Little’s Law Flow Analysis of Observation Unit Impact and Sizing

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Abstract

Expanding hospital capacity by developing an observation unit may be an important strategy in congested hospitals. Understanding the principles for evaluating the potential impact and appropriate sizing of an observation unit is important. The objective of this paper is to contrast two approaches to determining observation unit sizing and profitability, real options, and a flow analysis based on Little’s Law. Both methods have validity and use similar data sets. The Little’s Law approach has the advantage of providing an estimate of appropriate size for the unit and a natural internal consistency check on data. The benefits of an observation unit can depend critically on assumptions regarding backfill patients, and minor changes in data or assumptions can translate into significant changes in annual financial consequences. Using both the real options and the Little’s Law approaches provides some internal consistency checks on data and assumptions. Both are sufficiently simple to be easily mastered and conducted. Using these two simple and accessible methods in parallel for computing the size and financial consequences for an observation unit is recommended.

Observation care, as defined by the Centers for Medicare and Medicaid Services (CMS), is a well-defined set of specific, clinically appropriate services, which include ongoing short-term treatment, assessment, and reassessment before a decision can be made regarding whether patients will require further treatment as hospital inpatients or if they are able to be discharged from the hospital. Observation services are commonly ordered for patients who present to the emergency department (ED) and who then require a significant period of treatment or monitoring to make a decision concerning their admission or discharge.1 It is increasingly important for hospitals to place patients in the appropriate status (observation vs. inpatient) regardless of bed location.2 CMS defines observation as an outpatient service, and this can take place in the ED, in an inpatient unit (admitted under observation status), or in a dedicated outpatient observation unit. While there are CMS constraints for what is allowable for observation (e.g., minimum 8 hours of observation, excluded recovery periods for some procedures, exclusion of some diagnoses) there are also medical necessity criteria that determine inpatient status. Congested tertiary care hospitals without a dedicated observation unit may find a significant number of inpatient beds occupied by patients on observation status. Since additional bed capacity can help relive ED overcrowding,3 increase accepted transfers and throughput, and increase revenues and profits, a congested hospital with observation status patients in inpatient beds may benefit from the creation of a dedicated observation unit. Dedicated observation beds may be less costly to construct and staff than inpatient beds due to differing code requirements and patient characteristics. This has led to a recent focus on adding observation beds to a hospital’s inventory. In Certificate of Need states, where the approval of a state agency is required for increasing inpatient bed capacity but not observation unit capacity, increasing observation bed capacity has an additional administrative advantage. In this context the appropriate questions to ask are whether it is profitable to create a dedicated observation unit, and if so, what size of observation unit is appropriate?

The methods available to address these questions include real options4 and flow analysis using Little’s Law. The first method (real options) computes the expected value of placing a patient in an observation bed to await further information rather than admitting the patient directly. The term “real option” is used because of the similarity between the hospital context and the valuation of a financial option for traded securities where something is paid for the right to delay a purchase decision. In this case one pays for the right to delay an admission decision. The second method begins...
with a standard process flow diagram, uses Little’s Law to relate the various model parameters, and assesses financial consequences by attaching cash flows to patient flows. Both of these methods are easily grasped and implemented and use largely the same input data. Both are also sensitive to assumptions regarding how vacated beds are backfilled, so performing both and checking for consistency can reveal important implicit assumptions.

METHODS

This article presents an analytical methodology for the physical sizing and financial justification for an observation unit. Flow and financial data presented in the Appendix example (see Data Supplement S1, available as supporting information in the online version of this paper) are simulated data but consistent in magnitude with actual data. The setting for this analysis was a large, tertiary care academic hospital considering the construction of an observation unit to relieve overcrowding and respond to the different reimbursement rates for observation relative to admitted patients.

Real Options

What is the value of being able to delay the use of an inpatient bed for a patient on observation status? A fraction of these patients will have to be admitted anyway, but the remaining fraction will be discharged without ever occupying an inpatient bed. Clearly, there is a benefit to delaying the allocation of the bed until this uncertainty is resolved. Observation beds are less costly to install, staff, and maintain than inpatient beds, so adding observation beds should increase the average net benefit for patients admitted on observation status but then discharged and also for those admitted on observation status but then converted to inpatient status. These advantages can be computed from the relevant reimbursement rules for various types of care and payers. For example, if an observation status patient occupies an inpatient bed and is then discharged without being converted to inpatient status, the net benefit will be the revenues for an observation stay plus the revenues for the ED visit, minus the ED costs and the costs of keeping the patient in an inpatient bed on observation status. If, however, that observation status patient is converted to inpatient status, the net benefit will be the DRG (diagnosis-related group) payment for the patient minus the cost of the ED visit and the cost of the inpatient stay. The direct benefit for having an observation bed available is a weighted average (weighted by the fraction of observation status patients who are discharged versus admitted) of these two.

But that calculation alone is likely to significantly underestimate the value of an observation bed, because by diverting observation status patients to an observation unit, an inpatient bed is opened up and can be backfilled by an admitted patient who, at least in a congested hospital, would otherwise not be there. That is, the total throughput volume of admitted patients will increase. To estimate this benefit, one must 1) check that there are currently a sufficient number of refused transfers, or some other pool of patients currently not entering the hospital, such that a freed-up bed will actually be utilized and 2) understand the characteristics and average net benefit for backfill patients. When computing this benefit, one must account for the fact that observation status patients cannot be replaced one for one by backfill patients, because these two categories of patients will likely have different lengths of stay (LOS). Baugh and Bohan show how to compute the value of an observation bed using the above logic, and a worked example is shown in the Appendix (Data Supplement S1, available as supporting information in the online version of this paper). The options approach performs its calculations on a per-observation-patient basis and at the margin. That is, it considers the possible trajectories for a single arriving observation patient and how the value to the hospital might change if an admission decision can be delayed for that patient. The basic calculation is to compare the financial consequences of immediately placing an observation patient into an inpatient bed against the expected value of delaying the decision with known probabilities that the patient will be discharged rather than being converted to inpatient status. Once the cash flow consequences for each of these paths are determined, the final calculation is straightforward. See the Appendix for a detailed example.

The options method reveals the value to the hospital, per-observation-status-patient, for having an observation bed available. It does not directly address the question of an appropriate size for an observation unit (how many beds?). In the next section, we present an alternative method that can offer more insights into sizing and can be used in parallel to reveal implicit assumptions that may significantly affect the calculations.

Little’s Law

Another approach to analyzing the benefits of an observation unit is to map the patient flows in a process flow diagram as in a standard operational analysis and then attach cash flows to patient flows. The technique has intuitive appeal, since flows can be easily visualized in graphics, and it facilitates the use of some standard relationships in operations analysis. One of these is known as “Little’s Law,” a principle that dictates the relationship among flows, LOS, and census. Little’s Law says that regardless of how one defines the boundaries of a system (the ED, the observation unit, or the entire hospital; it does not matter), and regardless of whether flows are random, the long run average number of patients in any system (I = average patient inventory) equals the product of the long run average throughput rate for the system (R = patients/time) and the long run average time each patient spends in the system (T = LOS). The mathematical expression for Little’s Law is I = R · T.

It is important to emphasize that Little’s Law is a relationship among long-run averages and that in the short term the throughput rate, census, and LOS can vary considerably relative to one another, causing varying levels of congestion from one day to the next or one hour to the next. But if one looks at annual data, Little’s Law should not be significantly violated. Also in the long run, since people do not languish in the ED...

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for months, the “arrival rate” of patients to the ED will equal the throughput rate, so we call that common rate the “throughput rate” of patients through the ED. In the long run, patients who leave the ED without being seen can be accounted for by either considering them as balks from the system (they are not treated) or as having an LOS equal to zero (they may wait in the waiting room, but spend zero time in a bed). With either alternative the mathematical result will be the same. To demonstrate this long-run analysis, suppose 10,950 patients arrive over a year (365 days), so the average arrival rate over the year is 10,950/365 = 30 patients/day. If we started January 1 with eight patients in the ED and ended December 31 with 11 patients in the ED, then 8 + 10,950 – 11 = 10,947 patients were discharged over the year, for an average of 10,947/365 = 29.99 patients/day. If we did the calculations over a 2-year period the difference between arrival rate and discharge rate would be even less. In the long run, the arrival rate equals the discharge rate, and as noted we call that common rate the throughput rate \( R \). Likewise, although daily census may go up and down, in the long run there is one average number of patients in the system \( I \). We refer to \( I \) divided by the number of beds as the “utilization” of the beds. It is the fraction of beds that are occupied, on average.

In the short run we may have 60 patients arrive in 1 day to an ED that averages 30 patients/day, and this will cause high levels of congestion with all beds filled and patients backed up in the waiting room. On a different day the arrival rate might be much lower and only half the beds filled. Clearly the arrival rate, census, and LOS can vary relative to one another on a day-to-day basis. It is common to say the unit is highly utilized one day and less so another. But in the long run the averages must satisfy Little’s Law, and the utilization (as we have defined it) is a constant. The Little’s Law relationship is illustrated in Figure 1. The mathematical expression \( I = R \times T \) demonstrates some intuitive tradeoffs. If \( LOS \) \( (T) \) increases for the same throughput rate \( (R) \), then patient inventory \( (I) \) will increase. If the hospital is highly utilized with all beds filled so that patient inventory \( (I) \) cannot increase, then to increase throughput rate \( (R) \) the hospital must decrease the \( LOS \) \( (T) \).

Little’s Law can be used for a consistency check on hospital data. It is not guaranteed that hospital planning data are consistent, as they often come from a variety of sources, which may have idiosyncratic reporting practices and implicit assumptions. It is especially important to check this when, as here, minor variations in input data can translate into significant annual cash flow differences. If a hospital with \( N \) beds has reported occupancy rate (utilization) \( \rho \), average length of stay \( LOS \) days, and discharge rate \( R \) patients/day, then by Little’s Law it must be that \( I = \rho N = R \times LOS \) in the long run (over a year, say). If this does not hold, then at least one of the pieces of input data is incorrect. For example, if in a 500-bed hospital with 90% occupancy the average LOS and discharge rate are estimated at 6 days and 60 patients/day, respectively, then something is wrong with the data, since \( 0.9(500) = 450 \neq 60(6) = 360 \). Also, because Little’s Law holds for any system or subsystem, a consistency check can be performed for observation (or any other subset) patients only. For example, suppose plans call for a 17-bed observation unit that is intended to be 90% utilized to handle an input rate of 15 observation patients/day who stay in the unit on average 1.2 days. These data imply that the unit is planned for an average inventory of patients equal to \( (0.9)(17) = 15.3 \), but this does not equal the average number of patients who will be in the unit from Little’s Law, which is \( (15)(1.2) = 18 \), so something is wrong with the plans. Either the unit size should be increased, or the unit will not be able to handle the anticipated admission rate of 15 patients/day.

How to Use Patient Flow Diagrams and Little’s Law to Size and Evaluate an Observation Unit

**Patient Flows Without an Observation Unit.** Figure 2 shows the patient flows without an observation unit, where any ED patient who requires a bed (whether on observation or inpatient status) gets an inpatient bed. The box in the middle of the diagram represents the inpatient tower, and any patient requiring a bed finds a place there.

Patients arrive to the ED and some are discharged or transferred or in some way leave without occupying a bed in the inpatient tower. The remaining patients require beds (this is the flow of \( \lambda \) patients/day in Figure 2). This stream of patients is divided into three substreams: observation to discharge, observation to admission, and admitted directly. The fractions that go

![Figure 1. Little’s Law: \( I = R \times T \).](image-url)
into each of these substreams are shown on the branching arrows in Figure 2, their LOS are shown in the inpatient tower box, and the final throughput in patients/day for each type of patient are shown on the right hand side of Figure 2. Such a flow diagram is always the first step in a flow analysis. The hospital profits in dollars per day is the product of the throughput rate of patients ($k$ patients/day) and the average benefit per patient (which we will call $B$). $B$ is a weighted average of the benefits for each patient trajectory. For example, if $B_{oa}$ is the net benefit ($$/patient) for a patient that enters in inpatient bed on observation status but then is converted to admitted (inpatient) status, then multiplying this by the flow rate for such patients ($f_{oa}k$) yields the dollars per day profit flow from those patients. We can perform similar calculations for patients who enter on observation status and are then discharged and those who are admitted directly. The sum of the profits from these three sub-flows is the total hospital profits/day. A complete example is worked in the Appendix. All that we need to compute hospital profits before we add an observation unit is the current patient flow rates and the average benefit per patient in each flow.

**Patient Flows With an Observation Unit.** The patient flows with an observation unit are shown in Figure 3. The fractional breakdown for each type of patient is exactly the same as in Figure 2, because nothing medically has changed. However, there are two significant changes in the patient geography and resulting cash flows. The first is where the patients spend their time. Patients who are discharged from observation status never occupy an inpatient bed. If tests and diagnoses are completed while the patient is on observation status, patients who are put into an observation bed but then admitted may spend less time in an inpatient bed than somebody admitted directly. So, after the observation unit becomes operational, some patients who were occupying inpatient beds are no longer there, and the beds are freed up to be backfilled with additional patients. Because of these additional patients, the new total throughput of patients ($k'$ patients/day in Figure 3) will be greater than the old throughput ($k$ patients/day from Figure 2). Thus, there are two benefits to the hospital: the reduced cost of serving observation status patients in observation beds instead of inpatient beds, and the additional profits due to the increased throughput of new patients. The former are easily computed by overlaying the new costs and benefits on each patient flow, as before. Because the observation portion of each patient’s stay will be less costly, there will be a cost reduction with an operational observation unit. The additional profits from the new backfill patient flows will be discussed in more detail below, and a worked example appears in the Appendix. To achieve either of these benefits, an observation unit of appropriate size must be constructed, equipped, and staffed.

**How Large an Observation Unit? How Many New Patients?** From Little’s Law, the average number of patients on observation status in the hospital on any given day (call this $I_o$) must equal the throughput rate of observation patients ($f_o\lambda$) multiplied by their stay (on observation status, $LOS_o$, days, which can be computed as a weighted average of observation patients who are discharged and those who are admitted; see the Appendix). That is, $I_o = f_o\lambda LOS_o = f_o\lambda LOS_o/0.9$. For example, if 55.4 patients/day are put into beds and 20% of these

![Figure 2. Patient flows without an observation unit.](image-url)
are on observation status for which the average LOS is 1.14 days, then $I_o = (0.2)(55.4)(1.14) = 12.63$ (the census of observation patients on average) and an observation unit with a design utilization of 90% should have $12.63/0.9 = 14$ beds.

As mentioned above, the unit may still be congested some days and relatively empty on others, but on average a unit sized as described will be 90% utilized housing the average number of observation status patients currently in the hospital.

Removing $I_o$ patients from inpatient beds frees these beds up for new patients. How many new patients? Recall that Little’s Law holds for any system, so we can consider just backfill patients. There will be $I_o$ of these on average in the system since the new patients are replacing the removed observation patients. If backfill patients stay on average $LOS_{bkf}$ days, then from Little’s Law we must have a backfill flow rate (that is, an enhanced patient flow rate, call it $\lambda'$ patients/day) that satisfies $I_o = \lambda' \times LOS_{bkf}$. Given the data we already have in hand, the only additional information we need is the average LOS for backfill patients, which we can compute as another weighted average (see the Appendix) considering the mix of patient types that will be in the new patient stream. The easiest assumption is that these will look just like the current patient stream. For example, suppose the average census of observation patients is $I_o = 12.63$ patients and backfill patients will have an average LOS of 5.34 days, then by Little’s Law $12.63 = (\Delta\lambda)(5.34)$ so $\Delta\lambda = 2.37$ patients/day who will be added to the hospital’s throughput. We will want to be sure that current patient diversions and/or refused transfers are sufficient to support the assumption that patient flows could increase by that amount. If they can, then the new patient flow rate will be $\lambda' = \lambda + \Delta\lambda$ patients/day, or in our example $55.4 + 2.37 = 57.8$ patients/day.

Now we have all the information we need to build a business case for an observation unit. By incurring the fixed cost of building and equipping an observation unit of size 14 beds the hospital will realize benefits equal to the reduced cost of serving the $(0.2)(55.4) = 11$ patients/day who are put on observation status plus the enhanced profits from a backfill patient stream of 2.37 patients/day (see the Appendix for more details).

**Be Clear About Assumptions.** One reason it is important to do these calculations at least two ways (for example, real options and Little’s Law) is that seemingly insignificant changes in assumptions can have significant cash flow consequences. Note that in the previous section we assumed that the backfill patients will look just like current patients. That is, we assumed we would draw from the same overall patient pool with the same general characteristics. This seems reasonable enough. But note that currently there are no observation beds, so an observation status patient goes into an inpatient bed, and the profitability of the patient reflects this. By assuming that new patients look like old patients we are implicitly assuming that new observation-status patients in the new backfill flow go into inpatient beds. That is, although we removed the $I_o$ observation patients who currently occupy inpatient beds, the new flow will have its own share of patients who are put on observation status, and we did not size the unit to accommodate these patients.

Should we build a larger observation unit, sufficient to house both current and new observation-status patients? What would be the profitability consequences? How large a unit must we build to accomplish this? The profitability benefits are easily computed, since we simply apply the reduced cost scenario to all patients (current and new) when comparing with the current situation (no cost savings and current flows).
That is, the profitability benefit will be \( \lambda' \times (\text{profit/patient with observation unit}) - \lambda \times (\text{profit/patient without observation unit}) \). Since we have already estimated the enhanced profitability (due to the availability of observation beds) for patients requiring beds in the current patient flow, all we need is the new flow rate \( \lambda' \).

We can again use Little’s Law. We are assuming that we want to accommodate observation patients in both the current and additional flows. If the patient census in the inpatient tower remains constant between the two scenarios, we know that the (flow rate into inpatient beds) \( \times \) (average LOS in inpatient beds) must be the same before and after we build an observation unit. The “before” scenario includes no observation unit, so the flow rate going into the inpatient tower is the total flow rate \( \lambda \) (patients/day in Figure 2). The LOS can be computed as a weighted average over the various trajectories in Figure 2. The LOS in the inpatient tower in the “after” scenario is adjusted to include only admitted patients, because all observation patients will be in the observation unit. This gives us all the information we need to compute the new (“after”) flow rate. For example, if \( \lambda = 55.4 \) patients/day, the average LOS before we build an observation unit is 5.34 days, and the average LOS for admitted patients is 5.11 days, then we must have a new flow rate of \( \lambda' = (55.4/5.34)/(5.11) = 57.9 \) patients/day. This is slightly higher than the 57.8 patients/day we computed in the previous scenario, because here we are removing even more observation patients from the inpatient tower.

The profitability calculations are now straightforward. If the average benefit per patient \( B \) is $4,642 currently but will be $4,733 with an observation unit, then with a current flow of 55.4 patients/day and a new flow of 57.9 patients/day the hospital will benefit by

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(57.9)(4,733) - (55.4)(4,642) = $16,874/day.
\]

We can size the unit as before, by multiplying the new flow rate times the fraction that go on observation status to get the flow into the observation unit and multiplying this by the average LOS on observation status to get the average census. We divide that by the target utilization to get the number of beds. An example appears in the Appendix. It is important to approach these sorts of investment problems from at least two different directions to cross-check one’s assumptions, some of which are implicit and easily overlooked. In the Appendix we provide an example where the first scenario (assuming backfill patients have the same profitability as current patients) indicates that the hospital should build an observation unit with 14 beds and can expect enhanced profits of $16,032/day or $5.85 million/year. Using the second assumption we would build a unit with 15 beds and enhance profits by $16,847/day or $6.16 million/year. The difference is $307,330/year and just one observation bed. In general, minor changes in assumptions and calculations done on a per-day basis become major financial consequences when scaled up to a year. This reinforces the advantages of approaching the calculations in several ways and checking for consistency. Example calculations using the real options method and both backfill assumptions in the flows method appear in the Appendix.

In congested tertiary care hospitals faced with ED congestion, the addition of lower-cost observation status beds to a hospital’s inventory can relieve ED overcrowding and enhance the hospital’s bottom line. In many hospitals, data garnered from several different sources can suffer from inadvertent inconsistencies, and yet in high-volume facilities even minor changes in estimated costs or revenues per patient can translate into major annual cash flow consequences. This argues for approaching proposals for observation units from at least two directions and testing for consistency. The real options approach and flow analysis using Little’s Law both have validity and use similar data sets. Little’s Law approach has an advantage in providing some internal consistency checks and also suggests an appropriate size for the observation facility.

**DISCUSSION**

A limitation of both the real options and the Little’s Law approach is that they compute benefits using “average” patient data, so the benefits are essentially assumed to scale linearly with patient volumes. In reality, there are diminishing returns as one adds beds to a facility, as the potential patient population is increasingly exhausted, and there may be complex nonlinear effects due to high levels of variability in patient flows and needs. These effects can affect the relationship between unit size (capacity) and patient/cash flows. The real options approach does not address unit size directly, because it bases its computations on the “marginal” patient. Little’s Law approach uses a simple target utilization method for sizing observation units, which ignores the relationship between utilization and performance driven by variability. There are two alternative approaches to optimally sizing an observation facility. The first uses queuing theory (see Green and Nguyen for an application to sizing an inpatient unit). Queuing theory requires more data, and in particular requires data on the variability (in addition to the expected values) of patient arrival rates and LOS. If certain specific distributional assumptions are satisfied, some closed form solutions are available that will exhibit the appropriate diminishing returns to scale. The other approach is Monte Carlo simulation, which is very general but typically requires the expertise of a specialist to do well. The real options and Little’s Law approaches are simple, can be rapidly executed, use data that are likely to be available, and are readily accessible to physicians and administrators. The queuing theory and simulation approaches are more detailed, require input data unlikely to be readily available, require specialist knowledge, and are less accessible to the typical physician. However, by being more detailed, these more complicated methods can reveal more nuanced dynamics in their capacity analyses. Since simpler and more accessible models are better if they provide reasonably accurate results, one possible topic for future investigations is to test these four methods on standard data sets to suggest which is appropriate in different contexts.

Hospital capacity expansions receive close scrutiny because of a body of opinion that capacity is always
used, whether needed or not. This is known as “Roemer’s Law,” after health researcher Milton Roemer, who published an early paper showing a positive correlation between the number of hospital beds available and the number filled, leading to the conclusion that capacity might induce demand rather than respond to it. It is this logic that led state legislatures to consider Certificates of Need to limit construction. The calculations here are accurate at the hospital level, in that if new beds are filled with patients for whom reimbursements are at the assumed levels, the hospital will benefit financially as computed. We do not address the issue of appropriateness. That is, we assume throughout that patients who are discharged, admitted, or kept overnight on observation status are being treated appropriately. If this is not the case the calculations remain accurate for the hospital, but society at large may be paying more, or less, than is medically appropriate. In the specific case of congested, tertiary care hospitals, which would be the primary users of this methodology, perverse incentives for observation status patients are unlikely, since it is very costly to keep them in inpatient beds. If beds are a scarce resource, they are unlikely to be used frivolously. If they are not a scarce resource, it is unlikely that a hospital will incur construction costs to expand their inventory. The possibility of misusing the released inpatient beds is possible if the number released exceeds the number needed. We do not address this issue directly, however.

CONCLUSIONS

We present the concept of a flow analysis approach using Little’s Law to analyze the impact of a proposed hospital observation facility. Both the Little’s Law and the real options approaches are valid for the analysis of a proposed hospital observation facility, and both use similar data sets. The Little’s Law approach can provide some internal data consistency checks. In many hospitals, data garnered from several different sources can suffer from inadvertent inconsistencies, and yet in high-volume facilities, even minor changes in estimated costs or revenues per patient can translate into major annual cash flow consequences. Approaching proposals for observation units from at least two directions and testing for data consistency are important, as long as the methodology is manageable. The simplicity of both the Little’s Law and real options approaches suggests that both should be used in observation unit analyses, providing another cross check for the internal consistency of the assumptions and data.

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References


Supporting Information

The following supporting information is available in the online version of this paper:

Data Supplement S1. Example applications.
The document is in DOC format.

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