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The effect of frequency-mismatched spontaneous emission on atom-field entanglement

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Abstract
A Schrödinger representation approach is used to calculate the atom-field dynamics following spontaneous emission by an atom in its excited state to a superposition of its two ground-state sublevels, in the case where the frequency separation of the ground-state sublevels is large compared to the excited-state decay rate. The emitted radiation is incident on a broadband photodetector. Using a relatively simple model for the photodetector, we show how a measurement of a photo-signal leaves the atom in a coherent superposition of the two ground states. The relative phase between the two ground-state amplitudes can be interpreted in terms of the temporal phase acquired in the time interval between spontaneous emission (viewed as a quantum jump process) and detection. Alternatively, the phase can be associated with a spatial phase of the entangled atom-field system; the source atom is projected into a state containing this spatial phase when the emitted photon is detected.

1. Introduction

The theory of spontaneous emission in an atomic lambda system is generally well understood, but recent experiments demonstrating entanglement between the emitted photon's polarization and the resulting state of the atom have raised new questions about the nature of the entanglement between the spontaneously emitted radiation and the resulting state of the atom [1–3]. In these experiments, an atom or solid-state spin system having the level scheme shown in figure 1(a) is prepared in its excited electronic state. Spontaneous emission results in a superposition of the two ground-state sublevels of the lambda scheme that is entangled with the spontaneously emitted radiation. The direction, polarization and frequency of the emitted field may be correlated with the final state of the source atom. Atom-field entanglement has been demonstrated in experiments where the ground-state coherence of the lambda system is measured in correlation with the polarization and direction of the emitted photon [4, 5]. In the case of orthogonal optical transitions and nearly degenerate ground states, the entanglement between the spontaneously emitted photon and resulting source atom ground-state coherence is well understood [4]. On the other hand, it is not immediately obvious whether or not a post-selection measurement of the photon's polarization correlates with a specific source atom ground-state coherence when the frequency separation ω_{21} of the two ground states is much larger than its excited-state decay rate [5, 2]. We present a model system in which both the emission and detection processes are treated quantum mechanically in the Schrödinger picture. Using this model, we are able to follow the system's evolution and show that in certain limits, a fast photodetector can be used to exploit the latent atom–photon entanglement from a lambda system, even if the two transitions are frequency mismatched by an amount greater than the spontaneous emission rate. A similar result was reported by Economou *et al* in 2005; however, our model features a quantum mechanical treatment of the detection process which shows how the entanglement is preserved and gives insight into the quantum eraser like effect which leads to the entanglement [2, 5–7]. A related problem has been addressed by Metz and Barrett, where the authors analyse the case of interfering frequency-mismatched photons from two different atoms, each in a separate cavity [8].

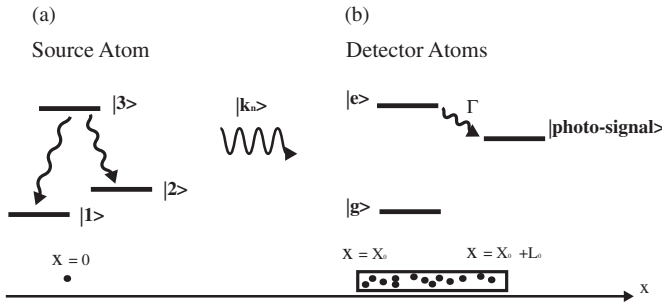

Figure 1. Level diagram depicting the eigenstates of the system. The source atom is taken to be at $X = 0$ and the detector atoms extend from $X = X_0$ to $X = X_0 + L_0$.

2. Preservation of atom-field entanglement

In order to understand how measurement-induced atom-field entanglement comes about in a lambda system, we construct a fully quantum mechanical model of both the photon emission and detection processes. The model consists of a source atom with ground states $|1\rangle$ and $|2\rangle$ which are both coupled via an optical dipole transition to excited state $|3\rangle$ (see figure 1(a)). These ground states can represent hyperfine levels of a single atom or electron spin eigenstates of a solid-state system. The source atom is initially prepared in the excited state $|3\rangle$ and then decays via spontaneous emission to the states $|1\rangle$ and $|2\rangle$. A single-photon state is emitted from the source atom. In the case of orthogonally polarized transitions, it is known that the polarization of the emitted photon and the resulting ground state of the source atom are entangled [2, 4, 5]. The photon's polarization state is rotated prior to reaching the detector, in effect, mixing the two orthogonal polarization components of the emitted radiation. The correlated state of the atom-field system is then demonstrated by a measurement of the ground-state atomic coherence, performed conditionally on the detection of a photon.

To model this process, regardless of the frequency separation of the ground-state sublevels, we take the detector to consist of an ensemble of atoms having ground state $|g\rangle$, dipole-coupled excited state $|e\rangle$, and a reservoir state to which the excited state $|e\rangle$ decays (see figure 1(b)). The excited-state decay occurs at the rate Γ and the decay to the reservoir is intended to represent a 'click' at the photodetector. It is assumed that the rate Γ is much larger than the excited-state decay rate γ_3 of the source atom as well as the frequency difference ω_{21} of the ground-state sublevels. The fast decay rate is crucial to the recovery of the ground-state coherence. Our detector model can be considered to represent a simple physical model for an avalanche photo-diode (APD) (or photomultiplier tube (PMT)) operating in the Geiger mode. In the case of an APD, a single photon creates a pair of carriers in the active region of the detector and $1/\Gamma$ is related to the timing resolution (jitter) of the detector. In principle, a narrow band detector could contain frequency which-path information about the spontaneously emitted radiation, which implies that any ground-state coherence created by the emission process is negligibly small. By choosing a large bandwidth of the detector, represented by the decay rate Γ, such which-path information is not present.

A measurement at the detector projects the source atom into a coherent superposition of its ground-state sublevels which is *independent* of the detection time, but depends on a spatial phase $\omega_{21}X_0/c$, where X_0 is the distance from the source atom to the entrance plane of the detector. In some sense, the effect is similar to a 'quantum-eraser' effect [5], where the which-path information of a decaying lambda quantum system can be erased by another pulse followed by the fast decay of the source atom [7]. Our result is unique in that it relies on a fast decay rate in the detector, not the source atom, and provides insight into the results observed in recent experiments studying spin–photon interfaces [5, 9]. This atomic coherence is the signature of atom-field entanglement, where the resulting phase associated with the ground-state coherence of the source atom is correlated with the projected measurement of the spontaneously emitted photon. Alternatively, the relative phase can be interpreted in terms of the *temporal* phase acquired in the time interval between spontaneous emission (viewed as a quantum jump process) and detection.

The calculation is carried out using the Schrödinger representation and parallels that given in previous work [10–12]. In order to focus on the effect of the frequency mismatch of the two decay channels, several simplifications are made. We consider an effective one-dimensional problem in which the radiation striking the detector is assumed to propagate in a single direction. In addition, the polarization of the emitted radiation on both transitions is assumed to be identical. If the polarizations are orthogonal, as is often the case, one would use a polarizer to couple the two modes into a single, linearly polarized field that is incident on the detector. Of course, radiation from a localized source atom is emitted in all directions. We are concerned here only with that portion of the emitted radiation that strikes the detector.

The Hamiltonian for the atom-field system is

$$\begin{aligned} H = & \sum_m \hbar\omega_d \sigma_{ee}^m + \sum_j \hbar\omega_j a_j^\dagger a_j \\ & + \sum_m \sum_j \hbar g_j \left[e^{ik_j X_m} \sigma_+^m a_j - a_j^\dagger e^{-ik_j X_m} \sigma_-^m \right] \\ & + \hbar\omega_3 \sigma_{33}^s + \hbar\omega_2 \sigma_{22}^s \\ & + \sum_j \sum_{s=1,2} \hbar g_{j,s} \left[e^{ik_j X} \sigma_+^s a_j - a_j^\dagger e^{-ik_j X} \sigma_-^s \right], \end{aligned} \tag{1}$$

where $k_j = \omega_j c$, ω_d is the transition frequency of the detector atoms, σ_{ee}, σ_{33} and σ_{22} are the population operators $|e\rangle\langle e|$, $|3\rangle\langle 3|$ and $|2\rangle\langle 2|$ for the source atom, respectively, $\sigma_\pm^s$ are raising and lowering operators for the source atom, $\sigma_\pm^m$ are raising and lowering operators for detector atom m, and $a_j^\dagger$ and a_j are the usual raising and lowering operators for the field, respectively. The zero of energy is taken as that associated with the source atom in state $|1\rangle$ and all the detector atoms in their ground states. As a consequence, $\hbar\omega_\alpha$ ($\alpha = 2, 3$) is the energy of the state α of the source atom and $\hbar\omega_d$ is the energy corresponding to one detector atom in its excited state and all the others in their ground states. The atom-field coupling strength for the source atom-field and detector atom-field interactions are given by

$$g_{j,s} = -i \left(\frac{\omega_j}{2\hbar\epsilon_0 AL} \right)^{1/2} \mu_{3s}; \quad s = 1, 2, \tag{2a}$$

$$g_j = -\mathrm{i}\left(\frac{\omega_j}{2\hbar\epsilon_0 AL}\right)^{1/2}\mu_d, \tag{2b}$$

where μ_{3s} is a source atom dipole matrix element (assumed real) in the direction of the field polarization between the states $\langle 3|$ and $|s=1,2\rangle$, μ_d is a detector atom dipole matrix element (assumed real) in the direction of the field polarization between its excited and ground state, A is the cross-sectional area of the detector, also equal to the cross-sectional area of the quantization volume of our field, and L is the length of the quantization volume. The source atom is taken to be at the origin and the detector atoms are located in a cylinder whose axis is the x-axis and whose end caps are located at X_0 and $X_0 + L_0$, with $X_0 > 0$.

The state vector of the system in an interaction representation is given by

$$\begin{aligned}|\Psi\rangle = {} & b_{3,G,0}(t)|3,G,0\rangle\,\mathrm{e}^{-\mathrm{i}\omega_3 t} + \sum_j b_{1,G,j}(t)|1,G,k_j\rangle\,\mathrm{e}^{-\mathrm{i}\omega_j t} \\ & + \sum_j b_{2,G,j}(t)|2,G,k_j\rangle\,\mathrm{e}^{-\mathrm{i}(\omega_2+\omega_j)t} \\ & + \sum_m b_{1,m,0}(t)|1,m,0\rangle\,\mathrm{e}^{-\mathrm{i}\omega_d t} \\ & + \sum_m b_{2,m,0}(t)|2,m,0\rangle\,\mathrm{e}^{-\mathrm{i}(\omega_2+\omega_d)t},\end{aligned} \tag{3}$$

where the first subscript of the bs labels the state of the source atom $(1,2,3)$, the second subscript labels the state of the detector atoms, $|G\rangle$ or $|m\rangle$ ($|G\rangle$ corresponds to all detector atoms in their ground state $|g\rangle$, and $|m\rangle$ to detector atom m in its excited state $|e\rangle$ and all other detector atoms in their ground states), and the third subscript corresponds to the mode of the single-photon radiation field, $|0\rangle$ or $|k_j\rangle$. The final state of the detector is not needed explicitly in the calculation at this stage, so it is not included in the state vector.

Substituting the wavefunction into Schrödinger's equation, we find the equations of motion for the state amplitudes,

$$\begin{aligned}\dot b_{1,G,j}(t) = {} & \mathrm{i}g_{j,1}\,\mathrm{e}^{-\mathrm{i}(\omega_{31}-\omega_j)t}b_{3,G,0}(t) \\ & + \mathrm{i}\sum_m g_j\,\mathrm{e}^{-\mathrm{i}k_j X_m}\,\mathrm{e}^{-\mathrm{i}(\omega_d-\omega_j)t}b_{1,m,0}(t),\end{aligned} \tag{4a}$$

$$\begin{aligned}\dot b_{2,G,j}(t) = {} & \mathrm{i}g_{j,2}\,\mathrm{e}^{-\mathrm{i}(\omega_{32}-\omega_j)t}b_{3,G,0}(t) \\ & + \mathrm{i}\sum_m g_j\,\mathrm{e}^{-\mathrm{i}k_j X_m}\,\mathrm{e}^{-\mathrm{i}(\omega_d-\omega_j)t}b_{2,m,0}(t),\end{aligned} \tag{4b}$$

$$\dot b_{1,m,0}(t) = -\mathrm{i}\sum_j g_j\,\mathrm{e}^{\mathrm{i}k_j X_m}\,\mathrm{e}^{-\mathrm{i}(\omega_j-\omega_d)t}b_{1,G,j}(t), \tag{4c}$$

$$\dot b_{2,m,0}(t) = -\mathrm{i}\sum_j g_j\,\mathrm{e}^{\mathrm{i}k_j X_m}\,\mathrm{e}^{-\mathrm{i}(\omega_j-\omega_d)t}b_{2,G,j}(t), \tag{4d}$$

$$\dot b_{3,G,0}(t) = -\mathrm{i}\sum_{s=1,2} g_{j,s}\sum_j \mathrm{e}^{-\mathrm{i}(\omega_j-\omega_{3s})t}b_{s,G,j}(t), \tag{4e}$$

where $\omega_{32} = \omega_3 - \omega_2$ and $\omega_{31} = \omega_3 - \omega_1$.

It is convenient to move to a continuum description of the radiation modes by replacing $b_j(t)$ with $\sqrt{\frac{2\pi}{L}}b(k,t)$, $\sum_j$ with $\left(\frac{L}{2\pi}\right)\int_{-\infty}^{\infty}\mathrm{d}k$ and ω_j with $\omega_k = kc$. With these substitutions, the evolution equations become

$$\begin{aligned}\dot b_{1,G}(k,t) = {} & \mathrm{i}\sqrt{\frac{L}{2\pi}}g_1\,\mathrm{e}^{-\mathrm{i}(\omega_{31}-\omega_k)t}b_{3,G,0}(t) \\ & + \mathrm{i}\sqrt{\frac{L}{2\pi}}\sum_m g_d\,\mathrm{e}^{-\mathrm{i}kX_m}\,\mathrm{e}^{-\mathrm{i}(\omega_d-\omega_k)t}b_{1,m,0}(t),\end{aligned} \tag{5a}$$

$$\begin{aligned}\dot b_{2,G}(k,t) = {} & \mathrm{i}\sqrt{\frac{L}{2\pi}}g_2\,\mathrm{e}^{-\mathrm{i}(\omega_{32}-\omega_k)t}b_{3,G,0}(t) \\ & + \mathrm{i}\sqrt{\frac{L}{2\pi}}\sum_m g_d\,\mathrm{e}^{-\mathrm{i}kX_m}\,\mathrm{e}^{-\mathrm{i}(\omega_d-\omega_k)t}b_{2,m,0}(t),\end{aligned} \tag{5b}$$

$$\dot b_{1,m,0}(t) = -\mathrm{i}g_d\sqrt{\frac{L}{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kX_m}\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_d)t}b_{1,G}(k,t)\,\mathrm{d}k, \tag{5c}$$

$$\dot b_{2,m,0}(t) = -\mathrm{i}g_d\sqrt{\frac{L}{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kX_m}\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_d)t}b_{2,G}(k,t)\,\mathrm{d}k, \tag{5d}$$

$$\dot b_{3,G,0}(t) = -\mathrm{i}\sqrt{\frac{L}{2\pi}}\sum_{s=1,2} g_s\int_{-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_{3s})t}b_{s,G}(k,t)\,\mathrm{d}k. \tag{5e}$$

In these equations, we have anticipated the Weisskopf–Wigner approximation by evaluating the frequencies in the various g's at the corresponding atomic resonance frequencies, namely

$$g_s = -\mathrm{i}\left(\frac{\omega_{3s}}{2\hbar\epsilon_0 AL}\right)^{1/2}\mu_{3s}; \qquad s=1,2, \tag{6a}$$

$$g_d = -\mathrm{i}\left(\frac{\omega_d}{2\hbar\epsilon_0 AL}\right)^{1/2}\mu_d. \tag{6b}$$

If equations (5*a*) and (5*b*) are integrated formally and substituted into equation (5*e*), and if the back action of the detector atoms on the source atom is neglected, equation (5*e*) becomes

$$\begin{aligned}\dot b_{3,G,0}(t) = {} & \frac{L}{2\pi}\sum_{s=1,2} g_s^2\int_0^t \mathrm{d}t'\int_{-\infty}^{\infty}\mathrm{d}k\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_{3s})(t-t')}b_{3,G,0}(t') \\ = {} & -(\gamma_3/2)b_{3,G,0}(t),\end{aligned} \tag{7}$$

where

$$\gamma_3 = \frac{1}{4\hbar\epsilon_0 Ac}\left(\omega_{31}\mu_{31}^2 + \omega_{32}\mu_{32}^2\right) \tag{8}$$

is the excited-state decay rate in our one-dimensional field model (see also [12]). Thus, the initial state amplitude decays as

$$b_{3,G,0}(t) = \mathrm{e}^{-\gamma_3 t/2}\Theta(t), \tag{9}$$

where $\Theta(t)$ is a Heaviside function.

The decay to the reservoir state of the detector atoms is now incorporated into the equations for $\dot b_{1,m,0}(t)$ and $\dot b_{2,m,0}(t)$ as

$$\begin{aligned}\dot b_{1,m,0}(t) = {} & -\mathrm{i}g_d\sqrt{\frac{L}{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kX_m}\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_d)t}b_{1,G}(k,t)\,\mathrm{d}k \\ & - \frac{\Gamma}{2}b_{1,m,0}(t),\end{aligned} \tag{10a}$$

$$\dot{b}_{2,m,0}(t) = -\mathrm{i}g_d\sqrt{\frac{L}{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kX_m}\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_d)t}b_{2,G}(k,t)\,\mathrm{d}k - \frac{\Gamma}{2}b_{2,m,0}(t). \tag{10b}$$

If the decay rate Γ is large compared with γ_3, the solution of these equations is given approximately by the quasi-static amplitudes,

$$b_{1,m,0}(t) \approx -\frac{2\mathrm{i}}{\Gamma}g_d\sqrt{\frac{L}{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kX_m}\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_d)t}b_{1,G}(k,t)\,\mathrm{d}k, \tag{11a}$$

$$b_{2,m,0}(t) \approx -\frac{2\mathrm{i}}{\Gamma}g_d\sqrt{\frac{L}{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kX_m}\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_d)t}b_{2,G}(k,t)\,\mathrm{d}k. \tag{11b}$$

Owing to the similarity of the equations for $b_{1,m,0}(t)$ and $b_{2,m,0}(t)$, only the details of the $b_{2,m,0}(t)$ calculation is presented. Inserting the expression for $b_{2,m,0}$ into equation (5*b*) and using equation (9), we obtain the equation of motion

$$\dot{b}_{2,G}(k,t) = \mathrm{i}\sqrt{\frac{L}{2\pi}}g_2\,\mathrm{e}^{-\mathrm{i}(\omega_{32}-\omega_k)t}\,\mathrm{e}^{-\gamma_3 t/2}\Theta(t) + \frac{2}{\Gamma}\frac{L}{2\pi}g_d^2 \times\sum_m\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}(k'-k)X_m}\,\mathrm{e}^{-\mathrm{i}(\omega_{k'}-\omega_k)t}b_{2,G}(k',t)\,\mathrm{d}k'. \tag{12}$$

If we define a Fourier space state amplitude by

$$B_{s,G}(X,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kX}b_{s,G}(k,t)\,\mathrm{d}k, \tag{13}$$

the equation of motion for the field amplitude $B_{2,G}(X,t)$ becomes

$$\dot{B}_{2,G}(X,t) = \mathrm{i}\sqrt{L}g_2\,\mathrm{e}^{-\mathrm{i}\omega_{32}t}\,\mathrm{e}^{-\gamma_3 t/2}\delta(X+ct)\Theta(t) + \frac{2}{\Gamma}g_d^2L\sum_m\delta(X-X_m+ct)B_{2,G}(X,t). \tag{14}$$

We convert the sum over m to an integral using

$$\sum_m \to NA\int_{X_0}^{X_0+L_0}\mathrm{d}X_m, \tag{15}$$

where N is the density of detector atoms. In the limit $L_0\to\infty$, we find

$$\dot{B}_{2,G}(X,t) = \mathrm{i}\sqrt{L}g_2\,\mathrm{e}^{-\mathrm{i}\omega_{32}t}\,\mathrm{e}^{-\gamma_3 t/2}\delta(X+ct)\Theta(t) - \frac{\alpha c}{2}\Theta(X-X_0+ct)B_{2,G}(X,t), \tag{16}$$

where

$$\alpha = \frac{2\omega_d N\mu_d^2}{\Gamma\epsilon_0 c\hbar} \tag{17}$$

is the absorption coefficient of the detector. The solution of equation (16), subject to the initial condition $B_{2,G}(X,0)=0$, is

$$B_{2,G}(X,t) = \frac{\mathrm{i}\sqrt{L}g_2}{c}\,\mathrm{e}^{\mathrm{i}\omega_{32}X/c}\,\mathrm{e}^{\gamma_3 X/2c}\Theta(-X)\Theta(X+ct) \times\exp\left[-\frac{\alpha c}{2}\left(t+\frac{X-X_0}{c}\right)\Theta(X-X_0+ct)\right]. \tag{18}$$

Returning to expression (5*d*) for $b_{2,m,0}(t)$, we find

$$b_{2,m,0}(t) = -\frac{2\mathrm{i}}{\Gamma}\sqrt{\frac{L}{2\pi}}g_2\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}kX_m}\,\mathrm{e}^{-\mathrm{i}(\omega_k-\omega_d)t}b_{2,G}(k,t)\,\mathrm{d}k = -\frac{2\mathrm{i}}{\Gamma}\sqrt{L}g_2\,\mathrm{e}^{\mathrm{i}\omega_d t}B_{2,G}(X_m-ct,t). \tag{19}$$

Inserting equation (18) into equation (19) and using the fact that $(X_m-X_0)>0$, we obtain

$$b_{2,m,0}(t) = -\frac{L}{\Gamma}\frac{g_d g_2}{c}\,\mathrm{e}^{\mathrm{i}(\omega_d-\omega_{32})t}\,\mathrm{e}^{\mathrm{i}k_{32}X_m}\,\mathrm{e}^{-\frac{\gamma_3}{2}(t-\frac{X_m}{c})} \times\,\mathrm{e}^{-\frac{\alpha}{2}(X_m-X_0)}\Theta(ct-X_m), \tag{20}$$

where $k_{i,j}=\omega_{i,j}/c$. A similar calculation for $b_{1,m,0}(t)$ yields

$$b_{1,m,0}(t) = -\frac{L}{\Gamma}\frac{g_d g_1}{c}\,\mathrm{e}^{\mathrm{i}(\omega_d-\omega_{31})t}\,\mathrm{e}^{\mathrm{i}k_{31}X_m}\,\mathrm{e}^{-\frac{\gamma_3}{2}(t-\frac{X_m}{c})} \times\,\mathrm{e}^{-\frac{\alpha}{2}(X_m-X_0)}\Theta(ct-X_m). \tag{21}$$

Equations (20) and (21) are consistent with the fact that the atom m cannot be excited for times $t<X_m/c$.

The state vector of the source–detector system is given by equation (3). The timing resolution of the detector is taken to be equal to Γ^{-1}, which is faster than all relevant time scales in the problem. Thus, a measurement of a detector signal at time t that results from decay into the reservoir from the detector atom m projects the source atom into a superposition of its ground states. Since we do not know which detector atom fired, we must average over all possibilities. The projected state must be normalized as well. In other words, the ij density matrix element of the source atom following the measurement is given by

$$\rho_{ij}(t) = \frac{\int_{X_0}^{X_0+L_0}\mathrm{d}X_m b_{i,m,0}(t)[b_{j,m,0}(t)]^*\,\mathrm{e}^{\mathrm{i}\omega_{ji}t}}{\sum_{s=1}^{2}\int_{X_0}^{X_0+L_0}\mathrm{d}X_m|b_{s,m,0}(t)|^2}, \tag{22}$$

where we have allowed for finite L_0. The resulting atomic ground-state coherence is

$$\rho_{12}(t) = \frac{\mu_{31}\mu_{32}}{\mu_{31}^2+\mu_{32}^2} \times\frac{\int_{X_0}^{X_0+L_0}\mathrm{e}^{-\alpha(X_m-X_0)}\,\mathrm{e}^{-\gamma_3(t-\frac{X_m}{c})}\,\mathrm{e}^{\mathrm{i}k_{21}X_m}\Theta(ct-X_m)\,\mathrm{d}X_m}{\int_{X_0}^{X_0+L_0}\mathrm{e}^{-\alpha(X_m-X_0)}\,\mathrm{e}^{-\gamma_3(t-\frac{X_m}{c})}\Theta(ct-X_m)\,\mathrm{d}X_m}, \tag{23}$$

where we have assumed that

$$\omega_{32}\approx\omega_{31}\approx\omega_d, \tag{24}$$

except where differences in these frequencies appear in phases. The atomic-state coherence is linked to the atom-field entanglement that was present just before the measurement at the photodetector. The degree of atomic coherence that is produced depends on the values of α and L_0. For large α (large absorption) or $k_{21}L_0\ll 1$, the phase factor $\mathrm{e}^{\mathrm{i}k_{21}X_m}$ is effectively constant over the range of X_m contributing to the integral. For example, in the limit when $\alpha L_0\gg 1$ and $\alpha\gg k_{21},\gamma_3$, we find

$$\rho_{12}(t) = \frac{\mu_{31}\mu_{32}}{\mu_{31}^2+\mu_{32}^2}\,\mathrm{e}^{\mathrm{i}k_{21}X_0}. \tag{25}$$

In some sense, the phase factor $\mathrm{e}^{\mathrm{i}k_{21}X_m}$ in equation (23) represents a memory of which-path information. If this factor

varies significantly over the integration range contributing in equation (23), it would diminish the source atom coherence. By choosing an optically dense medium for the detector and a sufficiently large Γ, any which-path information is 'erased.'

The off-diagonal density matrix element depends on the relative spatial phase of the fields emitted on each transition at the entrance plane of the detector, where most of the absorption occurs. This dependence can be viewed as resulting from the Einstein–Podolsky–Rosen-like correlation associated with the entangled atom-field state. When the measurement is made, the source atom is projected into a state in which the coherence ρ_{12} contains the spatial phase $k_{21}X_0$. Alternatively, the phase can be interpreted in terms of a quantum jump-type process. If we view spontaneous emission as a quantum jump process, detection near the entrance plane of the detector at time t implies that spontaneous emission (viewed as a quantum jump) occurred at time $t - X_0/c$. Therefore, in the time interval X_0/c between 'emission' and detection, the ground-state coherence accumulates a phase equal to $\omega_{21}X_0/c = k_{21}X_0$.

Experimentally, the ground-state atomic coherence can then be measured through standard quantum control techniques and time correlated with the detection time of the spontaneously emitted photon. Such a measurement is an indirect proof of the atom-field entanglement that existed just prior to the measurement. The signature of the atom–photon entanglement is a coincidence oscillation in time at the ground-state frequency ω_{21}.

For consistency of our model, the total probability to measure a click in the detector should equal $1/2$ when $\gamma_3 t \gg 1$. The factor of $1/2$ arises since emission can be either to the right or left in our one-dimensional field model. To verify this, we first calculate the probability $\mathrm{d}P$ to measure a click in the detector in the time interval between t and $t + \mathrm{d}t$. This probability for a successful measurement at time t is given by

$$\begin{aligned}\mathrm{d}P(t) &= NA\Gamma \mathrm{d}t \sum_{s=1}^{2} \int_{X_0}^{\infty} \mathrm{d}X_m |b_{s,m,0}(t)|^2 \\ &= \Gamma \left(\frac{NAL}{\Gamma}\frac{g_d}{c}\right)^2 \left(g_1^2 + g_2^2\right) \mathrm{d}t \int_{X_0}^{\infty} \mathrm{e}^{-\alpha(X_m - X_0)} \\ &\quad \times \mathrm{e}^{-\gamma_3\left(t - \frac{X_m}{c}\right)} \Theta(ct - X_m)\, \mathrm{d}X_m \\ &= \left(\frac{NAL^2}{\Gamma c^2}\right) g_d^2 \left(g_1^2 + g_2^2\right) \frac{\mathrm{e}^{-\gamma_3\left(t - \frac{X_0}{c}\right)} - \mathrm{e}^{-\alpha(ct - X_0)}}{\alpha - \gamma_3/c} \\ &\quad \times \Theta(ct - X_0)\, \mathrm{d}t. \end{aligned} \tag{26}$$

The total probability for long times is

$$P(\infty) = \int_0^{\infty} \mathrm{d}P(t) = \left(\frac{NAL^2}{\Gamma c^2}\right) \frac{g_d^2\left(g_1^2 + g_2^2\right)}{\alpha\gamma_3}. \tag{27}$$

Using equations (6), (17), (8) and (24), this reduces to $P(\infty) = 1/2$, as predicted. Had we used a three-dimensional model for field emission for an optically dense detector which is in the radiation zone of the source atom, we would find that both $\mathrm{d}P(t)$ and $P(\infty)$ are multiplied by $2A/4\pi X_0^2$, reflecting the fact that only a fraction $A/4\pi X_0^2$ of the emitted radiation strikes the detector.

3. Discussion

The theory of spontaneous emission in a lambda system has been thoroughly investigated in the past [1–3]. In this paper, we presented a model which includes the quantum mechanical detection process in order to bring new insight into the details of measurement and atom-field entanglement. Specifically, we studied a system in which the two transition frequencies of the lambda scheme, ω_{31} and ω_{32}, could be resolved owing to the condition $\omega_{21} > \gamma_3$. At first glance, it would appear that the ground-state coherence would be small following emission, owing to the very small overlap of the Lorentzian spectral profiles associated with each transition. However, our analysis shows that the fast detection of the spontaneously emitted radiation effectively erases the frequency information stored in the field, resulting in a significant ground-state coherence ρ_{12} of the source atom that is correlated with the measurement of a detector signal. The creation of the ground-state coherence following the measurement is connected with the atom-field entanglement that was present just *before* the measurement at the photodetector. Our result is applicable to experiments demonstrating entanglement between the polarization state of the spontaneously emitted photon and the resulting ground states of a lambda system and is consistent with the results reported thus far [2, 4, 5, 9].

Two specific examples of lambda systems in which the ground-state sublevel splitting is large compared to the excited-state spontaneous emission rate are the charged InAs quantum dot (QD) system and the diamond nitrogen vacancy (NV) centre system. Charged InAs QDs have typical spontaneous emission rates of the order $\gamma_3/2\pi \approx 0.20$ GHz, while the ground-state frequency splitting is of the order $\omega_{21}/2\pi \approx 10$ GHz, determined by an externally applied magnetic field that is typically chosen to be sufficiently large to allow for state initialization through resonant optical pumping [13]. In a recent work on the diamond NV centre system, the ground-state frequency splitting is chosen to be 122 MHz in order to perform frequency selective excitation, and the excited-state spontaneous emission rate is $\gamma_3/2\pi \approx 15$ MHz. As reported by the authors, a fast photodetector with ≈ 300 ps timing resolution is used to destroy the which-path frequency information in the emitted radiation [5].

We have presented a quantum mechanical model of the detection process which consists of an excitation of an atomic ensemble followed by fast decay at the rate Γ to another 'avalanche' state of the detector, intended to represent the triggering of an avalanche detector or PMT. It is assumed in the calculation that this decay rate Γ is fast relative to the relevant frequency scales in the problem (γ_3, ω_{21}). The fast decay from the excited state of the detector atoms is crucial to the result as it destroys all frequency information contained in the spontaneously radiated emission. On the other hand, the model does not require detailed information about the resulting state of the detector.

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