AIP Physics of Plasmas

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Citation: Phys. Plasmas **18**, 030702 (2011); doi: 10.1063/1.3555532
View online: http://dx.doi.org/10.1063/1.3555532
View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v18/i3
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PHYSICS OF PLASMAS **18**, 030702 (2011)

Heating of ions by low-frequency Alfvén waves in partially ionized plasmas

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(Received 22 October 2010; accepted 24 January 2011; published online 16 March 2011)

In the solar atmosphere, the chromospheric and coronal plasmas are much hotter than the visible photosphere. The heating of the solar atmosphere, including the partially ionized chromosphere and corona, remains largely unknown. In this letter, we demonstrate that the ions can be substantially heated by Alfvén waves with very low frequencies in partially ionized low-beta plasmas. This differs from other Alfvén wave related heating mechanisms such as ion-neutral collisional damping of Alfvén waves and heating described by previous work on resonant Alfvén wave heating. We find that the nonresonant Alfvén wave heating is less efficient in partially ionized plasmas than when there are no ion-neutral collisions, and the heating efficiency depends on the ratio of the ion-neutral collision frequency to the ion gyrofrequency. *© 2011 American Institute of Physics*.
[doi:10.1063/1.3555532]

The chromosphere, the region between the solar surface and the corona, is permeated by low-frequency (0.0014–0.0079 Hz) Alfvén waves with strong amplitudes (2.6 km/s in amplitude)[1] while the plasma beta value $[\beta=(v_p/v_A)^2$, where v_p and v_A are the thermal speed and the Alfvén speed, respectively] in this region is low (where β reaches a minimum with $\beta < 3\times 10^{-2}$).[2] The chromospheric and coronal plasmas are much hotter than the visible photosphere. The heating mechanisms in these regions, however, have not yet been fully understood.[1,3] Alfvén waves have long been considered to play a crucial role in heating of plasma in these two regions. Numerous theoretical and experimental papers have been published to investigate resonant heating of ions by Alfvén waves.[4–6] In these works, the cyclotron resonant condition is necessary for ion heating by the Alfvén waves, and in general, the frequencies of the applied Alfvén waves are comparable to the cyclotron frequency. However, the heating of ions by low-frequency Alfvén waves and related problems have triggered great interest in recent years.[7–15] In these works, the ions can be heated by Alfvén waves via nonresonant interactions, which means $\omega \ll \Omega_q$, where ω is the frequency of the Alfvén waves and Ω_q denotes the gyrofrequency for ion species labeled q. Researchers have used various simulation and theoretical methods in order to validate the heating mechanism of low-frequency Alfvén wave interactions; these include test-particle approaches,[8–10,12] hybrid simulations,[7,14] and kinetic theory.[11,14] More importantly, several of these works demonstrate quantitatively similar results even while employing different approaches. For example, the numerical and analytical results adopting the heating mechanism proposed by Wang and Wu [10,11] are quantitatively in accordance with the work of Voitenko and Goossens who examine cross-field heating by low-frequency kinetic Alfvén waves.[9]

Some recent works[15–17] indicate that Alfvén waves can propagate through the partially ionized solar chromosphere. They find that Alfvén waves with frequencies below 0.6 Hz will not be completely damped. Further, these works all show that Alfvén waves with frequencies below 0.01 Hz are undamped by ion-neutral collisions in the solar chromosphere, and are therefore available for damping by other mechanisms, such as the mechanism described in this letter.

We demonstrate that the low-frequency Alfvén waves propagating along the background magnetic field $\mathbf{B}_0=B_0\mathbf{i}_z$ can heat ions even in partially ionized plasmas. The heating process in the present work is due to a randomization of the spatial velocity distribution in the ion population, which is caused by the nonresonant Alfvén wave interactions. We find that the heating process becomes less efficient than the situation with no ion-neutral collisions. Moreover, the nonresonant heating process is only effective for low-beta plasmas[10] and Alfvén wave frequencies lower than 0.6 Hz; the most efficient heating occurs when $\omega \le 0.01$ Hz since low-frequency Alfvén waves are more abundant due to the damping of high frequency waves by ion-neutral collisions.[15–17]

We consider the Alfvén waves have a spectrum and the dispersion relation can be described as $\omega = kv_A$ (v_A: Alfvén speed; ω: wave angular frequency; k: wave number). This relationship is still appropriate even when the plasma is partially ionized, as described in the following paragraphs. Without loss of generality, we consider the left-hand circular polarization. The wave magnetic field vector $\delta\mathbf{B}_w$ and electric field vector $\delta\mathbf{E}_w$ can be expressed as

$$\delta\mathbf{B}_w = \sum_k B_k(\cos\phi_k\mathbf{i}_x - \sin\phi_k\mathbf{i}_y), \quad (1)$$

$$\delta\mathbf{E}_w = -\frac{v_A}{c}\mathbf{b}\times\delta\mathbf{B}_w, \quad \mathbf{b}=\frac{\mathbf{B}_0}{B_0}, \quad (2)$$

where $\mathbf{i}_x$ and $\mathbf{i}_y$ are unit directional vectors, $\phi_k = k(v_A t - z) + \varphi_k$ denotes the wave phase, and φ_k is the random phase for mode k. In the following, we pay attention to the protons only, whose equation of motion is described by

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$$m_i \frac{d\mathbf{v}}{dt} = q_i\left(\delta\mathbf{E}_w + \frac{\mathbf{v}}{c} \times (\mathbf{B}_0 + \delta\mathbf{B}_w)\right) + m_i \nu_{in}(\mathbf{u} - \mathbf{v}), \tag{3}$$

where $\mathbf{v}=dr/dt$ is the ion velocity, $\mathbf{u}$ is the bulk velocity of a background neutral fluid, and ν_{in} is the frequency for elastic collisions between ions and neutrals. Ion collisions with electrons are neglected in Eq. (3) because $m_i \gg m_e$. The collision frequency responsible for momentum transfer between species i and n is defined as

$$\nu_{in} = \frac{m_n}{m_i + m_n} n_n \sqrt{\frac{8k_B T}{\pi m_{in}}} \sigma_{in}, \tag{4}$$

with $m_{in}=(m_i m_n)/(m_i+m_n)$, σ_{in} the collisional cross-section for collisions between the two species. The wave phase speed v_{ph} here is defined by[16]

$$v_{\text{ph}} = \frac{\omega}{k} = v_A \sqrt{1 - i\frac{\rho_n}{\rho_{\text{tot}}}\frac{\omega}{\nu_{ni}}}, \tag{5}$$

in which ρ_n is the mass density of neutrals, ρ_{tot} is the total mass density of plasmas, and ν_{ni} is the neutral-ion collision frequency. Thus, the relationship $\rho_n/\rho_{\text{tot}}<1$ is necessarily valid. As shown by De Pontieu and Haerendel, for waves with $\nu \le 1$ Hz, the assumption $\omega \ll \nu_{ni}$ $(\omega/\nu_{ni} \ll 1)$ holds throughout the chromosphere.[16] Furthermore, most theories for the generation of Alfvén waves in the solar atmosphere predict typical frequencies below 1 Hz,[16] which has also been observed.[1] These allow us to simplify Eq. (5) to

$$v_{\text{ph}} = \frac{\omega}{k} = v_A \sqrt{1 - i\frac{\rho_n}{\rho_{\text{tot}}}\frac{\omega}{\nu_{ni}}} \approx v_A. \tag{6}$$

Hence, the relationship $\omega=kv_A$ is still appropriate.

Defining $v_\perp=v_x+iv_y$, $u_\perp=u_x+iu_y$, $v_\parallel=v_z$, $u_\parallel=u_z$, and $\delta B_\omega=\Sigma_k B_k e^{-i\phi_k}$, we are left with

$$\frac{dv_\perp}{dt} + (i\Omega_0 + \nu_{in})v_\perp = i(v_\parallel - v_A)\sum_k \Omega_k e^{-i\phi_k} + \nu_{in} u_\perp, \tag{7}$$

$$\frac{dv_\parallel}{dt} = -\operatorname{Im}\left(v_\perp \sum_k \Omega_k e^{i\phi_k}\right) + \nu_{in}(u_\parallel - v_\parallel), \quad \frac{dz}{dt} = v_\parallel, \tag{8}$$

where $\Omega_0=q_iB_0/m_ic$ (the proton gyrofrequency), $\Omega_k=q_iB_k/m_ic$. Im() denotes the imaginary part of its argument. As a first-order approximation, we can assume $v_\parallel \approx v_\parallel(0)$, where $v_\parallel(0)$ is the particle initial parallel velocity. The approximation is valid when $\Omega_k/\Omega_0=B_k/B_0$ is small enough and the frequencies of the Alfvén wave are sufficiently low to ensure that $|\Omega_0| \gg |k[v_\parallel(0)-v_A]|$. For simplicity, we assume the bulk velocities of the cold neutrals $u_x(t)=u_y(t)=u_z(t)\approx 0$ and ion-neutral collision frequency $\nu_{in}\approx$const, which provides a lower limit for the amount of ion heating. Since the ions and the background neutral fluid are both "cold" initially, it is reasonable to assume $\mathbf{u}(t)\approx 0$ due to the relatively short time scales (compared to the time required to heat the neutrals) considered in this letter and the high neutral fraction (detailed below).

The ion gyrofrequency in the region of 800–1500 km above the photosphere in solar chromosphere ranges from 1×10^5 to 5×10^5 Hz. The ion-neutral frequency in this region ranges from 5×10^3 to 5×10^5 Hz. These results are based on the solar atmospheric model VAL C (Ref. 18) for the densities and temperatures, and a magnetic flux tube model with 1500 G field strength in the photosphere and 10 G magnetic field strength in the corona.[19] We select two values of the ratio a $(a=\nu_{in}/\Omega_0)$, 0.1 and 0.5, to represent the region around 1000 km above the photosphere. According to VAL C model[18] and the magnetic flux tube model[19] described above, when $a=\nu_{in}/\Omega_0 \ge 0.05$, the ratio of neutral density to ion density $\lambda=n_n/n_i \gg 1$, which indicates that the amount of neutrals is much larger than that of ions. It is important to note that although the value of ratio $\lambda \gg 1$ indicates that ν_{in} is large [with respect to Alfvén wave frequency ω, refer to Eq. (4)], the value of Ω_0 can also be large (with respect to ω) due to the background magnetic field B_0 ($\Omega_0=q_iB_0/m_ic$), thus the value of ratio a can still be very small.

In the following, we focus on the perpendicular velocity component due to the fact that the ion kinetic temperature increase is more prominent along the perpendicular direction than the parallel direction.[9,10,12] We acknowledge that the anisotropy between $v_\perp$ and $v_\parallel$ should be alleviated after a sufficiently long period of time due to the collisional effects; however, this is not taken into account due to the relatively short time scales considered in this letter. Even over much longer time scales, where ion temperatures become isotropic, the fact that the ions are heated by Alfvén waves remains unchanged. With the initial condition $v_\perp=v_\perp(0)$ and $z=z(0)$, the solution of Eq. (7) can be written as

$$\begin{aligned} v_\perp &= v_\perp(0)e^{-(i\Omega_0+\nu_{in})t} + \frac{\nu_{in}u_\perp(\nu_{in}-i\Omega_0)}{\Omega_0{}^2+\nu_{in}{}^2}[1-e^{-i\Omega_0 t}e^{-\nu_{in}}] \\ &\quad - \frac{\Sigma_k\Omega_k[v_A - v_\parallel(0)](\Omega_0+i\nu_{in})}{\Omega_0{}^2+\nu_{in}{}^2}\{e^{-ik(v_At-z)-i\varphi_k} \\ &\quad - e^{i[kz(0)-\varphi_k]}e^{-(i\Omega_0+\nu_{in})t}\}. \end{aligned} \tag{9}$$

Here, we use the approximation that $\Omega_0-k[v_A-v_\parallel(0)]\approx\Omega_0$ and $z=z(0)+v_\parallel(0)t$. In order to verify that the analytical solution is correct, we set $\nu_{in}=0$. Then Eqs. (3), (7), and (8) are reduced to those found in previous work[8,10,12] where the analytical result is as follows:

$$\begin{aligned} v_\perp &= v_\perp(0)e^{-i\Omega_0 t} - v_A\frac{\Sigma_k B_k}{B_0}e^{-ik(v_At-z)-i\varphi_k} \\ &\quad + v_A\frac{\Sigma_k B_k}{B_0}e^{i[kz(0)-\varphi_k]}e^{-i\Omega_0 t}. \end{aligned} \tag{10}$$

The relationships below between the kinetic temperature and the velocity are based on plasma which consists of an ensemble of protons,

$$T_\perp(t) = \frac{1}{2}\sum_{i=1}^{N}\frac{m_i v_\perp^2(t)}{N}; \quad T_\parallel(t) = \sum_{i=1}^{N}\frac{m_i v_\parallel^2(t)}{N}, \tag{11}$$

where $T_\perp$ ($T_\parallel$) is the perpendicular (parallel) kinetic temperature and N denotes total number of the protons.

We present the simulation results using test-particle calculations that build upon previous works.[8,10,12] The test-particle simulation will be valid in partially ionized plasmas when no ion-neutral collisional damping occurs. We discretize the Alfvén wave number by $k_j = k_{\min} + (j-1)\frac{k_{\max}-k_{\min}}{J-1}$, for $j=1,\ldots,J$, where $k_{\min}=k_1=1\times 10^{-8}\Omega_0/v_A$ and $k_{\max}=k_J = 5\times 10^{-8}\Omega_0/v_A$. This range of wave numbers implies that we are considering $1\times 10^{-8}\Omega_0 < \omega < 5\times 10^{-8}\Omega_0$, so that the wave frequencies are much lower than the proton gyrofrequency and will be low enough to guarantee that no ion-neutral collisional damping occurs.[15–17] The field amplitudes of different wave modes are equal to each other, and they are constant. Here, we set two values of $\delta B_w^2/B_0^2=\zeta\sim 0.05$ and 0.12. The total number of test particles is 10^5, which are randomly distributed during the time interval $0<\Omega_0 t<2\pi$ and along the spatial range $0<z\Omega_0/v_A<3\times 10^9$. We vary the ratio $a=\nu_{in}/\Omega_0$ between 0.1 and 0.5, which is well within the acceptable range for describing the solar chromosphere as determined by the previous work.[18,19] The initial ion velocities are assumed to have a Maxwellian distribution with thermal speed v_p, which is less than v_A, to ensure that the cyclotron resonance condition cannot be satisfied.

In Fig. 1, we present the scatter plots in the $v_x - v_y$ space which illustrate the process of particle heating. Here, the velocity is normalized to the initial thermal speed v_p, which is set to be $v_p = 0.07 v_A$ [$\beta=(v_p/v_A)^2=4.9\times 10^{-3} \ll 1$, which is well within the range of chromosphere beta value determined by Gary[2]]. We compare nonresonant heating without and with neutral collisions in Fig. 1. The results for the noncollisional case, where $a=0$, shown in Figs. 1(a)–1(d), are in accordance with the former results.[10] We find that ion-neutral collisions reduce the amount of heating over time and are responsible for creating a ring distribution. Since $\mathbf{u}(t)\approx 0$ is valid in this letter as explained above, a balance is reached between the energy gained via Alfvén wave interactions and lost from collisions with cold neutrals. Thus, the velocity distribution evolves into a "thin" ring structure, with all of the ions evolving to a single speed. This result can also be analytically approximated, and will be discussed in detail in the following paragraphs.

Figure 2 shows the temporal evolution of the kinetic temperatures, where the results are based on two sets of input parameters $(\delta B_w{}^2/B_0{}^2, v_p/v_A)=(0.05,0.07)$ and (0.12, 0.07). The cases $a=0$, 0.1, and 0.5 are represented by the dotted line, the solid line, and the dashed line, respectively. The results again demonstrate that heating via nonresonant Alfvén wave interactions is possible, and that even in the presence of neutrals, heating still occurs and reaches a steady value. Figure 2 illustrates that the stronger Alfvén wave amplitude $\sqrt{\delta B_w{}^2/B_0{}^2}$ will result in the larger amounts of heating, which is also indicated by Eq. (9) and consistent with the previous works.[10–12] We also find that the ratio $a = \nu_{in}/\Omega_0$ will directly affect the heating process. When the ratio a is larger, the heating process becomes less efficient. However, the low-beta plasmas are still significantly heated, as indicated in Fig. 2, where the temperature $T_\perp(t)$ is much larger than the initial temperature T_i. This phenomenon may provide a partial explanation as to why the temperature of the chromosphere is higher than the photosphere while lower than the corona, as the fraction of ionized particles increases from the photosphere through the chromosphere and to the corona.[18] Furthermore, we can see from the evolution of the population in Figs. 1(e)–1(h) that there exists a dynamic equilibrium between the heating and the cooling of the ions, which is reached after a period of time. This is also demonstrated by the oscillations in temperature which decrease over time, as shown in Fig. 2 for the cases $a\neq 0$.

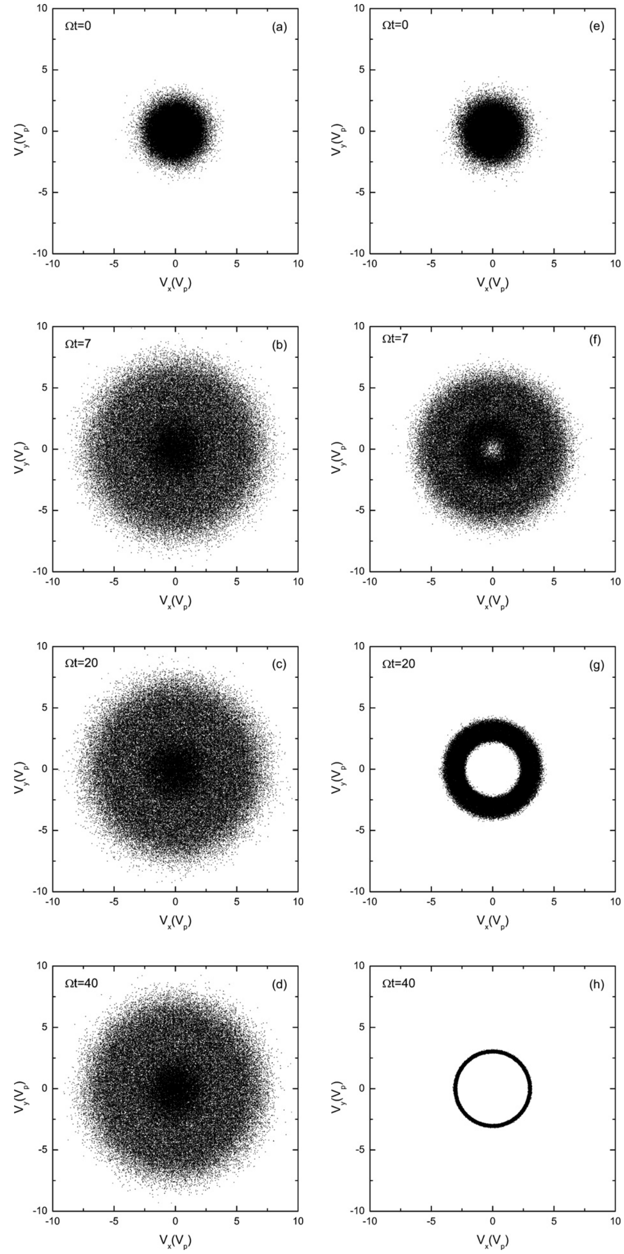

FIG. 1. Velocity scatter plots of the test particles in the $v_x - v_y$ space in the case $\nu_{in}=0$ (first column) and $\nu_{in}=0.1\Omega_0$ (second column) at time $\Omega t = 0, 7, 20$, and 40 for input parameters $(\delta B_w{}^2/B_0{}^2, v_p/v_A)=(0.05,0.07)$.

The results of the test-particle calculations are strongly consistent with our analytical predictions. The ratio of frequencies a in the analytic solution equation (9) is in the term $e^{-\nu_{in}t}$, where $\nu_{in}=a\Omega_0$; therefore, a larger value of a will result in a lower value of $e^{-\nu_{in}t}$, which will affect the value of ion kinetic temperatures, as shown in Fig. 2. Given the parameters $u(t)\approx 0$, $v_p=0.07v_A$, $\delta B_w^2/B_0^2=\zeta=0.05$, $a=0.1$, and $t=40$, the solution of Eq. (9) can be reduced to

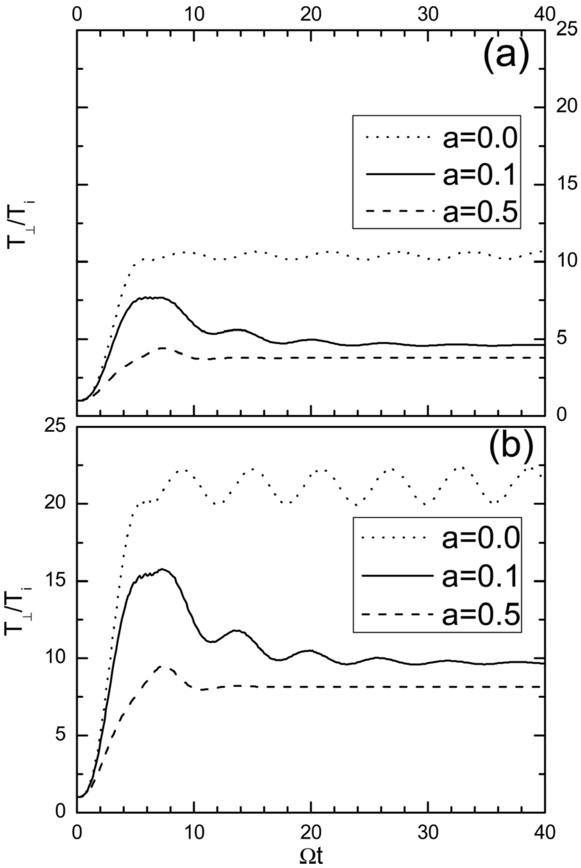

FIG. 2. The temporal evolution of the perpendicular kinetic temperatures normalized with respect to their initial values T_i. The various input parameters $(\delta B_w{}^2/B_0{}^2, v_p/v_A)=(0.05, 0.07)$ and $(0.12, 0.07)$ are shown in (a) and (b), respectively.

$$
\begin{aligned}
|v_\perp| &\approx \left| -\frac{\Sigma_k \Omega_k [v_A - v_\parallel(0)](\Omega_0 + i\nu_{in})}{\Omega_0{}^2 + \nu_{in}{}^2} e^{-ik(v_A t - z) - i\varphi_k} \right| \\
&\approx \left| \frac{\sqrt{\zeta}[v_A - v_\parallel(0)](1+ia)}{1+a^2} e^{-ik(v_A t - z) - i\varphi_k} \right| \\
&\approx \sqrt{\zeta} \times v_A |(1+ia) e^{-ik(v_A t - z) - i\varphi_k}| \\
&\approx \sqrt{\zeta} \times v_A \sqrt{1+a^2} \approx 3.2 v_p,
\end{aligned} \qquad (12)
$$

where we use the approximations $e^{-\varepsilon} \approx 0$ when $\varepsilon > 3$, $|v_A - v_\parallel(0)| \approx |v_A|$, and $1+a^2 \approx 1$. Thus, the stable value of velocity is $3.2v_p$, which is in accordance with the ring structure of velocity distribution shown in Fig. 1(h).

The physical mechanism of this heating can be described as follows: in low-beta plasma, nonresonant wave particle scattering by Alfvén waves can lead to rendomization of the particle motion transversely to the background magnetic field and thus effectively to heating of the ions. Due to the fact that the neutrals are given an approximately invariable bulk velocity, the heated ions will lose energy from ion-neutral elastic collisions throughout the process. This leads to an overall reduced efficiency of the ion heating relative to the simulation with no ion-neutral collisions.

In conclusion, we show that partially ionized low-beta plasmas can be heated by a spectrum of low-frequency Alfvén waves with large amplitude, which have been observed in the low-beta chromosphere.[1] This is contrary to the linear theory, according to which ions can only be heated through resonant interactions with Alfvén waves. In our model, the frequencies of the Alfvén waves are much lower than the ion cyclotron frequency, so the cyclotron resonant condition is not met. This also ensures that the waves will not be damped by ion-neutral collisions.[15–17] We find that the amount of heating depends on the ratio a, where less heating occurs when the importance of ion-neutral coupling is increased. However, in all cases, significant heating of the ions occurred. Furthermore, we show that the velocity distribution will form a ring structure during the heating process when elastic ion-neutral collisions are considered, which is consistent with our analytic approximation. Our results demonstrate significant heating of ions through nonresonant Alfvén wave interactions in partially ionized plasmas, which may provide an alternate source of heating in the solar chromosphere than previously considered.

The authors would like to thank two anonymous referees' helpful comments and suggestions. C.F.D. also appreciates many fruitful discussions with Prof. C. B. Wang, Dr. C. S. Wu, Dr. B. De Pontieu, Prof. T. D. Arber, Dr. M. L. Khodachenko, and Prof. Y. Li.

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