Online Appendix

Pricing Policy in a Supply Chain: Negotiation or Posted Pricing

In the appendix, we present the proofs under the assumption that the valuation distribution satisfies Assumption **(A)**. We should note that when the valuation distribution is uniform, all of our results can be reproduced with much shorter proofs by taking advantage of closed form expressions that the uniform distribution enables.

In Section A of this appendix, we first prove Propositions 1 through 7 assuming that the supply chain capacity is sufficiently large: $Q \geq a$. The proofs for the case with tight capacity ($Q < a$) are more involved since the sales volume can be bounded by the supply chain capacity at lower wholesale prices. Section B of this appendix extends the results established in Section A to the case with tight capacity case. The remaining results (Propositions 8–10) are also proved in Section B.

Note that, if the supply chain capacity exceeds the size of consumer population a, any additional capacity beyond a plays no role. For ease of exposition, we describe the case with $Q \geq a$ by omitting Q and adding a superscript u to represent the unlimited capacity case. For instance, define $\Pi^u_{\mathbf{RP}}(p, w)$ and $\Pi^u_{\mathbf{MP}}(w, p)$ to be the retailer's and the manufacturer's profits under posted pricing when $Q \geq a$, respectively:

$$\Pi^u_{\mathbf{RP}}(p, w) := \Pi_{\mathbf{RP}}(p, w, Q), \text{ and } \Pi^u_{\mathbf{MP}}(w, p) := \Pi_{\mathbf{MP}}(w, p, Q) \text{ for } Q \geq a. \tag{A-1}$$

Then, $p^u(w)$ and $w^u_{\mathbf{P}}$ (defined in Section 2.1) are given by:

$$\begin{aligned} p^u(w) &= \arg\max_p \Pi^u_{\mathbf{RP}}(p, w) = \arg\max_p \Pi_{\mathbf{RP}}(p, w, Q) \text{ for } Q \geq a \text{ and} \\ w^u_{\mathbf{P}} &= \arg\max_w \Pi^u_{\mathbf{MP}}(w, p^u(w)) = \arg\max_w \Pi_{\mathbf{MP}}(w, p^*(w, Q), Q) \text{ for } Q \geq a. \end{aligned}$$

Assumption **(A)** guarantees that $\Pi^u_{\mathbf{RP}}(p, w)$ and $\Pi^u_{\mathbf{MP}}(w, p^u(w))$ are well behaved. In particular, as shown in Lemma A.1, $\Pi^u_{\mathbf{RP}}(p, w)$ is unimodal in p and $\Pi^u_{\mathbf{MP}}(w, p^u(w))$ is unimodal in w.

Likewise, define $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ and $\Pi^u_{\mathbf{MN}}(w, q_{\min})$ to be the retailer's and manufacturer's profits under negotiation when $Q \geq a$, respectively:

$$\Pi^u_{\mathbf{RN}}(q_{\min}, w) := \Pi_{\mathbf{RN}}(q_{\min}, w, Q), \text{ and } \Pi^u_{\mathbf{MN}}(w, q_{\min}) := \Pi_{\mathbf{MN}}(w, q_{\min}, Q) \text{ for } Q \geq a. \tag{A-2}$$

Then, $q^u_{\min}(w)$ and $w^u_{\mathbf{N}}$ (defined in Section 2.2) satisfy

$$\begin{aligned} q^u_{\min}(w) &= \arg\max_{q_{\min}} \Pi^u_{\mathbf{RN}}(q_{\min}, w) = \arg\max_{q_{\min}} \Pi_{\mathbf{RN}}(q_{\min}, w, Q) \text{ for } Q \geq a \text{ and} \\ w^u_{\mathbf{N}} &= \arg\max_w \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) = \arg\max_w \Pi_{\mathbf{MN}}(w, q^*_{\min}(w, Q), Q) \text{ for } Q \geq a. \end{aligned}$$

Again, Assumption **(A)** guarantees that $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ and $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$ are well behaved. In particular, as shown in Lemma A.2, $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ is unimodal in $q_{\min}$ and $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$ is unimodal in w.

A. Proofs of Propositions 1-7 with sufficient supply chain capacity: Q $\geq a$

In this appendix, we present the proofs of Propositions 1–7 assuming that the supply chain capacity is sufficiently large. The proofs utilize technical lemmas (A.1 – A.4), which are stated and proved at the end of Appendix A.

Proofs of Propositions 1 and 2

The proof follows from the definition of $w^u_{\mathbf{P}}$ and $w^u_{\mathbf{N}}$. Notice that if $Q \geq a$, $\Pi_{\mathbf{MP}}(w, p^*(w, Q), Q) = \Pi^u_{\mathbf{MP}}(w, p^u(w))$, which is maximized at $w^u_{\mathbf{P}}$. A similar argument proves Proposition 2.

Proof of Proposition 3

Notice from equation (11) that the retailer's profit is a function of the total cost of negotiation, $c_{\mathbf{T}} = c_r + c_b$. Therefore, as long as $c_{\mathbf{T}}$ remains the same, the retailer's optimal cut-off valuation, $q^*_{\min}$, remains unchanged, and the result follows from equations (11) and (12).

Proof of Proposition 4

Define $\Delta^u_{\mathbf{R}}(w)$ to be the difference in the retailer's profits under posted pricing and negotiation for a given wholesale price w:

$$\Delta^u_{\mathbf{R}}(w) = \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w) \tag{A-3}$$

The proof utilizes Lemma A.3, which shows that $\Delta^u_{\mathbf{R}}(w)$ changes sign at most once, which is proved in parts (a) and (b) of Lemma A.3. Notice that, when part (a) of Lemma A.3 holds, $\Delta^u_{\mathbf{R}}(w) > 0$ for all $w \geq c$ and the retailer prefers posted pricing for any $w \geq c$. When part(b)-(i) of Lemma A.3 holds, $\Delta^u_{\mathbf{R}}(w) < 0$ for all $w \geq c$ and the retailer prefers negotiation for any $w \geq c$. The result holds trivially for these two cases. Now suppose that $\Delta^u_{\mathbf{R}}(w)$ changes sign at some $w > c$. Then, from part (b)-(ii) of Lemma A.3, there must exist a threshold wholesale price $\widehat{w}^u_{\mathbf{R}}$ such that $\Delta^u_{\mathbf{R}}(\widehat{w}^u_{\mathbf{R}}) = 0$, $\Delta^u_{\mathbf{R}}(w) < 0$ for $w < \widehat{w}^u_{\mathbf{R}}$ (the retailer prefers negotiation), and $\Delta^u_{\mathbf{R}}(w) > 0$ for $w > \widehat{w}^u_{\mathbf{R}}$ (the retailer prefers posted pricing).

Proof of Proposition 5

Recall that $\Delta^u_{\mathbf{R}}(w) = \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$. Also, recall that the definition of $\widehat{w}^u_{\mathbf{R}}$ implies that $\Delta^u_{\mathbf{R}}(w) < 0$ for $w < \widehat{w}^u_{\mathbf{R}}$ and $\Delta^u_{\mathbf{R}}(w) > 0$ for $w > \widehat{w}^u_{\mathbf{R}}$. In other words, $\Delta^u_{\mathbf{R}}(w)$ changes sign at $w = \widehat{w}^u_{\mathbf{R}}$.

Define $\Delta^u_{\mathbf{M}}(w)$ to be the difference in the manufacturer's profit under posted pricing and negotiation for a given wholesale price w:

$$\Delta^u_{\mathbf{M}}(w) = \Pi^u_{\mathbf{MP}}(w, p^u(w)) - \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)). \tag{A-4}$$

Then, from Lemma A.3(c), there must exist a $\widehat{w}^u_{\mathbf{M}} \geq \widehat{w}^u_{\mathbf{R}}$ such that $\Delta^u_{\mathbf{M}}(w) \leq 0$ for $w \leq \widehat{w}^u_{\mathbf{M}}$ and $\Delta^u_{\mathbf{M}}(w) \geq 0$ for $w \geq \widehat{w}^u_{\mathbf{M}}$. That is, $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \geq \Pi^u_{\mathbf{MP}}(w, p^u(w))$ for $w \leq \widehat{w}^u_{\mathbf{M}}$ and $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \leq \Pi^u_{\mathbf{MP}}(w, p^u(w))$ for $w \geq \widehat{w}^u_{\mathbf{M}}$. Thus, the result directly follows from $\widehat{w}^u_{\mathbf{M}} \geq \widehat{w}^u_{\mathbf{R}}$.

Proof of Proposition 6

First, when the retailer chooses negotiation at all wholesale prices $w \geq c$, it follows from Proposition 2 that the optimal wholesale price is simply $w^u_{\mathbf{N}}$. Likewise, when the retailer chooses posted pricing at all wholesale prices $w \geq c$, then, from Proposition 1, the optimal wholesale price is simply $w^u_{\mathbf{P}}$.

We now focus on the case where there exists $\widehat{w}^u_{\mathbf{R}}$ such that the retailer chooses negotiation when $w \leq \widehat{w}^u_{\mathbf{R}}$ and posted pricing when $w > \widehat{w}^u_{\mathbf{R}}$. For the purposes of this proof, temporarily define

$$G_{\mathbf{N}} := \max_{c \leq w \leq \widehat{w}^u_{\mathbf{R}}} \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \text{ and } w^o_{\mathbf{N}} = \arg\max_{c \leq w \leq \widehat{w}^u_{\mathbf{R}}} \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$$
$$G_{\mathbf{P}} := \sup_{w > \widehat{w}^u_{\mathbf{R}}, w \geq c} \Pi^u_{\mathbf{MP}}(w, p^u(w)) \text{ and } w^o_{\mathbf{P}} = \arg\sup_{w > \widehat{w}^u_{\mathbf{R}}, w \geq c} \Pi^u_{\mathbf{MP}}(w, p^u(w))$$

With these definitions, observe that the manufacturer's problem of choosing the wholesale price, given by (20), reduces to picking the wholesale price $w^o_{\mathbf{N}}$ if $G_{\mathbf{N}} \geq G_{\mathbf{P}}$ or the wholesale price $w^o_{\mathbf{P}}$ if $G_{\mathbf{N}} < G_{\mathbf{P}}$. Consider two cases: (1) $G_{\mathbf{N}} \geq G_{\mathbf{P}}$ and (2) $G_{\mathbf{N}} < G_{\mathbf{P}}$.

(1) $G_{\mathbf{N}} \geq G_{\mathbf{P}}$

Since $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$ is unimodal in w, the manufacturer's optimal wholesale price is $w^o_{\mathbf{N}} = \min\{\widehat{w}^u_{\mathbf{R}}, w^u_{\mathbf{N}}\}$ and negotiation is the supply chain's pricing policy toward consumers. The case with $w^o_{\mathbf{N}} = w^u_{\mathbf{N}}$ corresponds to part (a) of the proposition (ordinary negotiation) and the case with $w^o_{\mathbf{N}} = \widehat{w}^u_{\mathbf{R}}$ corresponds to part (b) of the proposition (reconciliatory negotiation).

(2) $G_{\mathbf{N}} < G_{\mathbf{P}}$

We will first prove that, if $G_{\mathbf{N}} < G_{\mathbf{P}}$, $w^u_{\mathbf{P}} > \widehat{w}^u_{\mathbf{R}}$. The proof is by contradiction. Suppose $G_{\mathbf{N}} < G_{\mathbf{P}}$, but $w^u_{\mathbf{P}} \leq \widehat{w}^u_{\mathbf{R}}$. Since $\Pi^u_{\mathbf{MP}}(w, p^u(w))$ is unimodal in w, $w^o_{\mathbf{P}}$ must be $\widehat{w}^u_{\mathbf{R}}$, and hence, $G_{\mathbf{P}} = \Pi^u_{\mathbf{MP}}(\widehat{w}^u_{\mathbf{R}}, p^u(\widehat{w}^u_{\mathbf{R}}))$. However, Proposition 5 implies that

$$G_{\mathbf{P}} = \Pi^u_{\mathbf{MP}}(\widehat{w}^u_{\mathbf{R}}, p^u(\widehat{w}^u_{\mathbf{R}})) \leq \Pi^u_{\mathbf{MN}}(\widehat{w}^u_{\mathbf{R}}, q^u_{\min}(\widehat{w}^u_{\mathbf{R}})) \leq G_{\mathbf{N}},$$

which is a contradiction to the assumption that $G_{\mathrm{N}} < G_{\mathrm{P}}$.

Now that we have shown $w^u_{\mathrm{P}} > \widehat{w}^u_{\mathrm{R}}$, it follows from the fact that w^u_{P} maximizes $\Pi^u_{\mathrm{MP}}(w, p^u(w))$ that $w^o_{\mathrm{P}} = w^u_{\mathrm{P}}$. At the wholesale price $w^u_{\mathrm{P}} \geq \widehat{w}^u_{\mathrm{R}}$, the retailer chooses posted pricing. This corresponds to part (c) of the proposition.

Proof of Proposition 7

Existence of $\overline{c_{\mathrm{T}}}$: We first prove that if posted pricing is the equilibrium at a given c_{T}, then posted pricing remains to be the equilibrium at higher c_{T}. If this result holds, once the equilibrium falls in the posted pricing regime, it will never switch back to negotiation as c_{T} increases. We will then conclude that there exists a unique $\overline{c_{\mathrm{T}}}$ such that the equilibrium pricing policy is negotiation for $c_{\mathrm{T}} \in [0, \overline{c_{\mathrm{T}}})$ and posted pricing for $c_{\mathrm{T}} \geq \overline{c_{\mathrm{T}}}$.

Suppose posted pricing is an equilibrium at a given $c_{\mathrm{T}} = c^o_{\mathrm{T}}$. It must be that the equilibrium wholesale price is w^u_{P} (from Proposition 6). We will divide the proof into two cases, depending on whether a threshold wholesale price $\widehat{w}^u_{\mathrm{R}}$ exists at c^o_{T}. The two cases are: (1) there does not exist $\widehat{w}^u_{\mathrm{R}} > c$ and the retailer chooses posted pricing for any $w \geq c$, and (2) there exists $\widehat{w}^u_{\mathrm{R}} > c$,.

(1) $\widehat{w}^u_{\mathrm{R}} > c$ does not exist

In this case, at c^o_{T}, the retailer is choosing posted pricing for any $w \geq c$, that is, $\Pi^u_{\mathrm{RP}}(p^u(w), w) \geq \Pi^u_{\mathrm{RN}}(q^u_{\min}(w), w)$ for $w \geq c$. We observe that the retailer's profit under negotiation, $\Pi^u_{\mathrm{RN}}(q^u_{\min}(w), w)$ decreases in c_{T} while the retailer's profit under posted pricing $\Pi^u_{\mathrm{RP}}(p^u(w), w)$, is unaffected by c_{T}. Hence, at $c_{\mathrm{T}} > c^o_{\mathrm{T}}$, we continue to have $\Pi^u_{\mathrm{RP}}(p^u(w), w) \geq \Pi^u_{\mathrm{RN}}(q^u_{\min}(w), w)$ for $w \geq c$, and the retailer will choose posted pricing no matter what the wholesale price is.

(2) $\widehat{w}^u_{\mathrm{R}} > c$

For the purposes of this proof, we temporarily make the dependence on c_{T} explicit. For this, temporarily define, for a given c_{T}:

$$
\begin{aligned}
G_{\mathrm{N}}(c_{\mathrm{T}}) &= \max_{c \leq w \leq \widehat{w}^u_{\mathrm{R}}} \Pi^u_{\mathrm{MN}}(w, q^u_{\min}(w)) \text{ and } w^o_{\mathrm{N}}(c_{\mathrm{T}}) = \arg \max_{c \leq w \leq \widehat{w}^u_{\mathrm{R}}} \Pi^u_{\mathrm{MN}}(w, q^u_{\min}(w)) \\
G_{\mathrm{P}}(c_{\mathrm{T}}) &= \sup_{w > \widehat{w}^u_{\mathrm{R}}, w \geq c} \Pi^u_{\mathrm{MP}}(w, p^u(w)) \text{ and } w^o_{\mathrm{P}}(c_{\mathrm{T}}) = \arg \sup_{w > \widehat{w}^u_{\mathrm{R}}, w \geq c} \Pi^u_{\mathrm{MP}}(w, p^u(w))
\end{aligned}
$$

Since posted pricing is the equilibrium pricing policy at c^o_{T}, it must be that $G_{\mathrm{N}}(c^o_{\mathrm{T}}) < G_{\mathrm{P}}(c^o_{\mathrm{T}})$. Suppose we increase c_{T} to $c^o_{\mathrm{T}} + \delta$ for some $\delta > 0$.

Observe from Lemma A.4(b) that $\widehat{w}^u_{\mathrm{R}}$ decreases in c_{T}, and this implies the feasible region of the optimization problem that determines G_{N} contracts when c_{T} increases. Hence, $G_{\mathrm{N}}(c^o_{\mathrm{T}}+\delta) \leq G_{\mathrm{N}}(c^o_{\mathrm{T}})$ (formally proved in Lemma A.4(e)). On the other hand, the feasible region of the optimization problem that determines G_{P} becomes larger when c_{T} increases. As a result, w^u_{P}, which maximizes the function $\Pi^u_{\mathrm{MP}}(w, p^u(w))$, remains feasible at $c^o_{\mathrm{T}}+\delta$. Thus, we conclude that $w^o_{\mathrm{P}}(c^o_{\mathrm{T}}+\delta) = w^o_{\mathrm{P}}(c^o_{\mathrm{T}}) = w^u_{\mathrm{P}}$

and $G_{\mathrm{P}}(c^o_{\mathrm{T}}+\delta) = G_{\mathrm{P}}(c^o_{\mathrm{T}})$. Consequently,

$$G_{\mathrm{N}}(c^o_{\mathrm{T}}+\delta) \le G_{\mathrm{N}}(c^o_{\mathrm{T}}) < G_{\mathrm{P}}(c^o_{\mathrm{T}}) = G_{\mathrm{P}}(c^o_{\mathrm{T}}+\delta),$$

and the manufacturer will choose to induce posted pricing at $c^o_{\mathrm{T}}+\delta$.

Combining cases (1) and (2), we conclude that if posted pricing is the equilibrium at a given c_{T}, then it remains to be the equilibrium at higher c_{T}. Hence, there exists a unique $\overline{c_{\mathrm{T}}}$ such the equilibrium is negotiation for $c_{\mathrm{T}} \in [0, \overline{c_{\mathrm{T}}})$ and posted pricing for $c_{\mathrm{T}} \ge \overline{c_{\mathrm{T}}}$.

Existence of $\underline{c_{\mathrm{T}}}$: It remains to show that $\underline{c_{\mathrm{T}}}$ exists and separates the regions where the equilibrium wholesale price is w^u_{N} versus $\widehat{w}^u_{\mathrm{R}}$. Focus now on the region $c_{\mathrm{T}} \in [0, \overline{c_{\mathrm{T}}})$. For any c_{T} in this region, we know from Proposition 6 that the equilibrium wholesale price must be either $\widehat{w}^u_{\mathrm{R}}$ or w^u_{N}. Consider two cases:

(1) There does not exist $c_{\mathrm{T}} \in [0, \overline{c_{\mathrm{T}}})$ such that the equilibrium wholesale price is $\widehat{w}^u_{\mathrm{R}}$

In this case, it must be that the equilibrium wholesale price is w^u_{N} for any $c_{\mathrm{T}} \in [0, \overline{c_{\mathrm{T}}})$, in which case we have $\underline{c_{\mathrm{T}}} = \overline{c_{\mathrm{T}}}$.

(2) There exists $\widetilde{c}_{\mathrm{T}} \in [0, \overline{c_{\mathrm{T}}})$ such that the equilibrium wholesale price is $\widehat{w}^u_{\mathrm{R}}$ at $\widetilde{c}_{\mathrm{T}}$

From Lemma A.4(d), for any $c_{\mathrm{T}} \in [\widetilde{c}_{\mathrm{T}}, \overline{c_{\mathrm{T}}})$, the manufacturer would choose $\widehat{w}^u_{\mathrm{R}}$. Hence, there exists $\underline{c_{\mathrm{T}}}$, given by the lowest such $\widetilde{c}_{\mathrm{T}}$, and the equilibrium wholesale price is $\widehat{w}^u_{\mathrm{R}}$ for any $c_{\mathrm{T}} \in [\underline{c_{\mathrm{T}}}, \overline{c_{\mathrm{T}}})$.

Technical Lemmas used in Appendix A

Lemma A.1. [Profit functions under posted pricing]

(a) The retailer's profit, $\Pi^u_{\mathrm{RP}}(p, w)$, is strictly unimodal in the posted price, p.

(b) Let $p^u(w)$ denote the optimal posted price, that is, the maximizer of $\Pi^u_{\mathrm{RP}}(p, w)$. Then, $p^u(w)$ is convex and strictly increasing in the wholesale price, w.

(c) Given that the retailer responds to the wholesale price w with the posted price $p^u(w)$, the manufacturer's profit, $\Pi^u_{\mathrm{MP}}(w, p^u(w))$, is strictly unimodal in w.

Lemma A.2. [Profit functions under negotiation]

(a) The retailer's profit, $\Pi^u_{\mathrm{RN}}(q_{\min}, w)$, is strictly unimodal in the retailer's cut-off valuation, $q_{\min}$.

(b) Let $q^u_{\min}(w)$ denote the optimal cut-off valuation, that is, the maximizer of $\Pi^u_{\mathrm{RN}}(q_{\min}, w)$. Then $q^u_{\min}(w)$ is convex and strictly increasing in w.

(c) Given that the retailer responds to the wholesale price w with the cut-off valuation $q^u_{\min}(w)$, the manufacturer's profit, $\Pi^u_{\mathrm{MN}}(w, q^u_{\min}(w))$, is strictly unimodal in w.

Proofs of Lemmas A.1 and A.2

We present the proof of Lemma A.2 in detail and omit the proof of Lemma A.1 since the proof of Lemma A.1 follows a similar (but algebraically simpler) sequence of arguments.

Proof of (a): We prove that $\Pi^u_{\mathbf{RN}}(q_{\min}, w) = a\int_{q_{\min}}^{\infty}[(1-\beta)x + \beta q_{\min} - w - c_r - c_b]f(x)dx$ is unimodal in $q_{\min}$ by showing: (i) $\left.\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}}\right|_{q_{\min}=w+c_r+c_b} \geq 0$, (ii) $\frac{\partial^2 \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q^2_{\min}} < 0$ whenever $\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}} = 0$, and (iii) $\Pi^u_{\mathbf{RN}}(q_{\min}, w) \to 0$ as $q_{\min} \to \infty$.

The first and second partial derivatives of $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ with respect to $q_{\min}$ are

$$\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}} = a(-q_{\min} + w + c_r + c_b)f(q_{\min}) + a\beta\overline{F}(q_{\min}) \text{ and} \tag{A-5}$$

$$\frac{\partial^2 \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q^2_{\min}} = -a(1+\beta)f(q_{\min}) + a(-q_{\min} + w + c_r + c_b)f'(q_{\min}). \tag{A-6}$$

Claim (i) follows from (A-5) while claim (iii) follows from $\overline{F}(q_{\min}) \to 0$ as $q_{\min} \to \infty$. To show claim (ii), note from (A-5) and (A-6)

$$\left.\frac{\partial^2 \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q^2_{\min}}\right|_{\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}}=0} = -a\frac{(1+\beta)f^2(q_{\min}) + \beta f'(q_{\min})\overline{F}(q_{\min})}{f(q_{\min})}. \tag{A-7}$$

Since F is IFR, $f'(\cdot)\overline{F}(\cdot) + f^2(\cdot) \geq 0$. Hence claim (ii) follows from (A-7), concluding the proof of unimodality of $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ in $q_{\min}$.

Proof of (b): From part (a), $q^u_{\min}(w)$ satisfies $(-q^u_{\min}(w)+w+c_r+c_b)f(q^u_{\min}(w))+\beta\overline{F}(q^u_{\min}(w)) = 0$. Implicit differentiation of this equality with respect to w yields

$$\left[(1+\beta)f(q^u_{\min}(w)) - (-q^u_{\min}(w) + w + c_r + c_b)\, f'(q^u_{\min}(w))\right]\frac{dq^u_{\min}(w)}{dw} - f(q^u_{\min}(w)) = 0. \tag{A-8}$$

Substituting $-q^u_{\min}(w) + w + c_r + c_b = \frac{-\beta\overline{F}(q^u_{\min}(w))}{f(q^u_{\min}(w))}$ from (A-5) in (A-8), we obtain

$$\frac{dq^u_{\min}(w)}{dw} = \frac{f^2(q^u_{\min}(w))}{(1+\beta)f^2(q^u_{\min}(w)) + \beta f'(q^u_{\min}(w))\overline{F}(q^u_{\min}(w))}, \tag{A-9}$$

Since F is IFR, we have $f'(\cdot)\overline{F}(\cdot) + f^2(\cdot) \geq 0$, which implies $\frac{dq^u_{\min}(w)}{dw} > 0$. Thus, $q^u_{\min}(w)$ strictly increases in w.

To prove $q^u_{\min}(w)$ is convex in w, we show $\frac{d^2 q^u_{\min}(w)}{dw^2} \geq 0$. Taking the second derivative of (A-9) with respect to w, we obtain

$$\begin{aligned}\frac{d^2 q^u_{\min}(w)}{dw^2} =& \frac{\beta f(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw}}{[(1+\beta)f^2(q^u_{\min}(w)) + \beta f'(q^u_{\min}(w))\overline{F}(q^u_{\min}(w))]^2} \\ &\times \Big\{f'(q^u_{\min}(w))\big(2f'(q^u_{\min}(w))\overline{F}(q^u_{\min}(w)) + f^2(q^u_{\min}(w))\big) - f''(q^u_{\min}(w))f(q^u_{\min}(w))\overline{F}(q^u_{\min}(w))\Big\}.\end{aligned}$$

Since the term in the braces is positive under Assumption **(A)**, we have $\frac{d^2 q^u_{\min}(w)}{dw^2} \geq 0$.

Proof of (c): We prove the unimodality of $\Pi^u_{\text{MN}}(w, q^u_{\min}(w)) = a(w-c)\overline{F}(q^u_{\min}(w))$ in w by showing: (i) $\left.\frac{d\Pi^u_{\text{MN}}(w,q^u_{\min}(w))}{dw}\right|_{w=c} \geq 0$, (ii) $\frac{d^2\Pi^u_{\text{MN}}(w,q^u_{\min}(w))}{dw^2} < 0$ whenever $\frac{d\Pi^u_{\text{MN}}(w,q^u_{\min}(w))}{dw} = 0$, and (iii) $\Pi^u_{\text{MN}}(w, q^u_{\min}(w)) \to 0$ as $w \to \infty$.

The first and second partial derivatives of $\Pi^u_{\text{MN}}(w, q^u_{\min}(w))$ with respect to w are

$$\frac{d\Pi^u_{\text{MN}}(w, q^u_{\min}(w))}{dw} = a\overline{F}(q^u_{\min}(w)) - a(w-c)f(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw} \quad \text{and} \tag{A-10}$$

$$\begin{aligned}\frac{d^2\Pi^u_{\text{MN}}(w, q^u_{\min}(w))}{dw^2} &= -2af(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw} \\ &\quad -a(w-c)\left[f(q^u_{\min}(w))\frac{d^2q^u_{\min}(w)}{dw^2} + f'(q^u_{\min}(w))\left(\frac{dq^u_{\min}(w)}{dw}\right)^2\right]\end{aligned} \tag{A-11}$$

Claim (i) follows from (A-10) while claim (iii) follows from $\overline{F}(q^u_{\min}(w)) \to 0$ as w, and hence $q^u_{\min}(w)$ approaches to infinity. To show claim (ii), note from (A-10) and (A-11)

$$\begin{aligned}&\left.\frac{d^2\Pi^u_{\text{MN}}(w, q^u_{\min}(w))}{dw^2}\right|_{\frac{d\Pi^u_{\text{MN}}(w,q^u_{\min}(w))}{dw}=0} \\ = \; & -2af(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw} \\ & -a\frac{\overline{F}(q^u_{\min}(w))}{f(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw}}\left[f(q^u_{\min}(w))\frac{d^2q^u_{\min}(w)}{dw^2} + f'(q^u_{\min}(w))\left(\frac{dq^u_{\min}(w)}{dw}\right)^2\right] \\ = \; & -a\frac{dq^u_{\min}(w)}{dw}\left[f(q^u_{\min}(w)) + \frac{f'(q^u_{\min}(w))\overline{F}(q^u_{\min}(w))}{f(q^u_{\min}(w))}\right] \\ & -af(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw} - a\frac{\overline{F}(q^u_{\min}(w))}{\frac{dq^u_{\min}(w)}{dw}}\frac{d^2q^u_{\min}(w)}{dw^2}.\end{aligned} \tag{A-12}$$

Since F is IFR, we have $f'(\cdot)\overline{F}(\cdot) + f^2(\cdot) \geq 0$, and the term in brackets is positive. Since $q^u_{\min}(w)$ strictly increases in w and $\frac{d^2q^u_{\min}(w)}{dw^2} \geq 0$ from part (b), all three terms are negative with the second term being strictly negative. Thus, claim (ii) follows, concluding the proof of unimodality of $\Pi^u_{\text{MN}}(w, q^u_{\min}(w))$ in w.

Lemma A.3. *Given the wholesale price w, let $\Delta^u_{\text{R}}(w)$ be the difference between the retailer's optimal profits under posted pricing and negotiation, that is, $\Delta^u_{\text{R}}(w) = \Pi^u_{\text{RP}}(p^u(w), w) - \Pi^u_{\text{RN}}(q^u_{\min}(w), w)$, and $\Delta^u_{\text{M}}(w)$ be the difference between the manufacturer's profits under posted pricing and negotiation, that is, $\Delta^u_{\text{M}}(w) = \Pi^u_{\text{MP}}(w, p^u(w)) - \Pi^u_{\text{MN}}(w, q^u_{\min}(w))$. Then:*

(a) If $\Delta^u_{\text{R}}(c) \geq 0$, then $\Delta^u_{\text{R}}(w) > 0$ for all $w > c$.

(b) If $\Delta^u_{\text{R}}(c) < 0$, then either:

(*i*) $\Delta^u_{\mathbf{R}}(w) < 0$ *for all* $w > c$ *and* $\Delta^u_{\mathbf{R}}(w)$ *is strictly increasing in* w*, or*

(*ii*) $\Delta^u_{\mathbf{R}}(w)$ *is strictly unimodal and changes sign once. If* $\Delta^u_{\mathbf{R}}(w)$ *changes sign, it crosses zero at a unique* $\widehat{w}^u_{\mathbf{R}} > c$ *such that* $\Delta^u_{\mathbf{R}}(\widehat{w}^u_{\mathbf{R}}) = 0$*,* $\Delta^u_{\mathbf{R}}(w) < 0$ *for* $w < \widehat{w}^u_{\mathbf{R}}$*, and* $\Delta^u_{\mathbf{R}}(w) > 0$ *for* $w > \widehat{w}^u_{\mathbf{R}}$*.*

(*c*) *If* $\Delta^u_{\mathbf{R}}(w)$ *changes sign at* $w = \widehat{w}^u_{\mathbf{R}}$ *, there must exist a unique* $\widehat{w}^u_{\mathbf{M}} \geq \widehat{w}^u_{\mathbf{R}}$ *such that* $\Delta^u_{\mathbf{M}}(w) \leq 0$ *for* $w \leq \widehat{w}^u_{\mathbf{M}}$*, and* $\Delta^u_{\mathbf{M}}(w) \geq 0$ *for* $w \geq \widehat{w}^u_{\mathbf{M}}$*.*

(*d*) $\Pi^u_{\mathbf{RP}}(p^u(w), w)$ *and* $\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ *are convex decreasing in* w*.*

Proof of Lemma A.3

Proofs of (a) and (b): We prove the result by showing (1) $\Delta^u_{\mathbf{R}}(w) \to 0$ as $w \to \infty$, and (ii) $\frac{d^2\Delta^u_{\mathbf{R}}(w)}{dw^2} < 0$ whenever $\frac{d\Delta^u_{\mathbf{R}}(w)}{dw} = 0$. Claim (2) implies that if a stationary point exists, it must be a maximum. Claim (2) thus implies that there exists at most one maximizer. (Otherwise, there must be a minimizer between two local maxima, which contradicts the claim that all stationary points are local maxima.) If claims (1) and (2) hold, the behavior of the function $\Delta^u_{\mathbf{R}}(w)$ must follow either part (a) or part (b) of this lemma, which is depicted in Figure 7. Any other behavior would contradict (1) and/or (2), therefore cannot exist.

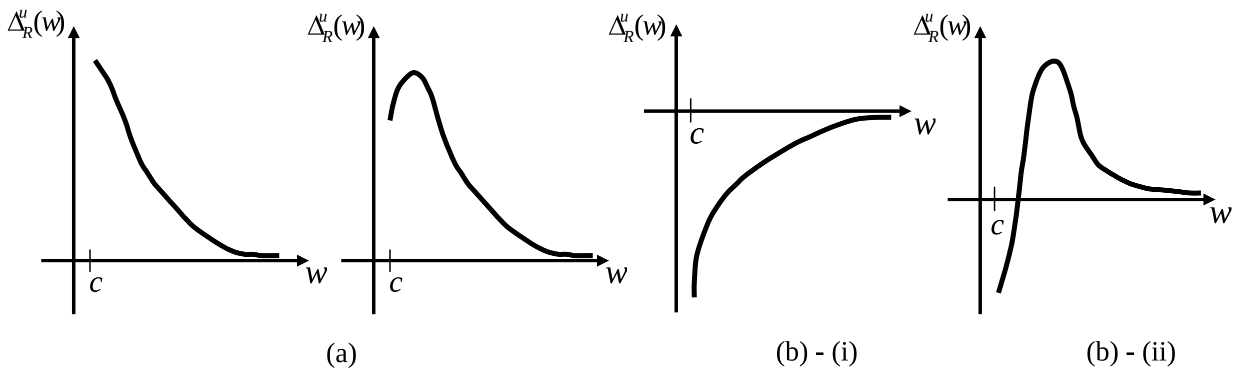

Figure 7: The figure illustrates the possibilities discussed in parts (a) and (b) of Lemma A.3.

We now prove claims (1) and (2) hold. From the facts that $p^u(w) \to \infty$ and $q^u_{\min}(w) \to \infty$ as $w \to \infty$, it can be shown that $\Pi^u_{\mathbf{RP}}(p^u(w), w)$ and $\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ both approach zero as $w \to \infty$. Hence, as $w \to \infty$, $\Delta^u_{\mathbf{R}}(w)$ approaches zero, which proves (1).

To show (2), recall that $p^u(w)$ is a solution to $\frac{\partial \Pi^u_{\mathbf{RP}}(p,w)}{\partial p} = 0$ and $q^u_{\min}(w)$ is a solution to $\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}} = 0$, respectively. Applying the envelope theorem, we have

$$\frac{d\Pi^u_{\mathbf{RP}}(p^u(w), w)}{dw} = \left.\frac{\partial \Pi^u_{\mathbf{RP}}(p, w)}{\partial w}\right|_{p=p^u(w)} = -a\overline{F}(p^u(w)), \text{ and} \tag{A-13}$$

$$\frac{d\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)}{dw} = \left.\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min}, w)}{\partial w}\right|_{q_{\min}=q^u_{\min}(w)} = -a\overline{F}(q^u_{\min}(w)) \tag{A-14}$$

Therefore:

$$\frac{d\Delta^u_{\mathrm{R}}(w)}{dw} = -a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w)). \tag{A-15}$$

Let $\tilde{w}$ be a wholesale price such that $\frac{d\Delta^u_{\mathrm{R}}(\tilde{w})}{dw} = 0$. Thus, at $\tilde{w}$, we have $\overline{F}(p^u(\tilde{w})) = \overline{F}(q^u_{\min}(\tilde{w}))$ and, hence, $p^u(\tilde{w}) = q^u_{\min}(\tilde{w})$. Using the expression for $\frac{dq^u_{\min}(\tilde{w})}{dw}$ given by equation (A-9) and obtaining a similar expression for $\frac{dp^u(\tilde{w})}{dw}$, we can write:

$$\begin{aligned} \frac{dq^u_{\min}(\tilde{w})}{dw} &= \frac{f^2(q^u_{\min}(\tilde{w}))}{(1+\beta)f^2(q^u_{\min}(\tilde{w})) + \beta f'(q^u_{\min}(\tilde{w}))\overline{F}(q^u_{\min}(\tilde{w}))} \\ &> \frac{f^2(p^u(\tilde{w}))}{2f^2(p^u(\tilde{w})) + f'(p^u(\tilde{w}))\overline{F}(p^u(\tilde{w}))} = \frac{dp^u(\tilde{w})}{dw}, \end{aligned} \tag{A-16}$$

where the inequality follows from the facts that $p^u(\tilde{w}) = q^u_{\min}(\tilde{w})$, F is IFR, and $0 < \beta < 1$.

Now, we can use (A-15) to write:

$$\begin{aligned} \left.\frac{d^2\Delta^u_{\mathrm{R}}(w)}{dw^2}\right|_{w=\tilde{w}} &= \frac{d}{dw}\left[-a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w))\right]\Big|_{w=\tilde{w}} \\ &= a\left(f(p^u(\tilde{w}))\frac{dp^u(\tilde{w})}{dw} - f(q^u_{\min}(\tilde{w}))\frac{dq^u_{\min}(\tilde{w})}{dw}\right) < 0, \end{aligned}$$

where the inequality is from $p^u(\tilde{w}) = q^u_{\min}(\tilde{w})$ and (A-16). Hence, (2) is proven, which concludes the proof of part (a) and (b).

Proof of (c): Our first goal is to prove that if $\Delta^u_{\mathrm{R}}(w)$ changes sign, then $\Delta^u_{\mathrm{M}}(w)$ changes sign exactly once by crossing zero from below. First, note that

$$\Delta^u_{\mathrm{M}}(w) = \Pi^u_{\mathrm{MP}}(w, p^u(w)) - \Pi^u_{\mathrm{MN}}(w, q^u_{\min}(w)) = a(w-c)\overline{F}(p^u(w)) - a(w-c)\overline{F}(q^u_{\min}(w)).$$

Hence, from (A-15), it follows that $\Delta^u_{\mathrm{M}}(w) = -(w-c)\frac{d\Delta^u_{\mathrm{R}}(w)}{dw}$. Therefore, it suffices to show that if $\Delta^u_{\mathrm{R}}(w)$ changes sign, then $\frac{d\Delta^u_{\mathrm{R}}(w)}{dw}$ also changes sign exactly once from positive to negative. Suppose now $\Delta^u_{\mathrm{R}}(w)$ changes sign. From the discussion in parts (a) and (b), we know that we must be in case (b)(ii): $\Delta^u_{\mathrm{R}}(w)$ crosses zero from below and is strictly unimodal with a peak at $w = \tilde{w}$ such that $\frac{d\Delta^u_{\mathrm{R}}(\tilde{w})}{dw} = 0$. Hence, $\frac{d\Delta^u_{\mathrm{R}}(w)}{dw}$ is positive for $w \le \tilde{w}$ and negative for $w \ge \tilde{w}$. It now follows that $\Delta^u_{\mathrm{M}}(w)$ changes sign exactly once, and the point where it changes sign, $\widehat{w}^u_{\mathrm{M}}$, is given by $\tilde{w}$ such that $\frac{d\Delta^u_{\mathrm{R}}(\tilde{w})}{dw} = 0$. Furthermore, observe from Figure 7 that, in case (b)(ii), the point at which $\Delta^u_{\mathrm{R}}(w)$ changes sign, $\widehat{w}^u_{\mathrm{R}}$, must come before $\widehat{w}^u_{\mathrm{M}} = \tilde{w}$.

Proof of (d): It immediately follows from (A-13) and (A-14) that both $\Pi^u_{\mathrm{RP}}(p^u(w), w)$ and $\Pi^u_{\mathrm{RN}}(q^u_{\min}(w), w)$ are decreasing in w. Furthermore, from (A-13) and (A-14), we obtain:

$$\frac{d^2\Pi^u_{\mathrm{RP}}(p^u(w), w)}{dw^2} = af(p^u(w))\frac{dp^u(w)}{dw}, \text{ and}$$
$$\frac{d^2\Pi^u_{\mathrm{RN}}(q^u_{\min}(w), w)}{dw^2} = af(q^u_{\min}(w))\frac{dq^u_{\min}(w)}{dw}$$

Since both $p^u(w)$ and $q^u_{\min}(w)$ increase in w (by Lemma A.1 and A.2, respectively), both $\Pi^u_{\mathbf{RP}}(p^u(w), w)$ and $\Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ are convex in w.

Lemma A.4. *Let $\Delta^u_{\mathbf{R}}(w)$ be the difference between the retailer's profits under posted pricing and negotiation, that is, $\Delta^u_{\mathbf{R}}(w) = \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$. Suppose there exists a $\widehat{w}^u_{\mathbf{R}}$ such that $\Delta^u_{\mathbf{R}}(\widehat{w}^u_{\mathbf{R}}) = 0$, $\Delta^u_{\mathbf{R}}(w) < 0$ for $w < \widehat{w}^u_{\mathbf{R}}$, and $\Delta^u_{\mathbf{R}}(w) > 0$ for $w > \widehat{w}^u_{\mathbf{R}}$. Consider the following optimization problem:*

$$\max_{c \le w \le \widehat{w}^u_{\mathbf{R}}} \Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w)) \tag{A-17}$$

Let $w^o_{\mathbf{N}}(c_{\mathbf{T}})$ denote the optimal solution to (A-17) *and $G_{\mathbf{N}}(c_{\mathbf{T}})$ be the optimal value of the objective function for a given $c_{\mathbf{T}}$. Then:*

(a) $w^o_{\mathbf{N}}(c_{\mathbf{T}}) = \min\{\widehat{w}^u_{\mathbf{R}}, w^u_{\mathbf{N}}\}$.

(b) $\widehat{w}^u_{\mathbf{R}}$ decreases in $c_{\mathbf{T}}$. Furthermore, $\frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} < -1$.

(c) $w^u_{\mathbf{N}}$ decreases in $c_{\mathbf{T}}$. Furthermore, $-1 \le \frac{dw^u_{\mathbf{N}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \le 0$.

(d) If $w^o_{\mathbf{N}}(c_{\mathbf{T}}) = \widehat{w}^u_{\mathbf{R}}$ for some $c_{\mathbf{T}} = c^o_{\mathbf{T}}$, then $w^o_{\mathbf{N}}(c_{\mathbf{T}}) = \widehat{w}^u_{\mathbf{R}}$ for $c_{\mathbf{T}} > c^o_{\mathbf{T}}$.

(e) $G_{\mathbf{N}}(c_{\mathbf{T}})$ decreases in $c_{\mathbf{T}}$.

Proof of Lemma A.4

Proof of (a): Recall that $w^u_{\mathbf{N}}$ is the unconstrained maximizer of $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$. Since w must be chosen in $[c, \widehat{w}^u_{\mathbf{R}}]$ and $\Pi^u_{\mathbf{MN}}(w, q^u_{\min}(w))$ is unimodal in w (by Lemma A.2), the optimal solution to (A-17) is the minimum of $w^u_{\mathbf{N}}$ and $\widehat{w}^u_{\mathbf{R}}$.

Proof of (b): To express explicit dependence, we write $q^u_{\min}(w)$, $\widehat{w}^u_{\mathbf{R}}$, and $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ as $q^u_{\min}(w, c_{\mathbf{T}})$, $\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})$, and $\Pi^u_{\mathbf{RN}}(q_{\min}, w, c_{\mathbf{T}})$, respectively. Recall that, by definition of $\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})$:

$$\Pi^u_{\mathbf{RN}}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}}), \widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}}) - \Pi^u_{\mathbf{RP}}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})), \widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})) = 0. \tag{A-18}$$

Implicit differentiation of (A-18) with respect to $c_{\mathbf{T}}$ yields:

$$\begin{aligned}
0 &= \frac{d\Pi^u_{\mathbf{RN}}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}}), \widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}})}{dc_{\mathbf{T}}} - \frac{d\Pi^u_{\mathbf{RP}}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})), \widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}))}{dc_{\mathbf{T}}} \\
&= \frac{dq^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}})}{dc_{\mathbf{T}}} \left.\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min}, w, c_{\mathbf{T}})}{\partial q_{\min}}\right|_{q_{\min}=q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}}), w=\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})} \\
&\quad + \frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \left.\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min}, w, c_{\mathbf{T}})}{\partial w}\right|_{q_{\min}=q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}}), w=\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})} + \left.\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min}, w, c_{\mathbf{T}})}{\partial c_{\mathbf{T}}}\right|_{q_{\min}=q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}}), w=\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})} \\
&\quad - \frac{dp^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}})}{dc_{\mathbf{T}}} \left.\frac{\partial \Pi^u_{\mathbf{RP}}(p, w)}{\partial p}\right|_{p=p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})), w=\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})} - \frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \left.\frac{\partial \Pi^u_{\mathbf{RP}}(p, w)}{\partial w}\right|_{p=p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})), w=\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}
\end{aligned} \tag{A-19}$$

Note that the first and fourth terms of (A-19) are zero since $q^u_{\min}$ and p^u satisfy the first-order conditions of $\Pi^u_{\mathbf{RN}}(q_{\min}, w, c_{\mathbf{T}})$ and $\Pi^u_{\mathbf{RP}}(p, w)$, respectively. Also, recall that

$$\begin{aligned}
\Pi^u_{\mathbf{RN}}(q_{\min}, w, c_{\mathbf{T}}) &= a \int_{q_{\min}}^{\infty} [(1-\beta)x + \beta q_{\min} - w - c_{\mathbf{T}}]\, f(x)dx, \\
\Pi^u_{\mathbf{RP}}(p, w) &= a(p-w)\overline{F}(p).
\end{aligned}$$

Taking the partial derivatives of these profit functions, we obtain:

$$\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min}, w, c_{\mathbf{T}})}{\partial w} = -a\overline{F}(q_{\min}), \quad \frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min}, w, c_{\mathbf{T}})}{\partial c_{\mathbf{T}}} = -a\overline{F}(q_{\min}), \quad \text{and} \quad \frac{\partial \Pi^u_{\mathbf{RP}}(p, w)}{\partial w} = -a\overline{F}(p).$$

Substituting the above partial derivatives in (A-19) and rearranging the terms, we obtain:

$$\frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} \left[\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}})) - \overline{F}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})))\right] + \overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}})) = 0.$$

Hence:

$$\frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} = -\frac{\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}}))}{\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}})) - \overline{F}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})))}.$$

To show that $\frac{d\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})}{dc_{\mathbf{T}}} < -1$, it suffices to show that $\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}})) > \overline{F}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})))$. Since $\Delta^u_{\mathbf{R}}(w)$ is crossing from negative to positive at $w = \widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})$ (by Lemma A.3(b)(ii), it follows that $\Delta^u_{\mathbf{R}}(w)$ must be strictly increasing in w at $w = \widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})$. (See b(ii) of Figure 7.) Using this fact, we obtain from (A-15) that $\overline{F}(q^u_{\min}(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}}), c_{\mathbf{T}})) > \overline{F}(p^u(\widehat{w}^u_{\mathbf{R}}(c_{\mathbf{T}})))$, which concludes the proof of (b).

Proof of (c): In preparation for the proof, we will first derive a few useful expressions. First, substituting the expression for $\frac{dq^u_{\min}(w)}{dw}$, given by (A-9), into the manufacturer's first-order condition, (A-10), and recalling that $w^u_{\mathbf{N}}$ is the solution to the manufacturer's first-order condition, we get the following identity:

$$\frac{\overline{F}(q^u_{\min}(w^u_{\mathbf{N}}))}{f(q^u_{\min}(w^u_{\mathbf{N}}))} - (w^u_{\mathbf{N}} - c)\frac{f^2(q^u_{\min}(w^u_{\mathbf{N}}))}{(1+\beta)f^2(q^u_{\min}(w^u_{\mathbf{N}})) + \beta f'(q^u_{\min}(w^u_{\mathbf{N}}))\overline{F}(q^u_{\min}(w^u_{\mathbf{N}}))} = 0. \tag{A-20}$$

Let $\phi(x) := \frac{f^2(x)}{(1+\beta)f^2(x)+\beta f'(x)\overline{F}(x)}$. As an aside, note that

$$\frac{d\phi(x)}{dx} = \frac{\beta f(x)[f'(x)(2f'(x)\overline{F}(x) + f^2(x)) - f''(x)f(x)\overline{F}(x)]}{[(1+\beta)f^2(x) + \beta f'(x)\overline{F}(x)]^2}. \tag{A-21}$$

We observe from (A-21) that $\phi(x)$ increases in x (since the numerator is non-negative by Assumption **A**). Using our definition of $\phi(x)$, we can rewrite (A-20) as

$$\frac{\overline{F}(q^u_{\min}(w^u_{\mathbf{N}}))}{f(q^u_{\min}(w^u_{\mathbf{N}}))} - (w^u_{\mathbf{N}} - c)\phi(q^u_{\min}(w^u_{\mathbf{N}})) = 0. \tag{A-22}$$

Here, to make explicit the dependence on c_{T}, we write $w^u_{\mathrm{N}}(c_{\mathrm{T}})$ instead of w^u_{N}. In addition, for notational convenience, we write $q^u_{\min}(c_{\mathrm{T}})$ to denote $q^u_{\min}(w^u_{\mathrm{N}}(c_{\mathrm{T}}))$. With these notational changes, (A-22) can be written as:

$$\frac{\overline{F}(q^u_{\min}(c_{\mathrm{T}}))}{f(q^u_{\min}(c_{\mathrm{T}}))} - (w^u_{\mathrm{N}}(c_{\mathrm{T}}) - c)\phi(q^u_{\min}(c_{\mathrm{T}})) = 0. \tag{A-23}$$

Now we are ready to prove the result. We first show that $\frac{dq^u_{\min}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \times \frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \leq 0$, that is, when c_{T} increases, $q^u_{\min}(c_{\mathrm{T}})$ and $w^u_{\mathrm{N}}(c_{\mathrm{T}})$ cannot both strictly increase or strictly decrease. We prove this by contradiction.

Suppose both $q^u_{\min}(c_{\mathrm{T}})$ and $w^u_{\mathrm{N}}(c_{\mathrm{T}})$ strictly increase in c_{T}. Then, $\frac{\overline{F}(q^u_{\min}(c_{\mathrm{T}}))}{f(q^u_{\min}(c_{\mathrm{T}}))}$ decreases in c_{T} (because F is IFR) and $\phi(q^u_{\min}(c_{\mathrm{T}}))$ increases in c_{T} (because $\phi(x)$ is increasing in x, as observed earlier). Hence, the left-hand side of (A-23) must be strictly decreasing in c_{T}, which is a contradiction since (A-23) must hold as an identity at any given c_{T}.

Next, suppose that both $q^u_{\min}(c_{\mathrm{T}})$ and $w^u_{\mathrm{N}}(c_{\mathrm{T}})$ strictly decrease in c_{T}. Then $\frac{\overline{F}(q^u_{\min}(c_{\mathrm{T}}))}{f(q^u_{\min}(c_{\mathrm{T}}))}$ increases in c_{T} (because F is IFR) and $\phi(q^u_{\min}(c_{\mathrm{T}}))$ decreases in c_{T}. Hence, the left-hand side of (A-23) must be strictly increasing in c_{T}, which again yields a contradiction. It is now proved that $\frac{dq^u_{\min}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \times \frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \leq 0$.

Next, we show that $-1 \leq \frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \leq 0$. Implicit differentiation of the retailer's first-order condition, given by (A-5), with respect to c_{T} yields

$$\frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} = \left[1 + \frac{\beta\big(f'(q^u_{\min}(c_{\mathrm{T}}))\overline{F}(q^u_{\min}(c_{\mathrm{T}})) + f^2(q^u_{\min}(c_{\mathrm{T}}))\big)}{f^2(q^u_{\min}(c_{\mathrm{T}}))}\right] \frac{dq^u_{\min}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} - 1, \tag{A-24}$$

If $\frac{dq^u_{\min}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} < 0$, then it follows from (A-24) that $\frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} < 0$. (To see why, note that the term in the brackets is positive, because F is IFR.) However, we would then obtain a contradiction to $\frac{dq^u_{\min}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \times \frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \leq 0$. Thus, it must be that $\frac{dq^u_{\min}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \geq 0$, and it follows that $\frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \leq 0$. Furthermore, since $\frac{dq^u_{\min}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \geq 0$, we observe from (A-24) that $\frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} \geq -1$. This concludes the proof of part (c).

Proof of (d): From part (a), we have $w^o_{\mathrm{N}}(c_{\mathrm{T}}) = \min\{\widehat{w}^u_{\mathrm{R}}, w^u_{\mathrm{N}}\}$. Hence, if $w^o_{\mathrm{N}}(c^o_{\mathrm{T}}) = \widehat{w}^u_{\mathrm{R}}$, it must be that $\widehat{w}^u_{\mathrm{R}} \leq w^u_{\mathrm{N}}$ at c^o_{T}. From parts (b) and (c), we know that $\frac{d\widehat{w}^u_{\mathrm{R}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}} < \frac{dw^u_{\mathrm{N}}(c_{\mathrm{T}})}{dc_{\mathrm{T}}}$. Hence, if c_{T} increases, $\widehat{w}^u_{\mathrm{R}}$ continues to be less than or equal to w^u_{N}, and $w^o_{\mathrm{N}}(c_{\mathrm{T}}) = \widehat{w}^u_{\mathrm{R}}$ continues to hold for $c_{\mathrm{T}} > c^o_{\mathrm{T}}$.

Proof of (e): We will show that for $c^o_{\mathrm{T}} < c'_{\mathrm{T}}$, $G_{\mathrm{N}}(c^o_{\mathrm{T}}) \geq G_{\mathrm{N}}(c'_{\mathrm{T}})$. In this proof, we will write $q^u_{\min}(w, c_{\mathrm{T}})$, $\widehat{w}^u_{\mathrm{R}}(c_{\mathrm{T}})$ and $\Pi^u_{\mathrm{MN}}(w, q_{\min}, c_{\mathrm{T}})$ instead of, respectively, $q^u_{\min}(w)$, $\widehat{w}^u_{\mathrm{R}}$ and $\Pi^u_{\mathrm{MN}}(w, q_{\min})$, to make the dependence on c_{T} explicit. It is not difficult to check that $\Pi^u_{\mathrm{MN}}(w, q^u_{\min}(w, c_{\mathrm{T}}), c_{\mathrm{T}})$ is decreasing in c_{T}. Hence:

$$G_{\text{N}}(c'_{\text{T}}) = \Pi^u_{\text{MN}}(w^o_{\text{N}}(c'_{\text{T}}), q^u_{\min}(w^o_{\text{N}}(c'_{\text{T}}), c'_{\text{T}}), c'_{\text{T}}) \leq \Pi^u_{\text{MN}}(w^o_{\text{N}}(c'_{\text{T}}), q^u_{\min}(w^o_{\text{N}}(c'_{\text{T}}), c^o_{\text{T}}), c^o_{\text{T}}). \tag{A-25}$$

Furthermore, note that when $c_{\text{T}} = c^o_{\text{T}}$, $w = w^o_{\text{N}}(c'_{\text{T}})$ is a feasible solution for the optimization problem in (A-17). To see why, note that $\widehat{w}^u_{\text{R}}(c_{\text{T}})$ decreases in c_{T}. Hence, $\widehat{w}^u_{\text{R}}(c^o_{\text{T}}) \geq \widehat{w}^u_{\text{R}}(c'_{\text{T}})$. It then follows that $w^o_{\text{N}}(c'_{\text{T}})$, which is feasible for the problem in (A-17) when $c_{\text{T}} = c'_{\text{T}}$, is also feasible when $c_{\text{T}} = c^o_{\text{T}}$. Therefore:

$$G_{\text{N}}(c^o_{\text{T}}) = \Pi^u_{\text{MN}}(w^o_{\text{N}}(c^o_{\text{T}}), q^u_{\min}(w^o_{\text{N}}(c^o_{\text{T}}), c^o_{\text{T}}), c^o_{\text{T}}) \geq \Pi^u_{\text{MN}}(w^o_{\text{N}}(c'_{\text{T}}), q^u_{\min}(w^o_{\text{N}}(c'_{\text{T}}), c^o_{\text{T}}), c^o_{\text{T}}). \tag{A-26}$$

Combining (A-25) and (A-26), we obtain $G_{\text{N}}(c^o_{\text{T}}) \geq G_{\text{N}}(c'_{\text{T}})$.

B. Proofs for the Capacitated Supply Chain

In this appendix, we prove the results for the case where the sales volume could be bounded by the supply chain capacity Q. We utilize Lemmas B.1 through B.3, stated and proved at the end of Appendix B.

Proofs of Propositions 1 and 2

Notice from the manufacturer's profit function under posted pricing given by (6) that $w^*_{\mathbf{P}}(Q)$, the maximizer of $\Pi_{\mathbf{MP}}(w, p^*(w,Q), Q)$, cannot be strictly less than $\overline{w}_{\mathbf{P}}(Q)$ (since $\Pi_{\mathbf{MP}}(w, p^*(w,Q), Q)$ is linearly increasing in w for $w \in [c, \overline{w}_{\mathbf{P}}(Q)]$). Now, for $w \geq \overline{w}_{\mathbf{P}}(Q)$, the supply chain capacity does not play a role: $\Pi_{\mathbf{MP}}(w, p^*(w,Q), Q)$ is equal to $\Pi^u_{\mathbf{MP}}(w, p^u(w))$, which itself is unimodal by Lemma A.1 and peaks at $w^u_{\mathbf{P}}$. Therefore, $w^*_{\mathbf{P}}(Q)$ is $w^u_{\mathbf{P}}$ or $\overline{w}_{\mathbf{P}}(Q)$, whichever is larger. The same line of arguments proves Proposition 2 as well.

Proof of Proposition 3

The proof is identical to the case with sufficient capacity, thus omitted.

Proof of Proposition 4

Define $\Delta_{\mathbf{R}}(w,Q) = \Pi_{\mathbf{RP}}(p^*(w,Q), w, Q) - \Pi_{\mathbf{RN}}(q^*_{\min}(w,Q), w, Q)$ to be the difference between the retailer's optimal profits under the two pricing policies at a given wholesale price, w. Parts (a) and (b) of Lemma B.1 together prove that either (1) $\Delta_{\mathbf{R}}(w,Q) \geq 0$ for all $w \geq c$ (in which case the retailer prefers posted pricing for all $w \geq c$), or (2) $\Delta_{\mathbf{R}}(w,Q) < 0$ for all $w \geq c$ (in which case the retailer prefers negotiation for all $w \geq c$), or (3) if $\Delta_{\mathbf{R}}(w,Q)$ crosses zero for some w, it does so only once and from below (in which case there exists a wholesale price below which the retailer prefers negotiation and above which the retailer prefers posted pricing).

Proof of Proposition 5

As in the proof of Proposition 4, define $\Delta_{\mathbf{R}}(w,Q) = \Pi_{\mathbf{RP}}(p^*(w,Q), w, Q) - \Pi_{\mathbf{RN}}(q^*_{\min}(w,Q), w, Q)$ to be the difference between the retailer's optimal profits under the two pricing policies at a given wholesale price, w. In addition, define $\Delta_{\mathbf{M}}(w,Q) = \Pi_{\mathbf{MP}}(w, p^*(w,Q), Q) - \Pi_{\mathbf{MN}}(w, q^*_{\min}(w,Q), Q)$ to be the difference between the manufacturer's optimal profits under the two pricing policies at a given wholesale price, w. Lemma B.1(e) shows that if there exists a threshold wholesale price $\widehat{w}_{\mathbf{R}}(Q) > c$, there must exist a $\widehat{w}_{\mathbf{M}}(Q) \geq \widehat{w}_{\mathbf{R}}(Q)$ such that $\Delta_{\mathbf{M}}(w,Q) \leq 0$ for $w \leq \widehat{w}_{\mathbf{M}}(Q)$ and $\Delta_{\mathbf{M}}(w,Q) \geq 0$ for $w \geq \widehat{w}_{\mathbf{M}}(Q)$. Hence, in the range of wholesale prices where the retailer prefers negotiation (i.e., $w \le \widehat{w}_{\mathbf{R}}(Q)$), we have $\Delta_{\mathbf{M}}(w,Q) \le 0$; the manufacturer also prefers negotiation.

Proof of Proposition 6

The proof is almost identical to that for the case with sufficient supply chain capacity (proved in Section A of this appendix), once we replace $q^u_{\min}(w)$, $p^u(w)$, $w^u_{\mathbf{N}}$ and $w^u_{\mathbf{P}}$ in the earlier proof with $q^*_{\min}(w,Q) = \max\{q^u_{\min}(w), \bar{p}(Q)\}$, $p^*(w,Q) = \max\{p^u(w), \bar{p}(Q)\}$, $w^*_{\mathbf{N}}(Q) = \max\{\overline{w}_{\mathbf{N}}(Q), w^u_{\mathbf{N}}\}$ and $w^*_{\mathbf{P}}(Q) = \max\{\overline{w}_{\mathbf{P}}(Q), w^u_{\mathbf{P}}\}$ here. Thus, we omit the proof.

Proof of Proposition 7

Again, the proof is almost identical to that for the case with supply chain capacity (proved in Section A of this appendix), once we replace $q^u_{\min}(w)$, $p^u(w)$, $w^u_{\mathbf{N}}$ and $w^u_{\mathbf{P}}$ in the earlier proof with $q^*_{\min}(w,Q)$, $p^*(w,Q)$, $w^*_{\mathbf{N}}(Q)$ and $w^*_{\mathbf{P}}(Q)$ here. The proof utilizes Lemma B.2 which is a counterpart for Lemma A.4 used in the case with sufficient supply chain capacity.

Proof of Proposition 8

The proof proceeds in two parts, the first part showing the existence of $\underline{Q}$ and the second part showing the existence of $\overline{Q}$.

Part 1: The existence of $\underline{\mathbf{Q}}$

We first show that if negotiation is the equilibrium pricing policy at a given Q (be it ordinary or reconciliatory negotiation) then the equilibrium pricing policy is still negotiation for a larger Q. Hence, the smallest Q at which the supply chain settles in negotiation yields $\underline{Q}$. This includes a special case where posted pricing is the equilibrium for all $Q < \infty$, in which case we set $\underline{Q} = \infty$.

Suppose that the supply chain uses negotiation as a pricing policy toward consumers at Q^o. From Proposition 6, the equilibrium wholesale price (which is the solution to problem (20)) must be $\min\{\widehat{w}_{\mathbf{R}}(Q), w^*_{\mathbf{N}}(Q)\}$. There are two cases to consider: Either $\widehat{w}_{\mathbf{R}}(Q) = \infty$ in which case the retailer prefers to use negotiation for all $w \ge c$, or there exists a finite $\widehat{w}_{\mathbf{R}}(Q) \ge c$ such that the retailer prefers to use negotiation for $w \in [c, \widehat{w}_{\mathbf{R}}(Q))$ and posted pricing when $w > \widehat{w}_{\mathbf{R}}(Q)$. We consider these two cases separately.

Case (1): $\widehat{w}_{\mathbf{R}}(Q) = \infty$ **at capacity** Q^o**:** In this case the retailer prefers negotiation for all $w \ge c$ when the supply chain capacity is Q^o. In other words,

$$\Delta_R(w,Q^o) = \Pi_{\mathbf{RP}}(p^*(w,Q^o), w, Q^o) - \Pi_{\mathbf{RN}}(q^*_{\min}(w,Q^o), w, Q^o) \le 0 \text{ for all } w \ge c.$$

We now show that the retailer continues to prefer negotiation for all $w \ge c$ at any $Q > Q^o$ (i.e., $\Delta_R(w,Q) \le 0$ for all $w \ge c$), which then implies that negotiation is used at $Q > Q^o$.

The proof is by contradiction. Pick a $Q' > Q^o$ and suppose that there exists a $w' \geq c$ such that $\Delta_R(w', Q') > 0$. Notice, from Proposition 4, that if the retailer prefers posted pricing at some wholesale price w', it continues to prefer posted pricing at a higher wholesale price. Also, notice that the sales quantities under both pricing policies decrease and converge to zero as w increases. Hence, there must exist a $w'' \geq w'$ at which $\Delta_R(w'', Q') > 0$, and the sales volumes under both policies are strictly less than Q^o. Notice also that at this wholesale price w'', reducing capacity from Q' to Q^o will not change the retailer's profits since the sales quantities do not exceed Q^o. Therefore, $\Delta_R(w'', Q') = \Delta_R(w'', Q^o) > 0$. This contradicts the fact that $\Delta_R(w, Q^o) \leq 0$ for all $w \geq c$. Hence, there does not exist w' at which $\Delta_R(w', Q') > 0$. In other words, for any $Q' > Q^o$, $\Delta_R(w, Q) \leq 0$ for all $w \geq c$, concluding the proof of Case (1).

Case (2): $\widehat{w}_{\mathbf{R}}(Q) < \infty$ at capacity Q^o: For the sake of exposition, we temporarily define the following functions, which correspond to the optimal solutions to the sub-problems in problem (20).

$$G_{\mathbf{N}}(Q) = \max_{c \leq w \leq \widehat{w}_{\mathbf{R}}(Q)} \Pi_{\mathbf{MN}}(w, q^*_{\min}(w, Q), Q) \text{ and } w^o_{\mathbf{N}}(Q) = \arg \max_{c \leq w \leq \widehat{w}_{\mathbf{R}}(Q)} \Pi_{\mathbf{MN}}(w, q^*_{\min}(w, Q), Q)$$

$$G_{\mathbf{P}}(Q) = \sup_{w > \widehat{w}_{\mathbf{R}}(Q), w \geq c} \Pi_{\mathbf{MP}}(w, p^*(w, Q), Q) \text{ and } w^o_{\mathbf{P}}(Q) = \arg \sup_{w > \widehat{w}_{\mathbf{R}}(Q), w \geq c} \Pi_{\mathbf{MP}}(w, p^*(w, Q), Q)$$

Since negotiation is the equilibrium at Q^o, it must be that $G_{\mathbf{N}}(Q^o) \geq G_{\mathbf{P}}(Q^o)$ and $\widehat{w}_{\mathbf{R}}(Q) \geq c$. We now show that, for any $Q' > Q^o$, $G_{\mathbf{N}}(Q') \geq G_{\mathbf{P}}(Q')$, which implies that the equilibrium remains to be negotiation at capacity Q'. We next state and prove a claim that will help us complete the proof:

- *Claim:* $G_{\mathbf{P}}(Q^o) \geq G_{\mathbf{P}}(Q')$.

  Consider two cases, depending on whether there exists a $\overline{w}_{\mathbf{P}}(Q^o) \geq c$ (i.e., whether there exists a feasible wholesale price at which posted pricing is constrained by the capacity Q^o).

  (2.a) There does not exist a $\overline{w}_{\mathbf{P}}(Q^o) \geq c$ (i.e., there does not exist a feasible wholesale price at which posted pricing is constrained by the capacity Q^o). In this case, at the higher capacity Q', there does not exist a $\overline{w}_{\mathbf{P}}(Q') \geq c$ either. Therefore, both $\Pi_{\mathbf{MP}}(w, p^*(w, Q^o), Q^o)$ and $\Pi_{\mathbf{MP}}(w, p^*(w, Q'), Q')$ are equal to the profit in the uncapacitated supply chain, $\Pi^u_{\mathbf{MP}}(w, p^u(w))$, which does not depend on the supply chain capacity. It follows that, for both $Q = Q^o$ and $Q = Q'$:

$$G_{\mathbf{P}}(Q) = \sup_{w > \widehat{w}_{\mathbf{R}}(Q), w \geq c} \Pi^u_{\mathbf{MP}}(w, p^u(w)).$$

Hence, $G_{\mathbf{P}}(Q^o)$ differs from $G_{\mathbf{P}}(Q')$ only because $\widehat{w}_{\mathbf{R}}(Q^o)$ differs from $\widehat{w}_{\mathbf{R}}(Q')$. Now, according to Lemma B.3(b), $\widehat{w}_{\mathbf{R}}(Q)$ is increasing in Q. Therefore, $\widehat{w}_{\mathbf{R}}(Q') \geq \widehat{w}_{\mathbf{R}}(Q^o)$, which allows us to conclude that $G_{\mathbf{P}}(Q^o) \geq G_{\mathbf{P}}(Q')$.

(2.b) There exists a $\overline{w}_{\mathbf{P}}(Q^o) \geq c$ (i.e., there exists a feasible wholesale price at which posted pricing is constrained by the capacity Q^o). Then, $\widehat{w}_{\mathbf{R}}(Q^o) > \overline{w}_{\mathbf{P}}(Q^o)$ by Lemma B.1(d). Therefore, for any $w > \widehat{w}_{\mathbf{R}}(Q^o)$, we have $\Pi_{\mathbf{MP}}(w, p^*(w, Q^o), Q^o) = \Pi^u_{\mathbf{MP}}(w, p^u(w))$. Recall that $\overline{w}_{\mathbf{P}}(Q)$ decreases in Q, and then disappears when Q becomes sufficiently large. On the other hand, $\widehat{w}_{\mathbf{R}}(Q)$ increases in Q from Lemma B.3(b). Thus, at any $Q' > Q^o$, either there does not exist $\overline{w}_{\mathbf{P}}(Q') \geq c$ or $\widehat{w}_{\mathbf{R}}(Q') > \overline{w}_{\mathbf{P}}(Q')$. Therefore, $\Pi_{\mathbf{MP}}(w, p^*(w, Q'), Q') = \Pi^u_{\mathbf{MP}}(w, p^u(w))$ for any $w > \widehat{w}_{\mathbf{R}}(Q')$. Consequently, we observe once again that $G_{\mathbf{P}}(Q^o)$ differs from $G_{\mathbf{P}}(Q')$ only because $\widehat{w}_{\mathbf{R}}(Q^o)$ differs from $\widehat{w}_{\mathbf{R}}(Q')$, and the result follows as in the previous case.

Based on the above claim and the fact that $G_{\mathbf{N}}(Q)$ increases in Q (by Lemma B.3(d)), we obtain:

$$G_{\mathbf{N}}(Q') \geq G_{\mathbf{N}}(Q^o) \geq G_{\mathbf{P}}(Q^o) \geq G_{\mathbf{P}}(Q').$$

We thus proved that $G_{\mathbf{N}}(Q') \geq G_{\mathbf{P}}(Q')$ for any $Q' > Q^o$, which concludes Part 1.

Part 2: The existence of $\overline{\mathbf{Q}}$

It suffices to show that if the ordinary negotiation is the equilibrium at some capacity level $Q^o \geq \underline{Q}$ (i.e., the equilibrium wholesale price is $w^*_{\mathbf{N}}(Q^o)$), then the ordinary negotiation remains to be the equilibrium at all $Q > Q^o$ with the equilibrium wholesale price, $w^*_{\mathbf{N}}(Q)$. This result directly follows from Lemma B.3(c).

Proof of Proposition 9

The proof follows as a direct corollary of Propositions 7 and 8.

Proof of Proposition 10

We first prove part (a) of the proposition. For ease of exposition, we suppress the dependence on the supply chain capacity Q throughout this proof. Suppose that the manufacturer chooses posted pricing in the manufacturer leadership model. Then, from equation (21), it must be the case that

$$\Pi_{\mathbf{MP}}(w^*_{\mathbf{P}}, p^*(w^*_{\mathbf{P}})) > \Pi_{\mathbf{MN}}(w^*_{\mathbf{N}}, q^*_{\min}(w^*_{\mathbf{N}})). \tag{B-1}$$

We show that the wholesale $w^*_{\mathbf{P}}$ induces posted pricing even with a discretionary retailer. Once we show that $w^*_{\mathbf{P}}$ induces posted pricing even with a discretionary retailer, we will have concluded the proof, because a manufacturer facing the discretionary retailer would set its wholesale price equal to w_{P}^* and obtain the same profits as in the manufacturer leadership model.

In preparation for the rest of the proof, define $\Delta_{\mathrm{M}}(w) = \Pi_{\mathrm{MP}}(w, p^*(w)) - \Pi_{\mathrm{MN}}(w, q_{\min}^*(w))$. Since w_{N}^* is a maximizer of $\Pi_{\mathrm{MN}}(w, q_{\min}^*(w))$, we have

$$\Pi_{\mathrm{MP}}(w_{\mathrm{P}}^*, p^*(w_{\mathrm{P}}^*)) > \Pi_{\mathrm{MN}}(w_{\mathrm{N}}^*, q_{\min}^*(w_{\mathrm{N}}^*)) \geq \Pi_{\mathrm{MN}}(w_{\mathrm{P}}^*, q_{\min}^*(w_{\mathrm{P}}^*)).$$

In other words, $\Delta_{\mathrm{M}}(w_{\mathrm{P}}^*) > 0$.

We divide the proof into three cases depending on the behavior of $\Delta_{\mathrm{R}}(w) = \Pi_{\mathrm{RP}}(p^*(w), w) - \Pi_{\mathrm{RN}}(q_{\min}^*(w), w)$, which crosses zero at most once and from below.

If $\Delta_{\mathrm{R}}(w) \geq 0$ for all $w \geq c$, the retailer already prefers posted pricing no matter what wholesale price. Hence, the equilibrium of the discretionary retailer model is posted pricing with the wholesale price w_{P}^*.

Now consider the case that there exists a $\widehat{w}_{\mathrm{R}} > c$ such that $\Delta_{\mathrm{R}}(w) \leq 0$ for all $w \leq \widehat{w}_{\mathrm{R}}$ and $\Delta_{\mathrm{R}}(w) > 0$ for all $w \geq \widehat{w}_{\mathrm{R}}$. From the part (e) of Lemma B.1, there must exist a wholesale price $\widehat{w}_{\mathrm{M}}$ such that $\widehat{w}_{\mathrm{M}} \geq \widehat{w}_{\mathrm{R}}$, $\Delta_{\mathrm{M}}(w) \leq 0$ for $w \leq \widehat{w}_{\mathrm{M}}$ and $\Delta_{\mathrm{M}}(w) \geq 0$ for $w \geq \widehat{w}_{\mathrm{M}}$. Given that $\Delta_{\mathrm{M}}(w_{\mathrm{P}}^*) > 0$, we have $w_{\mathrm{P}}^* \geq \widehat{w}_{\mathrm{M}}$. Therefore, $w_{\mathrm{P}}^* \geq \widehat{w}_{\mathrm{R}}$ as well. Thus, the wholesale price w_{P}^* will induce the retailer to use posted pricing.

Finally, consider the case that $\Delta_{\mathrm{R}}(w) \leq 0$ for all $w \geq c$. Observe, however, this case cannot occur, because $\Delta_{\mathrm{M}}(w_{\mathrm{P}}^*) > 0$ as shown above: If $\Delta_{\mathrm{R}}(w)$ were less than or equal to 0 for all $w \geq c$, then we should have $\Delta_{\mathrm{M}}(w) \leq 0$ for all $w \geq c$ by Lemma B.1(f).

As for part (b), it is not difficult to find examples. For instance, in the numerical example presented in Figure 1, if $c_{\mathrm{T}} = 1.5$, the equilibrium is ordinary negotiation in the manufacturer leadership model, but reconciliatory negotiation in the discretionary retailer model. However, at $c_{\mathrm{T}} = 1.62$, the equilibrium is ordinary negotiation in the manufacturer leadership model, but posted pricing in the discretionary retailer model.

Technical Lemmas used in Appendix B

Lemma B.1. *For a given wholesale price w and capacity Q, let $\Delta_{\mathrm{R}}(w, Q)$ be the difference between the retailer's optimal profits under posted pricing and negotiation, that is, $\Delta_{\mathrm{R}}(w, Q) = \Pi_{\mathrm{RP}}(p^*(w, Q), w, Q) - \Pi_{\mathrm{RN}}(q_{\min}^*(w, Q), w, Q)$. Likewise, let $\Delta_{\mathrm{M}}(w, Q)$ be the difference between the manufacturer's optimal profits under posted pricing and negotiation, that is, $\Delta_{\mathrm{M}}(w, Q) = \Pi_{\mathrm{MP}}(w, p^*(w, Q), Q) - \Pi_{\mathrm{MN}}(w, q_{\min}^*(w, Q), Q)$. Then:*

(a) If $\Delta_{\mathrm{R}}(c,Q) \geq 0$*, then* $\Delta_{\mathrm{R}}(w,Q) \geq 0$ *for all* $w > c$*.*

(b) If $\Delta_{\mathrm{R}}(c,Q) < 0$*, then either:*

(i) $\Delta_{\mathrm{R}}(w,Q) < 0$ *for all* $w > c$*, or*

(ii) There exists a $\widehat{w}_{\mathrm{R}}(Q) > c$ *such that* $\Delta_{\mathrm{R}}(\widehat{w}_{\mathrm{R}}(Q),Q) = 0$*,* $\Delta_{\mathrm{R}}(w,Q) \leq 0$ *for* $w < \widehat{w}_{\mathrm{R}}(Q)$*, and* $\Delta_{\mathrm{R}}(w,Q) \geq 0$ *for* $w \geq \widehat{w}_{\mathrm{R}}(Q)$*. In other words,* $\Delta_{\mathrm{R}}(w,Q)$ *crosses zero only once at* $\widehat{w}_{\mathrm{R}}(Q) > c$*.*

(c) For a given Q*, suppose that there exists* $\overline{w}_{\mathrm{P}}(Q) \geq c$ *but there does not exist* $\overline{w}_{\mathrm{N}}(Q) \geq c$*. Then,* $\Delta_{\mathrm{R}}(w,Q) \geq 0$ *for all* $w \geq c$*.*

(d) For a given Q*, suppose that there exist* $\overline{w}_{\mathrm{P}}(Q) \geq c$ *and* $\widehat{w}_{\mathrm{R}}(Q) > c$*. Then,* $\overline{w}_{\mathrm{N}}(Q) > \overline{w}_{\mathrm{P}}(Q)$ *and* $\widehat{w}_{\mathrm{R}}(Q) > \overline{w}_{\mathrm{P}}(Q)$*.*

(e) Suppose that $\Delta_{\mathrm{R}}(w,Q)$ *crosses zero from below at* $\widehat{w}_{\mathrm{R}}(Q) > c$*. Then, there must exist a wholesale price* $\widehat{w}_{\mathrm{M}}(Q)$ *such that* $\widehat{w}_{\mathrm{M}}(Q) \geq \widehat{w}_{\mathrm{R}}(Q)$*,* $\Delta_{\mathrm{M}}(w,Q) \leq 0$ *for* $w \leq \widehat{w}_{\mathrm{M}}(Q)$ *and* $\Delta_{\mathrm{M}}(w,Q) \geq 0$ *for* $w \geq \widehat{w}_{\mathrm{M}}(Q)$*.*

(f) Suppose that $\Delta_{\mathrm{R}}(w,Q) \leq 0$ *for all* $w \geq c$*. Then,* $\Delta_{\mathrm{M}}(w,Q) \leq 0$ *for* $w \geq c$*.*

Proof of Lemma B.1

Notice that under the posted pricing policy, the supply chain capacity Q will not play a role if $w \geq \overline{w}_{\mathrm{P}}(Q)$: $\Pi_{\mathrm{RP}}(p^*(w,Q),w,Q) = \Pi^u_{\mathrm{RP}}(p^u(w),w)$. Likewise, under the negotiation, the supply chain capacity Q will not play a role if $w \geq \overline{w}_{\mathrm{N}}(Q)$: $\Pi_{\mathrm{RN}}(q^*_{\min}(w,Q),w,Q) = \Pi^u_{\mathrm{RN}}(q^u_{\min}(w),w)$. Hence, equations (5) and (14) can be written as

$$\Pi_{\mathrm{RP}}(p^*(w,Q),w,Q) = \begin{cases} \Pi_{\mathrm{RP}}(\bar{p}(Q),w,Q) \text{ for } w \leq \overline{w}_{\mathrm{P}}(Q) \\ \Pi^u_{\mathrm{RP}}(p^u(w),w) \text{ for } w > \overline{w}_{\mathrm{P}}(Q) \end{cases} \quad \text{and}$$

$$\Pi_{\mathrm{RN}}(q^*_{\min}(w,Q),w,Q) = \begin{cases} \Pi_{\mathrm{RN}}(\bar{p}(Q),w,Q) \text{ for } w \leq \overline{w}_{\mathrm{N}}(Q) \\ \Pi^u_{\mathrm{RN}}(q^u_{\min}(w),w) \text{ for } w > \overline{w}_{\mathrm{N}}(Q) \end{cases}$$

These relationships between the retailer's profits in the capacitated and uncapacitated cases, described by the above equalities, will be used in the rest of the proof.

Proofs of (a) and (b): We consider four cases depending on whether there exist $\overline{w}_{\mathrm{N}}(Q) \geq c$ and/or $\overline{w}_{\mathrm{P}}(Q) \geq c$, that is, whether there exists a feasible wholesale price (i.e. greater than or equal to c) at which the quantity sold under negotiation and/or posted pricing is bounded by capacity. These four cases are: (1) neither $\overline{w}_{\mathrm{N}}(Q)$ nor $\overline{w}_{\mathrm{P}}(Q)$ exists, (2) both $\overline{w}_{\mathrm{N}}(Q)$ and $\overline{w}_{\mathrm{P}}(Q)$ exist, (3) only $\overline{w}_{\mathrm{P}}(Q)$ exists, and (4) only $\overline{w}_{\mathrm{N}}(Q)$ exists.

Case (1) neither $\overline{w}_{\mathrm{N}}(Q)$ nor $\overline{w}_{\mathrm{P}}(Q)$ exists: The retailer and manufacturer's profits are never bounded by capacity. The problem then collapses to the uncapacitated one. Thus, the results follow from Lemma A.3(a) and A.3(b).

Case (2) both $\overline{w}_{\mathbf{N}}(Q)$ and $\overline{w}_{\mathbf{P}}(Q)$ exist: We will divide the proof of case (2) into three mutually exclusive subcases:

$$\begin{cases} (2.a) & \overline{w}_{\mathbf{P}}(Q) \geq \overline{w}_{\mathbf{N}}(Q) \\ (2.b) & \overline{w}_{\mathbf{P}}(Q) < \overline{w}_{\mathbf{N}}(Q) \text{ and } \Delta_{\mathbf{R}}(\overline{w}_{\mathbf{P}}(Q), Q) \geq 0 \\ (2.c) & \overline{w}_{\mathbf{P}}(Q) < \overline{w}_{\mathbf{N}}(Q) \text{ and } \Delta_{\mathbf{R}}(\overline{w}_{\mathbf{P}}(Q), Q) < 0. \end{cases}$$

As we will prove next, in subcases (2.a) and (2.b), part (a) of this lemma holds. In subcase (2.c), part (b) of this lemma holds.

$(2.a) \quad \overline{w}_{\mathbf{P}}(Q) \geq \overline{w}_{\mathbf{N}}(Q)$

Since $\overline{w}_{\mathbf{P}}(Q) \geq \overline{w}_{\mathbf{N}}(Q) \geq c$, applying equations (5) and (14), we have

$$\begin{aligned} \Delta_{\mathbf{R}}(w, Q) &= \Pi_{\mathbf{RP}}(p^*(w, Q), w, Q) - \Pi_{\mathbf{RN}}(q^*_{\min}(w, Q), w, Q) \\ &= \begin{cases} \Pi_{\mathbf{RP}}(\bar{p}(Q), w, Q) - \Pi_{\mathbf{RN}}(\bar{p}(Q), w, Q) & \text{for } w \in [c, \overline{w}_{\mathbf{N}}(Q)], \\ \Pi_{\mathbf{RP}}(\bar{p}(Q), w, Q) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w) & \text{for } w \in [\overline{w}_{\mathbf{N}}(Q), \overline{w}_{\mathbf{P}}(Q)], \\ \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w) & \text{for } w \geq \overline{w}_{\mathbf{P}}(Q). \end{cases} \end{aligned} \tag{B-2}$$

From the definitions of $\overline{w}_{\mathbf{N}}(Q)$, $\overline{w}_{\mathbf{P}}(Q)$ and $\bar{p}(Q)$, it can be shown that $\Delta_{\mathbf{R}}(w, Q)$ is continuous and differentiable in w. To help with the proof, we substitute from (5) and (14) into (B-2), and take the derivative to obtain

$$\frac{d\Delta_{\mathbf{R}}(w, Q)}{dw} = \begin{cases} 0 & \text{for } w \in [c, \overline{w}_{\mathbf{N}}(Q)], \\ -Q + a\overline{F}(q^u_{\min}(w)) & \text{for } w \in [\overline{w}_{\mathbf{N}}(Q), \overline{w}_{\mathbf{P}}(Q)], \\ -a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w)) & \text{for } w \geq \overline{w}_{\mathbf{P}}(Q). \end{cases} \tag{B-3}$$

First, we show that $\Delta_{\mathbf{R}}(w, Q) \geq 0$ for any $w \in [c, \overline{w}_{\mathbf{P}}(Q)]$. We prove this by contradiction. Suppose there exists some $w^o \leq \overline{w}_{\mathbf{P}}(Q)$ such that $\Delta_{\mathbf{R}}(w^o, Q) < 0$. Notice that $a\overline{F}(q^u_{\min}(w)) \leq Q$ for $w \in [\overline{w}_{\mathbf{N}}(Q), \overline{w}_{\mathbf{P}}(Q)]$ (since $q^u_{\min}(w) \geq q^u_{\min}(\overline{w}_{\mathbf{N}}(Q)) = \bar{p}(Q)$ for $w \geq \overline{w}_{\mathbf{N}}(Q)$). Thus, $\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw} \leq 0$ for $w \in [c, \overline{w}_{\mathbf{P}}(Q)]$, and it must be that $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{P}}(Q), Q) < 0$. Also notice from (B-2) that, for $w \geq \overline{w}_{\mathbf{P}}(Q)$, capacity is no longer binding and the retailer's profits under both pricing policies are given by the profits in the uncapacitated problem: $\Delta_{\mathbf{R}}(w, Q) = \Delta^u_{\mathbf{R}}(w)$ for $w \geq \overline{w}_{\mathbf{P}}(Q)$. Combining the facts above, we must have

$$\Delta^u_{\mathbf{R}}(\overline{w}_{\mathbf{P}}(Q)) = \Delta_{\mathbf{R}}(\overline{w}_{\mathbf{P}}(Q), Q) < 0 \text{ and } \left.\frac{d\Delta^u_{\mathbf{R}}(w)}{dw}\right|_{w=\overline{w}_{\mathbf{P}}(Q)} = \left.\frac{d\Delta_{\mathbf{R}}(w, Q)}{dw}\right|_{w=\overline{w}_{\mathbf{P}}(Q)} \leq 0.$$

However, this contradicts Lemma A.3 since the function $\Delta^u_{\mathbf{R}}(w)$ cannot be decreasing at a w where $\Delta^u_{\mathbf{R}}(w)$ is strictly negative. (See Figure 7 for the possible behaviors of $\Delta^u_{\mathbf{R}}(w)$.) Therefore, $\Delta_{\mathbf{R}}(w, Q) \geq 0$ for any $w \in [c, \overline{w}_{\mathbf{P}}(Q)]$.

Second, we show that $\Delta_{\mathbf{R}}(w, Q) \geq 0$ for $w > \overline{w}_{\mathbf{P}}(Q)$. Recall that for $w \geq \overline{w}_{\mathbf{P}}(Q)$, $\Delta_{\mathbf{R}}(w, Q) = \Delta^u_{\mathbf{R}}(w)$ and we have shown above that $\Delta^u_{\mathbf{R}}(\overline{w}_{\mathbf{P}}(Q)) \geq 0$. Lemma A.3(a) and (b) together imply that once $\Delta^u_{\mathrm{R}}(w)$ is positive for some w, $\Delta^u_{\mathrm{R}}(w)$ remains positive for any larger w. Therefore, $\Delta_{\mathrm{R}}(w,Q) = \Delta^u_{\mathrm{R}}(w) \geq 0$ for $w > \overline{w}_{\mathrm{P}}(Q)$.

Combining the two intervals — $w \in [c, \overline{w}_{\mathrm{P}}(Q)]$ and $w > \overline{w}_{\mathrm{P}}(Q)$, we conclude that $\Delta_{\mathrm{R}}(w,Q) \geq 0$ for $w \geq c$.

(2.b) $\overline{w}_{\mathrm{P}}(Q) < \overline{w}_{\mathrm{N}}(Q)$ and $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}(Q), Q) \geq 0$

Since $\overline{w}_{\mathrm{P}}(Q) < \overline{w}_{\mathrm{N}}(Q)$, applying equations (5) and (14), we have

$$\begin{aligned}\Delta_{\mathrm{R}}(w,Q) &= \Pi_{\mathrm{RP}}(p^*(w,Q),w,Q) - \Pi_{\mathrm{RN}}(q^*_{\min}(w,Q),w,Q)\\ &= \begin{cases} \Pi_{\mathrm{RP}}(\bar{p}(Q),w,Q) - \Pi_{\mathrm{RN}}(\bar{p}(Q),w,Q) & \text{for } w \in [c, \overline{w}_{\mathrm{P}}(Q)],\\ \Pi^u_{\mathrm{RP}}(p^u(w),w) - \Pi_{\mathrm{RN}}(\bar{p}(Q),w,Q) & \text{for } w \in [\overline{w}_{\mathrm{P}}(Q), \overline{w}_{\mathrm{N}}(Q)],\\ \Pi^u_{\mathrm{RP}}(p^u(w),w) - \Pi^u_{\mathrm{RN}}(q^u_{\min}(w),w) & \text{for } w \geq \overline{w}_{\mathrm{N}}(Q).\end{cases}\end{aligned} \tag{B-4}$$

From the definitions of $\overline{w}_{\mathrm{N}}(Q)$, $\overline{w}_{\mathrm{P}}(Q)$ and $\bar{p}(Q)$, it can be shown that $\Delta_{\mathrm{R}}(w,Q)$ is differentiable in w. To help with the proof, we substitute from (5) and (14) into (B-4), and take the derivative to obtain

$$\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} = \begin{cases} 0 & \text{for } w \in [c, \overline{w}_{\mathrm{P}}(Q)],\\ -a\overline{F}(p^u(w)) + Q & \text{for } w \in [\overline{w}_{\mathrm{P}}(Q), \overline{w}_{\mathrm{N}}(Q)],\\ -a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w)) & \text{for } w \geq \overline{w}_{\mathrm{N}}(Q).\end{cases} \tag{B-5}$$

First, observe from (B-5) that $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} = 0$ for $w \in [c, \overline{w}_{\mathrm{P}}(Q)]$. Hence, given our assumption that $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}, Q) \geq 0$, it follows that $\Delta_{\mathrm{R}}(w,Q) \geq 0$ for $w \in [c, \overline{w}_{\mathrm{P}}(Q)]$.

Second, notice that $a\overline{F}(p^u(w)) \leq Q$ for $w \in [\overline{w}_{\mathrm{P}}(Q), \overline{w}_{\mathrm{N}}(Q)]$ (since $p^u(w) \geq p^u(\overline{w}_{\mathrm{P}}(Q)) = \bar{p}(Q)$ for $w \geq \overline{w}_{\mathrm{P}}(Q)$). Therefore, we observe from (B-5) that $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} \geq 0$ for $w \in [\overline{w}_{\mathrm{P}}(Q), \overline{w}_{\mathrm{N}}(Q)]$. Since $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}, Q) \geq 0$ by assumption, it follows that $\Delta_{\mathrm{R}}(w,Q) \geq 0$ for all $w \in [\overline{w}_{\mathrm{P}}(Q), \overline{w}_{\mathrm{N}}(Q)]$.

It remains to show that $\Delta_{\mathrm{R}}(w,Q) \geq 0$ for $w > \overline{w}_{\mathrm{N}}(Q)$. Notice from (B-4) that for $w \geq \overline{w}_{\mathrm{N}}(Q)$, $\Delta_{\mathrm{R}}(w,Q) = \Delta^u_{\mathrm{R}}(w)$. We have shown above that $\Delta^u_{\mathrm{R}}(\overline{w}_{\mathrm{N}}(Q)) \geq 0$. Lemma A.3(a) and (b) together imply that once $\Delta^u_{\mathrm{R}}(w)$ is positive for some w, $\Delta^u_{\mathrm{R}}(w)$ remains positive for any larger w. Therefore, $\Delta_{\mathrm{R}}(w,Q) = \Delta^u_{\mathrm{R}}(w) \geq 0$ for $w > \overline{w}_{\mathrm{N}}(Q)$.

Combining the three intervals — $w \in [c, \overline{w}_{\mathrm{P}}(Q)]$, $w \in (\overline{w}_{\mathrm{P}}(Q), \overline{w}_{\mathrm{N}}(Q)]$ and $w > \overline{w}_{\mathrm{N}}(Q)$, we conclude that $\Delta_{\mathrm{R}}(w,Q) \geq 0$ for $w \geq c$.

(2.c) $\overline{w}_{\mathrm{P}}(Q) < \overline{w}_{\mathrm{N}}(Q)$ and $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}(Q), Q) < 0$

Since $\overline{w}_{\mathrm{P}}(Q) < \overline{w}_{\mathrm{N}}(Q)$, $\Delta_{\mathrm{R}}(w,Q)$ and $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw}$ are given by (B-4) and (B-5), respectively. Observe that $\frac{d\Delta_{\mathrm{R}}(w,Q)}{dw} = 0$ for $w \in [c, \overline{w}_{\mathrm{P}}(Q)]$. Hence, given our assumption that $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}(Q), Q) < 0$, it must be that $\Delta_{\mathrm{R}}(w,Q) < 0$ for $w \in [c, \overline{w}_{\mathrm{P}}(Q)]$.

Next, we consider the behavior of $\Delta_{\mathrm{R}}(w,Q)$ for $w > \overline{w}_{\mathrm{P}}(Q)$ by examining two subcases: (2.c.i) $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}}(Q), Q) \geq 0$, and (2.c.ii) $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}}(Q), Q) < 0$.

$(2.c.i)$: $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q), Q) \geq 0$.

For $w \in (\overline{w}_{\mathbf{P}}(Q), \overline{w}_{\mathbf{N}}(Q)]$, observe from (B-5) that $\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw} > 0$ (since $p^u(w) > p^u(\overline{w}_{\mathbf{P}}(Q)) = \bar{p}(Q)$ for $w > \overline{w}_{\mathbf{P}}(Q)$). Since $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{P}}(Q), Q) < 0$ and $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q), Q) \geq 0$, it must be that $\Delta_{\mathbf{R}}(w, Q)$ crosses zero only once for some $w \in (\overline{w}_{\mathbf{P}}(Q), \overline{w}_{\mathbf{N}}(Q)]$.

As for $w \geq \overline{w}_{\mathbf{N}}(Q)$, observe from (B-4) that $\Delta_{\mathbf{R}}(w, Q) = \Delta^u_{\mathbf{R}}(w)$ when $w \geq \overline{w}_{\mathbf{N}}(Q)$. By Lemma A.3(a) and (b) together, once $\Delta^u_{\mathbf{R}}(w)$ crosses zero at some w, it stays strictly positive for larger w. Therefore, $\Delta_{\mathbf{R}}(w, Q) = \Delta^u_{\mathbf{R}}(w) > 0$ for $w > \overline{w}_{\mathbf{N}}(Q)$

Combining the three intervals — $w \in [c, \overline{w}_{\mathbf{P}}(Q)]$, $w \in (\overline{w}_{\mathbf{P}}(Q), \overline{w}_{\mathbf{N}}(Q)]$ and $w > \overline{w}_{\mathbf{N}}(Q)$, we conclude that $\Delta_{\mathbf{R}}$ crosses zero only once from below and stays strictly positive afterward (corresponding to part (b)(ii) of the lemma).

$(2.c.ii)$: $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q), Q) < 0$.

Recall that $\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw} = 0$ for $w \in [c, \overline{w}_{\mathbf{P}}(Q)]$ and $\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw} \geq 0$ for $w \in [\overline{w}_{\mathbf{P}}(Q), \overline{w}_{\mathbf{N}}(Q)]$ from (B-5). Therefore, given the assumption that $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q), Q) < 0$, it must be that $\Delta_{\mathbf{R}}(w, Q) < 0$ for $w \in [c, \overline{w}_{\mathbf{N}}(Q)]$.

For $w > \overline{w}_{\mathbf{N}}(Q)$, recall that $\Delta_{\mathbf{R}}(w, Q) = \Delta^u_{\mathbf{R}}(w)$. Since $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q), Q) = \Delta^u_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q)) < 0$, the behavior of $\Delta_{\mathbf{R}}(w, Q) = \Delta^u_{\mathbf{R}}(w)$ for $w > \overline{w}_{\mathbf{N}}(Q)$ must be the same as the behavior described in Lemma A.3(b).

Case (3) only $\overline{w}_{\mathbf{P}}(Q)$ exists: Consider now the case where $\overline{w}_{\mathbf{P}}(Q) \geq c$ exists, but $\overline{w}_{\mathbf{N}}(Q) \geq c$ does not exist. The quantity sold under negotiation is not bounded by capacity for any $w \geq c$. Hence, $\Pi_{\mathbf{RN}}(q^*_{\min}(w, Q), w, Q) = \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w)$ for all $w \geq c$. Given this fact and applying equation (5), we have

$$\begin{aligned}\Delta_{\mathbf{R}}(w, Q) &= \Pi_{\mathbf{RP}}(p^*(w, Q), w, Q) - \Pi_{\mathbf{RN}}(q^*_{\min}(w, Q), w, Q)\\ &= \begin{cases} \Pi_{\mathbf{RP}}(\bar{p}(Q), w, Q) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w) & \text{for } w \in [c, \overline{w}_{\mathbf{P}}(Q)],\\ \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w) & \text{for } w \geq \overline{w}_{\mathbf{P}}(Q).\end{cases}\end{aligned} \tag{B-6}$$

Notice that (B-6) is a special case of (B-2). Therefore, case (3) collapses to case (2.a), and $\Delta_{\mathbf{R}}(w, Q) \geq 0$ for all $w \geq c$.

Case (4) only $\overline{w}_{\mathbf{N}}(Q)$ exists: Consider now the case where $\overline{w}_{\mathbf{N}}(Q) \geq c$ exists, but $\overline{w}_{\mathbf{P}}(Q) \geq c$ does not exist. The quantity sold under posted pricing is not bounded by capacity for any $w \geq c$. Hence, $\Pi_{\mathbf{RP}}(p^*(w, Q), w, Q) = \Pi^u_{\mathbf{RP}}(p^u(w), w)$ for all $w \geq c$. Given this fact and applying equation (5), we have

$$\begin{aligned}\Delta_{\mathbf{R}}(w, Q) &= \Pi_{\mathbf{RP}}(p^*(w, Q), w, Q) - \Pi_{\mathbf{RN}}(q^*_{\min}(w, Q), w, Q)\\ &= \begin{cases} \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi_{\mathbf{RN}}(\bar{p}(Q), w, Q) & \text{for } w \in [c, \overline{w}_{\mathbf{N}}(Q)],\\ \Pi^u_{\mathbf{RP}}(p^u(w), w) - \Pi^u_{\mathbf{RN}}(q^u_{\min}(w), w) & \text{for } w \geq \overline{w}_{\mathbf{N}}(Q).\end{cases}\end{aligned} \tag{B-7}$$

Depending on whether $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}}(Q), Q) \geq 0$ or $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{N}}(Q), Q) < 0$, the result is the same as in case (2.c.i) or (2.c.ii), respectively.

Proof of (c): If $\overline{w}_{\mathrm{P}}(Q) \geq c$ exists, but $\overline{w}_{\mathrm{N}}(Q) \geq c$ doesn't, we must be in Case (3) discussed in the proof of parts (a) and (b). Then, the result that $\Delta_{\mathrm{R}}(w, Q) \geq 0$ for all $w \geq c$ follows immediately.

Proof of (d): Since $\overline{w}_{\mathrm{P}}(Q) \geq c$ exists, Cases (1) and (4) are ruled out. Furthermore, since $\widehat{w}_{\mathrm{R}}(Q) > c$ exists, Cases (2.a), (2.b), (3) are ruled out as $\Delta_{\mathrm{R}}(w, Q) \geq 0$ for all $w \geq c$ in these cases, which does not permit the existence of $\widehat{w}_{\mathrm{R}}(Q) > c$. Hence, we must be in case (2.c): both $\overline{w}_{\mathrm{P}}(Q) \geq c$ and $\overline{w}_{\mathrm{N}}(Q) \geq c$ exist, $\overline{w}_{\mathrm{P}}(Q) < \overline{w}_{\mathrm{N}}(Q)$, and $\Delta_{\mathrm{R}}(\overline{w}_{\mathrm{P}}(Q), Q) < 0$. In the proof of case (2.c), we have shown that $\Delta_{\mathrm{R}}(w, Q) < 0$ for $w \in [c, \overline{w}_{\mathrm{P}}(Q)]$. Hence, the wholesale price at which $\Delta_{\mathrm{R}}(w, Q) = 0$, i.e., $\widehat{w}_{\mathrm{R}}(Q)$, must be strictly greater than $\overline{w}_{\mathrm{P}}(Q)$.

Proof of (e): For ease of exposition, we prove the results when there exist market-clearing wholesale prices, $\overline{w}_{\mathrm{P}}(Q) \geq c$ and $\overline{w}_{\mathrm{N}}(Q) \geq c$ for a given Q. Notice that if neither $\overline{w}_{\mathrm{P}}(Q) \geq c$ nor $\overline{w}_{\mathrm{N}}(Q) \geq c$ exists, then, under any pricing policy, there would be no feasible wholesale price ($w \geq c$) that makes the sales volume bounded by the supply chain capacity. Thus, the problem reverts to the case with sufficient capacity, for which the result has already been established in Section A of this appendix. If only one of $\overline{w}_{\mathrm{P}}(Q) \geq c$ or $\overline{w}_{\mathrm{N}}(Q) \geq c$ exists, then there would be no feasible wholesale price that makes the sales volume bounded by the supply chain capacity under one of the pricing policies, and the result would follow as a special case of the proof we provide here.

First, note from Lemma B.1(d) that the existence of $\widehat{w}_{\mathrm{R}}(Q) > c$ implies that $\overline{w}_{\mathrm{N}}(Q) > \overline{w}_{\mathrm{P}}(Q)$ and $\widehat{w}_{\mathrm{R}}(Q) > \overline{w}_{\mathrm{P}}(Q)$. Since $\overline{w}_{\mathrm{N}}(Q) > \overline{w}_{\mathrm{P}}(Q)$, using the expressions for the manufacturer's profit functions, given by (6) and (15), we have

$$\begin{aligned}
\Delta_{\mathrm{M}}(w, Q) &= \Pi_{\mathrm{MP}}(w, p^*(w, Q), (Q)) - \Pi_{\mathrm{MN}}(w, q^*_{\min}(w, Q), Q) \\
&= \begin{cases} 0 & \text{for } w \in [c, \overline{w}_{\mathrm{P}}(Q)], \\ a(w-c)\overline{F}(p^u(w)) - (w-c)Q \leq 0 & \text{for } w \in [\overline{w}_{\mathrm{P}}(Q), \overline{w}_{\mathrm{N}}(Q)], \\ a(w-c)\overline{F}(p^u(w)) - a(w-c)\overline{F}(q^u_{\min}(w)) & \text{for } w \geq \overline{w}_{\mathrm{N}}(Q). \end{cases}
\end{aligned} \tag{B-8}$$

Notice from above that $\Delta_{\mathrm{M}}(w, Q) \leq 0$ for $w \leq \overline{w}_{\mathrm{N}}(Q)$. We next analyze the behavior of $\Delta_{\mathrm{M}}(w, Q)$ for $w > \overline{w}_{\mathrm{N}}(Q)$. Observe that, if $w \geq \overline{w}_{\mathrm{N}}(Q)$, then the sales quantity will not be bounded by the supply chain capacity Q under either pricing policy. Since capacity Q does not play any role under either pricing policy when $w \geq \overline{w}_{\mathrm{N}}(Q)$, $\Delta_{\mathrm{M}}(w, Q)$ is equal to $\Delta^u_{\mathrm{M}}(w)$ and $\Delta_{\mathrm{R}}(w, Q)$ is equal to $\Delta^u_{\mathrm{R}}(w)$ for $w \geq \overline{w}_{\mathrm{N}}(Q)$.

From (B-8), $\Delta_{\mathrm{M}}(w, Q) = a(w-c)\left(\overline{F}(p^u(w)) - \overline{F}(q^u_{\min}(w))\right)$ for $w \geq \overline{w}_{\mathrm{N}}(Q)$. Also, from (A-15),

$\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw} = \frac{d\Delta^u_{\mathbf{R}}(w)}{dw} = -a\overline{F}(p^u(w)) + a\overline{F}(q^u_{\min}(w))$ for $w \geq \overline{w}_{\mathbf{N}}(Q)$. Then, we have

$$\Delta_{\mathbf{M}}(w,Q) = \Delta^u_{\mathbf{M}}(w) = -(w-c)\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw} = -(w-c)\frac{d\Delta^u_{\mathbf{R}}(w)}{dw} \text{ for } w \geq \overline{w}_{\mathbf{N}}(Q) \tag{B-9}$$

Since $\Delta_{\mathbf{M}}(\overline{w}_{\mathbf{N}}(Q),Q) \leq 0$, $\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw} \geq 0$ at $w = \overline{w}_{\mathbf{N}}(Q)$. This observation will be used later in the proof.

To show that $\widehat{w}_{\mathbf{M}}(Q)$ exists and that $\widehat{w}_{\mathbf{M}}(Q) \geq \widehat{w}_{\mathbf{R}}(Q)$, we separately examine two cases: (1) $\widehat{w}_{\mathbf{R}}(Q) \leq \overline{w}_{\mathbf{N}}(Q)$ and (2) $\widehat{w}_{\mathbf{R}}(Q) > \overline{w}_{\mathbf{N}}(Q)$. Note that, by the definition of $\widehat{w}_{\mathbf{R}}(Q)$, $\widehat{w}_{\mathbf{R}}(Q) \leq \overline{w}_{\mathbf{N}}(Q)$ is equivalent to $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q),Q) \geq 0$, and $\widehat{w}_{\mathbf{R}}(Q) > \overline{w}_{\mathbf{N}}(Q)$ is equivalent to $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q),Q) < 0$.

(1) $\widehat{w}_{\mathbf{R}}(Q) \leq \overline{w}_{\mathbf{N}}(Q)$ (equivalently, $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q),Q) \geq 0$)

$\Delta_{\mathbf{R}}(w,Q) \geq 0$ at $w = \overline{w}_{\mathbf{N}}(Q)$ by the assumption of this case. Furthermore, as we showed above, $\Delta_{\mathbf{R}}(w,Q)$ is locally increasing in w at $w = \overline{w}_{\mathbf{N}}(Q)$. Given that $\Delta_{\mathbf{R}}(w,Q) = \Delta^u_{\mathbf{R}}(w)$ for $w \geq \overline{w}_{\mathbf{N}}(Q)$, we can now apply Lemma A.3(b) to conclude that the function $\Delta_{\mathbf{R}}(w,Q)$ is unimodal in w for $w \geq \overline{w}_{\mathbf{N}}(Q)$ and peaks at some $w^o \geq \overline{w}_{\mathbf{N}}(Q)$. This implies that $\frac{d\Delta_{\mathbf{R}}(w,Q)}{dw}$ changes sign from positive to negative at $w^o \geq \overline{w}_{\mathbf{N}}(Q)$, which in turn implies that $\Delta_{\mathbf{M}}(w,Q)$ changes sign from negative to positive precisely at this $w^o \geq \overline{w}_{\mathbf{N}}(Q)$ (see (B-9)). Hence, $\widehat{w}_{\mathbf{M}}(Q) = w^o \geq \overline{w}_{\mathbf{N}}(Q) \geq \widehat{w}_{\mathbf{R}}(Q)$.

(2) $\widehat{w}_{\mathbf{R}}(Q) > \overline{w}_{\mathbf{N}}(Q)$ (equivalently, $\Delta_{\mathbf{R}}(\overline{w}_{\mathbf{N}}(Q),Q) < 0$)

Now, applying Lemma A.3(c), there must exist a $\widehat{w}^u_{\mathbf{M}} \geq \widehat{w}_{\mathbf{R}}(Q) \geq \overline{w}_{\mathbf{N}}(Q)$ such that $\Delta^u_{\mathbf{M}}(w) \leq 0$ for $w \leq \widehat{w}^u_{\mathbf{M}}$, and $\Delta^u_{\mathbf{M}}(w) \geq 0$ for $w \geq \widehat{w}^u_{\mathbf{M}}$. Now, recalling that $\Delta_{\mathbf{M}}(w,Q) = \Delta^u_{\mathbf{M}}(w)$ for $w \geq \overline{w}_{\mathbf{N}}(Q)$, the value of $\widehat{w}^u_{\mathbf{M}}$ yields $\widehat{w}_{\mathbf{M}}(Q)$ in this case.

Proof of (f): The proof follows a similar argument to that of part (e).

Lemma B.2. *Define* $\Delta_{\mathbf{R}}(w,Q) = \Pi_{\mathbf{RP}}(p^*(w,Q),w,Q) - \Pi_{\mathbf{RN}}(q^*_{\min}(w,Q),w,Q)$. *Suppose there exists a unique* $\widehat{w}_{\mathbf{R}}(Q) > c$ *such that* $\Delta_{\mathbf{R}}(\widehat{w}_{\mathbf{R}}(Q),Q) = 0$, $\Delta_{\mathbf{R}}(w,Q) < 0$ *for* $w < \widehat{w}_{\mathbf{R}}(Q)$, *and* $\Delta_{\mathbf{R}}(w,Q) > 0$ *for* $w > \widehat{w}_{\mathbf{R}}(Q)$. *Consider the following optimization problem:*

$$\max_{c \leq w \leq \widehat{w}_{\mathbf{R}}(Q)} \Pi_{\mathbf{MN}}(w, q^*_{\min}(w,Q), Q) \tag{B-10}$$

Let $w^o_{\mathbf{N}}(Q,c_{\mathbf{T}})$ *denote the optimal solution to* (B-10) *and* $G_{\mathbf{N}}(Q,c_{\mathbf{T}})$ *be the optimal value of the objective function for a given capacity,* Q, *and a given cost of negotiation,* $c_{\mathbf{T}}$. *Then,*

(a) $w^o_{\mathbf{N}}(Q,c_{\mathbf{T}}) = \min\{\widehat{w}_{\mathbf{R}}(Q), w^*_{\mathbf{N}}(Q)\}$.

(b) $\overline{w}_{\mathbf{N}}(Q)$ *decreases in* $c_{\mathbf{T}}$. *Furthermore,* $\frac{d\overline{w}_{\mathbf{N}}(Q)}{dc_{\mathbf{T}}} = -1$.

(c) $\widehat{w}_{\mathbf{R}}(Q)$ *decreases in* $c_{\mathbf{T}}$. *Furthermore,* $\frac{d\widehat{w}_{\mathbf{R}}(Q)}{dc_{\mathbf{T}}} < -1$.

(d) If $w^o_{\mathbf{N}}(Q,c_{\mathbf{T}}) = \widehat{w}_{\mathbf{R}}(Q)$ *for some* $c_{\mathbf{T}} = c^o_{\mathbf{T}}$, *then* $w^o_{\mathbf{N}}(Q,c_{\mathbf{T}}) = \widehat{w}_{\mathbf{R}}(Q)$ *for* $c_{\mathbf{T}} > c^o_{\mathbf{T}}$.

(e) $G_{\mathbf{N}}(Q,c_{\mathbf{T}})$ *decreases in* $c_{\mathbf{T}}$.

Proof of Lemma B.2

The proof of Lemma B.2 is similar to that of Lemma A.4 and mostly algebraic, therefore omitted.

Lemma B.3.

(a) Suppose there exists a $\overline{w}_{\mathbf{N}}(Q) > c$ *at a given* Q*. Then,* $\frac{d\overline{w}_{\mathbf{N}}(Q)}{dQ} \leq 0$.

(b) Define $\Delta_{\mathbf{R}}(w,Q) = \Pi_{\mathbf{RP}}(p^*(w,Q),w,Q) - \Pi_{\mathbf{RN}}(q^*_{\min}(w,Q),w,Q)$*. Suppose there exists a unique* $\widehat{w}_{\mathbf{R}}(Q) > c$ *such that* $\Delta_{\mathbf{R}}(\widehat{w}_{\mathbf{R}}(Q),Q) = 0$*,* $\Delta_{\mathbf{R}}(w,Q) < 0$ *for* $w < \widehat{w}_{\mathbf{R}}(Q)$*, and* $\Delta_{\mathbf{R}}(w,Q) > 0$ *for* $w > \widehat{w}_{\mathbf{R}}(Q)$*. Then,* $\frac{d\widehat{w}_{\mathbf{R}}(Q)}{dQ} \geq 0$.

Consider now the following optimization problem:

$$\max_{c \leq w \leq \widehat{w}_{\mathbf{R}}(Q)} \Pi_{\mathbf{MN}}(w, q^*_{\min}(w,Q), Q) \tag{B-11}$$

Let $w^o_{\mathbf{N}}(Q)$ *denote the optimal solution to* (B-11) *and* $G_{\mathbf{N}}(Q)$ *be the optimal value of the objective function for a given* Q*. Then,*

(c) Suppose, for some $Q = Q^o$*,* $w^o_{\mathbf{N}}(Q^o) = w^*_{\mathbf{N}}(Q^o)$*. Then,* $w^o_{\mathbf{N}}(Q) = w^*_{\mathbf{N}}(Q)$ *for* $Q > Q^o$.

(d) $G_{\mathbf{N}}(Q)$ *increases in* Q.

Proof of Lemma B.3

Proof of (a): For a given capacity Q, the market-clearing wholesale price under negotiation, $\overline{w}_{\mathbf{N}}(Q)$, is defined so that $q^u_{\min}(\overline{w}_{\mathbf{N}}(Q)) = \bar{p}(Q)$ (see (13)). Hence, $q^u_{\min}(\overline{w}_{\mathbf{N}}(Q)) = \bar{p}(Q)$ will satisfy the first-order condition of the retailer's profit function under negotiation, $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$ at $w = \overline{w}_{\mathbf{N}}(Q)$. Using the expression for $\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min},w)}{\partial q_{\min}}$ from (A-5) and the fact that $q^u_{\min}(w)$ satisfies the first-order condition for $\Pi^u_{\mathbf{RN}}(q_{\min}, w)$:

$$\left.\frac{\partial \Pi^u_{\mathbf{RN}}(q_{\min}, w)}{\partial q_{\min}}\right|_{q_{\min}=q^u_{\min}(w)} = a(-q^u_{\min}(w) + w + c_{\mathbf{T}})f(q^u_{\min}(w)) + a\beta\overline{F}(q^u_{\min}(w)) = 0.$$

Substituting $w = \overline{w}_{\mathbf{N}}(Q)$ and $q^u_{\min}(\overline{w}_{\mathbf{N}}(Q)) = \bar{p}(Q)$ in the above equation, we obtain the following identity:

$$\big(-\bar{p}(Q) + \overline{w}_{\mathbf{N}}(Q) + c_{\mathbf{T}}\big)f(\bar{p}(Q)) + \beta\overline{F}(\bar{p}(Q)) = 0, \text{ or, } -\beta\frac{\overline{F}(\bar{p}(Q))}{f(\bar{p}(Q))} + \bar{p}(Q) = \overline{w}_{\mathbf{N}}(Q) + c_{\mathbf{T}}. \tag{B-12}$$

When Q increases, $\bar{p}(Q)$ decreases (since $a\overline{F}(\bar{p}(Q)) = Q$) and, thus, $\beta\frac{\overline{F}(\bar{p}(Q))}{f(\bar{p}(Q))}$ increases due to F being IFR. Therefore, the left-hand side of the above identity decreases in Q. Hence, $\overline{w}_{\mathbf{N}}(Q)$ must decrease in Q.

Proof of (b): We consider four cases depending on whether there exist $\overline{w}_{\mathbf{N}}(Q) \geq c$ and/or $\overline{w}_{\mathbf{P}}(Q) \geq c$, that is whether there exists a feasible wholesale price (i.e. greater than or equal to c) at which the quantity sold under negotiation and/or posted pricing is bounded by capacity. Four cases are: (1) both $\overline{w}_{\mathbf{N}}(Q)$ and $\overline{w}_{\mathbf{P}}(Q)$ exist, (2) only $\overline{w}_{\mathbf{N}}(Q)$ exists, (3) only $\overline{w}_{\mathbf{P}}(Q)$ exists, and (4) neither of them exists.

(1) both $\overline{w}_{\mathbf{P}}(Q)$ and $\overline{w}_{\mathbf{N}}(Q)$ exist

Note from Lemma B.1(d) that $\overline{w}_{\mathbf{P}}(Q) < \min\{\widehat{w}_{\mathbf{R}}(Q), \overline{w}_{\mathbf{N}}(Q)\}$. We will prove that $\frac{d\widehat{w}_{\mathbf{R}}(Q)}{dQ} \geq 0$ in the following two subcases: (1.a) $\overline{w}_{\mathbf{P}}(Q) < \widehat{w}_{\mathbf{R}}(Q) < \overline{w}_{\mathbf{N}}(Q)$, and (1.b) $\overline{w}_{\mathbf{P}}(Q) < \overline{w}_{\mathbf{N}}(Q) \leq \widehat{w}_{\mathbf{R}}(Q)$.

Consider the first subcase (1.a). By definition, $\widehat{w}_{\mathbf{R}}(Q)$ satisfies $\Delta_{\mathbf{R}}(\widehat{w}_{\mathbf{R}}(Q), Q) = 0$. Observe from the expressions for $\Pi_{\mathbf{RP}}$ and $\Pi_{\mathbf{RN}}$, given by (5) and (14) that

$$\Pi_{\mathbf{RP}}(p^*(\widehat{w}_{\mathbf{R}}(Q), Q), \widehat{w}_{\mathbf{R}}(Q), Q) = \Pi^u_{\mathbf{RP}}(p^u(\widehat{w}_{\mathbf{R}}(Q)), \widehat{w}_{\mathbf{R}}(Q)) \quad \text{(since } \overline{w}_{\mathbf{P}}(Q) < \widehat{w}_{\mathbf{R}}(Q)\text{), and}$$
$$\Pi_{\mathbf{RN}}(q^*_{\min}(\widehat{w}_{\mathbf{R}}(Q), Q), \widehat{w}_{\mathbf{R}}(Q), Q) = \Pi_{\mathbf{RN}}(\bar{p}(Q), \widehat{w}_{\mathbf{R}}(Q), Q) \quad \text{(since } \widehat{w}_{\mathbf{R}}(Q) < \overline{w}_{\mathbf{N}}(Q)\text{)}.$$

Therefore, at $w = \widehat{w}_{\mathbf{R}}(Q)$, the following identity must be satisfied:

$$\Pi_{\mathbf{RN}}(\bar{p}(Q), \widehat{w}_{\mathbf{R}}(Q), Q) - \Pi^u_{\mathbf{RP}}(p^u(\widehat{w}_{\mathbf{R}}(Q)), \widehat{w}_{\mathbf{R}}(Q)) = 0.$$

Implicit differentiation of the above identity with respect to Q yields:

$$\begin{aligned} 0 &= \frac{d\Pi_{\mathbf{RN}}(\bar{p}(Q), \widehat{w}_{\mathbf{R}}(Q), Q)}{dQ} - \frac{d\Pi^u_{\mathbf{RP}}(p^u(\widehat{w}_{\mathbf{R}}(Q)), \widehat{w}_{\mathbf{R}}(Q))}{dQ} \\ &= \tfrac{d\bar{p}(Q)}{dQ} \tfrac{\partial \Pi_{\mathbf{RN}}(q_{\min}, w, Q)}{\partial q_{\min}}\Big|_{q_{\min}=\bar{p}(Q), w=\widehat{w}_{\mathbf{R}}(Q)} + \tfrac{d\widehat{w}_{\mathbf{R}}(Q)}{dQ} \tfrac{\partial \Pi_{\mathbf{RN}}(q_{\min}, w, Q)}{\partial w}\Big|_{q_{\min}=\bar{p}(Q), w=\widehat{w}_{\mathbf{R}}(Q)} \\ &\quad - \tfrac{dp^u(\widehat{w}_{\mathbf{R}}(Q))}{dQ} \tfrac{\partial \Pi^u_{\mathbf{RP}}(p,w)}{\partial p}\Big|_{p=p^u(\widehat{w}_{\mathbf{R}}(Q)), w=\widehat{w}_{\mathbf{R}}(Q)} - \tfrac{d\widehat{w}_{\mathbf{R}}(Q)}{dQ} \tfrac{\partial \Pi^u_{\mathbf{RP}}(p,w)}{\partial w}\Big|_{p=p^u(\widehat{w}_{\mathbf{R}}(Q)), w=\widehat{w}_{\mathbf{R}}(Q)} \end{aligned} \quad \text{(B-13)}$$

Note that the third term on the right-hand side of (B-13) is zero since p^u satisfies the first-order condition of $\Pi^u_{\mathbf{RP}}(p, w)$. Recall that

$$\begin{aligned} \Pi_{\mathbf{RN}}(q_{\min}, w, Q) &= a \int_{q_{\min}}^{\infty} [(1-\beta)x + \beta q_{\min} - w - c_{\mathbf{T}}] f(x) dx, \\ \Pi^u_{\mathbf{RP}}(p, w) &= a(p - w)\overline{F}(p). \end{aligned}$$

Taking the partial derivatives of these functions, we obtain

$$\begin{aligned} \frac{\partial \Pi_{\mathbf{RN}}(q_{\min}, w, Q)}{\partial q_{\min}} &= a(-q_{\min} + w + c_{\mathbf{T}}) f(q_{\min}) + a\beta \overline{F}(q_{\min}), \\ \frac{\partial \Pi_{\mathbf{RN}}(q_{\min}, w, Q)}{\partial w} &= -a\overline{F}(q_{\min}), \text{ and} \\ \frac{\partial \Pi^u_{\mathbf{RP}}(p, w)}{\partial w} &= -a\overline{F}(p). \end{aligned}$$

Substituting the partial derivatives above in (B-13) and rearranging the terms, we obtain:

$$\frac{d\widehat{w}_{\mathbf{R}}(Q)}{dQ} \big(\overline{F}(\bar{p}(Q)) - \overline{F}(p^u(\widehat{w}_{\mathbf{R}}(Q)))\big) = \frac{d\bar{p}(Q)}{dQ} \big[\big(-\bar{p}(Q) + \widehat{w}_{\mathbf{R}}(Q) + c_{\mathbf{T}}\big) f(\bar{p}(Q)) + \beta \overline{F}(\bar{p}(Q))\big] \quad \text{(B-14)}$$

Note from (B-12) that $\left(-\bar{p}(Q)+\overline{w}_{\mathrm{N}}(Q)+c_{\mathrm{T}}\right)f(\bar{p}(Q))+\beta\overline{F}(\bar{p}(Q))=0$. Since $\widehat{w}_{\mathrm{R}}(Q)<\overline{w}_{\mathrm{N}}(Q)$, it follows that

$$\left(-\bar{p}(Q)+\widehat{w}_{\mathrm{R}}(Q)+c_{\mathrm{T}}\right)f(\bar{p}(Q))+\beta\overline{F}(\bar{p}(Q))<0.$$

Furthermore, $\frac{d\bar{p}(Q)}{dQ}\leq 0$ since $\bar{p}(Q)$ is such that $a\overline{F}(\bar{p}(Q))=Q$. Hence, the right-hand side of (B-14) is positive. We then consider the left-hand side of (B-14). Note that, since $\overline{w}_{\mathrm{P}}(Q)<\widehat{w}_{\mathrm{R}}(Q)$, it follows that $p^u(\widehat{w}_{\mathrm{R}}(Q))>p^u(\overline{w}_{\mathrm{P}}(Q))=\bar{p}(Q)$, where the equality is by definition of $\overline{w}_{\mathrm{P}}(Q)$. Hence, $\overline{F}\big(\bar{p}(Q)\big)>\overline{F}\big(p^u(\widehat{w}_{\mathrm{R}}(Q))\big)$. Since the right-hand side of (B-14) is positive, we now conclude $\frac{d\widehat{w}_{\mathrm{R}}(Q)}{dQ}\geq 0$.

Subcase (1.b) can be proven similarly by implicit differentiation of the same identity.

(2) only $\overline{w}_{\mathrm{N}}(Q)$ exists

We consider two separate subcases: (2.a) $\widehat{w}_{\mathrm{R}}(Q)<\overline{w}_{\mathrm{N}}(Q)$ and (2.b) $\overline{w}_{\mathrm{N}}(Q)\leq\widehat{w}_{\mathrm{R}}(Q)$. The proof of (2.a) is similar to case (1.a), and (2.b) is similar to case (1.b).

(3) only $\overline{w}_{\mathrm{P}}(Q)$ exists

Note that if $\overline{w}_{\mathrm{P}}(Q)$ exists but $\overline{w}_{\mathrm{N}}(Q)$ does not exist, Lemma B.1(c) shows that $\Delta_{\mathrm{R}}(w,Q)\geq 0$ for all $w\geq c$. Therefore, $\widehat{w}_{\mathrm{R}}(Q)$ does not exist, and this case cannot occur.

(4) both $\overline{w}_{\mathrm{P}}(Q)$ and $\overline{w}_{\mathrm{N}}(Q)$ do not exist

The analysis is similar to case (1.b).

Proof of (c): Suppose that $w^o_{\mathrm{N}}(Q^o)=w^*_{\mathrm{N}}(Q^o)$ for some Q^o. Pick a capacity level Q' such that $Q'>Q^o$. We consider three cases depending on whether there exists a feasible wholesale price at which the quantity sold under negotiation will be capacity-constrained at each capacity level, Q^o and Q': (1) both $\overline{w}_{\mathrm{N}}(Q^o)\geq c$ and $\overline{w}_{\mathrm{N}}(Q')\geq c$ exist, (2) neither of them exists, and (3) $\overline{w}_{\mathrm{N}}(Q^o)\geq c$ exists, but $\overline{w}_{\mathrm{N}}(Q')\geq c$ does not exist. (The case that $\overline{w}_{\mathrm{N}}(Q^o)\geq c$ does not exist and $\overline{w}_{\mathrm{N}}(Q')\geq c$ exists cannot occur since $\overline{w}_{\mathrm{N}}(Q)$ decreases in Q, which is proven in part (a) of this lemma.)

(1) $\overline{w}_{\mathrm{N}}(Q^o)\geq c$ and $\overline{w}_{\mathrm{N}}(Q')\geq c$

Note that $w^o_{\mathrm{N}}(Q)=\min\{\widehat{w}_{\mathrm{R}}(Q),w^*_{\mathrm{N}}(Q)\}$ (from Lemma B.2(a)) and that $w^*_{\mathrm{N}}(Q)=\max\{\overline{w}_{\mathrm{N}}(Q),w^u_{\mathrm{N}}\}$ (from Proposition 2). Therefore, given that $w^o_{\mathrm{N}}(Q^o)=w^*_{\mathrm{N}}(Q^o)$, it must be that $\widehat{w}_{\mathrm{R}}(Q^o)\geq w^*_{\mathrm{N}}(Q^o)=\max\{\overline{w}_{\mathrm{N}}(Q^o),w^u_{\mathrm{N}}\}$. Observe that w^u_{N} is constant with respect to Q and, from part (a) of this lemma, $\overline{w}_{\mathrm{N}}(Q)$ decreases when Q increases. Also observe from part (b) of this lemma, $\widehat{w}_{\mathrm{R}}(Q)$ increases as Q increases. Therefore,

$$\widehat{w}_{\mathrm{R}}(Q')\geq\widehat{w}_{\mathrm{R}}(Q^o)\geq\max\{\overline{w}_{\mathrm{N}}(Q^o),w^u_{\mathrm{N}}\}\geq\max\{\overline{w}_{\mathrm{N}}(Q'),w^u_{\mathrm{N}}\},$$

and $w^o_{\mathrm{N}}(Q')=w^*_{\mathrm{N}}(Q')$.

(2) neither of them exists

In this case, there does not exist a feasible wholesale price at which the quantity sold under negotiation is capacity-constrained at either Q^o or Q'. Hence, $w^*_{\mathrm{N}}(Q^o) = w^u_{\mathrm{N}}$ and $w^*_{\mathrm{N}}(Q') = w^u_{\mathrm{N}}$. It follows that

$$w^o_{\mathrm{N}}(Q^o) = \min\{\widehat{w}_{\mathrm{R}}(Q^o), w^u_{\mathrm{N}}\} \text{ and } w^o_{\mathrm{N}}(Q') = \min\{\widehat{w}_{\mathrm{R}}(Q'), w^u_{\mathrm{N}}\}.$$

Then, $w^o_{\mathrm{N}}(Q') = w^*_{\mathrm{N}}(Q')$ follows since $w^o_{\mathrm{N}}(Q^o) = w^*_{\mathrm{N}}(Q^o) = w^u_{\mathrm{N}}$ and $\widehat{w}_{\mathrm{R}}(Q') \geq \widehat{w}_{\mathrm{R}}(Q^o)$ (from part (b) of this lemma).

(3) only $\overline{w}_{\mathrm{N}}(Q^o) \geq c$ exists

In this case, $w^*_{\mathrm{N}}(Q') = w^u_{\mathrm{N}}$. The result follows from the following set of inequalities:

$$\widehat{w}_{\mathrm{R}}(Q') \geq \widehat{w}_{\mathrm{R}}(Q^o) \geq \max\{\overline{w}_{\mathrm{N}}(Q^o), w^u_{\mathrm{N}}\} \geq w^u_{\mathrm{N}}$$

where the first inequality comes from part (b) of this lemma and the second inequality comes from the fact that $w^o_{\mathrm{N}}(Q^o) = w^*_{\mathrm{N}}(Q^o) = \max\{\overline{w}_{\mathrm{N}}(Q^o), w^u_{\mathrm{N}}\}$. Hence, $w^o_{\mathrm{N}}(Q') = w^*_{\mathrm{N}}(Q') = w^u_{\mathrm{N}}$.

Proof of (d): It is not difficult to check that $\Pi_{\mathrm{MN}}(w, q^*_{\min}(w, Q), Q)$ increases in Q. Therefore, if $Q^o < Q'$, then

$$G_{\mathrm{N}}(Q^o) = \Pi_{\mathrm{MN}}(w^o_{\mathrm{N}}(Q^o), q^*_{\min}(w^o_{\mathrm{N}}(Q^o), Q^o), Q^o) \leq \Pi_{\mathrm{MN}}(w^o_{\mathrm{N}}(Q^o), q^*_{\min}(w^o_{\mathrm{N}}(Q^o), Q'), Q'). \quad \text{(B-15)}$$

Furthermore, since $\widehat{w}_{\mathrm{R}}(Q)$ increases in Q, $w^o_{\mathrm{N}}(Q^o)$ must be feasible for the optimization problem (B-11) at $Q = Q' > Q^o$. Therefore,

$$G_{\mathrm{N}}(Q') = \Pi_{\mathrm{MN}}(w^o_{\mathrm{N}}(Q'), q^*_{\min}(w^o_{\mathrm{N}}(Q'), Q'), Q') \geq \Pi_{\mathrm{MN}}(w^o_{\mathrm{N}}(Q^o), q^*_{\min}(w^o_{\mathrm{N}}(Q^o), Q'), Q'). \quad \text{(B-16)}$$

Combining (B-15) and (B-16), we obtain $G_{\mathrm{N}}(Q') \geq G_{\mathrm{N}}(Q^o)$.