industries, the industrial distribution of the same economic activity on an establishment basis can differ significantly from that on a company basis.

This is very important because multi-establishment corporations (such as Multinational corporations or MNCs) can operate in industries that are radically different from their establishments (or branches); however, for tax purposes, corporate tax filings are prepared by the parent company, and generally allocated to industries on a company basis. For purposes of calculating tax allowances and the welfare impact of corporate taxation, we need to focus on industry classifications from tax filings, which may radically differ from filings in NIPA. Nevertheless, the distribution found in NIPA accounts has at least one advantage over that found in tax filings. Tax filings are based on consolidated returns, including not only domestic corporations and their domestic subsidiaries, but also their foreign subsidiaries. By contrast, NIPA accounts only cover domestic operations. For our purposes of accurately calculating depreciation allowances and welfare impact of taxation, we are only interested in domestic corporations.

Fortunately, the BEA's Survey of Current Businesses also collects information on the corporate status of the companies surveyed. Corporations—including parents and their subsidiaries–are separate entities filing taxes separately from their parent or their own subsidiaries. By contrast, branches are not separated from their parent, and non-corporate businesses (such as partnerships) do not pay the corporate tax. In this paper, we start by separating investment between the corporate and the non-corporate sector. Then we distribute corporations across industries based on NIPA accounts by assuming that intangible assets have the same distribution across sectors than equipment and software assets. The latter is obtained from the Bureau of Economic Analysis (BEA)'s current cost net stocks and investments in physical assets (as explained below); however, when industry classification of corporate investment is also available from the IRS/SOI, we use data from the IRS/SOI. This is the case for two intangible assets: research and development or R&D spending and advertising.

We recognize that the distribution of investment across industries is still subject to misclassification but we believe that our approach limits the industry distribution error as much as possible. In Table 3.10 we present a simple comparison of the impact of measuring intangible investments from the corporate part of the BEA's Survey of Current Businesses as compared to IRS/SOI Tax Filings in the case of R&D, for which we can compare R&D spending from tax filings to corporate expenditures in R&D from the corporate sector of the BEA. The table shows that although the distribution of R&D expenditures across industries is not precisely the same when BEA/NSF accounts are used as when IRS tax filings are used, the relative importance of industries is preserved. For example, in both the IRS and the BEA's distribution, the share of R&D spending is the largest for the manufacturing industry, followed by information and finance.

Investment in physical assets (also referred to as tangible, or fixed assets) is also obtained from the BEA's NIPA accounts.

In order to accurately estimate the stock and the depreciation of intangible assets, whenever possible, we measure investment in intangible assets over as many years as their economic lives. When investment data in intangible assets is not available for all years along their economic life, we extend the data over time based on each industry's growth rate of gross domestic value added, obtained from the BEA (see details below).

We obtain data for six broad types of intangible assets, including computerized information, scientific and non-scientific R&D, firm-specific human capital, organizational skills, and brand equity. None of these assets (except for software) is included in NIPA accounts. Instead they are directly expensed for accounting purposes, generally because they are difficult to measure.

Table 3.11 and 3.12 describe in detail data sources used to measure various types of intangible assets, and the methodology used to measure the part of these assets that creates long-term revenue (and therefore can be considered as investment). CHS (2005) provide more details on the reason why these data provide a comprehensive measure of detailed intangible assets available. The first columns of Table 3.11 and 3.12 list the non-NIPA intangible asset. The second column lists the data sources. The third column defines the asset. While some intangible assets could be directly measured for the corporate sector only, for other intangible assets, the disaggregation between the corporate and the non-corporate sectors is based on NIPA investment and stocks share of physical assets between the corporate and the non-corporate sectors, and specified in column 4. This method is also used to separate corporate and non-corporate physical assets in each industry.

Table 3.13 shows the total value and the average annual growth rate of investment in intangible assets by industry from 1998 to 2006. Over this period, investment in intangible assets amounted to $7.2 trillion, represented about 45 percent of total corporate investment, and grew at an average annual rate of 3.7 percent. Almost 47 percent of this investment was concentrated in 3 industries: finance and insurance, metals, machinery, electronic, electrical, and transportation equipment manufacturing, and information.

To obtain the stock of intangible assets, we use the data obtained for investment in intangible assets and assumes that these assets depreciate according to the perpetual inventory method (PIM). The PIM is also used by NIPA accounts to age the stock of physical assets, and is explained in detail in Meinen et al. (1998). The net stock of any asset in year t and in year t prices is defined as:

$$NCS_{t,t} = \sum_{i=0}^{d-1} \left(I_{t-i} P^I_{t-i,t} - \sum_{j=0}^{i} CC_{t-j} \right),$$

where d is the recovery period of the asset, I_t is the amount invested in the asset in current dollars, $P^I_{t-i,t}$ is the price index of year t with base year t-i, and CC_t is the consumption of the capital asset in year t. Assuming straight line depreciation of intangible assets, we have

$$CC_t = \frac{1}{d} \left(\frac{GCS_{t,t} + GCS_{t,t-1}}{2} \right), \tag{3.15}$$

where

$$GCS_{t,t} = \sum_{i=0}^{d-1} I_{t-i} P_{t-i,t}.$$

Equation (3.15) assumes that investment is made throughout the year, while the gross capital stock in year t and in year t prices ($GCS_{t,t}$) is generally obtained in December.

Table 3.14 to 3.16 shows the assumed values of economic depreciation of various assets. For intangible assets, we follow previous literature (CHS, 2005; Fraumeni, 1997). Table 3.17 shows the total stock of intangible assets over the period 1998-2006 and the compounded annual growth rate. The stock of intangible assets was about \$10.6 trillion, or 11 percent of total assets, which is relative terms is the much smaller as a share of total assets than investment in intangible assets. The reason is essentially that in spite of its fast rate of growth, intangible assets stocks started from a small share of total assets (about 7 percent in 1998) and depreciate at a faster rate than most structures and many equipment assets.

BEA tables

- BEA value added table: “GDPbyInd_VA_NAICS_1998-2006.xls”- industry value added, gross output and intermediate inputs, and components of value added, in current dollars and corresponding quantity and price indexes (2005=100) for 1998-2006.

Physical Assets

To isolate investments and stocks of physical and intangible assets by industry, assets, and over time, we use BEA's Physical Asset tables (listed below). First, we use the stocks and flows of non-residential (tables 4.1 and 4.7) and residential (Tables 5.1. and 5.7) physical assets by legal form of organization to isolate corporate stock and investment in equipment and structures for each year. Second, for each year, we distribute the corporate amounts of investment and stocks in these broad asset types—obtained from step one–across detailed asset types, using BEA tables 2.1 and 2.7, which provide detailed stocks and flows of private physical assets for 75 detailed asset types. Third, for each year and each asset, we distribute these detailed corporate asset investment amounts (stock and flows) across industries, using the 1997 BEA's capital flow data, based on the Survey of Current Businesses. This implies that the total distribution across assets of investment and stocks of corporate stock physical assets varies not only over time (step two), but also across industries (step 3). We obtain one matrix for each year showing the distribution of corporate stock (respectively investment) across detailed physical assets and two-digit industries: 9 matrices (one for each year from 1998 to 2006) showing the distribution across assets and within industries of industrial corporate physical asset stocks, and 9 matrices showing the distribution across assets and within industries of industrial corporate physical asset flows.

- MS[ms_a,i,t] = matrix showing total stocks (or levels) ms in physical assets a (a=1-A1), by industry I (i=1 to N) at time t (t=1998-2006). A1 is the number of tangible assets and N is the number of industries.

- MF[mf_a,i,t] = matrix showing total investment (or flow) mf in physical assets a (a=1-A1), by industry I (i=1 to N) at time t (t=1998-2006).

Tables A3 and A5 show the total amount of physical assets by type and industry from 1998 to 2006. Total investment in physical assets was $8.8 trillion over 9 years, representing 55 percent of total assets, three fourth of which were in equipment and software. The stock of physical assets was $82.1 trillion over 9 years, or 89 percent of all capital stock, two third of which were in structure assets, due to their longer recovery period.

BEA tables

- Table 2.1. Current-Cost Net Stock of Private Physical Assets, Equipment and Software, and Structures by Type
- Table 2.7. Investment in Private Physical Assets, Equipment and Software, and Structures by Type
- Table 4.1. Current-Cost Net Stock of Private Nonresidential Physical Assets by Industry Group and Legal Form of Organization
- Table 4.7. Investment in Private Nonresidential Physical Assets by Industry Group and Legal Form of Organization
- Table 5.1. Current-Cost Net Stock of Residential Physical Assets by Type of Owner, Legal Form of Organization, Industry, and Tenure Group
- Table 5.7. Investment in Residential Physical Assets by Type of Owner, Legal Form of Organization, Industry, and Tenure Group
- Capital Flows: table 4-"NIPAx123EqSoft"-Capital flow table, in purchasers' prices, with NIPA equipment and software categories as rows, with 123 columns

of using industries, and table 5-"NIPAx123Struc"-Capital flow table, in purchasers' prices, with NIPA structures categories as rows, with 123 columns of using industries.

The distribution of corporate investment and stock by asset, industry, and over time

The previous subsection provides two groups of matrices: (i) 9 matrices distributing physical asset stocks by asset and across industries, one for each year from 1998 to 2006, and (ii) the same as (i) for physical asset investment. Appendix A1 provides the stocks and flows of corporate intangible assets over time and by industry, for 6 types of intangible. We update the annual matrices of physical assets from appendix A2 with intangible assets. This provides 9 new matrices (one for each year from 1998 to 2006) showing the distribution, across assets and within industries, of industrial corporate physical and intangible asset stocks, and 9 similar matrices for corporate physical and intangible asset investment flows.

- NS[ns_a,i,t] = matrix showing total stocks (or levels) ns in physical assets a (a=1-A2), by industry I (i=1 to Ni) at time t (t=1998-2006), where A2 is the number of physical and intangible assets.

- NF[nf_a,i,t] = matrix showing total investment (or flow) nf in physical assets a (a=1-A2), by industry I (i=1 to Ni) at time t (t=1998-2006).

These matrices permit to calculate the weight of each assets within industries, which are critical in order to calculate the present value of depreciation allowances of $1 of investment in industry i, which is explained in the next subsection.

3.7.2 Tax Parts

We follow Cummins et al. (1994) and House and Shapiro (2008) to construct the tax parts. Many changes in the treatment of depreciable assets have been passed in the last decade: in 2002 to 2004. All of these changes were temporary investment tax

Calendar Year	MACRS	BD 30%	BD 50%
1998-2001	365 or 366		
2002		365	
2003		125	240
2004			366
2005-6	365		

Table 3.9: Policy Calculations

MACRS = modified accelerated recovery system under current law. BD = first-year bonus depreciation allowance. The Job Creation and Worker Assistance Act of 2002 created 30 percent bonus depreciation for qualified capital put in place after September 11, 2001. However, because the Act was passed in March 2002, investors sitting in 2001 did not make their investment decisions based on the reduced asset cost for that year. The Jobs and Growth Tax Relief Reconciliation Act of 2003 provided 50 percent first-year depreciation allowance was for capital put in place after May 28.

incentives with different effects for different asset types. The depreciation allowances allowed in 1998-2006 only applied to short-lived investment assets, which is defined as equipment and structure assets with a recovery period of 20 years or less.

The calculation of the present value of tax depreciation allowances takes account of the fact that the periods covering bonus depreciation were not always the same as the calendar year. In this case, the PV of depreciation allowance for a given asset and a given calendar year is calculated as the weighted average of the PV of depreciation allowances available for that year, weighted by the number of days of the applicable policy:

$$DA_a = \frac{\#days_1}{365} DA_{a,1} + \frac{\#days_2}{365} DA_{a,2},$$

where where $DA_{a,1}$ and $\#days_1$ (respectively $DA_{a,2}$ and $\#days_2$ are respectively the present values of depreciation allowances and the number of calendar days when they are available under policies 1 (respectively policy 2). Table 3.14 to 3.16 shows the shows PV of depreciation allowances of each asset and under the alternative policies in place during the 1998-2006 period. Table 3.9 shows the number of days when a given policy has been applicable in a given year.

For instance, as shown in Table 3.14, the present value of depreciation allowances of software, which has a tax life of 5 years, is 0.933 under 30 percent bonus depreciation and 0.952 under 50 percent bonus depreciation. Because both policies overlap in year 2003, the PV of depreciation allowances of software in 2003 is given by (125/365)* 0.933 +(240/365)* (0.952), or 0.945. The present value of depreciation allowances for each physical asset is calculated based on the applicable MACRS rule, with mid-year convention (IRS, 2010). A discount rate of 5 percent is assumed, which is roughly the average of the rate on 10-year treasury bonds over the 9 years considered. Finally, the present value of depreciation allowance for physical assets in a given industry and a given year ($DA_{i,t}$) is measured as the weighted average of depreciation allowances of each types of physical assets in the industry, weighted by investment in the asset:

$$DA_{i,t} = \sum_{a=1}^{A} w_{i,a,t} DA_{a,t},$$

where $w_{i,a,t} = I_{i,a,t}/I_{i,t}$ is the proportion of investment in asset a and industry i in year t.

In this paper, since we are interesting in explaining investment in physical assets, calculations of the tax term of the cost of capital disregard intangible assets, implying that the denominator of $w_{i,a,t}$ only includes total investment in fixed assets. Using the matrix MF, this gives

$$w_{i,a,t} = \frac{mf_{i,a,t}}{\sum_{k=1}^{A_1} mf_{i,k,t}}.$$

Table 3.18 shows summary statistics of depreciation allowances of each industry during 1998-2006. The tax term of each group of assets is assets is

$$TaxTerm_{i,t} = 1 - \frac{DA_{i,t}\tau}{1-\tau},$$

where τ is the statutory top corporate tax rate, consistently 35 percent over the period considered.

When intangible assets are not included in total investment, the average PV of depreciation allowances for \$1 of investment over all industries in 1998-2006 was \$0.68 for the equipment and software part and \$0.11 for the structures part. When intangible assets are included in total investment, the PV of depreciation allowances for \$1 of investment is \$0.38 for the equipment and software part, \$0.06 for the structures part, and \$0.44 for the intangible assets part. The PV of depreciation allowances for the intangible assets part is largest in industries that are intangible intensive: food and apparel manufacturing, finance and insurance, and business management services.

3.7.3 Macroeconomic Variables

The real growth rate of GDP is obtained from the 2011 Economic Report of the President (ERP, table 1: "Current-Dollar and Real Gross Domestic Product, 2005 Price). Price indices (PPI and CPI) are obtained from tables 64 and 68 or the ERP. The unemployment rate is from table 42 of the ERP.

The 10-year federal funds rate is taken from Board of Governors of the Federal Reserve System, table H.15 (seasonally adjusted).

3.7.4 Proofs

Proof of Proposition 1

Proof. We shall first show that the ex-dividend value of the firm P_t is equal to the value of tangible assets and intangible assets under the assumption of constant return to scale in the production technology and adjustment costs:

$$P_t = q_t^m K_{t+1}^m + q_t^u K_{t+1}^u, \tag{3.16}$$

where q_t^m and q_t^u are the marginal value of K_{t+1}^m and K_{t+1}^u respectively.

To show this, let V_t denote the cum-dividend market value. V_t is the sum of the firm's ex-dividend market value plus dividend payout:

$$V_t = P_t + D_t, \tag{3.17}$$

where dividend D_t is given by

$$\begin{aligned} D_t &= (1-\tau_t)\left(F\left(K^m_{t+s}, K^u_{t+s}, X_{t+s}\right) -_{i=\{m,u\}} \Psi\left(I^i_{t+s}, K^i_{t+s}\right)\right) \\ &\quad - \sum_{i=\{m,u\}} \left(1 - k^i_{t+s} - \tau_{t+s} z^i_{t+s}\right) I^i_{t+s}. \end{aligned}$$

Profit maximization in (3.1) implies

(3.18)
$$V_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta_{ts} \left[\begin{array}{c} (1-\tau_{t+s})\left(F\left(K^m_{t+s}, K^u_{t+s}, X_{t+s}\right) -_{i=\{m,u\}} \Psi\left(I^i_{t+s}, K^i_{t+s}\right)\right) \\ -_{i=\{m,u\}} \left(1 - k^i_{t+s} - \tau_{t+s} z^i_{t+s}\right) I^i_{t+s} \\ +_{i=\{m,u\}} q^i_t \left[\left(1-\delta^i\right) K^i_{t+s} + I^i_{t+s} - K^i_{t+s+1}\right] \end{array} \right] \right\}.$$

Substituting the first order conditions (3.3) gives

$$V_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta_{ts} \left[\begin{array}{c} (1-\tau_{t+s})\left(F\left(K^m_{t+s}, K^u_{t+s}, X_{t+s}\right) -_{i=\{m,u\}} \Psi\left(I^i_{t+s}, K^i_{t+s}\right)\right) \\ -_{i=\{m,u\}} \left(1 - k^i_{t+s} - \tau_{t+s} z^i_{t+s}\right) I^i_{t+s} \\ +_{i=\{m,u\}} q^i_{t+s} \left[\left(1-\delta^i\right) K^i_{t+s} - K^i_{t+s+1}\right] \\ +_{i=\{m,u\}} \left[1 - k^i_{t+s} - \tau_{t+s} z^i_{t+s} + (1-\tau_{t+s})\, \psi \frac{I^i_{t+s}}{K^i_{t+s}}\right] I^i_{t+s} \end{array} \right] \right\}.$$

Linear homogeneity of adjustment costs implies $\Psi\left(I^i_{t+s}, K^i_{t+s}\right) = \frac{\partial \Psi\left(I^i_{t+s}, K^i_{t+s}\right)}{\partial I^i_{t+s}} I^i_{t+s} + \frac{\partial \Psi\left(I^i_{t+s}, K^i_{t+s}\right)}{\partial K^i_{t+s}} K^i_{t+s}$, which simplifies to $\Psi\left(I^i_{t+s}, K^i_{t+s}\right) = \psi \frac{I^i_{t+s}}{K^i_{t+s}} I^i_{t+s} - \frac{\psi}{2}\left(\frac{I^i_{t+s}}{K^i_{t+s}}\right)^2 K^i_{t+s}$ under parameterization $\Psi\left(I_{it+s}, K_{it+s}\right) = \frac{\psi}{2} K_{it}\left(\frac{I_{it}}{K_{it}}\right)^2$. So V_t simplifies to

$$V_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta_{ts} \left[\begin{array}{c} (1-\tau_{t+s})\left(F\left(K^m_{t+s}, K^u_{t+s}, X_{t+s}\right) +_{i=\{m,u\}} \frac{\psi}{2}\left(\frac{I^i_{t+s}}{K^i_{t+s}}\right)^2 K^i_{t+s}\right) \\ +_{i=\{m,u\}} q^i_{t+s} \left[\left(1-\delta^i\right) K^i_{t+s} - K^i_{t+s+1}\right] \end{array} \right] \right\}.$$

Recursively substituting the first order conditions (3.4) gives

$$V_t = (1-\tau_t)\left(F\left(K^m_t, K^u, X_t\right) +_{i=\{m,u\}} \frac{\psi}{2}\left(\frac{I^i_t}{K^i_t}\right)^2 K^i_t\right) +_{i=\{m,u\}} q^i_t\left(1-\delta^i\right) K^i_t$$

Equating (3.17) and (3.18) and collecting terms gives

$$\begin{aligned} P_t &= \sum_{i=\{m,u\}} \left[1 - k_t^i - \tau_t z_t^i + (1-\tau_t)\psi \frac{I_t^i}{K_t^i}\right] I_t^i +_{i=\{m,u\}} q_t^i \left(1-\delta^i\right) K_t^i \\ &= \sum_{i=\{m,u\}} q_t^i \left[I_t^i + \left(1-\delta^i\right) K_t^i\right] \\ &= q_t^m K_{t+1}^m + q_t^u K_{t+1}^u \end{aligned}$$

where the first equality follows from first order conditions (3.3) and the last equality follows from the law of motions (3.2).

Finally, by definition $q_t = \frac{P_t}{K_{t+1}^m + K_{t+1}^u}$, it follows that

$$q_t = q_t^m \frac{K_{t+1}^m}{K_{t+1}^m + K_{t+1}^u} + q_t^u \frac{K_{t+1}^u}{K_{t+1}^m + K_{t+1}^u}.$$

□

	1998	1999	2000	2001
NSF/BEA				
FIRE*	1,792	1,699	4,172	2,631
	1.2%	1.0%	2.2%	1.3%
Information	10,054	11,633	11,109	12,069
	7,9%	8.7%	9.5%	12.7%
Manufacturing (incl.)	79,475	85,547	96,078	101,099
Chemical	16.0%	16.0%	15.6%	16.5%
Computer and electronic product	21.4%	19.0%	21.4%	23.2%
Transportation equipment	14.2%	14.8%	12.7%	11.6%
incl. aerospace	4.4%	3.3%	2.2%	2.2%
IRS/SOI				
FIRE*	1,318	1,424	1,612	1,791
	1.2%	1,2%	1.3%	1.5%
Information	12,161	14,908	18,427	25,131
	9.1%	10.1%	9.2%	10.4%
Manufacturing (incl.)	86,428	88,234	92,304	86,279
Chemical	25.4%	24.7%	22.0%	23.7%
Computer and electronic product	18.5%	18.3%	16.5%	19.1%
Transportation equipment	22.0%	20.0%	18.6%	18.7%
incl. aerospace	3.2%	2.5%	2.5%	3.1%

Table 3.10: Research and Development Expenditures by Industry: BEA/NSV vs. IRS/SOL (Mil Dollars and Percentage of All Industries)

IRS/SOI: R&D tax credit claims and U.S. corporate tax returns claiming the credit, by selected NAICS industry; and National Science Foundation (used by the BEA): Table 5.1: Investment in RD. *Finance, insurance, and real estate.

Type of Intangible Assets	Source	Definition
1. Computerized Information[5]	SAS (see CHS 2005, 2009), and NIPA accounts I-O use tables for industry use. NIPA accounts for the corporate share.[1]	Total revenue from subscription to "Online directories, databases, and other collections of information" from publishers of databases (NAICS 51114); does not include print directories, databases, other collections of information (other media directories, databases, and other).
2. Innovative Property		R&E qualified expenditures reported by corporations. Does not include mineral exploration, because this is included in NIPA.
2.1. R&D	IRS Form 6567. Corporate share from source.	Innovation expenses lead to new copyrights and licenses. Cost of developing original performances or products in the arts and entertainment industry. Includes development costs in (1) the motion picture industry and (2) the radio and television, sound recording, and book publishing industries. No estimate for the arts.
2.2. Copyright & license costs	CHS (2005, 2009) NIPA accounts for the corporate share[2]	
2.2. Other productuct development, design, and research expense	Bureau of Economic Analysis (BEA) and accounts and SAS (see CHS 2005, 2009); NIPA accounts I-O use tables for industry use.NIPA accounts for the corporate share.[1]	These costs do not necessarily lead to new patents or copyrights. Covers non-scientific R&D in finance and services industries. Includes (i) New product development costs in the financial services industries, crudely estimated as 20% of intermediate purchases (from BEA).[3] (ii) Costs in new architectural and engineering designs, estimated as half of industry revenue for taxable employers in architecture and engineering services (NAICS 5413), including geophysical and mapping surveys, and (iii) R&D in social sciences and humanities, estimated as twice industry revenue for taxable employers in R&D in the social sciences and the humanities (NAICS 54172).[4]

Table 3.11: List of Corporate Intangible Assets Not-Included in NIPA Accounts, Definitions and Sources. (Part I)

(1) Input-Output tables from NIPA over time are used to distribute professional services across industries based on their use. (2) CHS (2005, 2009) use data from the Motion Picture Association of America (MPAA). (3) Intermediate purchases for finance industries (NAICS 521,523,525) from BEA's GDP-by-industry data; (4) SAS, Table 6.1 for professional, scientific and technical services (NAICS 54). (5) This only includes development of software because software is included in NIPA since 1998.

Type of Intangible Assets	Source	Definition
3. Economic Competencies		
3.1. Firm-specific human capital	Based on a broad survey of employer-provided training, conducted by the Bureau of Labor Statistics (BLS) in 1994 and 1995. NIPA accounts for the corporate share.	Cost of developing labor force skills
3.1.1. On-the-job training	BLS and BEA (see CHS, 2005, 2009). NIPA accounts for the corporate share.[2]	Direct firm expenses (Wages and salaries of in-house trainers, payments to outside trainers, tuition reimbursement, and contributions to outside training funds)
3.1.2. Payment for job-related education	BLS and BEA (see CHS, 2005, 2009). NIPA accounts for the corporate share.[3]	Tuition payments for job-related education. Wage and salary costs of employee time in formal and informal training.
3.2. Organizational capital	SAS, and Occupational Employment Statistics (OES) Surveys from the Bureau of Labor Statistics. (see CHS, 2005, 2009). NIPA accounts for the corporate share.	(i) Purchased "organizational" or "structural" capital, estimated using SAS data on the revenues of the management consulting industry, and distributed across industries using BEA I/O use tables, and (ii) Own-organizational skills, estimated as one fifth of the value of executive time using BLS data on employment and wages in executive occupations (OES).[2]
3.3. Brand equity		
3.3.1. Advertising	Internal Revenue Service. Corporate share from the source.	Expenditures on advertising services, from IRS/SOI data on corporate expenses on advertising.[4]
3.3.2. Marketing research	SAS (see CHS 2005, 2009), and NIPA accounts I-O use tables for industry use. NIPA accounts for the corporate share.[1]	Market research for the development of brands and trademarks, estimated as twice industry purchased services (revenues of the market and consumer research industry as reported in SAS), and distributed across industries based on BEA I/O use tables.

Table 3.12: List of Corporate Intangible Assets Not-Included in NIPA Accounts, Definitions and Sources (Part II)

(1)Input-Output tables from NIPA over time are used to distribute professional services across industries based on their use. (2) Estimates for other years were derived from (i) industry per employee costs reported in BLS surveys in 1994 (Tab. 9 for Selected expenditures by industry), and (ii) trends in the use of education by industry (from BEA I/O use table). (3) Estimates for other years were derived from (i) industry per employee costs reported in BLS surveys in 1994 (Tab. 11), and (ii) trends in aggregate FTE employment by industry (from BEA). (4) IRS, Returns of Active Corporations (Tab. 6).

Industry	Investment				AAGR			
	Total ($)	Equip. software (%)	Structure (%)	Intangibles (%)	Total ($)	Equip. software (%)	Structure (%)	Intangibles (%)
Agriculture, forestry, fishing, hunting	308	86.0	12.0	2.0	0.6	0.02	5.1	1.4
Mining	543	33.2	57.8	9.0	10.4	4.5	15.2	7.8
Utillities	522.36	51.6	43.2	5.2	5.8	2.0	9.9	5.7
Construction	622.11	67.7	1.4	30.8	0.4	1.6	4.3	1.9
Food, beverage, tobacco, textile, apparel, leather	577.59	27.5	3.1	69.4	0.5	0.6	5.7	0.4
Wood, paper, printing, petroleum, chemical	1,144	43.4	4.4	52.3	2.1	0.9	5.8	3.0
Metal, machinery, computer, electronic	2,040	38.0	4.2	57.9	1.4	0.9	5.7	1.4
Wholesale	708	33.2	5.1	61.8	2.6	0.1	1.4	4.1
Retail	1,020	25.8	21.6	52.6	1.1	0.2	1.4	2.0
Transportation, couriers, warehousing	875	77.7	11.1	11.2	1.4	0.1	6.7	0.0
Information	1,969	37.8	8.3	53.9	2.7	0.7	4.3	4.1
Finance and insurance	1,673	25.1	5.6	69.3	5.0	1.8	1.4	6.5
Real estate, rental & leasing	518.49	62.9	14.2	22.8	0.0	2.7	1.6	6.2
Profession & technical services	803	49.5	3.5	47.0	3.8	2.8	1.4	4.9
Management of companies & enterprises	246.49	21.9	3.2	74.9	3.7	2.2	1.4	4.3
Administrative & waste services	325.26	48.4	5.2	46.4	2.3	0.9	0.8	3.8
Educational services	303.98	25.0	27.9	47.1	4.5	3.4	4.5	5.3
Health care & socail assistance	900	52.2	23.5	24.4	5.7	5.1	5.3	7.5
Arts, entertainment & recreation	142.25	27.7	40.2	32.2	2.6	1.1	2.5	4.8
Accommodation & food services	570.32	26.5	40.7	32.9	2.9	0.4	2.5	6.3
Other services, excluding public administration	292.7	34.9	34.4	30.8	1.6	0.9	0.8	5.6
US Totals	16,106	41.5	13.4	45.1	2.8	1.0	5.3	3.7

Table 3.13: Investment and Average Annual Growth Rate of Corporate Physical and Intangible Assets, 1998 to 2006 (Bil. Dollars)

				NPV of tax depreciation allowances		
Category in purchasers' prices	Tax life	Declining balance	Depreciation	MACRS	BD 30%	BD 50%
(4), (5), (9) Computers, peripheral, software, office, accounting equipment	5	200	0.312	0.904	0.993	0.952
(6) Communication equipment	5	200	0.150	0.904	0.993	0.952
(7a), (7b) Nonmedical and medical instrument and related equipment	7	200	0.135	0.840	0.905	0.920
(8) Photocopy and related equipment	5	200	0.180	0.904	0.933	0.952
(11) Fabricated metal products	7	200	0.092	0.840	0.905	0.920
(12) Engines and turbines	15	150	0.052	0.694	0.786	0.847
(13) Metalworking machinery	7	200	0.123	0.840	0.905	0.920
(14) Special industry machinery, n.e.c.	7	200	0.103	0.840	0.905	0.920
(15) General industrial, incl. materials, equipment	7	200	0.107	0.840	0.905	0.920
(16) Electrical transmission, distribution & industrial apparatus	7	200	0.050	0.840	0.905	0.920
(18a), (18b) Light and other trucks, buses and trailers	5	200	0.123	0.904	0.933	0.952
(19) Autos	5	200	0.165	0.904	0.933	0.952
(20) Aircraft	7	200	0.110	0.840	0.905	0.920
(21) Ships and boats	10	200	0.061	0.781	0.868	0.890
(22) Railroad equipment	7	200	0.059	0.840	0.905	0.920
(24) Furniture and fixtures	7	200	0.138	0.840	0.905	0.920
(26*) Agricultural machinery, including tractors	7	150	0.145	0.840	0.905	0.920
(27*) Construction machinery, including tractors	5	200	0.163	0.904	0.933	0.952
(28) Mining and oilfield machinery	7	200	0.150	0.840	0.905	0.920
(29) Service industry machinery	7	200	0.165	0.840	0.905	0.920
(30) Electrical equipment, n.e.c.	7	200	0.183	0.840	0.905	0.920
(31) Other nonresidential equipment	7	200	0.147	0.840	0.905	0.920
(33) Residential (landlord durables)	5	200	0.118	0.904	0.933	0.952

Table 3.14: Rate of Economic Depreciation and Present Value of Tax Depreciation Allowances (Part I)

Category in purchasers' prices	Tax life	Declining balance	Depreciation	NPV of tax depreciation allowances MACRS	BD 30%	BD 50%
(6) Commercial buildings	39	SL	0.022	0.395	0.395	0.395
(11) Hospital and institutional buildings	39	SL	0.019	0.395	0.395	0.395
(12) Other nonresidential buildings, excluding farm	39	SL	0.025	0.395	0.395	0.395
(5) Industrial buildings	39	SL	0.031	0.395	0.395	0.395
(5) Industrial buildings	20	150	0.021	0.622	0.811	0.811
(16) Electric light and power	15	150	0.024	0.694	0.847	0.847
(15) and (17) Gas and Telecommunications	15	150	0.075	0.694	0.847	0.847
(21a) Petroleum and natural gas–wells	5	200	0.045	0.904	0.952	0.952
(22) Other mining construction	39	SL	0.019	0.395	0.395	0.395
(9), (10) Religious buildings and Educational buildings	20	150	0.028	0.622	0.811	0.811
(19) Farm nonresidential structures	20	150	0.024	0.622	0.811	0.811
(23) Other nonresidential non-building structures	39	SL	0.023	0.395	0.395	0.395
(30a) - (32) Single & multi family structures, nonfarm, Manufactured homes	27.5	SL	0.014	0.504	0.504	0.504
(33) Improvements	15	150	0.023	0.694	1	1
(34) Other	27.5	N/A	0.023	0.504	0.504	0.504

Table 3.15: Rate of Economic Depreciation and Present Value of Tax Depreciation Allowances (Part II)

				NPV of tax depreciation		
Category in purchasers' prices[2,3]	Tax life	Declining balance	Depreciation	MACRS	BD 30%	BD 50%
Intangible assets	N/A	N/A	0.33	1	1	1
Computerized Information	N/A	N/A	0.20	1	1	1
Scientific and non scientific R&D, firm-specific human capital, organizational capital	N/A	N/A	0.40	1	1	1
Brand Equity	N/A	N/A	0.60	1	1	1

Table 3.16: Rate of Economic Depreciation and Present Value of Tax Depreciation Allowances (Part III)

(1) The half-year convention is assumed. (i.e., investment is assumed to be installed in the middle of the first year, and therefore depreciates during half of the first year. The present value of tax depreciation allowances is calculated with a discount rate of 6% (approximately equal to the average nominal federal fund rate on 10-year Treasury bonds over the period). Under MACRS, assets with recovery periods above 20 years are depreciated based on straight-line. For assets with recovery periods of 20 years or less, the tax depreciation allowances each year is the maximum between straight line and declining balance. (2) The numbers in parenthesis preceding asset types are conform to the ones used in BEA physical assets tables. (3) The rates of economic depreciation of intangible assets are the same as in CHS (2005), except for computerized information, which is based on estimates by Fraumeni (1997).

Industry	Stock Total ($)	Stock Equip. software (%)	Stock Structure (%)	Stock Intangibles (%)	CAGR Total ($)	CAGR Equip. software (%)	CAGR Structure (%)	CAGR Intangibles (%)
Agriculture, forestry, fishing, hunting	3,168	49.0	51.0	0.29	3.17	3.31	3.03	5.54
Mining	4,721	21.5	77.1	1.5	0.7	0.9	1.6	70.7
Utillities	8,602	23.6	75.9	0.5	0.1	0.6	0.1	17.4
Construction	2,489	79.2	9.9	10.9	0.1	0.3	0.1	31.7
Food, beverage, tobacco, textile, apparel, leather	1,945	52.1	27.3	20.5	0.3	0.4	3.1	1.5
Wood, paper, printing, petroleum, chemical	5,383	56.0	27.7	16.3	0.5	0.8	1.6	6.6
Metal, machinery, computer, electronic	8,757	49.7	29.0	21.3	0.2	0.3	0.6	1.2
Wholesale	2,336	44.6	33.7	21.7	0.1	0.2	0.5	0.6
Retail	6,583	18.4	73.6	8.1	0.4	0.9	1.1	1.3
Transportation, couriers, warehousing	8,453	55.2	43.3	1.5	0.1	0.8	0.1	4.6
Information	8,477	39.6	38.4	22.0	3.3	2.3	13.1	84.0
Finance and insurance	5,403	21.7	39.0	39.3	0.2	0.5	0.2	0.7
Real estate, rental & leasing	3,387	43.6	51.9	4.5	0.1	0.3	0.4	0.4
Profession & technical services	2,331	49.7	26.6	23.7	0.4	1.8	0.5	7.8
Management of companies & enterprises	557.84	28.3	31.0	40.8	0.4	0.7	1.3	1.4
Administrative & waste services	1,281	51.8	33.5	14.6	1.3	5.4	4.4	4.3
Educational services	1,921	14.7	76.5	8.8	1.1	2.0	2.8	5.8
Health care & socail assistance	6,999	27.9	68.1	4.0	0.3	3.7	0.4	7.6
Arts, entertainment & recreation	1,374	14.0	81.8	4.1	0.1	0.3	0.2	2.8
Accommodation & food services	5,593	14.6	81.7	3.7	0.6	4.0	0.7	20.2
Other services, excluding public administration	2,967	16.5	79.8	3.7	0.1	0.9	0.2	4.5
US Totals	92,628	36.2	52.3	11.4	0.3	2.1	0.4	8.8

Table 3.17: Stock and Compounded Annual Growth Rate of Corporate Physical and Intangible Assets, 1998 to 2006 (Bil. Dollars)

Industry	Equip. software Mean	Std. Dev.	Structure Mean	Std. Dev.
Agriculture, forestry, and fishing	0.79	0.03	0.07	0.01
Mining	0.42	0.01	0.38	0.03
Utilities	0.49	0.02	0.26	0.02
Construction	0.88	0.02	0.01	0.00
Food, beverage, tobacco, textile, apparel, leather	0.76	0.03	0.05	0.00
Wood, paper, printing, petroleum, chemical	0.77	0.03	0.05	0.00
Metal, machinery, computer, electronic	0.77	0.03	0.05	0.00
Wholesale	0.78	0.02	0.05	0.00
Retail	0.48	0.02	0.18	0.00
Transportation, couriers and warehousing	0.76	0.02	0.09	0.01
Couriers and warehousing	0.85	0.02	0.02	0.00
Information	0.77	0.02	0.10	0.01
Finance and insurance	0.73	0.02	0.08	0.00
Real estate, rental & leasing	0.76	0.02	0.08	0.00
Professional & technical services	0.84	0.02	0.03	0.00
Management of businesses and enterprises	0.78	0.02	0.05	0.00
Administrative & waste services	0.80	0.02	0.04	0.00
Educational services	0.41	0.01	0.21	0.00
Health care & social assistance	0.60	0.02	0.12	0.00
Arts, entertainment, & recreation	0.33	0.01	0.24	0.00
Accommodation and food services	0.32	0.01	0.25	0.00
Other services, excluding public administration	0.46	0.01	0.19	0.00
US Totals	0.68	0.02	0.11	0.00

Table 3.18: Summary Statistics of the Present Value of Depreciation Allowances, 1998-2006

Depreciation allowances for each industry are weighted average of each industry-asset depreciation allowances (weighted by investment in a given asset). Depreciation allowances for equipment and software and structures is calculated without regards of intangible assets (i.e., the weights only includes physical assets.

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