Auxilliary Material for

# Observations of Core-Mantle Boundary Stoneley Modes

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## Introduction

The auxilliary material contains our measured splitting function coefficients and center frequencies for CMB Stoneley modes and fundamental spheroidal modes, as described in the text. In addition, the text contains more information regarding the calculation of the misfit and F-test statistics.

The measured splitting function coefficients are given in the table "ts01.txt" and plots of the degree 2 coefficients in the PDF file "fs01.pdf". The measured center frequencies and quality factors are compared with those of the PREM model and presented in the PDF file "fs02.pdf".

1. ts01.txt: Self coupled splitting function coefficients in micro Hz.

1.1 The first line contains the $n$, $l$ of the mode.

1.2 There are two lines for each angular order $s$ of the splitting function, starting with $s = 2$.

1.3 The first contains the $c_{st}$ coefficients and the second the corresponding error values.

1.4 The coefficients are ordered $c_{s0}$, Re($c_{s1}$), Im($c_{s1}$), Re($c_{s2}$), Im($c_{s2}$), ... etc.

2. fs01.pdf (Figure S1): Splitting function coefficients for $s = 2$ for our new normal mode measurements along with the predictions for crust structure using model Crust5.1 and mantle structure using models S20RTS and S40RTS. CMB Stoneley modes are indicated by shading.

3. fs02.pdf (Figure S2): Normal mode center frequency and quality factors for our new measurements compared to PREM values. We plot both the absolute value (a-b) as the difference with the PREM values (c-d).

## Misfit measure and F-test statistics

We define the misfit for the splitting function measurements as the difference between the data $\mathbf{d_i}$ and synthetics $\mathbf{u_i}(\mathrm{c_{st}})$, normalized by the norm of the data (Deuss et al., 2013):

$$\text{misfit} = \frac{1}{N} \sum^{N} \frac{\sum_{i=1}^{n} \left(\mathbf{d}_i - \mathbf{u}_i(c_{st})\right)^2}{\sum_{i=1}^{n} (\mathbf{d}_i)} \tag{1}$$

where n are the number of data points in each spectral segment and N are the total number of spectral segments for a specific mode. We take both the amplitude and phase into account in calculating the misfit.

Performing the joint measurement for the fundamental mode and CMB Stoneley mode increases the number of splitting function coefficients we are trying to obtain relative to the fundamental mode measurement. To establish whether the misfit reductions observed are statistically significant given this increase in parameters, we use F-test statistics. We calculate the values of the F-statistic under the null hypothesis that the misfit reductions are not significant compared to only the fundamental mode measurement. We then compare the F-statistic values to critical values of the F-distribution to establish whether the null hypothesis holds. If the value of our F-statistic exceeds the critical value, the null hypothesis can be rejected and the observed misfit reduction is significant.

We set the level of statistical confidence for calculating the critical F-values to 0.01, which means that if the critical F-value is exceeded, the null hypothesis can be rejected with 99 % confidence. We find that for most of our measurements the null hypothesis can be rejected at a 99 % confidence level, except for ${}_1S_{13}$, ${}_2S_{25}$ and ${}_3S_{26}$ in which case it is 90 % due to less available data.

## Splitting function coefficients

We present splitting function maps for examples of our new normal mode measurements (Figure 3 in the main paper). These resemble predictions for mantle and crust structure using model S20RTS (Ritsema et al., 1999) and Crust5.1 (Mooney et al., 1998). However, splitting function maps often look surprisingly similar even though individual coefficients are substantially different. Therefore, we present in Figure S1 splitting function coefficients for our new measurements along with the predictions for S20RTS+Crust5.1 structure. We also compare these to predictions for mantle model S40RTS (Ritsema et al., 2011) and Crust5.1 structure.

Our measurements are run either starting from PREM (Dziewonski and Anderson, 1981), corresponding to zero splitting, or from predictions for S20RTS+Crust5.1 structure. Figure S1a shows that especially the $c_{20}$ coefficients are markedly different from the starting model (differences up to 5 $\mu$Hz, well outside the uncertainties) and similarly for the $Re(c_{22})$ coefficient. These coefficients are the main contributors to the "Ring around the Pacific" pattern and for almost all CMB Stoneley modes the measurements require larger splitting than the predictions. For the upper mantle sensitive modes (${}_1S_{15}$, ${}_1S_{16}$ and ${}_2S_{14}$), the measurements match the predictions for S40RTS structure better than S20RTS structure, even though we start the measurement from S20RTS.

## Center frequencies and quality factors

In addition to showing the degree 2 coefficients, we also present the values of the center frequencies and quality factors of our new measurements compared to PREM values in Figure S2. The deviations of the center frequencies are very small, up to 3 $\mu$Hz. The frequency shift is generally more negative for the Stoneley modes. The measured quality factors match the trends of the PREM values, but are significantly lower for some of the Stoneley modes.

## References

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