Essays on Law and Economics, Economics of Education and Public Economics

by

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to my mom

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1.1 Introduction and Discussion of Literature

The choice of a cost apportionment rule and standard of care in a tort liability rule determine not only how prudent individual actors are when they engage in an inherently risky activity but also how frequently they choose to engage. The relationship in the bilateral accident setting is simple. Given a liability rule, a would-be injurer and victim each choose a level of precaution, which in equilibrium determines how likely each is to be found liable for damages sustained by the victim were an accident to occur. This expected liability, i.e., each party's expected share of the social cost, in turn becomes part of the calculus as the potential tortfeasors weigh the benefits and cost of engagement in the activity.

The question of how potential injurers and victims will adjust their levels of precaution when facing different cost apportionment rules has received plenty of attention in both perfect and imperfect information settings[1]; and there is a copious amount of scholarly work on the externality issues inherent in the level of engagement problem. However, the questions are often modeled and addressed separately despite being intimately related. This paper considers the problems simultaneously, focusing first on the question of efficient care, then extending the results to the more general problem of care and activity level. Two important questions are considered: Can any mechanism induce efficient care under fairly general conditions and what is the effect of uncertainty on activity

[1] Calfee and Craswell (1986); Haddock and Curran (1985); Ulen and Cooter (1984)

level?

Before turning to the problem with uncertainty, a discussion of the problem with perfect information provides the necessary context. The question of efficient care has traditionally been modeled as the solution to a cost minimization problem, with the social planner's objective being minimization of the aggregate private and social cost of the agents engaged in the activity. The problem naturally consists of dueling incentives from the perspective of the individual actors; while precaution is costly, it reduces the likelihood of an accident. Economic analysis of the problem shows that when parties have perfect information about the legal standard and juries are able to perfectly observe their behavior, any of three cost apportionment rules, namely simple, contributory or comparative negligence, will induce socially efficient levels of care from both parties in equilibrium.[2] This result dates back to Shavell's seminal paper on the subject.

Since the objective is not to do away altogether with accidents as complete prevention may be too costly, models of the problem assume some solution to the social planner's objective function where the optimal level of care results in a non-zero probability of an accident. This implies a positive social cost at the optimum. Therefore, a corollary to the finding is that at the equilibrium where efficient levels of cautiousness are exercised by both parties, repeated engagement increases total social cost. The effect of repeated engagement is considered in the more general model where individual agents derive some positive utility from the activity. Assuming that courts cannot monitor the frequency of engagement (and accordingly establish legal standards or quotas), these decisions can be judicially regulated only to the extent that the tort liability rule indirectly affects them. This question has been addressed in the perfect information setting.

The Shavell Theorem[3] shows that there exists no legal mechanism that can provide incentives to both injurer and victim to choose both the efficient level of care and the efficient level of engagement in the activity. Assuming that each chooses the efficient level of care in equilibrium, injurer will always over-engage.[4] This result is the by-product of the perfect information setting. Under any

[2] This result of course assumes that the legal standard of care is correctly assigned.

[3] Shavell (1987)

[4] As I show later in the paper, in the perfect information setting, in any pure strategy equilibrium, injurers will choose the efficient level of care while victims will be overly cautious whenever social cost is increasing the the injurer's level of activity.

of the three negligence rules, in equilibrium the injurer knows with certainty that meeting the legal standard of care bars the victim from recovery.[5] Therefore, her only cost consideration is the reduction in the private cost of care, as her share of the social cost is zero.

While the perfect information setting provides important insights into the efficient design of tort law, in the real world individuals face some form of judicial uncertainty. Uncertainty in the bilateral accident setting can come in the form of imperfect observation of care by juries or alternatively uncertainty about the legal standard of care on the part of individuals engaged in the activity. This paper considers uncertainty as the by-product of the ”reasonable person” standard, the prevalent standard against which the negligence of individuals is measured in American courts.

One of the elements that must be established by a victim seeking recovery for damages in any tort suit is a showing that injurer failed to meet the standard of care owed to the victim. Whether the standard of care has been breached by injurer is typically decided on the level of prudence a reasonable person in the injurer's situation would have exercised. The reasonable person, when engaging in an activity, is to consider the forseeable risk of harm her actions create, the extent of this risk, the likelihood that the risk will cause harm to another person, and alternatives that reduce the risk. The subjective nature of the standard naturally leads to a degree of uncertainty from the perspective of the individual agents, and with it several economic problems arise.

Amongst others, the introduction of uncertainty raises the following key questions:

1. How will would-be injurers and victims adjust their level of care in response to uncertainty under various legal mechanisms?[6]

2. Which mechanism is most efficient in this setting?

3. Can any mechanism induce the efficient level of care under general conditions despite the presence of uncertainty?

4. How will the distortions in level of care affect agents' decisions on how frequently to engage in

[5]Technically, if the standard of care is set above the efficient level of care with a contributory or comparative negligence rule there is no pure strategy equilibrium, therefore parties do not always face probability 1 or 0. However, Endres and Querner (1995) show that any mixed strategy equilibrium under contributory negligence must be socially inefficient.

[6]Throughout, the paper uses the terms legal mechanism and cost apportionment rule interchangeably.

the risky activity?

5. Is the reasonable person standard, i.e., uncertainty, desirable when the general problem is considered?

The first two questions have been the focus of the literature on the cost minimization problem with imperfect information. Considered are simple negligence, which assigns liability to the injurer when he is negligent, negligence with a defense of contributory negligence, which assigns liability to the injurer when he is negligent and the victim is not, and comparative negligence. Comparative negligence assigns all liability to the injurer when he is negligent and the victim is not and some portion of the liability when both parties are negligent according to a "division of liability" rule.[7] Given the different incentives that each rule provides to injurer and victim, one would certainly expect the rules to have different distortionary affects on equilibrium choices of care by injurer and victim.

Ulen and Cooter[8] addressed the possibility that one rule is generally more efficient, arguing essentially that since simple negligence causes the largest distortion in injurer's behavior and smallest distortion in victim's and contributory negligence does the opposite, comparative negligence must be most efficient since it causes average levels of distortions for both. This notion was shown to be incorrect. Bar-Gill and Ben-Shahar (2003) shows that for a given division of liability rule, each of the three cost apportionment rules can be most efficient depending on the accident prevention technology and the level and distribution of uncertainty. Specifically, the paper showed numerically that for a fixed standard of care and division of liability rule, contributory and comparative negligence outperform simple negligence at low levels of uncertainty while simple negligence is most efficient at high levels.

Two caveats are necessary. The objective of that paper was to show the absence of a general conclusion on the matter, so only one division of liability rule was considered, i.e., a 50/50 rule.

[7]The division of liability rule varies amongst US states, with most states adopting a 50/50 division rule. Other versions of the rule include a division that depends on how negligent injurer and victim, with a larger proportion of liability assigned to the party who was more negligent.

[8]Cooter, Robert C., and Thomas S. Ulen. 1986. "An Economic Case for Comparative Negligence," 61 New York University Law Review 1067110.

Additionally, the numerical simulation fixed the standard of care at the socially efficient level without uncertainty. As others have shown in the literature, the optimal standard of care differs across liability rules with the introduction of uncertainty.[9]. The conclusion of the paper, however, is supported even when the standard of care and division of liability rule are allowed to vary.

This paper proves that for a general class of accidents, comparative negligence is most efficient in that it induces the socially efficient outcome. It considers activities where the effort of either party reduces the likelihood of an accident independently of the other's, finding that contributory and simple negligence cannot induce the efficient equilibrium. Under comparative negligence however, there always exists some sufficiently small level of uncertainty and division of liability rule to induce such an equilibrium.

1.1.1 The Possibility of an Efficient Care Equilibrium and the Welfare Implications of Uncertainty

The introduction of uncertainty to the more general problem, where both care and activity level are considered, generates a trade-off. Generally speaking, uncertainty generates deadweight loss as one or both parties choose an inefficient level of care. The magnitude of this additional social cost depends of course on the particulars of the problem, including the level of uncertainty. However, uncertainty has the secondary effect of leaving the injurer with some non-zero ex-ante probability of bearing the cost of the accident. This in turn, induces the injurer to internalize some fraction of the social cost of her behavior and to adjust her level of activity accordingly. The trade-off between the increase in social cost from distortion in care and the increase in aggregate utility stemming from a more efficient level of activity is ambiguous.[10]

What is unambiguously true however is that a cost allocation mechanism that induces efficient levels of caution from both potential injurer and victim yet never guarantees either party fully escape liability should be preferred to one where injurer is guaranteed not to bear any portion of the social cost. No such rule exists in pure strategy equilibria in a perfect information setting.

[9]Bar-Gill and Ben-Shahar (2003); Edlin (1997); Shavell (1987)

[10]For example, an individual who drives once a year but chooses to drive at 200 miles per hour may be preferred to one who drives every day but always at 55 miles per hour, or he may not. Clearly, the choice reduces to the increased risk of driving 200 miles per hour versus 55 and the additional utility drivers get from the excess speed.

The paper simultaneously contemplates the social planner's cost minimization problem and the more general welfare maximization problem in a setting with evidentiary uncertainty. Section 2 formally describes the cost minimization problem and its solutions under contributory and simple negligence where parties face no evidentiary uncertainty. For standards of care at or below the efficient level, as identified in Section 2, Section 3 characterizes the set of possible equilibria for sufficiently low levels of uncertainty under both contributory and simple negligence. Throughout Section 3, the assumption is that a party's actions affects the likelihood of an accident independently of her counterpart's, i.e., an additively separable accident avoidance technology. Section 4 contains the main contribution of the paper with relation to the cost minimization problem: for the class of accidents considered there exists some sufficiently small level of uncertainty to support an equilibrium under comparative negligence where both parties choose the socially efficient level of care despite evidentiary uncertainty. Sections 3.2 through 4.2 contain the proof of this sufficiency result.

Using numerical simulations of the model, Section 5 demonstrates the results of the previous sections. Section 6 contemplates the general welfare maximization problem with no uncertainty, showing that the problem always results in over-engagement in the activity by potential injurers and under-engagement by the victim, and may result in over-cautiousness by the victim whenever the social cost of the activity is increasing in the injurer's level of activity. Section 6 also contains the contribution of the paper in relation to the welfare maximization problem, showing that given the findings in Section 4, uncertainty can be unambiguously welfare improving as parties choose the efficient level of care and potential injurers internalize some non-zero portion of the social cost they impose, thereby adjusting their level of engagement in a welfare improving way. Section 7 concludes.

1.2 The Cost Minimization Problem With No Evidentiary Uncertainty

The problem involves a potential injurer and victim that engage in some inherently risky activity. Risk takes the form of the possibility of an accident, which in turn causes the victim to incur monetary damages. Each agent chooses some level of precaution when engaging in the activity and while care is costly, additional care on the part of either injurer or victim reduces the risk of an accident according to some exogenous accident avoidance technology. The injurer and victim are identical in every respect until an accident occurs, at which point victim attempts to recover monetary damages from the injurer under tort law. Their choices of cautiousness have a symmetric effect on the likelihood of an accident and they face the same level of exogenous evidentiary uncertainty or absence thereof. In the absence of a cost apportionment rule, victim would always bear the cost of the accident as it is presumed throughout that injurer and victim cannot form a contractual agreement for allocation of damages.

Throughout, the paper assumes that the exogenous accident avoidance technology and the ratio of that the cost of a unit of cost of care and the monetary damages sustained in an accident are such that the socially efficient outcome demands that each agent exercises some positive level of care.[11] The symmetry of the problem in the level of care of the two parties implies that each should choose the same non-zero level of care at the optimum.

Injurer and victim generally have three incentives to consider in choosing how cautiously to engage in the activity. Firstly, each considers her private cost of care, preferring to expend as little effort as possible. Secondly, given the other party's level of care, each has an incentive to reduce total social cost, to the extent that she will bear some portion or all of it, by exercising more care. Finally, given the other party's level of care, each party has an incentive to reduce her expected share of the social cost. In a setting with perfect information, each agent knows with certainty her share of the social burden given the cost apportionment rule, the standard of care, and the level of caution exercised by both parties.

This is not the case when evidentiary uncertainty is introduced. Rather than knowing the standard of care with certainty, each agent takes the standard as the mean of the symmetric distribution of all possible standards that a jury will apply. This is an alternative way of thinking about the lack of perfect observation of precautionary effort by juries or the effects of the reasonable person standard. Agents are unaware of the exact standard of care (they have some idea that the legal standard is on average the standard applied by juries) as the standard is governed by the reasonable

[11]The reader should note here that in the absence of a liability rule, injurer would exercise no care while engaging in the activity, resulting in social inefficiency. In the degenerate case where zero care by one of the parties is optimal, the questions addressed by this paper are moot.

care criteria.

The social planner's objective is to design a liability rule, consisting of a cost apportionment rule and standard of care, that minimizes the sum of the agents' private and social costs. Upon announcement of the apportionment rule and standard of care, each party simultaneously chooses a level of care that minimizes the sum of her private cost and her share of damages. Since each party knows that the other has full information, in equilibrium the level of care of each party must minimizes those costs for the level of care of the other party. Formally, the problem can be written as

$$\begin{aligned} & \min_{\bar{x},\Gamma} && \hat{x}_n + \hat{x}_v + \lambda \cdot L(\hat{x}_n, \hat{x}_v) \;\; s.t. \\ \hat{x}_n &= \operatorname*{argmin}_{x_n} && x_n + \lambda \cdot \Gamma \cdot L \\ \hat{x}_v &= \operatorname*{argmin}_{x_v} && x_v + \lambda \cdot (1 - \Gamma) \cdot L, \end{aligned}$$

where x_i is the level of care of party i, λ measures the ratio of monetary damages sustained by the victim in the event of an accident to the cost of a unit of care, $L \equiv L(x_n, x_v)$ is a continuous, twice differentiable function, that describes the accident avoidance technology. L is decreasing and convex in both arguments. Throughout its remainder, the paper considers only the class of accidents where precaution by one party is a substitute for precaution from her counterpart. This implies that $L_n \equiv L_n(x_n)$ and that $L_v \equiv L_v(x_v)$, where L_i is the effect of additional caution by agent i on the likelihood of an accident. Finally, these conditions imply that $L_{ij} = 0$, where L_{ij} is the cross partial derivative of the accident avoidance function.

$\Gamma \equiv \Gamma(x_n, x_v; \bar{x})$ is an indicator function that assigns liability based on the level of care jointly exercised by the injurer and the victim and the standard of care $\bar{x}$. The indicator function differs across cost apportionment rules and will be defined individually for each of the rules considered.

The assumptions of linear cost and a convex accident avoidance technology imply that there must be some $(x_n, x_v) = (x^*, x^*)$ which is the interior solution to the social planner's problem. For the remainder of the paper, this will be referred to as the "socially efficient level of care". References to the efficient equilibrium throughout the paper will be to this equilibrium and behavioral distortions when uncertainty is introduced in Section 3 will be measured by deviations from this equilibrium.

At the optimum, each party chooses a level of care such that $L_i = -\frac{1}{\lambda}$. The literature shows that in the absence of uncertainty, when the standard of care $\bar{x}$ is set at the socially efficient level of care, each of the three cost apportionment rules that will be considered here result in a unique pure strategy equilibrium where both injurer and victim choose $\bar{x} = x^*$. Additionally, for any $\bar{x} < x^*$, injurer chooses $x_n = \bar{x}$, as at that level she does not bear the cost of the accident, and victim chooses $x_v = x^*$, as he is solving the social planner's problem.

1.2.1 Negligence and liability distinguished

The terms negligent and liable should be distinguished. Where negligence simply relates a party's level of care to the standard of care, liability depends also on the cost apportionment rule. A party to an accident is considered negligent whenever his level of care falls below the legal standard. Alternatively, liability is assigned on the basis of each party's level of care, the legal standard, and the cost apportionment rule. Table 1.1 summarizes the way in which liability is assigned under each of the three cost apportionment rules considered.

Table 1.1: Assignment of Liability Based on Relative Care

Injurer	Victim	Simple Negligence	Contributory Negligence	Comparative Negligence
Negligent	Negligent	Injurer	Victim	Injurer pays share α
Negligent	Not Negligent	Injurer	Injurer	Injurer
Not Negligent	Negligent	Victim	Victim	Victim
Not Negligent	Not Negligent	Victim	Victim	Victim

1.3 The Introduction of Evidentiary Uncertainty and Characterization of Equilibria Under Simple and Contributory Negligence

In what follows, the paper characterizes the set of possible equilibria under both a simple and contributory negligence rule for small levels of uncertainty.[12] For each rule, it considers first the parties' equilibrium behavior when the legal standard of care is set at the efficient level, i.e., x^*, finding that under contributory negligence, in equilibrium injurer and victim will both be overly cautious as each party has some incentive to escape liability in the event of an accident. At sufficiently

[12] The reader will note throughout the discussion that the level of uncertainty sufficient for existence of an efficient equilibrium is a function of the accident avoidance technology, monetary damages in the event of an accident and the form of the distribution of uncertainty.

small levels of uncertainty, this incentive overshadows the parties' incentives to reduce the private cost of care and the likelihood of an accident.

This, however, is not the case under simple negligence, where injurer will be overly cautious and victim will not be cautious enough. Victim's only incentive is to minimize the sum of private cost of care and the likelihood of an accident, weighted by the probability that he will be liable for damages as he is barred from recovery whenever injurer is non-negligent, irrespective of his level of precaution.

Given this equilibrium, the paper next considers the effect of progressive reductions in the standard of care, noting that at the optimum the effect of a change in the standard of care induces a continuous set of new equilibria. Under contributory negligence, injurer and victim will both be less cautious for progressively lower legal standards. Intuitively, injurer can reduce her private cost and maintain the same probability of being found non-negligent. As for increases in the likelihood of an accident, since injurer will be highly unlikely to be found liable in the first place, the incentive to reduce the likelihood of an accident will be overshadowed by her desire to avoid payment in the event an accident occurs. Under a simple negligence regime, injurer will reduce her care with a reduction in the legal standard. This in turn has an ambiguous affect on the victim's behavior.

In doing so, the paper identifies the set of feasible equilibria when the legal standards is set below the efficient level. Under contributory negligence, no equilibrium is feasible where injurer is overly cautious relative to the efficient level and victim is less than efficiently cautious. Under simple negligence, no equilibrium is feasible where victim is overly cautious relative to the efficient level, irrespective of injurer's behavior. Using the characterization of equilibria in the parties' strategy space, Section 4 identifies feasible equilibria under a comparative negligence rule for all division of liability rules.

Before deriving the sufficient conditions for the existence of an efficient equilibrium under comparative negligence, a numerical example of the problem illustrates the absence of a general result.

Figures 1.1 and 1.2 show, for a fixed level of uncertainty, equilibria generated by each of the three negligence regimes (comparative negligence for all division of liability rules between 0 and 1) for varying legal standards and different accident avoidance technologies, respectively. The equilibria are plotted in the player's strategy space. In each figure the x and y axes represents the injurer's and victim's level of care, respectively. The origin represents the efficient equilibrium. The level of uncertainty in each simulation is fixed at a ratio of 1 with the mean of the distribution of possible legal standards, which is distributed normally for purposes of the simulation. Figure 1.1 represents an accident avoidance technology where the level of care of each party is a substitute for the other's, whereas Figure 1.2 represents a situation where they are complements.[13]

The set of points that comprise the upper boundary in each figure represent equilibria under a contributory negligence regime for varying levels of the legal standard of care, and the set of points that make up the lower boundary represent equilibria under a simple negligence regime. Each point in the shaded region represents an equilibrium under a comparative negligence regime for a legal standard, division of liability rule pair. The reader will note that while comparative negligence is clearly most efficient in the numerical simulation in Figure 1.1 (as there is some legal standard, division of liability rule pair that supports the efficient equilibrium), contributory negligence is relatively more efficient in the simulation in Figure 1.2.[14]

1.3.1 Simple Negligence

In this subsection, the paper characterizes the set of equilibria that can be induced by the social planner under a simple negligence cost apportionment rule for small levels of uncertainty. Not surprisingly, irrespective of the accident avoidance technology, distributional form of uncertainty, and level of uncertainty, there is no standard of care that would induce the victim to choose a level of care at or above the efficient level. As the victim's cautiousness does not affect a finding of liability by a jury, his only incentives are his private cost of care and reducing the likelihood of an accident. The injurer on the other hand will be responsive to changes in the standard of care for small enough levels of uncertainty, as the cost of reducing his expected share of liability diminishes with a decrease

[13]Formally, the accident avoidance technology used to generate the equilibria in Figure 1 is the function $L(x_n, x_v) = \left(\frac{1}{1+x_n} + \frac{1}{1+x_v}\right)$ where $\lambda = 9$ is the ratio of monetary damage to a unit of cost of care, and x_i is party i's level of care. The accident avoidance technology function in Figure 2 is $L(x_n, x_v) = 9\left(\frac{1}{1+x_n x_v}\right)$.

[14]The reader should consider the distance between any ordered pair and the origin as the measure of inefficiency

in uncertainty.[15]

Sufficiently small levels of exogenous uncertainty imply that the social planner can effectively influence the injurer's behavior and induce equilibria that range from injurer being not cautious enough to overly cautious, while victim is always under-cautious. Starting with the standard of care set at the socially efficient level with no uncertainty, which implies an equilibrium with $(x_n, x_v) = (x^*, x^*)$, the paper introduces a small level of uncertainty and predicts the behavior of each agent, characterizing some features of the new equilibrium and asking how injurer will respond to a change in the standard of care. Through this line of reasoning, a subset of equilibria that the social planner can induce in the neighborhood of the socially efficient level is mapped out.

The social planner's problem under a simple negligence regime with evidentiary uncertainty can be expressed as

$$\begin{aligned} & \min_{\bar{x}} && \hat{x}_n + \hat{x}_v + \lambda \cdot L(\hat{x}_n, \hat{x}_v) \;\; s.t. \\ \hat{x}_n &= \operatorname*{argmin}_{x_n} && {c_n}^{SN} = x_n + \lambda \cdot (1 - \Phi_n) \cdot L \\ \hat{x}_v &= \operatorname*{argmin}_{x_v} && {c_v}^{SN} = x_v + \lambda \cdot \Phi_n \cdot L, \end{aligned}$$

where ${c_n}^{SN}$ and ${c_v}^{SN}$ are injurer and victim's individual cost functions. $\Phi_n(\cdot) \equiv \Phi_n(x_n; \bar{x}, \sigma)$ is a twice differentiable, symmetric, and unimodal cumulative distribution function with full support with mean $\bar{x}$ and variance σ. Φ_n represents the likelihood that Injurer will be found negligent by a jury given the standard of care $\bar{x}$ and her level of care x_n. Suppose that the activity in question is driving and the legal standard of care is the speed limit. The Injurer views the speed limit as the mean of the distribution of all possible speed limits that a jury will apply in a tort suit. If the speed limit were 55, for example, half of the randomly drawn juries will find the injurer negligent if she

[15] The injurer's response however is generally ambiguous. Analysis of the second order conditions of the problem reveal that there exist, in equilibrium, functional forms of the accident technology and high enough dispersion of uncertainty that could induce the injurer and victim to jointly stop reacting to an increase in the standard of care as their only incentive is to minimize the likelihood of an accident. The injurer may have no incentive to keep up with progressively increasing standards of care as the probability of liability does not change and an increase in care on his part would induce a decrease in care from victim, which would serve only to increase the likelihood of the accident. Therefore, there it is not generally true that the social planner can induce equilibria where $x_n > x^*$. The intuition behind this is that with enough evidentiary uncertainty (which diminishes the value of additional care in reducing share of accident), progressive increases in the standard of care that induce less care from the victim may actually make it beneficial for injurer to reduce his level of care in response as he cares only about minimizing private plus social cost and not his share of the cost.

observes the speed limit, while 50 percent of the time a jury will impose a stricter standard of care as they believe that the reasonable person standard dictates more prudence. As such, additional care on the part of the injurer will reduce the probability that she is found negligent, and therefore liable under simple negligence.

The problem leads to the following first order conditions:

$$\frac{\partial {c_n}^{SN}}{\partial x_n} = 1 + \lambda \cdot (1 - \Phi_n) \cdot L_n - \lambda \cdot \phi_n \cdot L = 0 \tag{1.1}$$

$$\frac{\partial {c_v}^{SN}}{\partial x_v} = 1 + \lambda \cdot \Phi_n \cdot L_v = 0, \tag{1.2}$$

where $\phi_n \equiv \phi_n(x_n; \bar{x}, \sigma)$ is the probability distribution function of Φ_n. The first term in the Injurer's first order condition is the marginal cost of a unit of care, which the paper assumes increases at a constant rate with care without loss of generality.[16] As indicated, the second term represents the marginal reduction in the Injurer's share of the social cost of his activity for an increase in care. The second term represents the measure of the Injurer's incentive for care based on the benefit of reducing the likelihood of the accident with additional care. Finally, the third term represents the marginal reduction in the likelihood of being found negligent in the event of an accident. This term measures the Injurer's incentive to escape a finding of negligence by a jury.

The victim's first order condition on the other hand consists of only two terms. The first term is the marginal cost of care and the second is the marginal reduction in his share of the social cost of his activity for an increase in care. Since the simple negligence rule ignores the victim's behavior in assigning liability (see Table 1.1), the victim here has no secondary incentive to increase care as any such increase would not affect the likelihood that the injurer is found negligent.

1.3.1.1 Effect of evidentiary uncertainty on level of care

For any social cost function, distributional form of uncertainty, and level of evidentiary uncertainty, there exists no equilibrium where victim's level of care is greater than the efficient level x^*:

[16] As the paper examines the behavior of the parties in the neighborhood of the efficient level of care for small levels of uncertainty, the parties essentially ignore all incentives other than the incentive to escape liability. Therefore the curvature of the private cost function becomes irrelevant.

Let $(\hat{x}_n, \hat{x}_y)$ be an equilibrium. Condition (2) above must hold in equilibrium, implying that $L_v(\hat{x}_v) = -\frac{1}{\lambda \cdot \Phi_n(\hat{x}_n)} > -\frac{1}{\lambda}$ since $\Phi_n < 1$ by definition. This in turn implies that $x_v < x^*$ as L_v is decreasing in care.

Therefore, victim will always reduce his level of care with the introduction of uncertainty. Intuitively, the introduction of any uncertainty gives victim some chance of escaping liability, which induces a reduction in care.

For any social cost function and distributional form of uncertainty, there is some level of evidentiary uncertainty small enough to induce injurer to chooses a level of care above the standard of care.

By assumption, $L(x_n, x_v) > 0$ for any action pair. For any standard of care at or below the socially efficient level, i.e., $\bar{x} \leq x^*$, in equilibrium with no uncertainty injurer will choose $\bar{x}$ in equilibrium. Evaluating condition (16) at such an equilibrium shows that $(1 - \Phi_n) = \frac{1}{2}$ and that $\lambda \cdot L_n \leq -1$. Let $\Theta = \lambda \cdot (1 - \Phi_n) \cdot L_n \leq -\frac{1}{2}$. Fixing $\tilde{x}_v < \bar{x}$, any equilibrium, if it exists, must meet the following condition:

$$\left.\frac{\partial {c_n}^{SN}}{\partial x_n}\right|_{(x_n, \tilde{x}_v)} = 1 - \Theta - \lambda \cdot \phi_n \cdot L(x_n, \tilde{x}_v) \qquad (1.3)$$

For any standard of care below the efficient level, if $\Theta < -1$, injurer will increase care. Intuitively, if the standard of care is set too low, the likelihood of an accident may be great and since injurer is responsible for half of the cost of an accident if it occurs (assuming that she does not respond), she may have a strong incentive to increase her level of care. Supposes that $\frac{1}{2} < \Theta < 1$. Since $\lambda L > 0$ for any action pair, injurer would only increase her care if she could sufficiently reduce the probability of bearing the cost of the accident. This incentive is captured by the value of ϕ_n at the equilibrium. But given the symmetric, unimodal properties of the distributional form of evidentiary uncertainty, a decrease in uncertainty increases the value of ϕ_n at the mean without bound, implying that there must be some σ small enough to induce Injurer to increase care for any action of the victim.

To show that an equilibrium exists such that injurer exercises too much caution and victim exercises too little, consider the problem dynamically. Injurer's increase in care incudes a decrease in care from victim, which in turn induces an increase from injurer, etc. To show that the system is dynamically stable, i.e., is increases in care by injurer and decreases in care by victim get sequentially smaller. Whenever $x_n > \bar{x}$, the Hessian $\begin{pmatrix} \frac{\partial^2 c_n}{\partial x_n^2} & \frac{\partial^2 c_n}{\partial x_n \partial x_v} \\ \frac{\partial^2 c_v}{\partial x_n \partial x_v} & \frac{\partial^2 c_v}{\partial x_v^2} \end{pmatrix}$ is positive semi-definite, i.e., that $\frac{\partial^2 c_n}{\partial x_n^2} > 0$ and that the Determinant of the Hessian is positive. The relevant second derivatives, given that $L_{nv} = 0$, are

$$\frac{\partial^2 {c_n}^{SN}}{\partial x_n^2} = \lambda \cdot (1 - \Phi_n) \cdot L_{nn} - 2 \cdot \lambda \cdot \phi_n \cdot L_n - \lambda \cdot \frac{\partial \phi_n}{\partial x_n} \cdot L \tag{1.4}$$

$$\frac{\partial^2 {c_n}^{SN}}{\partial x_n \partial x_v} = -\lambda \cdot \phi_n \cdot L_v < 0 \tag{1.5}$$

$$\frac{\partial^2 {c_v}^{SN}}{\partial x_v^2} = \lambda \cdot \Phi_n \cdot L_{vv} > 0 \tag{1.6}$$

$$\frac{\partial^2 {c_v}^{SN}}{\partial x_v \partial x_n} = \lambda \cdot \phi_n \cdot L_y < 0 \tag{1.7}$$

Note that the sign of $\frac{\partial^2 c_n}{\partial x_n^2}$ is ambiguous as $\frac{\partial \phi_n}{\partial x_n}$ can not generally be signed. However, $x_n > \bar{x}$ and the symmetric, unimodal nature of Φ_n, imply that $\frac{\partial^2 c_n}{\partial x_n^2} < 0$ and that in turn $\frac{\partial^2 c_n}{\partial x_n^2} > 0$.

Given that this general result holds for any standard of care, it follows that whenever $\bar{x} = x^*$, for a sufficiently small level of uncertainty, there exists an equilibrium where $x_n > x^*$ and $x_v < x^*$. The paper next shows that there must also exist some $\bar{x} < x^*$ that induces injurer to take less caution than is socially efficient. This is not trivially true (although intuition would suggest that lowering the standard of care progressively should induce smaller and smaller levels of effort from injurer) as injurer may always want to choose a level of care greater than the efficient level. However, small enough levels of uncertainty guarantee against that.

At the equilibrium with no uncertainty for a standard of care lower than the efficient level, the value of $L(x_n, x_v) > L(x^*, x^*)$. This implies that everything else being equal (including the level of uncertainty), injurer has a stronger incentive to increase his level of care above the standard of care relative to when $\bar{x} < x^*$.[17] Decreases in the standard of care have a dual effect on injurer's behavior. This duality is demonstrated numerically below.

Let $L = \frac{1}{1+x_n} + \frac{1}{1+x_v}$, $\Phi \sim N(2, \sigma)$,$\lambda = 9$, and $x_v = 2$. Evaluating $\frac{\partial c_n}{\partial x_n}$ when injurer choosing the standard of care, for $\sigma = 0.2$ and $\sigma = 1$, I find

[17]This assertion is based on condition (1).

$$\left.\frac{\partial c_n}{\partial x_n}\right|_{\sigma=0.2} = -11.83$$
$$\left.\frac{\partial c_n}{\partial x_n}\right|_{\sigma=1} = -1.893$$

In order to satisfy the first order condition when $\sigma = 0.2$ and $\sigma = 1$, injurer chooses $x_n = 2.44$ and $x_n = 3.2408$, respectively. So while injurer is more likely to be over-cautious for progressively lower levels of uncertainty, his response is smaller, fixing victim's action. In addition, at the lower level of uncertainty, the smaller increase in care ensures a smaller probability of being found negligent. In the above example $(1-\Phi(2.44; 2, 0.2)) = .0137$ while $(1-\Phi(3.2408; 2, 1)) = .1073$. Since victim's response to injurer is smaller whenever he faces a greater probability of bearing the cost of the accident (which in turn affects injurer's subsequent deviation, etc), we are ensured that in equilibrium injurer chooses a smaller deviation from the mean for progressively smaller levels of uncertainty since we have already shown that the system is dynamically stable for any standard of care and uncertainty that induces additional care from injurer.[18]

It therefore follows that for any standard of care below the efficient level, there is some $(\bar{x}, \sigma)$ pair such that injurer chooses to be overcautious but less than efficiently cautious in equilibrium.

These results collectively imply that whenever evidentiary uncertainty is distributed with infinite support, i.e., $\Phi_n < 1$ for all x_n, the socially efficient outcome is unattainable under the simple negligence regime as the victim chooses to be less cautious than the socially efficient level of care. More importantly, for sufficiently low levels of uncertainty, the social planner can induce equilibria where the injurer is both overly cautious, optimally cautious and not cautious enough by reducing the standard of care below the socially efficient standard.

1.3.2 Contributory negligence

In what follows the set of possible equilibria under a contributory negligence rule is characterized. Considered first is the equilibrium when the legal standard is set at the socially efficient level. When

[18]This dual effect is present for any symmetric, uni-modal distribution

uncertainty is sufficiently small, both parties choose to be overly cautious. This finding is based on a formal characterization of the parties' best response functions for low levels of uncertainty. However, intuition would lead to a similar conclusion; when a small amount of additional care, which is almost costless at very small levels of uncertainty, can almost guarantee that injurer does not bear any of the cost of the accident, she will certainly expend the additional effort. From the victim's perspective, given that there is always some possibility that injurer is found negligent, a small enough level of uncertainty provides a strong enough incentive to exercise care so that he will not be found negligent under such a scenario. Additionally, victim has a strong incentive to choose some level of care near the efficient level as he will bear the burden of the accident with a high probability, implying that he should minimize the likelihood of one.

Considered next is the set of all possible equilibria induced by continuous reductions in the standard of care, noting that continuous reductions in the standard of care will induce a continuous set of equilibria as the parties' best response functions are continuous in $\bar{x}$.[19] The central finding of this section is that certain equilibria cannot be sustained under a contributory negligence rule. Specifically, Section 3.2.2 shows that there exist no equilibria, relative to the socially efficient equilibrium (x^*, x^*) where

- Injurer is overly cautious and victim is efficiently cautious,
- Injurer and victim are both efficiently cautious, or
- Injurer is efficiently cautious and victim is not cautious enough

As demonstrated in this section, since the set of equilibria induced by continuous reductions in the legal standard can not include any of these three regions of the players' strategy space, it also can not include any equilibrium where injurer is overly cautious and victim is not cautious enough as the set would have to include one of the three types of equilibria above due to its continuity.

The social planner's problem under a contributory negligence regime can formally be expressed as follows:

[19] Application of the Theorem of The Maximum guarantees this assertion.

$$\min_{\bar{x}} \quad \hat{x}_n + \hat{x}_v + \lambda \cdot L(\hat{x}_n, \hat{x}_v), \quad s.t.$$

$$\hat{x}_n = \underset{x_n}{\text{argmin}} \quad c_n = x_n + \lambda \cdot L(x_n, x_v) \cdot (1 - \Phi_n) \cdot \Phi_v \tag{1.8}$$

$$\hat{x}_v = \underset{x_v}{\text{argmin}} \quad c_v = x_v + \lambda \cdot L(x_n, x_v)\Big(1 - \Phi_v \cdot (1 - \Phi_n)\Big), \tag{1.9}$$

where $\Phi_i \equiv \Phi_i(x_i; \bar{x}, \sigma)$, an everywhere twice differentiable (in all arguments) cumulative distribution function with mean $\bar{x}$ and variance σ, measures the likelihood that victim is found non-negligent by a jury given his level of care and the legal standard $\bar{x}$.[20] σ measures the evidentiary uncertainty faced by both parties and is exogenously determined.[21][22]

The problem generates the following first order conditions:

$$\frac{\partial c_n}{\partial x_n} = 1 + \lambda \cdot (1 - \Phi_n) \cdot \Phi_v \cdot L_n - \lambda \cdot \phi_n \cdot \Phi_v \cdot L = 0 \tag{1.10}$$

$$\frac{\partial c_v}{\partial x_v} = 1 + \lambda \cdot (1 - \Phi_v(1 - \Phi_n)) \cdot L_v - \lambda \cdot \phi_v \cdot (1 - \Phi_n) \cdot L = 0 \tag{1.11}$$

Letting $\tau = (1 - \Phi_n) \cdot \Phi_v$ and solving for $L(x_n, x_v)$ in (10) and (11) gives

$$L(x_n, x_v) = \frac{1 + \lambda \cdot \tau \cdot L_n}{\lambda \cdot \phi_n \cdot \Phi_v} \tag{1.12}$$

$$L(x_n, x_v) = \frac{1 + \lambda \cdot (1 - \tau) \cdot L_v}{\lambda \cdot \phi_v \cdot (1 - \Phi_n)} \tag{1.13}$$

Setting the two terms equal and rearranging provides the following necessary condition for an equilibrium under contributory negligence:

$$\frac{1 + \lambda \cdot \tau \cdot L_n}{1 + \lambda \cdot (1 - \tau) \cdot L_v} = \frac{\frac{\phi_n}{(1 - \Phi_n)}}{\frac{\phi_v}{\Phi_v}} \tag{1.14}$$

Figure 1.3 shows the relative values of $\frac{\phi_n}{(1-\Phi_n)}$ and $\frac{\phi_v}{\Phi_v}$ for any symmetric distribution. The reader will note, for example, that the right hand side of condition (14) takes on a value of one when injurer and victim each observe the legal standard of care or symmetrically deviate from the standard in

[20]Note that the subscript does not imply that the injurer and victim face different distributions, Φ_v is used as short-hand for $\Phi(x_v)$.

[21]Injurer and victim face the same level of uncertainty and standard of care as they are engaging in the same activity.

[22]Section 6 suggests welfare improvement from endogenous adjustments of σ.

opposite directions.

1.3.2.1 Equilibrium at $\bar{x} = x^*$

For $\bar{x} = x^*$, consider the possibility of an equilibrium in the highlighted regions of the (x_n, x_v) space of Figure 1.4.

Note that when both victim and injurer are observing the standard of care, the RHS=1. This is also true for deviations from the standard of care that are (a) identical or (b) symmetric if in opposite directions. When injurer's deviation from the standard of care is greater than the victim's, the RHS is less than 1, and finally the RHS is greater than 1 whenever victim's deviation is greater than injurer's.

No equilibrium can exist in Regions II and III:

At all points in those regions, $x_n > x_v$, which implies that $|\lambda \cdot L_n| < |\lambda \cdot L_v|$ and $\tau < \frac{1}{2}$. Therefore, the value of the LHS must be greater than 1. However, as Figure 1.3 shows, the value of the RHS in that region must be less than 1 as victim's deviation from the efficient point is larger.

No equilibrium can exist in (a) Region I or (b) at any point where $x_n \geq x^*$ and $x_v = x^*$:

The following constraints must be satisfied in order for such an equilibrium to exist:

$$\lambda \cdot L_v \leq -1 \quad \leq \quad \lambda \cdot L_n < 0 \tag{1.15}$$

$$\Phi_v \leq \frac{1}{2} \quad \leq \quad \Phi_n < 1 \tag{1.16}$$

$$\phi_n \quad \leq \quad \phi_v \tag{1.17}$$

$$\frac{\phi_v}{\Phi_v} \quad \leq \quad \frac{\phi_n}{(1 - \Phi_n)} \tag{1.18}$$

Solving this system of inequalities numerically shows that condition (14) can not be satisfied for the set of solutions to (15) through (18).[23]

These findings imply that if a locally unique equilibrium where $x_n > x^*$ and $x_v > x^*$ exists, then it must be globally unique unless injurer has a profitable deviation to $x_n \leq x^*$ (since there is no possible equilibrium in Regions I, II and III, i.e., victim would not deviate to a level of care below

[23] The system is solved using Mathematica's Reduce command. The solution to the system of equations, including condition (14) is the null set.

x^*). In what follows, the paper shows that there must exist some equilibrium where $x_n > x^*$ and $x_v > x^*$ and that injurer would not deviate to $x_n \leq x^*$.[24]

Best response functions:

Consider injurer's best response to $x_v = x^*$ relative to the efficient point.

$$\left.\frac{\partial c_n}{\partial x_n}\right|_{(x^*,x^*)} = \frac{3}{4} - \frac{1}{2}\lambda \cdot L(x^*, x^*) \cdot \phi_n \tag{1.19}$$

Since $L(x_n, x_v) > 0$ for any action pair, there must exist some σ small enough such that (19) is negative, implying that when victim chooses x^*, injurer would choose $\hat{x}_n > x^*$. Since Φ_n is unimodal and symmetric, the value of the probability density function is unbounded at the mean and such a σ must exists as Φ_n is continuous in σ.

The best response function,BR_n, of injurer is defined implicitly by the identity

$$\frac{\partial c_n(BR_n(x_v), x_v)}{\partial x_n} \equiv 0$$

Differentiating with respect to x_n and solving for BR_n' gives

$$BR_n' = -\frac{\frac{\partial^2 c_n}{\partial x_n \partial x_v}}{\frac{\partial^2 c_n}{\partial x_v^2}}$$

Differentiating with respect to x_n and x_v, the slope of injurer's best response function at $(\hat{x}, x^*)$ can be expressed and evaluated as

$$BR_n'(x_n, x_v, \bar{x}, \sigma) = -\frac{\lambda \cdot L_n \cdot (1 - \Phi_n) \cdot \phi_v - \lambda \cdot L_v \cdot \Phi_v \cdot \phi_n - \lambda \cdot L \cdot \phi_v \cdot \phi_n}{\lambda \cdot L_{nn} \cdot (1 - \Phi_n) \cdot \Phi_v - 2\lambda \cdot L_n \cdot \Phi_v \cdot \phi_n - \lambda \cdot L \cdot \Phi_v \cdot \frac{\partial \phi_n}{\partial x_n}}, \tag{1.20}$$

where BR_n is injurer's best response function. The denominator of (20) is always positive when $x_i > \bar{x}$, since $\frac{\partial \phi_i}{\partial x_i} < 0$. The numerator can be negative or positive. The numerator is negative, implying a positively sloped best response function, whenever

$$|\lambda \cdot L_n \cdot (1 - \Phi_n) \cdot \phi_v| > |\lambda \cdot L_v \cdot \Phi_v \cdot \phi_n| \tag{1.21}$$

[24] This implications is implicit in the construction of the injurer's best response function below

Since $x_v = x^*$, there must be some σ small enough such that this is true.

The slope of BR_n is not monotonic in x_v, as the first and third terms of the numerator are decreasing in x_v and the second term may increase.[25] However, in constructing BR_n it is sufficient to note that the sign of the slope will change at most once.

Next, as $x_v \to \infty$ relative to the efficient point, the additive separability of $L(x_n, x_v)$ implies that $\lim_{x_v \to \infty} L(x_n, x_v) = \underline{L} > 0$.

$$\left.\frac{\partial c_n}{\partial x_n}\right|_{(x^*, x_v)} = \frac{1}{2} - \lambda \underline{L} \phi_n \tag{1.22}$$

Finally, it is trivially true that as $x_v \to 0$, $\frac{\partial c_n}{\partial x_n} \to 1$, implying that injurer will choose some level of care less than x^*, for any level of uncertainty.

The preceding imply together that there must be some level of uncertainty small enough such that (19) and (22) are negative and that $|\lambda \cdot L_n \cdot (1 - \Phi_n) \cdot \phi_v| > |\lambda \cdot L_v \cdot \Phi_v \cdot \phi_n|$.

Considering next the victim's best response function, similar reasoning shows the following features of victim's best response function for small levels of uncertainty:

- above the x-axis when $x_n \to 0$
- above the x-axis when $x_n = x^*$
- Asymptotically goes to $x*$ as $x_n \to \infty$

Figure 1.5 illustrates the injurer's and victim's best response functions for a sufficiently small level of uncertainty. The dotted line represents the value to which injurer's best response function asymptotes, i.e., the value of x_n that solves condition (22).

The intersection of the best response functions represents the equilibrium level of care of injurer and victim. Injurer and victim are both inefficiently too cautious at low levels of uncertainty whenever the standard of care is distributed symmetrically and unimodally. In what follows, the paper examines equilibria for reductions in the standard of care below x^*.

[25]Whether the absolute value of the second term increases before decreasing depends on the relative curvature of the accident technology function and the cumulative distribution function. For small values of σ, the second term will increase as Φ_v increases in value rapidly around the mean.

1.3.2.2 Effect of decrease in standard of care

At each new equilibrium for a given standard of care below x^*, (a) either injurer and victim are both overly cautious relative to the efficient level of care, (b) victim is overly cautious and injurer is at or below x^*, or (c) victim is at or below x^* and injurer is below x^*. That is, there exists no equilibrium where injurer is overly cautious and injurer is at or below x^*. Turning once again to the players' action space, starting from point A (which represents the equilibrium at $\bar{x} = x^*$) in Figure 1.6, the set of equilibria represented by the dotted curves is impossible. That is, a reduction in the standard of care, while it reduces the level of care for both injurer and victim, can not result in any equilibrium where injurer is overly cautious and injurer is at or below x^*.

Since both parties' best response functions are continuous in the standard of care (since it is assumed that Φ is everywhere twice differentiable), continuous changes in the standard of care will induce a continuous set of equilibria. Given this continuity, that there are regions in the action space where an equilibrium cannot exist as an equilibrium in one of the regions highlighted in Figure 1.7 would necessarily have to be supportable under a contributory negligence cost apportionment rule.

Consider the efficient point, where $x_n = x_v = x^*$. For any $\bar{x} < x^*$, $\Phi_n = \Phi_v = \Phi > \frac{1}{2}$, $\phi_n = \phi_v = \phi$, and $\lambda L_n = \lambda L_v = -1$.

Substituting these conditions into (14) gives

$$
\begin{aligned}
\frac{(1-(1-\Phi)\Phi)}{(1-(1-(1-\Phi)\Phi))} &= \frac{\frac{\phi}{(1-\Phi)}}{\frac{\phi}{\Phi}} \\
\frac{(1-(1-\Phi)\Phi)}{(1-(1-(1-\Phi)\Phi))} &= \frac{\Phi}{(1-\Phi)} \\
\frac{1-\Phi+\Phi^2}{\Phi-\Phi^2} &= \frac{\Phi}{(1-\Phi)} \\
1-2\Phi+\Phi^2 &= 0 \\
\Phi &= 1,
\end{aligned}
$$

which contradicts the property that $\Phi_i < 1$ for all actions of injurer and victim.

Consider next some equilibrium where $x_n = x_v < x^*$, i.e., an equilibrium in Set 2 identified in Figure 1.7. At any equilibrium in that set, $\Phi_n = \Phi_v = \Phi$, $\phi_n = \phi_v = \phi$ and $\lambda \cdot L_n = \lambda \cdot L_v < -1$.

Letting $\lambda \cdot L_i = \gamma$ and substituting these conditions into condition (14) gives

$$\begin{aligned}
\frac{1+\gamma(1-\Phi)\Phi}{1+\gamma(1-\Phi+\Phi^2)} &= \frac{\frac{\phi}{(1-\Phi)}}{\frac{\phi}{\Phi}} \\
\frac{1+\gamma(1-\Phi)\Phi}{1+\gamma(1-\Phi+\Phi^2)} &= \frac{\Phi}{(1-\Phi)} \\
1-\Phi+\gamma(1-\Phi)^2\Phi &= \Phi+\gamma(1-\Phi+\Phi^2)\Phi \\
1-\Phi+\gamma(\Phi-2\Phi^2+\Phi^3) &= \Phi+\gamma(\Phi-\Phi^2+\Phi^3) \\
1-\Phi-2\gamma\Phi^2 &= \Phi-\gamma\Phi^2 \\
3\gamma\Phi^2+2\Phi-1 &= 0,
\end{aligned}$$

which implies that an equilibrium can exist only when $-\frac{1}{3} \leq \gamma < 0$, contradicting the condition that $\lambda \cdot L_n = \lambda \cdot L_v < -1$.

Finally, consider some equilibrium where $x_v = x^*$ and $x_n > x^*$, i.e., an equilibrium in Set 1. The following constraints must all hold at such an equilibrium:

$$\begin{aligned}
-1 < \lambda \cdot L_n &< 0 \\
\lambda \cdot L_v &= -1 \\
\frac{1}{2} < \Phi_v &< \Phi_n \\
\phi_n &< \phi_v \\
\frac{\phi_v}{\Phi_v} &< \frac{\phi_n}{(1-\Phi_n)}
\end{aligned}$$

Using numerical solution software, the paper finds no real values that simultaneously satisfy condition (14) and the constraints above.

1.3.3 Summary and numerical example

The foregoing results are illustrated numerically in the player's action space in the Figures 1.8 and 1.9.

Figure 1.8: Contributory Negligence

Using the functional forms and parameter values from the previous example, the problem is mapped into the (x_n, x_v) space, holding fixed the level of uncertainty. The two loci represent the set of equilibria for high and low levels of uncertainty (specifically $\sigma = 1$ and $\sigma = 3$), as marked for various legal standards $\bar{x} \leq x^*$. The origin represents the socially efficient outcome $(x^*, x^*) = (2, 2)$ for the given accident prevention technology and level of damages. The x-axis represents the Injurer's level of care in equilibrium and the y-axis represents same for Victim.

The figure demonstrates the findings in the previous subsection. For sufficiently low levels of uncertainty, the social planner can choose the standard of care such that (i) injurer and victim are both overly cautious (e.g., the reader will note that when $\bar{x} = x^*$, both are inefficiently over-cautious), (ii) injurer is efficiently cautious but victim is overly cautious, (iii) injurer is not cautious enough and victim is overly cautious, (iv) injurer is not cautious enough and victim is efficiently cautious and (v) neither is cautious enough. In contrast, when uncertainty is high, the injurer will not be sufficiently cautious as the payoff to care is weak. Injurer knows that additional care will have a small impact on a finding of negligence, and would rather lower her private cost of care. The victim will choose to be either overly cautious or not cautious enough, depending on $\bar{x}$.

An additional feature of Figure 1.8 which the reader should note is the players' responsiveness to changes in the standard of care. While both are quite responsive to changes in the legal standard when uncertainty is low, changes in the legal standard do not affect equilibrium care with the same magnitude when signals of the standard are weak from the individuals' perspective.

Figure 1.9: Simple Negligence

Using the same functional forms and parameter values from the numerical example above, the problem is mapped into the (x_n, x_v) space, holding fixed the level of uncertainty. The locus of points in each graph represents the equilibria induced by various standards of care. The Figure illustrates the set of equilibria that the social planner can induce under simple negligence for $\sigma = 0.5$ and $\sigma = 10$:

For sufficiently low levels of uncertainty, the social planner can choose the standard of care such that Injurer is overly cautious (e.g., the reader will note that when $\bar{x} = x^*$, she is inefficiently over-cautious), not cautious enough and perfectly cautious by lowering the legal standard of care.

In contrast, when uncertainty is high, the injurer will not be sufficiently cautious as the payoff to care is weak. Injurer knows that additional care will have a small impact on a finding of negligence, and would therefore lower her private cost of care. The victim will choose to be under-cautious irrespective of the level of uncertainty.

An additional feature of Figure 1.9 which the reader should note is the players' responsiveness to changes in the standard of care. While injurer is quite responsive to changes in the legal standard when uncertainty is low, changes in the legal standard do not affect equilibrium care with the same magnitude when signals of the standard are weak from the individuals' perspective.

1.4 Equilibrium under comparative negligence

In this section, the paper examines, for a fixed standard of care and level of uncertainty, the equilibrium level of care of injurer and victim relative to both contributory and simple negligence for any division of liability rule. As expected, ceteris peribus the injurer is most cautious under a simple negligence regime and least cautious under contributory negligence, while the victim is most cautious under contributory negligence and least cautious under simple negligence.[26]

Having shown the relative equilibria for a fixed standard of care and level of uncertainty, the paper combines this result from the findings of Sections 3.1 to 3.3, in which the set of equilibria for sufficiently small levels of uncertainty are characterized. Given the set of equilibria induced under contributory and simple negligence relative to the efficient point and the relation of comparative negligence to those rules, there must exist some standard of care and division of liability rule that induces the efficient outcome.

1.4.1 Comparative negligence relative to contributory and simple negligence

The social planner's problem under a comparative negligence cost apportionment rule, where α is the division of liability rule that determines each party's share of liability in the event both are found negligent, follows:

[26] This ordering is what led to conclusions that comparative negligence must be most efficient. However, this is only part of the story as relative efficiency depends on the equilibria induced in the first place. This result must be combined some characterization of the possible set of equilibria for a given level of uncertainty for any relative efficiency arguments to be valid.

$$\min_{\bar{x}} \qquad \hat{x}_n + \hat{x}_v + \lambda L(\hat{x}_n, \hat{x}_v), \quad s.t.$$

$$\hat{x}_n = \underset{x_n}{\operatorname{argmin}} \quad {c_n}^{PN} = x_n + \lambda L(\cdot)(1-\Phi_n)\Big[\Phi_v + \alpha(1-\Phi_v)\Big] \tag{1.23}$$

$$\hat{x}_v = \underset{x_v}{\operatorname{argmin}} \quad {c_n}^{PN} = x_v + \lambda L(\cdot)\Big[1 - (1-\Phi_n)[\Phi_v + \alpha(1-\Phi_v)]\Big] \tag{1.24}$$

Differentiating (23) and (24) with respect to x_n and x_v respectively leads to

$$\frac{\partial c_n}{\partial x_n}^{PN} = 1 + \lambda L_n(1-\Phi_n)\Big[\Phi_v + \alpha(1-\Phi_v)\Big] - \lambda L\phi_n\Big[\Phi_v + \alpha(1-\Phi_v)\Big] \tag{1.25}$$

$$\frac{\partial c_v}{\partial x_v}^{PN} = 1 + \lambda L_v\Big[1 - (1-\Phi_n)[\Phi_v + \alpha(1-\Phi_v)]\Big] - \lambda L(1-\Phi_n)(1-\alpha)\phi_v, \tag{1.26}$$

where (a) $\Phi_v + \alpha(1-\Phi_v) > \Phi_v$ since $\alpha(1-\Phi_v) > 0$ and (b) $\Phi_v + \alpha(1-\Phi_v) = \Phi_v(1-\alpha) + \alpha < 1$ since $0 < \alpha < 1$.

Equilibrium under comparative negligence relative to simple negligence for any $(\bar{x}, \sigma)$:

Suppose that for some standard of care $\bar{x}$, in equilibrium injurer and victim choose $({x_n}^{SN}, {x_v}^{SN})$ under a simple negligence regime. The following first order conditions must be satisfied in equilibrium:

$$1 + \lambda L_n({x_n}^{SN})(1-\Phi_n) - \lambda L({x_n}^{SN}, {x_v}^{SN})\phi_n = 0 \tag{1.27}$$

$$1 + \lambda L_v({x_v}^{SN})\Phi_n = 0 \tag{1.28}$$

Comparing the second and third terms of (25) and (27), we note that since $\Phi_v + \alpha(1-\Phi_v) < 1$, and since both terms of negative, $\frac{\partial c_n}{\partial x_n}^{PN}\Big|_{({x_n}^{SN}, {x_v}^{SN})} > 0$, implying that *everything else being equal injurer will choose a lower level of care under comparative negligence relative to simple negligence.* Intuitively, this formalizes the notion that injurer has a weaker incentives for care under comparative negligence because there is some non-zero chance that victim will be negligent even if she is herself negligent. Under similar reasoning, Victim will be more cautious under comparative negligence relative to simple negligence.

Equilibrium under comparative negligence relative to contributory negligence for any $(\bar{x}, \sigma)$:

Suppose that for some standard of care $\bar{x}$, in equilibrium injurer and victim choose $({x_n}^{CN}, {x_v}^{CN})$ under a contributory negligence regime. It must follow that the following conditions are met at the equilibrium:

$$1 + \lambda L_n({x_n}^{CN})(1 - \Phi_n)\Phi_v - \lambda L({x_n}^{CN}, {x_v}^{CN})\Phi_v \phi_n = 0 \quad (1.29)$$

$$1 + \lambda L_v({x_v}^{CN})(1 - (1 - \Phi_n)\Phi_v) - \lambda L({x_n}^{CN}, {x_v}^{CN})(\phi_v - \Phi_n \phi_v) = 0 \quad (1.30)$$

Given that $0 < \alpha < 1$, we note that $\alpha \lambda L_n (1 - \Phi_n)(1 - \Phi_v) < 0$ and $\alpha \lambda L (1 - \Phi_v)\phi_n > 0$ for all (x_n, x_v), implying that if $({x_n}^{CN}, {x_v}^{CN})$ solves (31), $\left.\frac{\partial c_n}{\partial x_n}\right|_{({x_n}^{CN}, {x_v}^{CN})}$ must be negative.

Therefore, injurer will always be more cautious under comparative negligence. The reader will also note that the additional caution is increasing in α, everything else being equal. Under similar reasoning, victim will be less cautious under comparative negligence.

1.4.2 Efficiency under comparative negligence

The main result of this paper can best be illustrated graphically. Consider Figure 1.10, where the origin represents the point where both parties exercise the socially efficient level of care:

The figure illustrates the set of equilibria that can be sustained for values of $\bar{x} \leq x^*$ in the neighborhood of the efficient point under both contributory and simple negligence for sufficiently small levels of uncertainty. The set of points below the origin represent equilibria under a simple negligence rule. At $\bar{x} = x^*$, injurer is overly cautious and victim is not cautious enough. Reductions in the standard of care induce injurer to reduce his level of care and victim never exercises the efficient level of caution. The set of points above the origin represent equilibria under a simple negligence regime. At $\bar{x} = x^*$, injurer and victim are both overly cautious. Reductions in the standard of care induce injurer to reduce his level of care and victim is always overly cautious whenever injurer is at least as cautious as the efficient level.

The dotted curve represent the set of possible equilibria under comparative negligence for all values of α, for an arbitrary standard of care. We note that for any standard of care the equilibrium point moves down and to the right, relative to the contributory negligence equilibrium, in the (x_n, x_v) space. Given the continuity of care in α and the standard of care, there must be some $(\bar{x}, \alpha)$ pair that supports an equilibrium at (x^*, x^*).

1.5 Numerical simulation: Who pays what share in equilibrium?

In this section, the theoretical results of sections 3 and 4 are illustrated using numerical solutions of the model. First, using the functional form used throughout, the paper solves for the optimal $(\bar{x}, \alpha)$ pair for various exogenous levels of uncertainty and summarizes the results. Next, more generally the effects of uncertainty, as it asymptotes from zero to infinity, on the injurer's share of the accident under three cost apportionment rules is graphed.

Table 1.2 illustrates, for a specified level of uncertainty, the optimal standard and division of liability rule in addition to the deadweight loss and the injurer's share of damages at the optimum. The reader should note that for low to intermediate levels of uncertainty, injurer and victim choose the efficient level of care (thus, there is no deadweight loss), but also that victim is not barred from complete recovery in expectation. The last column of the table shows injurer's expected share of damages for the various levels of uncertainty.

Table 1.2: Numerical Illustration Of Optimal α And Associated Deadweight Loss For Various Levels Of Uncertainty

σ	α	Optimal Standard of Care	Deadweight Loss	Injurer's Share of Damages
0.1	0.005832	1.747992	0	0.005832
0.2	0.012771	1.554376	0	0.012771
0.5	0.036967	1.115136	0	0.0369672
1	0.085265	0.678773	0	0.085265
2	0.215338	0.801088	0	0.215338
3	0.417716	1.793386	0.0275	0.351022
4	0.551794	1.536766	0.1092	0.376828

In the absence of evidentiary uncertainty, Victim always bears the entire social cost in equilibrium as it is incentive compatible for Injurer to choose the standard of care assigned by the social planner.[27] With the introduction of uncertainty however, Injurer cannot avoid some probability of bearing the

[27] Of course if the standard of care is greater than x^*, Injurer will choose x^* and bear the burden of the social cost while Victim chooses to exercise no care, but $\bar{x} > x^*$ is suboptimal from the social planner's perspective and will not be the standard of care.

cost of the accident.[28]

Next, the paper identifies the share of the damages that Injurer pays in expectation as evidentiary uncertainty goes from zero and to infinity. Plotting the effect of increasing levels of uncertainty on the Injurer's equilibrium expected share of social cost under three regimes (simple negligence, contributory negligence and comparative negligence with a 50/50 division of liability rule), Figure 1.11 illustrates the relationship of evidentiary uncertainty to the likelihood that Injurer will bear, in equilibrium, the social cost in the event of an accident for all regimes when $\bar{x}$ is not fixed, but set optimally for each given level of uncertainty. That is, given a fixed level of evidentiary uncertainty and cost allocation regime, the social planner chooses the standard of care that in equilibrium will induce the minimum social cost.

The implications of the the injurer's share of liability in equilibrium are discussed in Section 6.

1.6 The social planner's welfare maximization problem

In his seminal paper on the subject, Shavell proved the non-existence of a rule of law which can achieve both the optimal level of care and optimal level of activity. Essentially, if the standard of care is set optimally, it will induce both Injurer and Victim to choose the level of care in equilibrium. However, doing so will guarantee the injurer is not the residual bearer of the cost of an accident. As such, the injurer will fail to internalize the externality generated by excesses engagement of the activity.

The objective of this section is to show that a sufficiently small levels of uncertainty can be welfare improving under the assumptions on the accident technology and distributional form of uncertainty made throughout. Formally, the social planner's problem can be written as

$$\min_{\bar{x},\Gamma} \quad W(a_n, a_v, x_n, x_v) = U_n(a_n) + U_v(a_v) - a_n * \hat{x_n} - a_v * \hat{x_v} - F(a_n, a_v)\Big[\lambda L(\hat{x}_n, \hat{x}_v)\Big], \quad s.t.$$

$$\begin{aligned} \hat{x}_n &= \operatorname*{argmin}_{x_n, a_n} \quad W_n = U_n(a_n) - a_n * x_n - F(a_n, a_v)\Big[\lambda L(x_n, x_v)\Gamma(x_n, x_v; \bar{x})\Big] \\ \hat{x}_v &= \operatorname*{argmin}_{x_v, a_v} \quad W_v = U_v(a_v) - a_v * x_v - F(a_n, a_v)\Big[\lambda L(x_n, x_v)(1 - \Gamma(x_n, x_v; \bar{x}))\Big], \end{aligned}$$

[28] This is true if the uncertainty is distributed according to a distribution with infinite support such that $0 < \Phi_n < 1$ for all x_n.

where a_i is party i's level of activity, U_i is party i's utility function and $F(a_n, a_v)$ is a function that measures the aggregate frequency of the level of activity.[29] Assume that agents have identical preferences over the activity level, U_i is concave, i.e., $\frac{\partial U_i}{\partial a_i} < 0$, and the marginal utility of the activity approaches ∞ as activity approaches zero. These assumption on the utility function of the parties essentially guarantees that at the optimum $a_i > 0$. Further, for analytical tractability the paper assumes that $F(a_n, a_v) = a + b$. Although the injurer and victim do not independently impose a risk of an accident, the additive function is a proxy for the fact that a additional engagement in the activity by either increases the chance that an accident will occur. Differentiating W with respect to a_n, a_b, x_n and x_v gives

$$\frac{\partial W}{\partial a_n} = U_n' - x_n - \lambda L \quad = \quad 0 \tag{1.31}$$
$$\frac{\partial W}{\partial a_v} = U_v' - x_v - \lambda L \quad = \quad 0 \tag{1.32}$$
$$\frac{\partial W}{\partial x_n} = -a_n - (a_n + a_v)\Big[\lambda L_n\Big] \quad = \quad 0 \tag{1.33}$$
$$\frac{\partial W}{\partial x_v} = -a_v - (a_n + a_v)\Big[\lambda L_v\Big] \quad = \quad 0 \tag{1.34}$$

Given the concave, symmetric nature of the problem, there must be some $(a_n^*, a_v^*, x_n^*, x_v^*)$ that solves the social planner's problem.

In the absence of uncertainty, injurer and victim will never both choose the efficient level of care if cost is increasing in the activity level of injurer. For an equilibrium where both parties choose the efficient level of care to exist, let $x_n = \bar{x}$ and show that victim will never choose $x_n = \bar{x}$, as injurer would not choose the optimal level of activity whenever he is non-negligent. Let $x_n = \bar{x}$. Injurer's problem can now be written as

$$\max_{a_n} \qquad W_n(a_n) = U_n(a_n) - a_n \cdot \bar{x}, \tag{1.35}$$

Assuming that victim is behaving optimally, i.e., choosing $x_v = \bar{x}$, we note that injurer is no longer solving the social planner's problem. In fact, his first order condition is

$$\frac{\partial W_n}{\partial a_n} = U_n' - \bar{x} = 0 \tag{1.36}$$

[29] I assume a utilitarian social welfare function without loss of generality.

The paper notes that the first order condition for the social planner's problem, (32), differs in that λL is always positive, implying that injurer will choose a higher level of activity in equilibrium than the level of activity that solves the social planner's problem.

Turning to victim's problem, given injurer's choice of $x_n = \bar{x}$ and $a_n > a^*$, victim knows with certainty that he will bear the cost of any accident, leading to the following problem:

$$\max_{a_v, x_v} \qquad W_v(a_v, x_v) = U_n(a_n) - a_v * x_v - (a_n + a_v) * \Big[\lambda L(x^*, x_v)\Big] \tag{1.37}$$

Holding x_n fixed at $\bar{x}$, allow a_n to vary. Differentiating (38) with respect to x_v and a_v gives

$$\frac{\partial W_v}{\partial a_v} = U_v'(a_v) - x_v - \Big[\lambda L(x^*, x_v)\Big] \quad = \quad 0 \tag{1.38}$$

$$\frac{\partial W_v}{\partial x_v} = a_v - (a_n + a_v) * \Big[\lambda L_v\Big] \quad = \quad 0 \tag{1.39}$$

Comparing these first order conditions to (33) and (35), Figure 1.12 shows the plot of these two first order conditions when $a_n = a^*$. Given that assumption, the victim would be solving the social planner's problem since $x_n = x^*$, and therefore represent the point where both parties choose efficient level of care and activity.

Firstly, given the conditions on the social planner's problem we know that there is a unique intersection between graphs of the two first order conditions. The intersection between the solid lines represents the equilibrium where both injurer and victim choose the efficient level of care and activity. The dashed line represents the shift in the victim's problem for $a_n > a^*$. Since injurer's activity level only affects one of the two first order conditions, there is no change in the graph of (39) and as the Figure shows, in equilibrium victim will always choose a higher level of care. In other words, *victim anticipates the inefficiently high level of activity by injurer and given that he bears all the cost of the accident in equilibrium he compensates by increasing his level of care and decreasing his level of activity relative to the efficient levels.* If injurer chose the optimal level of activity, victim would behave optimally as well.

1.6.1 The introduction of uncertainty: Discussion and numerical example

The results of Sections 3 and 4 imply that for sufficiently small levels of uncertainty, there exists some comparative negligence rule that would induce efficient levels of care from both injurer and victim in equilibrium. Turning once again to the injurer's problem and letting $x_n = x^*$[30] gives

$$\max_{a_n} \qquad W_n(a_n) = U_n(a_n) - a_n \cdot \bar{x} - \rho F(a_n, a_v) \cdot \lambda L(x^*, x^*), \tag{1.40}$$

where ρ is some fraction of the damages that injurer expects to pay in expectation. Differentiating (41) with respect to a_n, we have

$$\frac{\partial W_n}{\partial a_n} = U_n' - \bar{x} - \rho \cdot \lambda L(x^*, x^*) = 0 \tag{1.41}$$

Comparting (32),(37) and (42) clearly implies that injurer will choose to engage in the activity less frequently relatively to the problem with no uncertainty, but more frequently than the solution to the social planner's problem. Given the inefficiently high level of engagement, uncertainty has an unambiguously positive welfare affect from the perspective of the injurer's behavior as she internalizes some portion ρ of the social cost of her activity.

Turning to the victim's equilibrium behavior, Figure 1.14 below compares the victim's first order conditions for the social planner's problem, the problem with no uncertainty and the problem with uncertainty. Comparing the figure to Figure 1.13, the dashed line is the shift in the victim's first order conditions relative to the social planner's problem for some $0 < \rho < 1$. Figure 1.14 shows that in solving the problem with uncertainty, the victim will choose a level of caution and level of activity closer to the efficient level, both of which are unambiguously welfare improving.

Numerical Example: Table 1.3 presents numerical solutions to the general welfare problem for various levels of uncertainty using the following functional forms: $U_i = 50{a_i}^{\frac{1}{2}} - 15a_i$,$F(a_n, a_v) = a_n + a_v$, $\lambda = 9$, $L(x_n, x_v) = \frac{1}{1+x_n} + \frac{1}{1+x_v}$, Φ_i is a normal distribution.

Solving the social planner's problem, the optimal level of activity is $a_i = 1.23618$ and the optimal level of care is $x_i = 3.24264$, for net utility of 55.5919. The table contains the solution to the problem

[30] This assumes that the standard of care and division of liability are set such that injurer chooses the efficient level of care. As shown Sections 3 and 4, such a rule must exist for sufficiently small levels of uncertainty whenever the parties' care reduces the likelihood of an accident independently of the others'

first with no uncertainty followed by various levels of uncertainty.

Table 1.3: Numerical Example of Social Planner's Maximization Problem

Variance	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
-	-	-	3.24264	3.80157	0	1.87804	1.20258	54.17185
1	1.7694	0.503	3.24257	3.24205	0.06789	1.81301	1.27266	54.34784
2	1.3271	0.549	3.24257	3.24653	0.15624	1.74202	1.31653	54.59134
3	1.766	0.481	3.24221	3.24224	0.26102	1.66927	1.36759	54.78747

(a) Optimal standard of care, (b) Division of liability rule, (c) Injurer level of care, (d) Victim level of care, (e) Injurer expected share of damages, (f) Injurer level of activity, (g) Victim level of activity, (h) Net welfare

The reader should note the correlation between column (e), the injurer's share of damages, to the decrease in activity level of injurer. As analytically shown in the previous section, the introduction of uncertainty incentivizes injurer to internalize some portion of the externality generated by repeated engagement in the risky activity.

1.7 Conclusion

The existing literature on the bilateral accident problem focuses either on the cautionary behavior of parties in the face of uncertainty or the externality created by the frequency of their engagement in the activity. For a better understanding of the optimal design of tort laws, both problems should be considered simultaneously as such rules affect agents' behavior on both dimensions.

The possibility of a cost apportionment rule that induces injurers and victims to take the efficient level of care and simultaneously internalize some portion of the externality of repeated engagement in the activity has been ruled out in the perfect information setting. The problem has not been addressed in a setting with evidentiary uncertainty because uncertainty generally creates inefficiency in care, generating a trade-off between costly distortions in the parties' level of care and possible gains from more efficient levels of activity. As this paper proves, however, for a general class of accidents efficient care can be induced for sufficiently low levels of uncertainty if the standard of care and division of liability under comparative negligence are optimally chosen. Moreover, under such a rule, in equilibrium injurer is in expectation liable for some non-zero portion of damages, inducing her to internalize a fraction of the social cost of her behavior when she otherwise does not do so with perfect information. As such, the paper shows that less rigid legal standards of care, such as the reasonable care standard, combined with an optimally designed comparative negligence rule are unambiguously welfare improving relative to the perfect information setting.

1.8 Figures

Figure 1.1: Numerical Simulation: Set of Possible Equilibria With Efficient Equilibrium Attainable

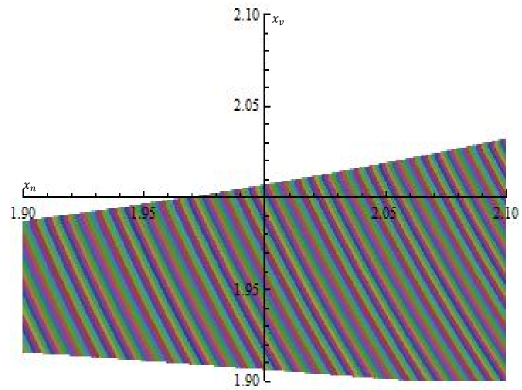


Figure 1.2: Numerical Simulation: Set of Possible Equilibria With Efficient Equilibrium Unattainable

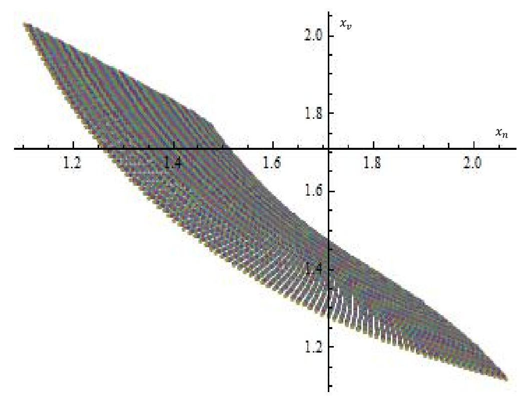


Figure 1.3: Plot of Condition (14)

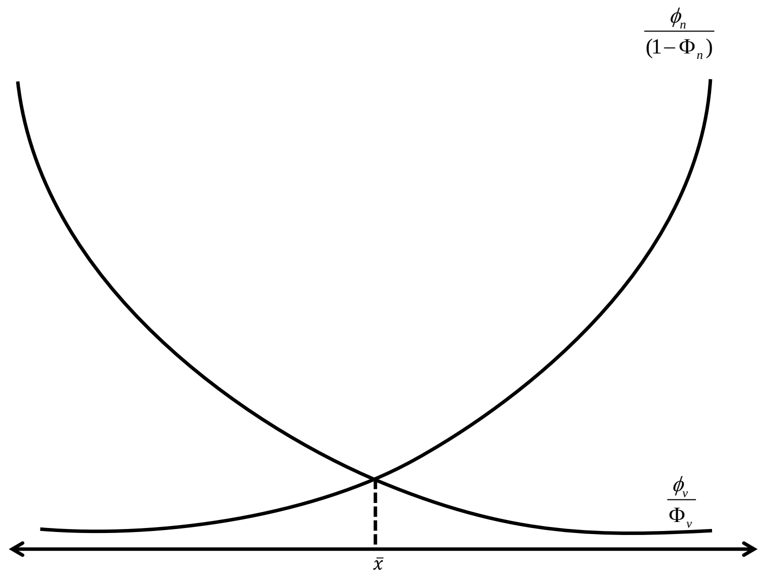


Figure 1.4: Strategy Space By Region

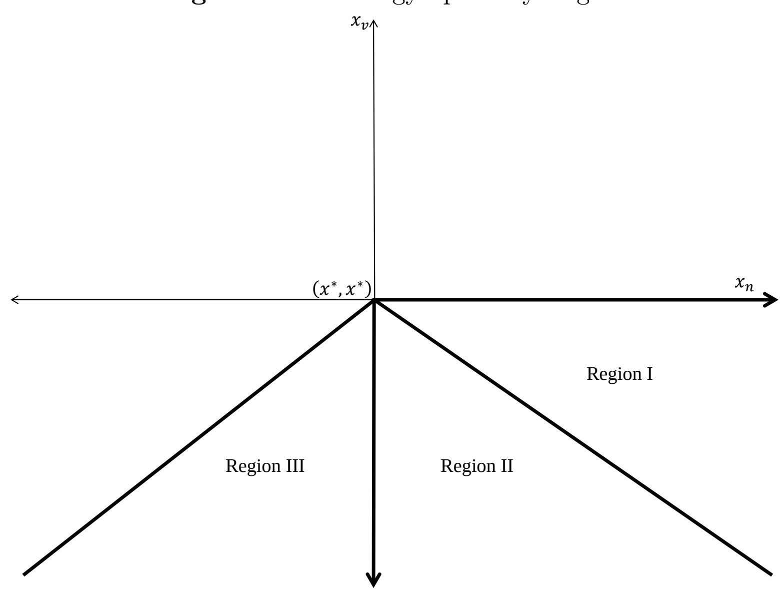


Figure 1.5: Best Response Functions For Injurer And Victim

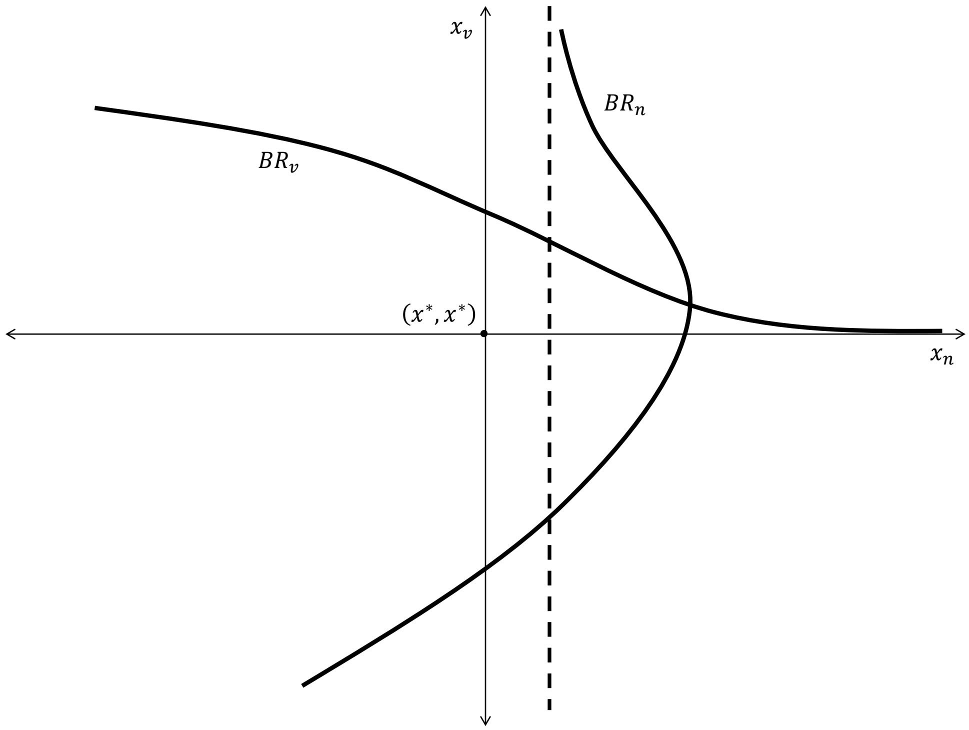


Figure 1.6: Effect Of Decrease In Standard Of Care

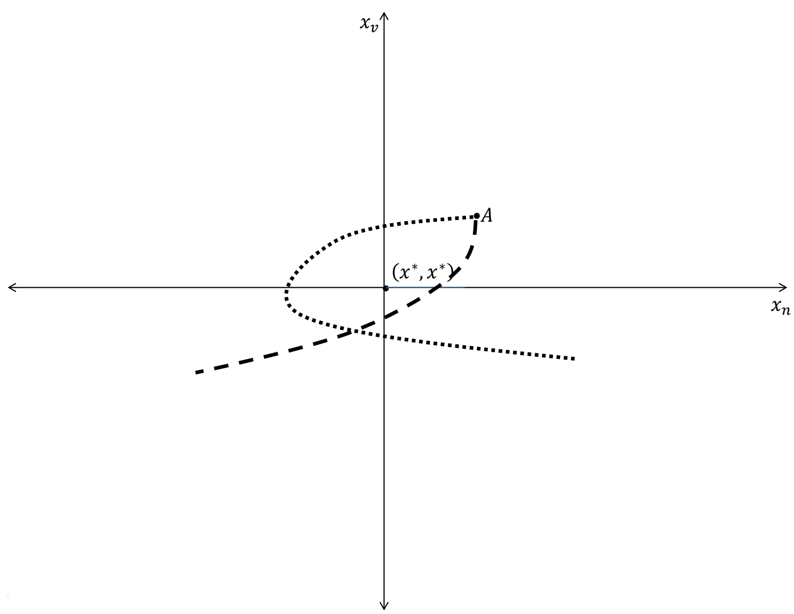


Figure 1.7: Regions In Strategy Space Where Equilibrium Cannot Exist

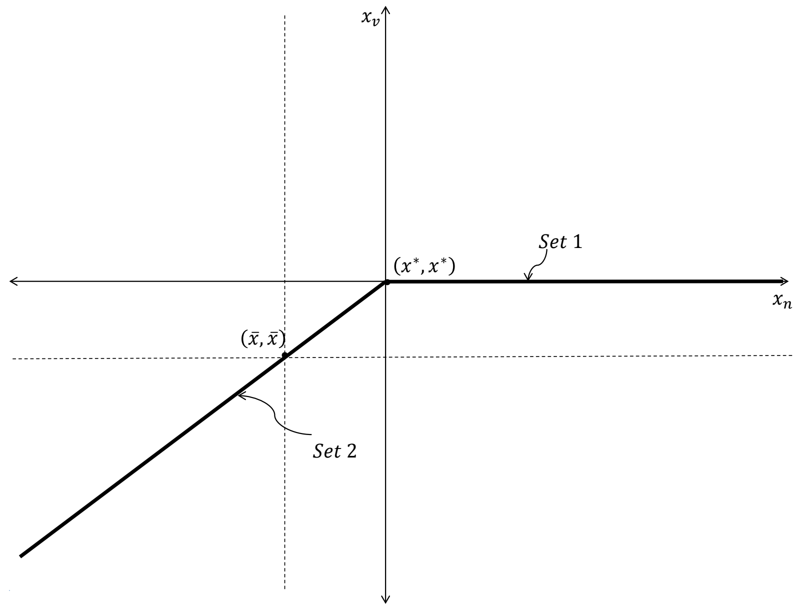


Figure 1.8: Numerical Simulation Showing Set Of Possible Equilibria Under Contributory Negligence For High And Low Levels Of Uncertainty

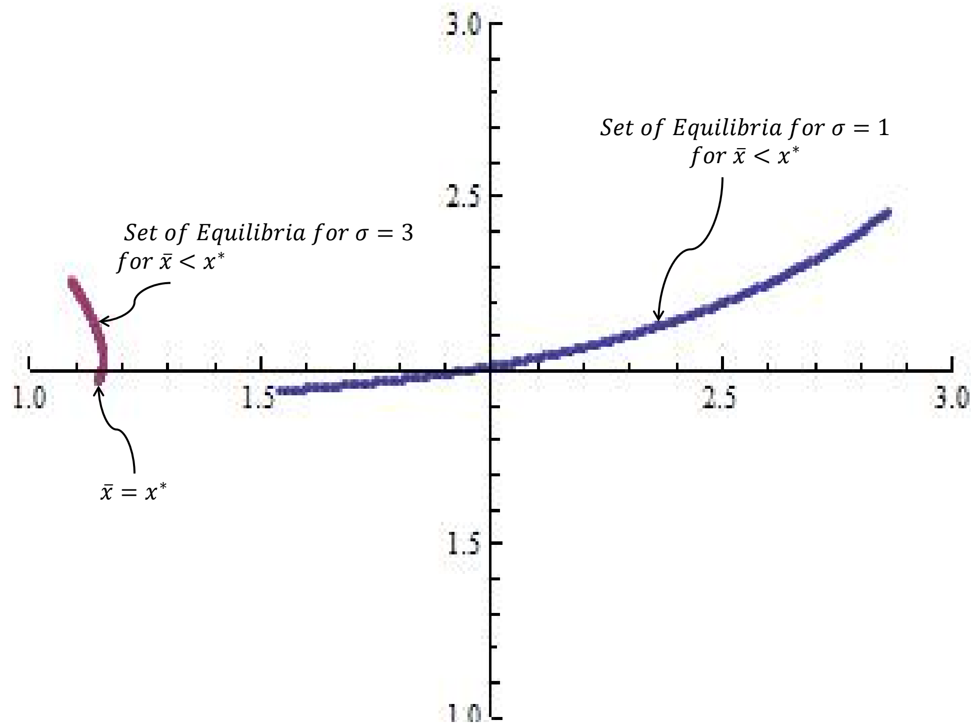


Figure 1.9: Numerical Simulation Showing Set Of Possible Equilibria Under Simple Negligence For High And Low Levels Of Uncertainty

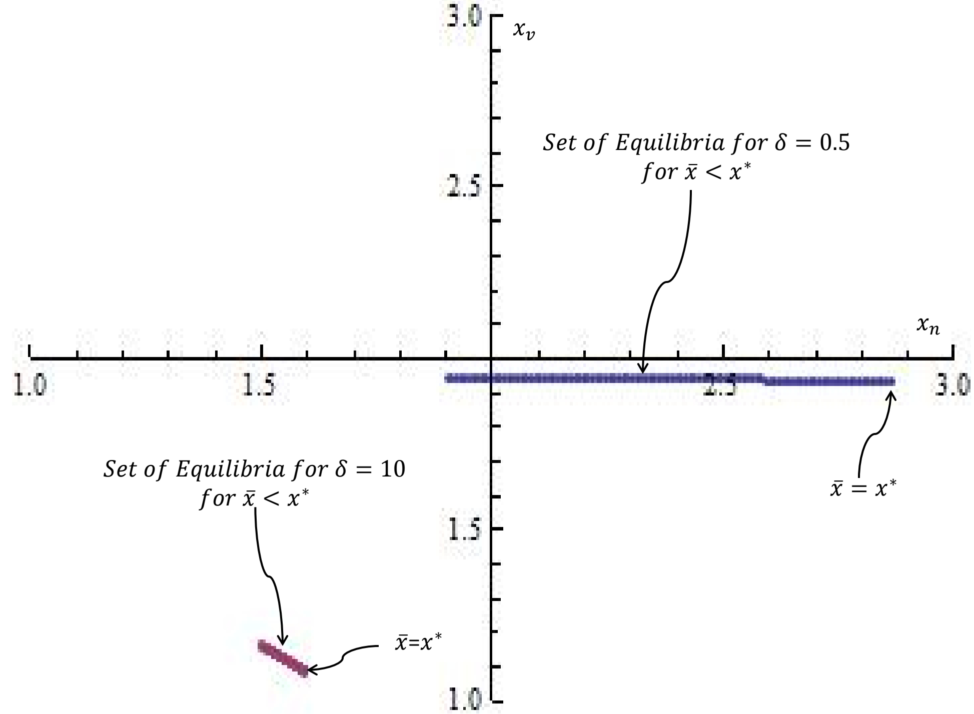


Figure 1.10: Existence Of Efficient Equilibrium Under Comparative Negligence

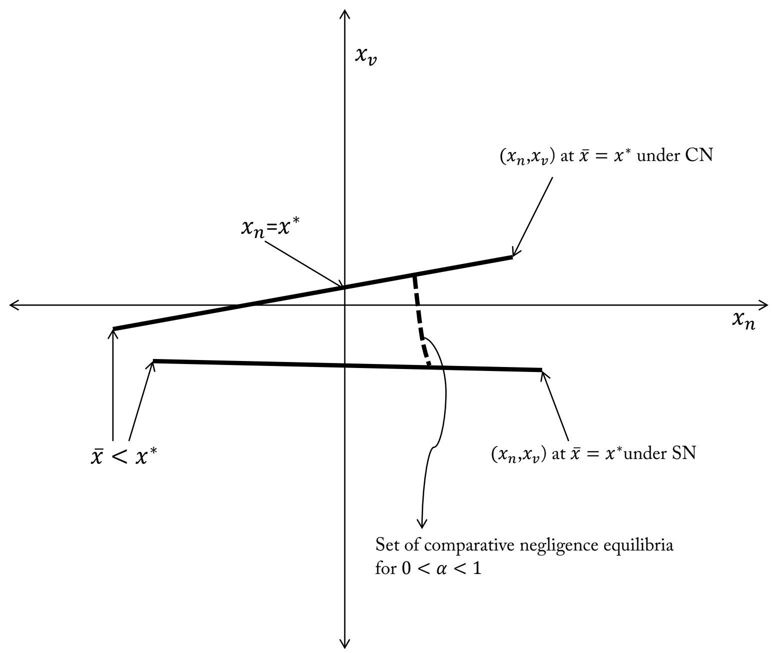


Figure 1.11: Numerical Simulation of Injurer's Share Of Liability As A Function of σ

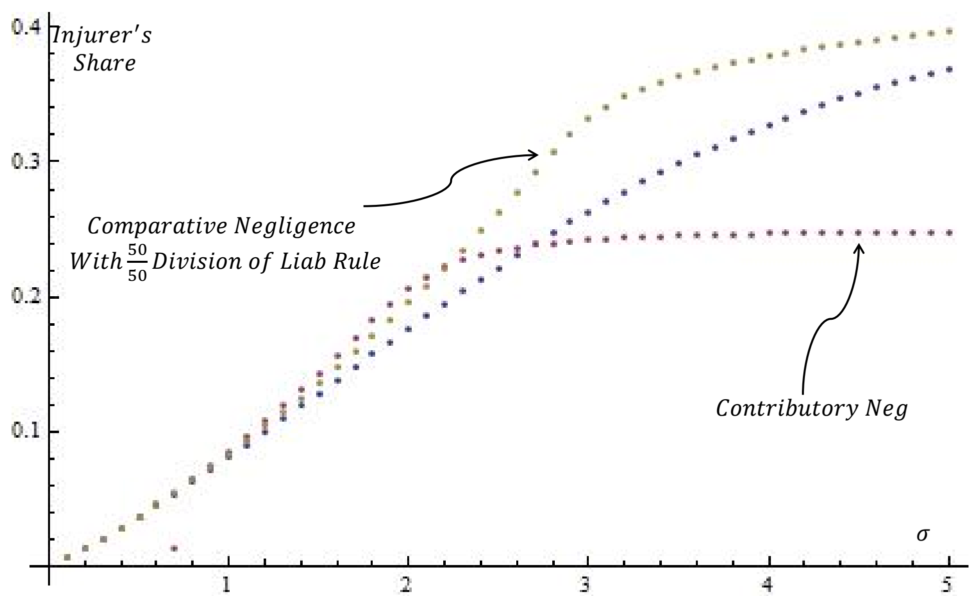


Figure 1.12: First Order Conditions For Victim's Choice Of Activity Level And Care When Injurer Chooses $x_n = x^*$

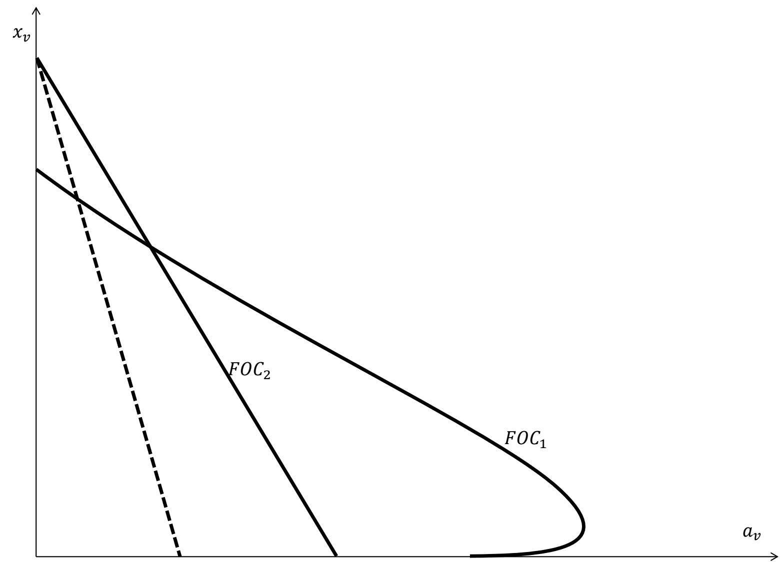
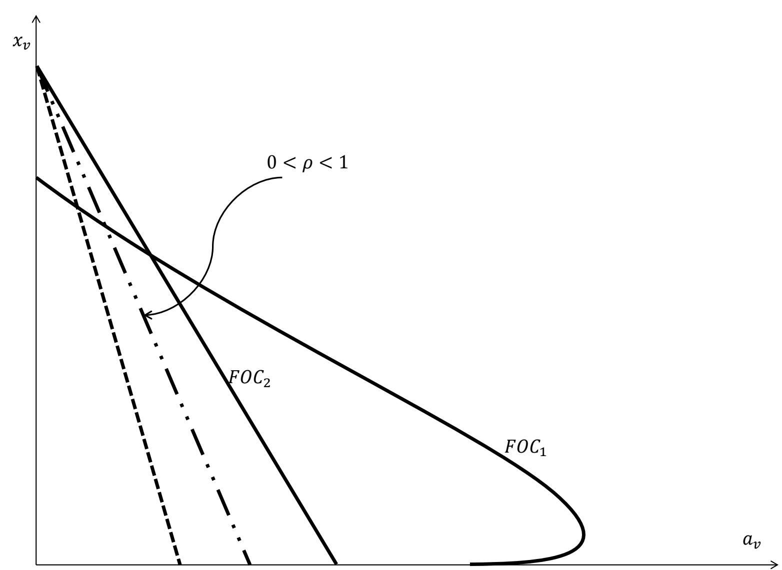


Figure 1.13: Victim's First Order Conditions With And Without Uncertainty



CHAPTER II

A Study of Synchronous Online Versus Traditional Classroom Learning

2.1 Introduction and Literature Review

There should be little doubt to the most casual of observers that the landscape of education in the United States has changed drastically in recent decades. That shift is a by-product of technological advances in computer and internet technology, facilitating the emergence of distance learning as a viable alternative to traditional classroom education. These rapid changes in modes of education have heightened the importance of the question of the effectiveness of distance learning, a topic which has received some attention in the last 20 years.

The popularity of distance or online learning has grown annually at all levels of education. A recent study by The Sloan Consortium, a private institution that collects data on online education, highlights the pervasiveness of distance learning as of the January of 2013. According to the study, the number of students enrolled in at least one online class at degree-granting postsecondary institutions in the United States increased from 1.6 million (9.6% of total enrollment at such institutions) in the Fall of 2002 to 6.7 million (32% of total enrollment) in the Fall of 2011, with some increase in enrollment in every year between. Additionally, the study found that 62.4% of degree-granting postsecondary institutions offered complete online programs (compared to 34.5% in 2002) as of the date of the study.[1]

[1]It is unclear from the report issued by the Sloan Consortium or any available source, however, whether online courses are crowding out traditional courses or attracting students who would not otherwise pursue a postsecondary education

In addition to the study by the Sloan Consortium, others have found increasing use of online tools in course design in many fields of study, including economics[2]. Despite the significant growth in both the number of institutions that offer online classes and degrees and the number of students, there is little convincing empirical evidence of the relative efficiency of the two modes of learning.

The evidence that does exist has been compiled into several meta-studies, which purport to mitigate the empirical inaccuracies of individual, small studies by aggregating their results. In 2010, the U.S. Department of Education released a report summarizing the state of research on the question of the effectiveness of online learning[3]. Having reviewed thousands of studies of varying econometric rigor, the study concluded that students who take all or part of their course online on average perform better than those taking the same courses using traditional face-to-face instruction. A second meta-analysis of studies of distance versus traditional education from 1990 to 2009 reported similar findings, concluding that in 70 percent of the studies reviewed students taking distance education courses outperformed their traditionally instructed counterparts[4].

Many of the individual studies in the literature find no significant difference in student performance between the online and in-person modes of instruction. Wagner, Garippo, and Lovaas (2011) found no difference in performance on a course in business application software taught by the same instructor over the course of ten years, while Ary and Brune (2011) found that delivery method for personal finance students made little difference in final exam performance. Bennett, Padgham, McCarty and Carter (2007) found that 406 traditional and 92 online microeconomics and macroeconomics students had similar averages in the two courses combined.[5]

A large majority of the studies have samples with only a dozen or so observations and rarely control for student selection into distance versus traditional learning. In fact, many of the studies cited in these meta-studies are not published in peer reviewed journals, often appearing in magazines and workshop presentations.

The reliability of these findings, however, is up for debate. An excerpt from Bernard et. al

[2]Navarro (2000). Coates and Humphreys (2003).

[3]U.S. Department of Education. 2010. Evaluation of Evidence-Based Practices in Online Learning: A Meta-Analysis and Review of Online Learning Studies

[4]Shachar M. & Neumann, Y (2010)

[5]Interestingly, however, online students fared significantly better in macroeconomics while traditional students outperformed their online counterparts in microeconomics.

(2009)summarizes well the state of the empirical literature on distance versus face-to-face instruction: "In this article, Bernard, et al. suggest three conclusions to be drawn by researchers. First, the wide variation in comparative effect sizes is pervasive in the literature, but some aspects of the DE modalities appear to be related positively to student learning outcomes. Second, the research designs and methods used for most projects are poorly conceived and executed. Lack of controls via experimental or quasi-experimental design introduces confounding factors that impede the ability to make causal inferences about the relationship between variables and render all other findings automatically subject to skepticism..."

There are, however, some recent studies which use econometric methodology similar to those employed in this paper. Coates et al. (2004) compare scores of online versus face-to-face classroom students on the Test of Understanding College Economics exam, and find that online students fared worse than face-to-face students even after controlling for selection into online courses using commute time as an instrument for taking the online class. The study found that online students performed poorly on the TUCE relative to students in face-to-face classes under OLS, IV and endogenous switching regression estimation, with IV estimation indicating a larger gap between online and face-to-face students. The study, like many of the studies included the meta-analyses mentioned suffered from small sample size issues and was unclear about the exact mode of instruction for "online students". The authors state that the online students were made up of those who took both synchronous and asynchronous online classes, but do not report differences amongst the two groups.

Trawick, Lile, and Howsen find very similar results when looking at undergraduate students taking in-person and online courses in principles of macroeconomics. While it is unclear exactly how online classes are conducted, that paper finds that online students on average perform 6% worse than students enrolled in regular classes. Instrumental variable estimation shows, as in Coates, that the differential is larger when the endogeneity of the online dummy is controlled for. As in the Coates paper however, the findings are based on 120 observations, only 30 of which are online students.

Gratton-Lavoie and Stanley (2007) used data from 58 students in online sections and 98 students in hybrid sections of an introductory economics course to study the efficacy of internet learning. Students in the online sections were provided with lecture materials on a course website and given access to synchronous chat sessions where they could pose questions about homework and the material. They found that an average student would fair better in a hybrid rather than pure online class, based on the performance of both groups on the final exam of the course.

Having broadly surveyed some of the studies included in the aforementioned meta-studies, this paper finds that they typically suffer from one or more problems. Firstly, the studies often do not have well-defined classifications of various modes of instruction. For example, classes categorized as online classes in these studies are often college courses in which the instructor provides supplemental resources to students on a course web page and/or courses where lectures are broadcast using telecommunication or internet technology in addition to being offered in-person. So while some studies claim that distance learning results in superior outcomes, the results could very well be driven by a comparison of students who have access to both in-person lectures and online resources to those who do not have the additional online resources.

Much of the econometrically rigorous literature on education, on the other hand, is related to standard classroom learning, focusing on the effects of allocation of resources on various outcome measures such as labor market wages, standardized test scores, and school attendance. This is likely because of a shortage of reliable and rich data on online learning, especially synchronous online learning, which has become possible quite recently with the advent of internet technology that allows students and instructors to connect in virtual classrooms.

This paper takes advantage of a unique learning setting provided by a standardized test preparation company that offers both in-person and synchronous online classes. While several studies claim to be measuring the effects of synchronous online classes, it is unclear whether the online platforms studied provided a truly "live" classroom experience where students can interact with one another as well as with the instructors. For example, it is unclear whether a video broadcast of a live lecture should be classified as a synchronous online class.

Using a cross-section of approximately 1,300 students provided by the company, the paper measures the differential Graduate Management Aptitude Test (the GMAT) outcomes of students enrolled in the two types of classes. Like their in-person counterparts, live on-line classes meet once a week for 3 hours, according to a pre-determined class schedule, for 9 weeks, and online and in-person students have access to identical material. The institutional details of the online setting are discussed in Section 2.

The paper assumes a simple theory of learning, that it is some function of ability, effort, and the mode through which information is presented to students. While the paper abstracts from the underlying mechanisms that may influence more or less "learning"[6] through use a of a particular mode of instruction, there are plausible reasons to believe there may be a difference in the effectiveness of classroom learning versus learning through synchronous online classes. Foremost amongst those reasons must be the variation in the level of engagement experienced by the students in the two settings.

Any study of the effectiveness of modes of learning must control for the characteristics of the students. Sample statistics show that the students who take online classes are very similar to those who take in-person classes in terms of observable characteristics. Most notably, the two groups have near identical diagnostic scores, which this paper takes as a proxy for ability. Two statistically significant differences between the two groups are that online students are more likely to have previously taken the test and are less likely to be from New York City, Chicago, Los Angeles, or Boston, the 4 largest cities by origin of students in the cross-section. These differences are discussed further in the paper and controlled for in the various specifications.

The distinguishing features of this paper, relative to the existing literature, are that the difference in mode of instruction between the control and treatment groups in the study is well defined, the sample size is relatively large, the outcome variable is an objective measure of learning, and the control for ability directly measures the set of skills measured by the outcome variable. The paper proceeds as follows. Section 2 discusses the institutional details of the study and provides sample statistics, Section 3 provides the empirical analysis for ordinary least squares, instrumental variable

[6]The paper presumes throughout that test scores on the GMAT reflect how much a particular student has learned relative to her diagnostic score, and are not influenced by the psychology of the test-taker. More precisely, the paper presumes that any such phenomenon, while clearly present, is not systematically correlated with whether the student took an online or in-person class

and endogenous switching regression estimations, and Section 4 estimates the results for students that live in either New York, Los Angeles, Boston or Chicago. Section 5 concludes.

2.2 Study Design and Sample Statistics

2.2.1 Details of Study

The GMAT is a standardized test required by admissions committees of MBA programs world-wide and consists of a verbal and quantitative section. The company in question offers online and in-person 9 weeks courses to students interested in a rigorous test preparation program. As part of the course fee, students receive a set of instructional texts designed by the company, online resources such as practice question sets, flash cards, and instructional video labs, and 27 hours of classroom instruction. At the time of data collection, the company offered in-person classes in 20 US cities, Montreal, Toronto, London and Paris. The company also began offering online classes in 2007.

Online classes are conducted through a web site that provides virtual white-boards and video and audio communications technology to users. Students are provided with microphones and headsets and participate in written and oral instruction. Instructors use slides provided by the company and have access to various writing and highlighting tools to supplement the materials on the slides. A student can post questions, comments and answers to instructor questions using a built-in chat mechanism or use her microphone to speak to the entire class. Classes are staffed with two instructors in the event of internet or computer malfunction, but also to accommodate quicker responses to student comments and questions via chat.

Each student in the cross-section self-selected into either a synchronous online class or traditional in-person class. During the period, online and in-person classes were taught by instructors drawn from the same pool. Instructors are assigned by the administration based on time availability and location (for the in-person classes). All instructors are interviewed and hired by the corporate office, mitigating the possibility of local variation in the hiring process, and thus systematic quality differences. They undergo a rigorous 2-month probationary training period after which they conduct a final teaching test. Additionally, the class curriculum is standard across instructors, as slides for each of the 9 three hour class sessions (with the same slides used in both in-person and online classes) are prepared and distributed to instructors by the curriculum development team, and timing guidelines for coverage of the material is standardized across instructors. As such, there is little variation in the content taught by the different instructors.

The data set contains information on 1293 students who were enrolled in either an online or in-person class from 2007 to 2010, with 304 online and 989 in-person students. For each student, the data set includes overall GMAT score (Score), the number of students enrolled in the class (ClassSize), an indicator variable for whether she took an online or in-person class, the student's gender (Gender=1 indicates that student was male), diagnostic test score (Diagnostic), diagnostic verbal sub-score (VerbalDiag), diagnostic quantitative sub-score (QuantDiag), physical address, an indicator for whether the student had previously taken the test (GMAT=1 indicates that student had previously taken the test), and an indicator for whether student took a weekday or weekend class (WeekDay=1 indicates a weekday class).

Controls included in the regressions are variables which may affect the student's learning experience. For each student in the cross-section, diagnostic score serves as a proxy for ability. Class size is included as it may have an impact on learning through various channels, such as peer effects and the quantity of personal attention. Included also is an indicator variable for whether the person has previously taken the GMAT. The impact of having taken the test previously may have one of two effects: a student may have reached some threshold level of preparedness beyond which learning may be impractical or the student may be a faster learner due to familiarity with the material. Finally, the regressions include dummy variables for students who live in large metropolitan cities. There are a significant number of students from New York City, Los Angeles, Chicago and Boston. Students from these cities may be characteristically different in terms of learning ability and effort, for example through access to outside resources. This possibility in Section 4.

2.2.2 Sample Statistics

Table 2.1 provides sample means for various measures included in the study.

Score is the student's self-reported final test score, Diagnostic is the student's diagnostic test score[7], GMAT=1 indicates that a student had previously taken the GMAT, Gender=1 indicates that the student is male, VerbalDiag is the student's verbal sub-score on her diagnostic test, QuantDiag is the student's quantitative sub-score on the diagnostic exam, and ClassSize is the number of student enrolled in her class, be it online or in-person.

T-tests of the difference in means of the various variables show that the mean difference in Score, the GMAT indicator variable, and class size are significantly different across online an in-person students at the 95% confidence level, while differences in means of the remaining variables are not.

The summary statistics do not seem to suggest characteristic differences between students who take the online course and regular course with respect to initial ability. Figure 2.1 separately plots the density function of the sample distribution of diagnostic scores for students in both online and in-person. While the online students display a bit more dispersion, the distributions are quite similar. Analysis of the distributions of diagnostic scores on the verbal and quantitative sections of the test suggest that while the two groups are nearly identical in terms of initial "verbal" ability, the online students have a small head start on the quantitative section of the test. To the extent that different types of starting ability affect learning and in particular affect the student's receptiveness to different modes of learning, the paper controls for initial verbal and quantitative scores. See Figures 2.2 and 2.3 for sample distributions.

2.3 Empirical Analysis

2.3.1 Ordinary Least Squares Estimation

The paper first estimates the parameters of the following equation using Ordinary Least Squares estimation:

$$Score = \beta_0 + \beta_1 Online + \beta X + \mu, \tag{2.1}$$

where *Score* is student's GMAT score, *Online* is an indicator variable which equals 1 if the student

[7]Students enrolled in either an online or in-person class are required to take a diagnostic test during the first week of class. Student diagnostic tests were individually examined. Observations were selected on the following criteria: (1) diagnostic completed within 4 hours, (2) both verbal and quantitative section of diagnostic completed, (3) diagnostic exam taken during first week of class. 700 observations were dropped for failing one or more of these requirements

took an online, X is a vector of controls which includes the student's diagnostic score (Diagnostic), verbal diagnostic score (VerbalDiag), quantitative diagnostic score (QuantDiag), the number of students enrolled in the student's class (ClassSize, SizeSq), indicator variables for whether the student resided in New York, Los Angeles, Boston, or Chicago based on the student's address (NY,LA,BOS,CHI), an indicator variable for whether the student had previously taken the test (GMAT), a gender dummy (Gender), and a dummy variable for whether the student took a weekday or weekend class (WeekDay).

Column (1) of Table 2.2 shows the estimates of the parameters of Equation (1). The coefficient of interest $\hat{\beta}_1 = -13.8$, indicating that conditional on the controls, on average students who take in-person class students outscore online students by roughly 14 fewer points [8] In addition, the regression shows that score outcomes are highly correlated with initial ability and that students who had previously taken the exam fared significantly worse than those who had not, suggesting that additional classroom instruction is not as effective for these individuals.

Given the relatively small sample size and the possibility of influential observations affecting the OLS estimate of $\hat{\beta}_1$, Column (2) of Table 2.2 displays the estimates of the parameters of a quantile regression at the median of the distribution. The results suggest that the OLS results are not driven by outliers.

2.3.2 Selection into Online classes

The underlying theoretical model employed by the paper is that learning is some function of initial ability, learning ability, effort and the mode with which information is transmitted to students. The error term in the OLS estimate is theoretically capturing unobserved learning ability and effort, as initial ability and mode of instruction are observed[9]. Given self-selection into the treatment group, however, the error term could potentially be capturing systematic differences in effort and learning ability between online and in-person students. To measure the effectiveness of classroom instruction alone, random assignment of students to online and in-person classes would be required. In the

[8] The GMAT is scored on a 200-800 point scale, with a median score a 540. A 14 point difference corresponds roughly to a 4 to 5 percentile point difference, depending on the part of the distribution.

[9] To the extent diagnostic scores are a good measure of initial ability

absence of random assignment, the paper uses the minimum distance from each student's residence to an in-person classroom location as an instrument that measures the likelihood that the student took an online class. Presumably physical location from class locations is uncorrelated with effort exerted by the student, implying that the IV estimation captures the difference in instructional effectiveness only.[10]

The direction of the selection bias, if any, was unclear at the outset of the empirical exercise, therefore the paper is theoretically agnostic about the direction of any endogeneity (that is, $E(\mu|Online)$ could be positive or negative). Students who self-select into online classes may be systematically less likely to exert effort, an unobservable captured by the error term, suggesting that the OLS estimator β_1 overstates the effectiveness of traditional classroom learning relative to live online learning. Alternatively, students who are more comfortable with computer technology may be more likely to take an online class, given the conveniences afforded by that mode of instruction. This may in turn be correlated with the ability to learn, again an unobservable captured by the error term in the OLS specification. If so, the OLS estimator would tend to understate the effectiveness of traditional classroom learning relative to live online learning.

To account for the possible endogeneity of the variable of interest, this paper contemplates two specifications, an two stage instrumental variable estimation model (a linear probability and probit first stage are reported) and an endogenous switching regression model.

2.3.3 Instrumental Variable Estimation

The paper first considers potential instruments for online classes. To be a valid instrument, the variable must be correlated with the decision to take an online class, but not to the error term in equation (1). Table 2.3 shows the correlation between taking an online class and some potential instruments.

Gender and previous experience with the test, while correlated with choice of mode of instruction, are ruled out as instruments because they may be correlated with effort or otherwise correlated with the error term. Distance from classroom locations and square root of distance are both correlated

[10] Section B of the appendix address the possibility that distance is to some extent correlated with learning ability by using OLS and IV regressions on a sub-sample of students

with whether a student enrolls in an online or traditional class and presumably uncorrelated with effort. Any correlation between distance and effort would be related to characteristic differences amongst students who live in large metropolitan areas and those who do not (for example, access to additional resources, better learning ability conditional on ability observed in diagnostic exam). In addition to being significantly more correlated with taking an online class, first stage linear regressions of the Online indicator variable on both distance and square root of distance indicate that square root of distance, as specified in the following section, indicate that square root of distance is a stronger instrument.

2.3.3.1 Linear First Stage

This section considers a two stage least squares estimation of the differential effects of an online and in-person class on test scores where the first stage estimates are derived using ordinary least squares. In the first stage, the following linear equation is estimated:

$$Online_i = \beta_0 + \beta_1 \sqrt{Distance_i} + \beta X_i + \epsilon, \tag{2.2}$$

where $\sqrt{Distance}$ is the square root of minimum distance from an in-person class location offered by the company. X is a vector of controls.

Equation (3) below is then estimated for the predicted value of $Online$, namely $\widehat{Online}_{RootDist}$ with the coefficient of interest being δ_1:

$$Score = \delta_0 + \delta_1 \widehat{Online} + \delta X + \mu \tag{2.3}$$

Table 2.4 summarizes the results. Column (1) reports the results of the estimation of Equation (2) while Columns (2) reports the results of the second stage.

First, the F-statistics from the the estimation of equations (2) suggests that jointly with the other regressors, square root of minimum distance is a strong instrument for whether a student takes an online class. The first stage regression also indicates that conditional on distance, gender, and GMAT experience, students in the sample in New York City, Los Angeles, Chicago and Boston were less likely to take an online class, and that being in an online class is strongly correlated with class size, with online classes having significantly more students.

Given $\widehat{Online}$, Column (2) reports the results of the second stage. The variable of interest in Column (2) is OHAT_OLS_ROOTDIST, which is the predicted value of *Online* from Equation (2). According to the model, the marginal effect of taking an in-person class, conditional on ability and other observable characteristics, is 45 points on the exam. The estimate suggests that if randomly assigned to online versus in-person classes, conditional on the marginal effects of ability, class size and other controls, students in in-person classes would outperform those in online classes by 45 points, as opposed to the marginal effect of 14 points suggested by the OLS estimation. Stated differently, a comparison of the OLS and IV regressions imply that students in the sample enrolled in online classes were systematically better learners (either through effort or learning ability), thereby biasing the OLS estimating toward zero.

2.3.3.2 Tests for Endogeniety of Regressor Online

The Durbin-Wu-Hausman test, which estimates the equation $Score = \beta_0 + \beta_1 Online + \beta_2 X + \rho\eta + \mu$, where η is the error from the first stage equation, suggests that the null hypothesis that the regressor *Online* is exogenous cannot be rejected for any p-value less than 0.1.

Furthermore, despite the substantial difference in the OLS and IV estimates from the previous section, a test of whether the OLS and IV estimates are significantly different is conducted. A Hausman test, where the statistic $T_H = \frac{(\hat{\beta_{IV}} - \hat{\beta_{OLS}})^2}{\hat{V}(\hat{\beta_{IV}} - \hat{\beta_{OLS}}})$ is computed. The results of the Hausman test suggest that the null hypothesis that the difference in OLS and IV coefficients from the previous section are not systematically different cannot be rejected.

2.3.3.3 Probit First Stage

The probit model predicts the probability that a student takes an online class given a set a characteristics. The latent variable interpretation is that the observed Online indicator is a function of an underlying choice based on a set of characteristics. More formally, suppose that

$$y^* = x'\beta + \mu,$$

where y^* is not observed, but instead we observe

$$y = \begin{cases} 1 & if y^* > 0 \\ 0 & if y^* \leq 0 \end{cases} \tag{2.4}$$

Given (2) and (3), we have $Pr(y = 1) = Pr(x'\beta + \mu) > 0 = Pr(-\mu < x'\beta) = F(x'\beta)$ where $F(\cdot)$ is the c.d.f. of $-\mu$. Per Wooldridge (2002), directly using predicted values of probability of being in an online class will result in erroneous standard errors. This issue can be sidestepped by using the prediction of probability of being in an online class in a two-step IV estimation as in the previous section.[11]

Table 2.5 summarizes the results of the two stage estimation, where once again the first stage is a probit regression on the observable characteristics of each student and the square root of her minimum distance from an in-person class. According to this model, the marginal effect of taking an online class is approximately 28 points.[12]. In the section that follows, the paper considers the two estimation methods, concluding that the probit first stage regression provides a more economically intuitive measures of predicted online score.

2.3.3.4 Linear versus Probit First Stage

Given the drastically different estimates of Sections 3.2.1 and 3.2.2, the paper investigates the two models a bit further. Examination of scatter plots of the fitted values for both first stage estimations tell a compelling story. Figure 2.4 shows the scatter plot of predicted values of $\widehat{Online}_{OLS}$ (small scatters) and $\widehat{Online}_{Probit}$ (large scatters) against the square root of minimum distance for all observations where the minimum distance from a classroom is greater than 50 miles. While the probit model essentially assigns a probability of 1 to observations beyond a certain threshold distance, the OLS model assigns vastly different fitted values to observations that are clearly factually alike in terms of likelihood to enroll in an online class. For example, realistically, a person who lives 500 miles from a classroom location is no less likely to be enrolled in an online class than one who lives 5,000 miles. The OLS first stage, however, does make the distinction by assigning too much weight

[11]See Wooldridge, Prodecure 18.11

[12]The regressions in this section were conducted using both distance and square root of distance. Unlike the linear first stage estimation of the previous section, the estimate using a first stage probit is not very sensitive to choice of functional form of distance

to other characteristics, namely class-size. To provide more context, a calculation (based on the set of co-efficients in Column (1) of Table (4)) reveal that being in a class of 14 and residing 2,800 miles from the nearest in-person class would make a student half as likely to be enrolled in a an online class as one in a class of 22 and residing 4,800 miles away. Given the small sample size and the potential effect of influential observations, the OLS model does not provide as sensible a measure as the probit model.

2.3.4 Endogenous Switching Regression Model

In this section, the paper considers the following model:

$$O_i = 1 \quad if \quad \gamma Z_i + \mu_i > 0 \tag{2.5}$$

$$O_i = 0 \quad if \quad \gamma Z_i + \mu_i \leq 0$$

$$Score_{Oi} = \beta_O X_{Oi} + \epsilon_{Oi} \quad if \quad O_i = 1 \tag{2.6}$$

$$Score_{Pi} = \beta_P X_{Pi} + \epsilon_{Pi} \quad if \quad O_i = 0, \tag{2.7}$$

where individual i chooses a regime (online or in-person class) based on a set of decision criteria Z_i. The criteria used are minimum distance from an in-person class offered and a vector of regressors that capture other characteristic differences. The subscripts O and P refer to online and in-person.

Assume that μ_i, ϵ_{Oi}, and ϵ_{Pi} have a trivariate normal distribution, with mean vector zero and covariance matrix:

$$\Omega = \begin{pmatrix} \sigma_u^2 & \cdot & \cdot \\ \sigma_{21} & \sigma_1^2 & \cdot \\ \sigma_{31} & \cdot & \sigma_2^2 \end{pmatrix}, \tag{2.8}$$

where σ_u^2 is a variance of the error term in (9), σ_1^2 and σ_2^2 are the variances of the error terms in (10) and (11), and σ_{21} and σ_{31} are the covariances of (9) and (10) and (9) and (11), respectively. According to Maddala(1983), given these assumptions, the logarithmic likelihood function of the system of equations (9) through (11) is:

$$lnL = \{O_i w_i[ln(F(\eta_{1i}) + ln(f(\epsilon_{1i}/\sigma_1)/\sigma 1) + (1 - O_i)w_i[ln(1 - F(\eta_{2i})) + ln(f(\epsilon_{2i}/\sigma_2)/\sigma_2]\}, \quad (2.9)$$

where F is the c.d.f. of a normal distribution, f is the corresponding p.d.f., w_i is an optional weight for observation i and $\eta_{ji} = \frac{(\gamma Z_i + \rho_j \epsilon_{ji}/\sigma_j)}{\sqrt{1-\rho_j^2}}$, where $\rho_1 = \frac{\sigma_{21}^2}{\sigma_\mu \sigma_1}$ is the correlation coefficient between ϵ_1 and μ and $\rho_2 = \frac{\sigma_{31}^2}{\sigma_\mu \sigma_2}$ is the correlation coefficient between ϵ_{Pi} and μ.

The unconditional expectations, equations (12) and (13), and conditional expectations, equations (14) through (17), below can be calculated by estimating the parameters of the model. This paper uses the user-written `movestay` command in Stata developed by Michael Lokshin and Zurab Sajaia of the World Bank. The results are summarized in Table 2.6.

$$E(Score_{Oi}|X_{Oi}) = \beta_O X_{Oi} \quad (2.10)$$

$$E(Score_{Pi}|X_{Pi}) = \beta_P X_{Pi} \quad (2.11)$$

$$E(Score_{Oi}|O_i = 1, X_{Oi}) = \beta_O X_{Oi} + \sigma_1 \rho_1 f(\gamma Z_i)/F(\gamma Z_i) \quad (2.12)$$

$$E(Score_{Oi}|O_i = 0, X_{Oi}) = \beta_P X_{Oi} - \sigma_1 \rho_1 f(\gamma Z_i)/(1 - F(\gamma Z_i)) \quad (2.13)$$

$$E(Score_{Pi}|O_i = 1, X_{Pi}) = \beta_O X_{Pi} + \sigma_2 \rho_2 f(\gamma Z_i)/F(\gamma Z_i) \quad (2.14)$$

$$E(Score_{Pi}|O_i = 0, X_{Pi}) = \beta_P X_{Pi} - \sigma_2 \rho_2 f(\gamma Z_i)/(1 - F(\gamma Z_i)) \quad (2.15)$$

Given the estimates of the parameters, Table 2.6 summarizes the conditional and unconditional expectations:

The results suggest that a randomly selected person would score roughly 28 points higher by taking an in-person class rather than an online class. The advantage of this model relative to the IV estimation of Section 3.2.1 to 3.2.3 is that it allows for differential effects of student characteristics in the two regimes. The IV regressions of assume, for example, that having taken the GMAT in the past has a particular effect on final score, irrespective of the mode of instruction. Suppose however, that the online setting is more conducive for someone who has a certain level of familiarity with the test, but not for someone who is seeing the material for the first time, then the IV estimates may unreasonably force the same effect on both groups.

2.4 Empirical Analysis of Students in Metropolitan Areas

One concern about the exogeneity of distance is addressed here. It is possible that distance from an in-person classroom location is simply capturing the differential in learning abilities of students who live in large cities versus those that do not. Using a sub-sample of 627 observations (students residing in New York City, Los Angeles, Boston or Chicago), the paper estimates Equation (1). Although not statistically significant at the 95% confidence interval, the estimate of the coefficient of interest mirrors the estimates from the entire sample. Table 2.8 summarizes the results of the OLS regression.

To test whether the OLS estimate is biased by selection into online classes, I use IV estimation with a probit first stage, using square root of distance as an instrument for having enrolled in an online class. The IV Estimates are reported in Table 2.8. The IV estimates suggest that within the sub-sample of students who lived in large metropolitan cities, online students tended to be better learners, as the coefficient of interest increases from -13 in the OLS estimates to -67.

2.5 Conclusion

The study of the effectiveness of online education is a topic of empirical study that is growing in popularity. This paper uses a unique learning environment whereby students study identical materials from identical curricula, but in different settings. The paper finds strong evidence that in-person learning, in the context of a test preparation class, is more effective than synchronous online learning. The effectiveness varies slightly with the estimation method, but in general it appears that students in in-person classes out score their online counterparts by between 13 and 28 points on the Graduate Management Aptitude Test after controlling for selection bias. Interestingly, the differences in OLS and other estimations that control for selection tend to suggest that students in the sample who selected an online class tended to be better learners. One possibility is that any study comparing these modes of education should consider the possibility that comfort with technology may be a decision criteria for students who have online and traditional options.

2.6 Figures

Figure 2.1: Kernel Density Plots of Diagnostic Score By Regime

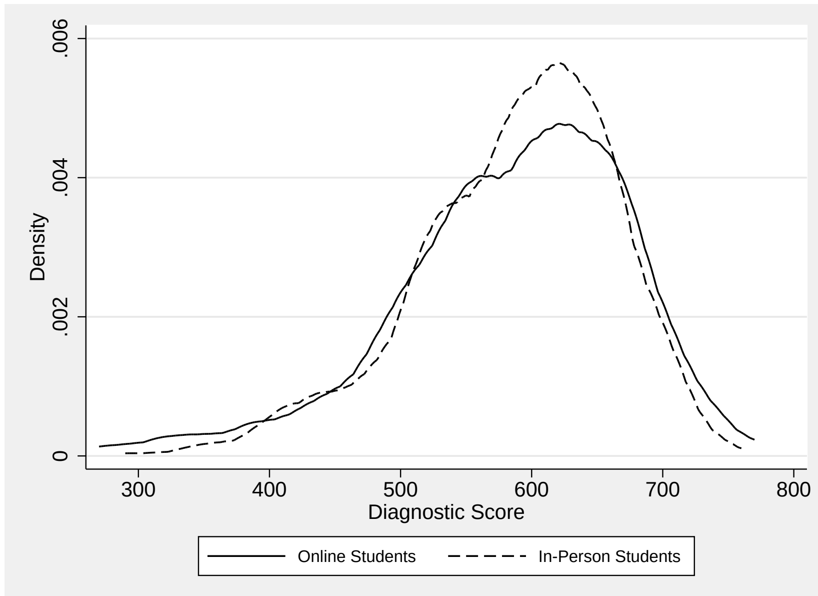


Figure 2.2: Kernel Density Plots of Verbal Diagnostic Score By Regime

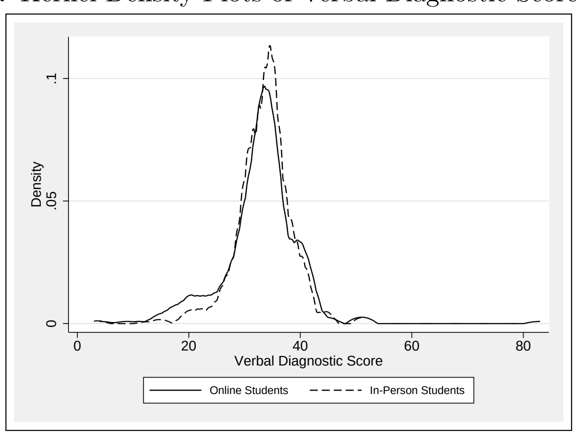


Figure 2.3: Kernel Density Plots of Quantitative Diagnostic Score By Regime

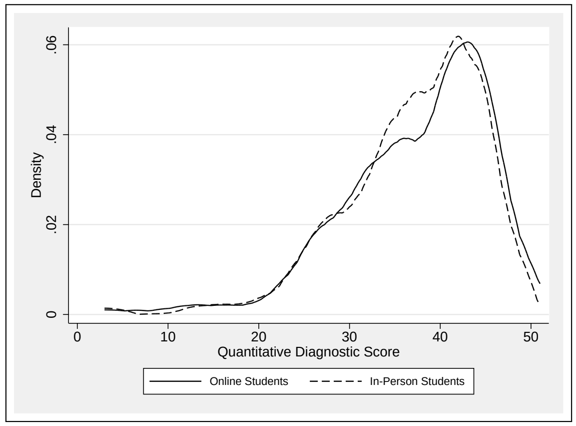
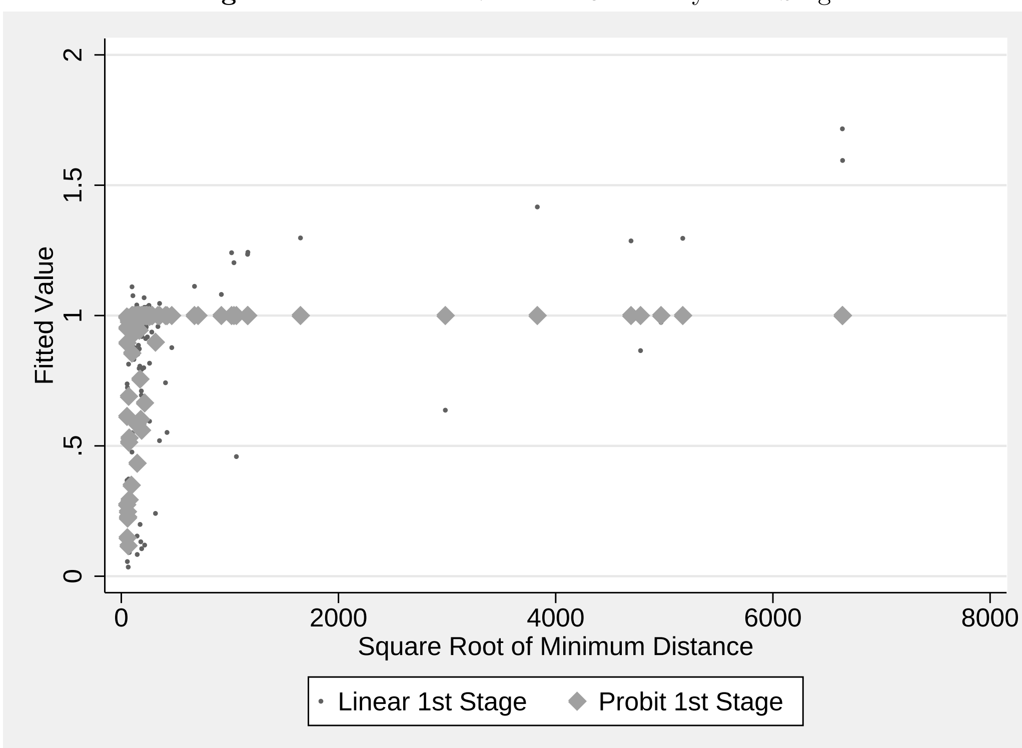


Figure 2.4: Predicted Values of $\widehat{Online}$ by First Stage



2.7 Tables

Table 2.1: Summary Statistics by Class Type

Class Type	Score	Diagnostic	GMAT	Gender	VerbalDiag	QuantDiag	ClassSize
In-Person	648.87	588.55	0.094	0.624	33.45	37.25	13.71
Online	630.03	586.02	0.135	0.671	32.96	37.52	22.46
Total	644.45	587.96	0.104	0.635	33.34	37.31	15.76

Table 2.2: OLS and Quantile Regression Results

	(1) OLS	(2) QREG
Online	-13.8414*	-14.1828
	(5.7655)	(8.0392)
Diagnostic	0.8077***	0.9921***
	(0.1238)	(0.1643)
VerbalDiag	0.5550	-0.8843
	(0.9288)	(1.2193)
QuantDiag	-0.5927	-1.6679
	(0.9204)	(1.2073)
GMAT	-31.3428***	-32.8710***
	(4.7131)	(7.0712)
Gender	1.5113	1.5311
	(3.1628)	(4.4789)
NY	12.7424***	9.3035
	(3.7525)	(5.2645)
LA	3.4919	-4.7156
	(6.5354)	(8.9055)
CHI	-0.3694	9.3858
	(6.7854)	(9.0439)
BOS	5.2329	-2.6305
	(6.6716)	(10.0528)
WeekDay	1.7474	3.5244
	(3.0841)	(4.3546)
ClassSize	0.5245	0.9056
	(1.4889)	(2.0777)
SizeSq	-0.0067	-0.0146
	(0.0502)	(0.0700)
_cons	166.8998***	148.8041***
	(18.2702)	(25.2963)
N	1286	1286
r2	0.5829	
F	128.5973	

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.3: Cross-correlation table

Variables	Online	Distance	RootDist	Gender	GMAT
Online	1.000				
Distance	0.215	1.000			
RootDist	0.388	0.906	1.000		
Gender	0.057	0.002	0.005	1.000	
GMAT	0.042	0.047	0.075	-0.059	1.000

Table 2.4: IV Estimate With Linear First Stage

	(1) OLS_FIRST_STAGE	(2) OLS_SECOND_STAGE
RootDist	0.0108***	
	(0.0019)	
Diagnostic	0.0002	0.8166***
	(0.0007)	(0.1292)
VerbalDiag	-0.0022	0.4514
	(0.0051)	(0.9754)
QuantDiag	-0.0021	-0.6792
	(0.0051)	(0.9605)
GMAT	0.0158	-30.9624***
	(0.0208)	(4.7118)
Gender	-0.0238	0.9452
	(0.0149)	(3.1890)
NY	-0.0821***	8.9919*
	(0.0181)	(4.5529)
LA	-0.0729	0.4061
	(0.0390)	(6.8726)
CHI	0.0153	-0.8764
	(0.0312)	(6.8233)
BOS	-0.0752***	1.9334
	(0.0214)	(7.0503)
WeekDay	-0.0345*	0.6579
	(0.0146)	(3.1897)
ClassSize	-0.0837***	-2.2173
	(0.0061)	(2.3135)
SizeSq	0.0041***	0.1285
	(0.0002)	(0.1013)
OHAT_OLS_ROOTDIST		-45.5769*
		(21.9678)
_cons	0.4851***	183.5207***
	(0.0812)	(21.6090)
N	1286	1286
r2	0.6393	0.5824
F	165.4614	128.0687

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.5: IV Estimation with Probit First Stage

	(1)
	PROBIT_FIRST_STAGE_IV
Online	-28.0858*
	(12.2257)
Diagnostic	0.8117***
	(0.1179)
VerbalDiag	0.5085
	(0.8756)
QuantDiag	-0.6315
	(0.8668)
GMAT	-31.1721***
	(5.0758)
Gender	1.2572
	(3.2197)
NY	11.0590**
	(3.9867)
LA	2.1068
	(6.4758)
CHI	-0.5969
	(6.4919)
BOS	3.7519
	(7.3002)
WeekDay	1.2584
	(3.1466)
ClassSize	-0.7061
	(1.7578)
SizeSq	0.0540
	(0.0681)
_cons	174.3601***
	(19.0096)
N	1286
r2	0.5810
F	

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.6: Estimation results : Endogenous Switching Regression Model

Variable	**Coefficient**	**(Std. Err.)**
Equation 1 : Score_1		
Diagnostic	1.005	(0.164)
VerbalDiag	-0.398	(1.070)
QuantDiag	-1.918	(1.298)
GMAT	-23.318	(9.515)
Gender	-6.462	(6.916)
NY	18.310	(10.464)
LA	9.557	(16.352)
BOS	28.621	(54.842)
CHI	1.616	(15.808)
WeekDay	12.257	(6.415)
ClassSize	0.916	(6.873)
SizeSq	-0.021	(0.185)
Intercept	115.477	(68.046)
Equation 2 : Score_0		
Diagnostic	0.661	(0.199)
VerbalDiag	1.471	(1.554)
QuantDiag	0.266	(1.414)
GMAT	-33.996	(6.183)
Gender	2.597	(3.767)
NY	4.808	(4.354)
LA	-3.213	(7.315)
BOS	-2.759	(7.765)
CHI	-0.564	(7.376)
WeekDay	-4.292	(3.735)
ClassSize	-7.141	(2.022)
SizeSq	0.362	(0.077)
Intercept	233.793	(24.235)
Equation 3 : Online		
Diagnostic	0.001	(0.004)
VerbalDiag	-0.009	(0.026)
GMAT	0.152	(0.181)
BOS	-0.839	(0.650)
CHI	0.341	(0.228)
WeekDay	-0.194	(0.111)
ClassSize	-0.238	(0.055)
SizeSq	0.013	(0.002)
QuantDiag	-0.001	(0.026)
Gender	-0.191	(0.116)
NY	-0.025	(0.146)
LA	-0.366	(0.219)
RootDist	0.193	(0.028)
Intercept	-1.559	(0.701)
Equation 4 : lns1		
Intercept	3.994	(0.041)
Equation 5 : lns2		
Intercept	4.055	(0.026)
Equation 6 : r1		
Intercept	-0.019	(0.184)
Equation 7 : r2		
Intercept	1.383	(0.265)

Table 2.7: Endogenous Switching Regression Model Results

Expectation	Value
$E(Score_{Oi}\|X_{Oi})$	638.79
$E(Score_{Pi}\|X_{Pi})$	667.24
$E(Score_{Oi}\|O_i = 0, X_{Oi})$	641.25
$E(Score_{Pi}\|O_i = 1, X_{Pi})$	726.87

Table 2.8: Linear and Quantile Regressions: Students in Metropolitan Areas

	(1)	(2)
	OLS_Metro	QREG_Metro
Diagnostic	0.3002	0.1967
	(0.2541)	(0.3754)
VerbalDiag	3.8238*	3.6194
	(1.8980)	(2.8350)
QuantDiag	2.8334	4.2914
	(1.8767)	(2.7143)
Gender	1.4343	1.1112
	(4.4939)	(6.8467)
GMAT	-32.9199***	-41.0661***
	(6.9968)	(11.7700)
ClassSize	1.1286	2.8912
	(2.1024)	(3.3967)
SizeSq	-0.0071	-0.0505
	(0.0754)	(0.1201)
WeekDay	2.1434	6.2771
	(4.4639)	(6.8128)
Online	-13.9235	-16.0475
	(10.5689)	(14.8474)
_cons	228.9912***	231.5093***
	(28.8480)	(45.0607)
N	627	627
r2	0.5308	
F	71.3219	

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2.9: Estimation results : IV

Variable	**Coefficient**	**(Std. Err.)**
Equation 1 : Score		
Diagnostic	0.326	(0.252)
VerbalDiag	3.699	(1.902)
QuantDiag	2.587	(1.822)
Gender	0.537	(4.604)
GMAT	-32.981	(7.892)
ClassSize	-3.955	(3.013)
SizeSq	0.224	(0.121)
WeekDay	1.484	(4.575)
Online	-67.680	(23.107)
Intercept	252.955	(31.611)
Equation 2 : Online		
RootDist	0.062	(0.104)
Diagnostic	0.003	(0.010)
VerbalDiag	-0.007	(0.074)
QuantDiag	-0.023	(0.071)
Gender	-0.136	(0.185)
GMAT	-0.115	(0.332)
ClassSize	-0.275	(0.099)
SizeSq	0.014	(0.003)
WeekDay	-0.068	(0.181)
Intercept	-1.676	(1.362)
Equation 3 : athrho		
Intercept	0.572	(0.261)
Equation 4 : lnsigma		
Intercept	4.016	(0.035)

Table 2.10: Estimation results : ivregress

Variable	**Coefficient**	**(Std. Err.)**
Online	-26.700	(11.545)
Diagnostic	0.806	(0.118)
VerbalDiag	0.559	(0.879)
QuantDiag	-0.565	(0.870)
GMAT	-30.549	(5.124)
Gender	1.851	(3.227)
NY	9.539	(4.590)
LA	-0.060	(6.776)
BOS	-0.306	(8.102)
CHI	-3.995	(6.870)
WeekDay	0.730	(3.222)
Intercept	178.251	(15.899)

CHAPTER III

Intertemporal Income Shifting by Firm: Evidence from The Tax Reform Act of 1986

3.1 Introduction

This paper investigates whether and to what extent firms engage in tax avoidance through timing of income recognition. Economic theory would suggest that given income from its business activities, a firm would seek to minimize the tax-cost of doing business through tax planning whenever the benefits of doing so outweigh the costs. The particular aspect of tax planning of interest for this paper involves the allocation of income to future tax periods in which the firm faces a lower statutory tax rate. Ideally, firms would prefer to recognize a given unit of income in years with relatively a lower statutory tax rate; they are, however, constrained in their choice by the costs of doing so. These costs include administrative and implementation costs, managerial compensation plan timing issues and costs associated with the possibility of an audit and underreporting penalties, and the political costs of reporting less income on their financial statements.[1]

While income earned in a given income period is subject to the corporate income tax for that period, firms have a certain degree of legal latitude as to which tax period a particular income item is reported to the Internal Revenue Service. In addition, firms can engage in tax avoidance behavior even in the event that income shifting is not legally justified under a reasonable interpretation of the tax code. This paper does not distinguish between the two types of behavior and looks for evidence

[1]See Scholes, Wolfson, Erickson, Maydew and Shevlin, pages 141-147 for a discussion of conflicts between financial reporting and tax planning.

of either.

While firms have an incentive to under-report taxable income in any year with a relatively high statutory tax rate, the paper makes the simplifying assumption that they have an incentive to report high levels of income on their financial statements in any given year.[2]. This contrast between incentives is the basis of the identification strategy employed in this paper.

Theoretically, how actively or aggressively a given firm engages in the behavior of interest should be a function of the potential tax savings associated with the activity, which can in turn be measured by the difference in the statutory tax rates across different tax years. The Tax Reform Act of 1986 provides an opportunity to measure the extent to which firms engage in aggressive tax behavior. Among the dozens of major changes to the corporate income tax, The Act changed the top statutory corporate tax rate from 46 percent for income years ending before July 1, 1987 to 34 percent for income years beginning on or after July 1, 1987 [3]. The tax rate for income years including July 1, 1987 was a blended rate, ranging from 40 to 34 percent depending on a firm's fiscal year end. Firms whose 1986 tax year did not include July 1, 1987 faced a 46 percent statutory tax rate in 1986, while firms with July through November year ends faced a lower tax rate[4]. Finally, all firms faced a 34 percent tax rate for the 1988 tax year. Table 3.1 summarizes the statutory rates faced by firms with various fiscal year ends for the 1986, 1987 and 1988 tax years.

Consider a firm with a fiscal year that goes from August 1 to July 31. Its 1986 fiscal year is defined as the period from August 1, 1986 to July 31, 1987 and includes July 1, 1987. Its 1986 tax rate is therefore determined by blending the rate of 46 percent, which it faced for the first 11 months, and the rate of 34 percent, which it faced for the last month of that fiscal year, for a blended rate of 45 percent for the 1986 tax year. Its 1987 tax year is completely within the new tax rate of 34 percent.

This unique statutory tax rate assignment provides firms with different fiscal year ends with differential incentives for deferring taxable income. For example, a firm with a November year end

[2]This paper abstracts from the issue of management incentives that is the focal point of most of the accounting literature and assumes that in general shareholder and manager incentives are aligned and that firms report the highest level of income justified under GAAP rules

[3]Joint Committee on Taxation, *General Explanation of The Tax Reform Act of 1986* (1987), 271-274

[4]A firm's 1986 tax year is defined as the accounting period beginning on or after January 1, 1986.

has a strong incentive to defer income from its 1985 fiscal year to its 1986 year, while a firm with a December year end has no incentive to do so. The paper exploits the exogenous differential incentives to examine the extent to which firms engage in intertemporal income shifting on their tax returns by treating July through November year end firms as the treatment group and the remaining firms in the panel as the control group.

A major empirical hurdle of this paper is measurement of the behavior of interest. Measuring income shifting across tax years by firms amounts to identifying the difference in the taxable income reported to the IRS and the actual taxable income earned during the same period. In the absence of tax return information for firms, neither is observable. In lieu of this data, this paper measures the extent of income deferral activity using information from firms' balance sheets. Use of financial statement information to learn about behavioral responses to taxation is common in the accounting and earnings management literature. Guenther(1994) tests whether larger firms and highly leveraged firms were less likely to defer income in response to TRA of 1986 by looking at accrued receivables and payables accounts, taking the approach that firms reduce their financial statement income to take advantage of tax savings opportunities and finds that larger firms and highly leveraged firms behave as predicted. However, the paper finds that firms with June year ends were no more likely to reduce income than other firms in the panel.

Unlike Guenther, this paper uses annual changes in each firm's deferred tax liability account as the measure of the difference between actual income and income reported to the IRS for a given year[5]. Guidelines established by the Generally Accepted Accounting Principles, specifically Financial Accounting Standards Board Publication 109 and its predecessors, provide that any income included in the firm's income statement which is not reported for income tax purpose with the knowledge that it will be taxed in some future period must be accounted for in the firm's balance sheet. For example, if a firm generates an income item of \$100 in Yr 1 and recognizes such income on its financial statements but not its income tax return for year 1, a deferred tax liability of \$100 x the statutory tax rate in Yr 1 must be recorded on its balance sheet.

[5]The use of deferred tax liability is not unique to this paper. See Phillips, Pincus, and Rego (2003)

A major complication to the measurement issue is that activities which are undertaken by all firms in the ordinary course of business generate deferred tax liability entries. The most common of these activities is the purchase and depreciation of assets. Due to the differences in rules, firms generate different depreciation expense amounts for financial statement and income tax purposes. Specifically, while firms are restricted to using straight line depreciation on their financial statements, they are allowed to use accelerated depreciation methods in arriving at taxable income. This temporary difference[6] in book versus tax income must be accounted for with a deferred tax liability entry. Therefore, two firms which are identical in terms of tax avoidance behavior may have different deferred tax liability accounts due to differences in their asset holdings. A simple example illustrates the point.

Assume that Firms A and B purchase a \$100 asset and a \$200 asset, respectively, in Year 1. Assuming they both face a 40% statutory tax rate and use double declining depreciation for income tax purposes, Firm A will generate deferred tax liability of \$20 in Year 1 while Firm B will generate deferred tax liability of \$40 in Year 1. Therefore, the paper cannot naively assume that any differential in deferred tax liability across firms is a signal of tax avoidance behavior. In Section 2 the paper discusses in detail the complex relationship in a given year between a firm's asset position, its capital acquisition activity and its deferred tax liability account. The measurement problem is somewhat mitigated by looking at firms with increasing deferred tax liability and net property, plant & equipment as empirically the deferred tax liability accounts of those firms is better explained by the control variables used in the estimations. In addition, the paper assumes that conditional on the firm's asset holdings and capital expenditure, any variation in the deferred tax liability is not correlated with its fiscal year end, which is what determines its statutory tax rate for the relevant years.

A second complication is discussed in Section 3, where a simple model of firm behavior illustrates the possibility of observing a larger shift of income from one period to the next by a firm with a relatively smaller anticipated tax rate reduction. Finally, a third complication involves the other

[6]This difference is temporary in that ultimately the sum of annual depreciation deductions for a given asset through the useful life of the asset will be the same for book and tax purposes. Tax depreciation will be greater than book depreciation in the early period of the asset's useful life, while the opposite will be true in the latter years

provisions of the Act. Specifically, the Act enacted substantive changes to tax depreciation rules, effective for assets placed in service on or after January 1, 1987, effecting firms with different year ends differentially.

Given the latter two issues, the paper restricts the analysis to the 1985 tax year as the source of identification, the year in which a subset of firms, those with July through November year ends faced some reduction in the statutory tax rate from their 1985 to 1986 tax years, while the other firms in the panel faced no reduction. The model would predict that income deferral activity should only be observed for those firms in the 1985 tax year. Additionally, the new depreciation provisions had not come into effect, implying that all firms faced the same depreciation rules during the entirety of the tax year.

The paper proceeds as follows: Section 2 presents some details of the Tax Reform Act of 86 and discusses the existing literature on the effects of the Act on tax planning. Section 3 presents a formal model of the firm's problem and highlights an empirical hurdle. Section 4 discusses the deferred tax liability account and identifies the complexity in parsing out liability account items related to depreciation and those that are otherwise recorded. Section 5 presents the empirical results and Section 6 concludes.

3.2 Institutional Background and Previous Literature

A brief history of the Tax Relief Act of 1986 and its provisions precedes a discussion of the choice of 1985 (as opposed to all years in which there was exogenous variation in tax rates) as the year in which income avoidance activity in measured in the paper. The genesis of the Tax Relief Act of 1986 is the Treasury Department's report to the President in November of 1984. In May of 1985, President Reagan submitted a tax reform proposal to Congress based on the Treasury Department Report. The Ways and Means Committee filed a report in December of 1985 and subsequently the House of Representatives passed H.R. 3838 on December 17, 1985. The Senate Committee on Finance then filed a report in favor of H.R. 3838 and the Senate ultimately voted in favor of the bill on June 24, 1986. The Tax Relief Act was signed into law by President Reagan on October 22, 1986.

It is unclear when firms initially reacted to the reforms proposed and later adopted by H.R. 3838. Cutler (1998) studied the effect of news of President Reagan's proposal to congress and H.R. 3838 on the stock market. Specifically, he tested for market reaction from May 5 to May 12 and again from December 16 to December 23, dates around which the President submitted his report and the House of Representatives ultimately passed the bill. Cutler found limited evidence of a stock market reaction to the tax news. While this can hardly be interpreted as evidence of an absence of anticipatory behavior on the part of firms, it also does not provide a compelling reason to believe that firms reacted to the provisions of the Act prior to its ultimate adoption in October 1986. Furthermore, there do not appear to be any empirical studies which test for firms' reaction to news preceding the adoption of the Act. In either case, Table 1 shows that all firms in with reduced tax rates in 1986 had 10-K filing dates after October of 1986. Additionally, since most firms extend their tax filing deadlines, tax returns for the 1985 tax year were also due after the bill passed.

The provisions of the Tax Relief Act which are relevant to this paper are of course changes in the statutory corporate tax rates and changes to asset depreciation rules to the extent that such rules affect book-tax differences and as such deferred tax liability accounts. While the Act enacted numerous changes to the tax code, the paper assumes throughout that changes other than the tax rate and depreciation rules did not affect firms with different fiscal year ends differentially. Since the empirical strategy is to measure the variation in activity as a function of firm year ends, other changes are assumed to be neutral for the control and treatment groups. This assumption does not hold, however, for changes in depreciation rules. Depreciation rules for assets placed in service as of January 1, 1987 differed from those placed in service on or before December 31, 1986[7]. The act re-classified the useful life of types of assets and added categories for 7 and 20 year assets. In essence, The Act curtailed the depreciation expense deduction for firms.

The implication of the changes in deduction rules is best demonstrated by the following example. Suppose that Firm A (a July year end firm facing an 11% reduction in tax rate from 1986 to 1987)

[7]Joint Committee on Taxation, *General Explanation of The Tax Reform Act of 1986* (1987), 89-110

and Firm B (a December year end firm facing a 6% reduction) are identical in every respect at the end of their respective 1985 fiscal years. Firm A purchases an asset on January 1, 1987, which generates a $x temporary book tax difference on its 1986 financial statement. Firm A also decides to defer income from 1986 to 1987, generating an additional $y in deferred tax liability. Firm B on the other hand does not engage in income deferral activity, but places the same asset into service prior to January 1, 1987, which because of the old depreciation rules generates $(x+y) of temporary book tax difference. Observationally, these two firms would look identical to the econometrician despite income deferral activity by one firm and not the other.

The relationship of statutory corporate income tax rates and earnings management has been a popular subject in the accounting literature. Two papers that exploit the unique tax rate assignment of the Act of 1986 are discussed. Scholes, Wilson and Wolfson (1992) used the statutory tax variation generated by The Act of 1986 to identify evidence of earnings management by firms. In the paper, they sought evidence of income shifting across tax years by way of delay of delivery of goods and services, structuring of installment sales, and other vehicles that would delay income recognition. The found that firms in fact changed their mode of business to take advantage of declining tax rates in future periods, despite the non-tax costs of doing so. The paper took advantage of the differential tax benefits of firms with different fiscal year ends to measure the extent of the activity, as does this paper.

Maydew (1994) looks at the effects of the same statutory tax variation to test whether firms with different fiscal year ends differentially allocated net operating loss to pre-1986 tax years to reduce overall tax liability across years. The paper finds that firms that faced lower tax rates in a given period accelerated expenses and delayed gross margin, both meant to increase net operating losses in years where the firm was subject to a higher tax rate, resulting in a larger tax refund.

This paper takes the approach used in Phillips, Pincus and Rego (2003). Ignoring the mechanisms/technology used by firms to reduce overall tax liability given the business activity of the firm, it investigates the extent to which they report different measures of net income on their financial statements and tax returns, as reflected in the deferred tax liability account.

3.3 A Model Of Firm Behavior

In the following model of the firm's problem, a firm generates income in year 1 and has the option of reporting all of its income in year 1 or deferring some portion of the income to the following tax year.[8] It's objective is to minimize the total cost of tax payments plus any costs associated with deferral of income. It earns income W in period 1 and nothing in period 2. The firm chooses to report income w_i in period $i = 1, 2$. Income is taxed at a flat rate of τ_i in period $i = 1, 2$. It faces both a marginal cost of $MC(w_2) > 0$ and a fixed cost $FC = \Lambda > 0$ whenever it decides not to report all of its income in period 1.

The function $MC(w_2)$ contemplates the non-tax costs of deferring income, costs such as the probability of an audit and the resulting potential underreporting penalties, whereas the function Λ captures the technology cost of deferred income, that is, the cost of tax planning, which may involve fees to third parties for analysis and documentation of the income deferral strategy, but is independent of the amount of income deferred.

The firm's minimization problem is formalized in Equation (1):

$$\underset{\bar{w_2}}{\text{Min}} \left\{ \tau_1 W, \tau_1 w_1 + \tau_2 w_2 + MC(w_2) + \Lambda \right\}, \quad s.t. \tag{3.1}$$

The firm chooses $w_2 > 0$ whenever

$$\begin{aligned} \tau_1 W &\geq \tau_1 w_1 + \tau_2 w_2 + MC(w_2) + \Lambda \\ \tau_1 w_1 + \tau_1 w_2 &\geq \tau_1 w_1 + \tau_2 w_2 + MC(w_2) + \Lambda \\ (\tau_1 - \tau_2) w_2 &\geq MC(w_2) + \Lambda \end{aligned}$$

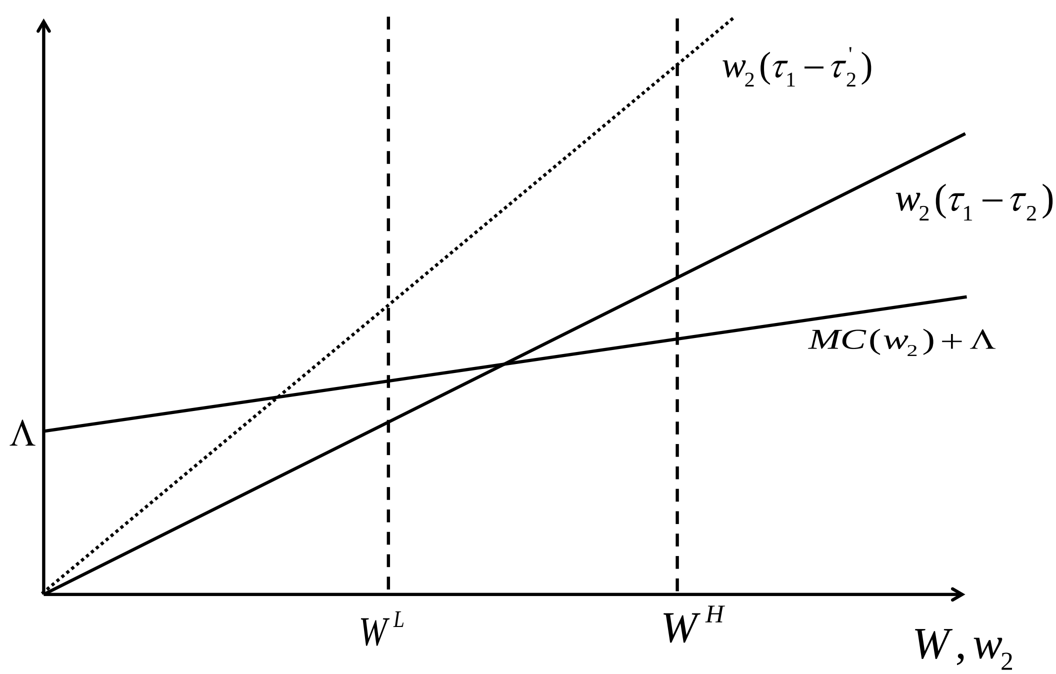
The last condition states that firms will defer income, i.e., choose $w_2 > 0$ as a function of the rate differential between periods 1 and 2, to the extent the total reduction in tax $(\tau_1 - \tau_2)w_2$ exceeds the non-tax cost of doing so. Assuming that $MC'(w_2) = \beta$, Figure 3.1 below illustrates the firm's problem, also highlighting an empirical hurdle faced by the paper.

[8] In reality, firms may have the option to defer income for more than one period. However, for purposes of the identification strategy employed in the paper, the assumption is that firms only have the technology to defer income by one period.

Consider two firms which face identical income deferral cost functions, where $MC(w_2) = \beta w_2$. The slope of the cost line is β. Firm 1 faced a reduction in tax rate from period 1 to period 2 of $(\tau_1 - \tau_2)$ and Firm 2 faces a smaller reduction, $(\tau_1 - \tau_2^{'})$, where $\beta < (\tau_1 - \tau_2^{'}) < (\tau_1 - \tau_2)$. This ensures that there exists some income level for which each firms finds it profitable to defer income to period 2.

Figure 3.1 shows that conditional on a random level of income W, Firm 1 is more likely to defer income. For example, for income level W^L, represented by the vertical line in Figure 3.1, Firm 1 will defer income while Firm 2 will find it too costly. However, the reader will note that at income level W^H, each firm will defer income.

Figure 3.1: Firm's Cost Minimization Problem



Suppose now that Firm 1 has income of W^L and Firm 2 has income of W^H. If the econometrician observes only income deferred in Year 1, τ_1,τ_2, and $\tau_2^{'}$, it would appear that the firm with the smaller anticipated drop in statutory tax engages in greater income deferral activity.

To ensure proper identification of the effect of a drop in statutory tax rate, this paper looks at the behavior of firms for the 1985 tax year, where only a subset of firms faced a non-zero anticipated change in statutory tax rate.

3.4 Measurement of Income Deferral

Given the absence of information on reportable and reported taxable income for each firm, identifying actual income deferred for a given tax period is a major econometric challenge. To overcome this challenge, the paper uses the deferred tax liability account reported on firms' balance sheets.

The Deferred Tax Liability Account is a balance sheet item which tracks temporary differences between book (or financial) income and tax income. Temporary differences arise where the tax treatment of an item is temporarily different from its financial accounting treatment. The major component of firms' deferred tax liability account for a given fiscal year is the difference in sum of the book-tax basis differences for each of the assets the firm holds at the end of the fiscal year. Because tax law allows for accelerated depreciation of assets than financial accounting guidelines, the depreciation deduction for an asset is greater for tax purposes early in the life of the asset and greater for book purposes later in its life. The difference in basis multiplied by the prevailing statutory tax rate must be accounted for through the deferred tax liability account.

Before, discussing the various measures used in the paper, it is important to look at the relationship of deferred tax liability to the firm's asset position and asset acquisitions in a given year. The relationship is not a simple one, as the example in Figure 3.2 illustrates.

The firm in the example 1 purchases assets, ranging from \$100 to \$200 in fair market value, in years 1 to 5. Each asset has a useful life of 5 years. The asset is depreciated using straight-line depreciation for financial statement purposes and double-declining (the choice of accelerated depreciation method is irrelevant to the point of the example) depreciation for tax purposes. An examination of the relationship between capital acquisition (CE), property, plant & equipment (either gross or net), and deferred tax liability highlights the empirical issue. In Year 2, the firm's capital acquisition, property, plant & equipment and deferred tax liability all increase relative to

Year 1 as the firm's asset holdings represent 2 assets which are both in the first half of their lives. In Year 4, however, while capital acquisition and property, plant and equipment increase relative to Year 3, deferred tax liability decreases. This is a function of the age of the mix of assets held by the firm in Year 4.

Example 2 illustrates the effects of longer lived assets on the relationship between these variables. The reader should note the smaller variation in the ratio of deferred tax liability to net property, plant & equipment in the second example. A distinguishing feature of the years in which variation is smallest is that as both net PP&E are increasing, the variation tends to be smaller. Of course, this is an artifact of the numerical example, therefore the paper test the empirical relationships within the sample of firms in the panel.

Table 3.2 summarizes the results of estimations of the following two equations for the panel of firms. 68% of the firms in the panel have positive changes in both deferred tax liability and net PP&E during the 1985 tax year, and the results of the estimations show that change in net PP&E, capital expenditure and lag of capital expenditure explain a large majority of the variation in change in deferred tax liability.

$$\Delta DTL_{1985} = \beta_0 + \beta_1 \Delta PPEN_{1985} + \beta_2 CE_{1985} + \beta_3 CE_{1984} + \epsilon \quad (3.2)$$

$$\Delta DTL_{1984} = \beta_0 + \beta_1 \Delta PPEN_{1984} + \beta_2 CE_{1984} + \beta_3 CE_{1983} + \epsilon, \quad (3.3)$$

where DTL is deferred tax liability, PPEN is net property, plant & equipment, and CE is capital expenditure. Columns (1) and (2) estimates the parameters of Equations (2) and (3) respectively for all the firms in the panel. The reader should note the F-statistics of the two regressions.

3.5 Empirical Analysis

3.5.1 Summary Statistics By Fiscal Year End

The dataset contains financial statement information of 555 publicly traded North American firms for the 1983 through 1985 fiscal years[9], as reported on each firm's annual 10-K forms, as filed with the

[9]The original panel consisted of information through the 1988 tax year. However, due to the identification issues discussed above, only 1983 through 1985 data were used

Figure 3.2: Example of Temporary Differences In Book v Tax Depreciation

Example 1: Assets with 5-Year Useful Life

Book

Asset	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9
100	20	20	20	20	20				
120		24	24	24	24	24			
140			28	28	28	28	28		
160				32	32	32	32	32	
200					40	40	40	40	40
CE	100	120	140	160	200	0	0	0	0
Depriciation	20	44	72	104	144	124	100	72	40
Gross Prop, Plant & Equipment	100	220	360	520	720	620	500	360	200
Net Prop, Plant & Equipment	80	176	288	416	576	496	400	288	160

Tax

Asset	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9
100	40	24	14.4	10.8	10.8				
120		48	28.8	17.28	12.96	12.96			
140			56	33.6	20.16	15.12	15.12		
160				64	38.4	23.04	17.28	17.28	
200					80	48	28.8	21.6	21.6
Depreciation	40	72	99.2	125.68	162.32	99.12	61.2	38.88	21.6
Deferred Tax Liability	20	28	27.2	21.68	18.32	-24.88	-38.8	-33.12	-18.4
Ratio of DTL to NetPP&E	0.25	0.1591	0.0944	0.0521	0.0318	-0.0502	-0.097	-0.115	-0.115

Example 2: Assets with 20-Year Useful Life

Book

Asset	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9
100	5	5	5	5	5	5	5	5	5
120		6	6	6	6	6	6	6	6
140			7	7	7	7	7	7	7
160				8	8	8	8	8	8
200					10	10	10	10	10
CE	100	120	140	160	200	0	0	0	0
Depriciation	5	11	18	26	36	36	36	36	36
Gross Prop, Plant & Equipment	100	220	360	520	720	720	720	720	720
Net Prop, Plant & Equipment	95	209	342	494	684	684	684	684	684

Tax

Asset	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9
100	10	9	8.1	7.29	6.561	5.9049	5.3144	4.783	4.3047
120		12	10.8	9.72	8.748	7.8732	7.0859	6.3773	5.7396
140			14	12.6	11.34	10.206	9.1854	8.2669	7.4402
160				16	14.4	12.96	11.664	10.498	9.4478
200					20	18	16.2	14.58	13.122
Depreciation	10	21	32.9	45.61	61.049	54.944	49.45	44.505	40.054
Deferred Tax Liability	5	10	14.9	19.61	25.049	18.944	13.45	8.5047	4.0542
Ratio of DTL to NetPP&E	0.0526	0.0478	0.0436	0.0397	0.0366	0.0277	0.0197	0.0124	0.0059

Securities Exchange Commission. The data includes all income statement and balance sheet items. While COMPUSTAT includes footnote information from financial statements for later years, for the years in the panel, footnotes are not available. Additionally, while field do exist for accumulated depreciation and property plant and equipment figures for various types of assets (machinery and equipment versus land, etc), for the years at issue actual data is quite sparse (and almost non-existent for accumulated depreciation). As such, each firm's exact asset mix is unavailable.

In constructing the data set, firms which changed their fiscal year ends during the 1983 through 1985 years were dropped from the panel, as were firms for whom deferred tax liability, PP&E and capital expenditure figures were coded as zero for either 1984 or 1985, under the assumption that either the firm ceased doing business or there was a coding error.

Given the theoretical prediction that firms facing that face a decrease in statutory tax rate from year t to year $t+1$ are more likely to defer income to year $t+1$, the paper looks at summaries of the variables of interest for 1985 by fiscal year end.

Table 3.3 shows absolute the number of observations, mean DTL_{1985}, mean $DTL_{1984}/Sales_{1984}$, mean $DTL_{1985}/Sales_{1985}$, and the percent change in the ratio of DTL to $Sales$ from 1984 to 1985. The firms are ordered according to the difference in statutory tax rate faced from 1985 to 1986 tax years.

The summary statistics do not tell a compelling story, but this may be due to the fact that deferred tax liability, without controlling for a firm's asset position, does not provide any sort of measure of income deferral. The last column, which reports the percent change in the ratio of DTL to sales from 1984 to 1985 seems to suggest that firms that faced a bigger reduction from 1985 to 1986 may have behaved differently than their control group counterparts.

3.5.2 OLS Estimations of Income Deferral

In this section, the paper uses ordinary least squares estimation to estimate the following equations for the various measures of income deferral activity:

$$\Delta DTL_{1985} = \beta_0 + \beta_1(\tau_{85} - \tau_{86}) + \beta X + \mu \tag{3.4}$$

$$\Delta DTL_{1985} = \beta_0 + \beta_1 Treatment + \beta X + \mu, \tag{3.5}$$

where $Treatment$ is a dummy variable equal to 1 for firms with fiscal year ends ending in July through November, i.e., firms that faced a positive tax rate decrease from their 1985 to their 1986 tax years. X is a vector of controls which include change in net property plant and equipment, capital expenditure, lag of capital expenditure, and operating income before depreciation and taxes.

The results are summarized in Table 3.4, where each column reports the estimates for the two equations, respectively.

Examination of the OLS estimation results indicate that there was no differential effect on deferred tax liability as a function of the anticipated tax rate reduction from 1985 to 1986 amongst the firms in the panel. The coefficients of interest are statistically insignificant at the 90% confidence level and the signs suggest some evidence that in fact firms that faced a tax reduction deferred less income conditional on the temporary book versus tax depreciation differences and operating income.

A plausible explanation for the absence of a finding is that firms that faced a decrease in statutory tax rates decreased both book and tax income in the 1985 year. If so, operating income, conditional on sales and cost of goods sold should be lower for such firms. The paper estimates the following equations for evidence:

$$OIBD = \delta_0 + \delta_1 Treatment + \delta_2 Sales + \delta_3 COGS + \mu \tag{3.6}$$

$$OIBD = \delta_0 + \delta_1(\tau_{1985} - \tau_{1986}) + \delta_2 Sales + \delta_3 COGS + \mu, \tag{3.7}$$

where once again $Treatment$ is a dummy variable for firms with July through November fiscal year ends, and $OIBD$ is operating income before depreciation expenses. The results are summarized in Table 6, indicating the absence of a systematic reduction in book income reporting by firms in the treatment group.

The null hypothesis that firms facing reductions in statutory tax rates in 1985 did not systematically report less book income cannot be rejected on the basis of the estimation.

A second plausible explanation for the absence of any evidence of income deferral is that since Equations (4) and (5) measure income deferral in absolute terms, firm size heterogeneity within the sample is confounding the results. Even if all firms defer an equal fraction of income annually, larger firms would have higher deferred tax liability in absolute terms, thereby driving the results of the estimation of those equations. If large firms systematically tend to be in the control group of firms that did not face a reduction in rate from 1985 to 1986 and simultaneously more likely to defer income due to the costs associated with tax planning, the OLS estimations of Equations (3) and (4) would be biased.

One method to mitigate this effect would be to measure income deferral activity by the percentage change in deferred tax liability from 1984 to 1985. However, this would naturally lead to the possibility that smaller firms may drive the results. Once again, if firm size is correlated with fiscal year end and smaller firms tend not to defer as much income (due to fixed costs) per dollar of income, the results would be biased.

The paper mitigates the effects of firm size on the measure of interest by weighting equations (4) and (5) with a measure of firm size by estimating the following equations:

$$\Delta DTL_{1985} = \beta_0 Sales + \beta_1 Sales * (\tau_{85} - \tau_{86}) + \beta Sales * X + \mu \tag{3.8}$$

$$\Delta DTL_{1985} = \beta_0 Sales + \beta_1 Sales * Treatment + \beta Sales * X + \mu, \tag{3.9}$$

where $Sales$ serves as a proxy for firm size. The estimation results are reported in Table 3.7. Column (1) of Table 3.7 suggests that firms on average increase deferred tax liability by roughly cents more for every dollar of revenue for a 1 percent decrease in tax rate in the subsequent year. Column (2) provides similar evidence, suggesting that firms that faced some decrease in statutory tax rate from 1985 to 1986 increased deferred tax liability by 1.4 cents more per 10 dollar of revenue than firms that did not face any decreases.[10]. The reader will note however, that the coefficients of interest are

[10]These estimates are sensible given that the average firm in the treatment group faced a tax rate decrease of 2.9 percent

not significant even at the 0.1 level.

3.5.3 Fixed Effects Estimation

Since the panel data includes information for firms from 1983 through 1985, the paper looks at the behavior of within firm variation in deferred tax liability as a function of controls and the variable of interest, anticipated tax rate decrease (which is zero for the 1984 tax year for all firms) for period from 1983 to 1984 versus 1984 to 1985. This would eliminate the possibility that the OLS regressions are biased due to correlation of firm asset positions and fiscal year end, possibly because firms within similar industries have similar assets. The equation that is estimated is:

$$DTL_{it} - D\bar{T}L_i = \beta(X_{it} - \bar{X}_i) + (\eta_{it} - \bar{\eta_i}), \tag{3.10}$$

where X is net PP&E, capital expenditure, operating income before depreciation, and anticipated tax rate reduction. The results are reported in Table 3.8. As with the OLS models of the previous section, there is no evidence that firms that an anticipated reduction in the statutory income tax rate induced firms to report different income figures for book versus tax purposes.

3.6 Conclusion

Using financial statement panel data for the 1983 through 1985 tax years for 555 firms, this paper tests whether firms engage in income deferral for tax purposes while truthfully reporting income on their financial statements. Using each firm's deferred tax liability account, which captures temporary differences in book versus tax depreciation and other income deferral items, as a measure of income deferral activity, the paper compares the level of such activity between firms that faced a reduction in the statutory corporate income tax rate from the 1985 to 1986 fiscal tax year. Ordinary least squares, quantile, and fixed effects estimations of absolute measure of deferred tax liability provide little evidence of additional income deferral activity amongst July through November year end firms, suggesting that firms face significant costs of intertemporal income shifting, though it is possible that the activity is not captured by the identification strategy due to the confounding factors discussed throughout. Alternative estimation of income deferral as a fraction of sales offers somewhat stronger evidence of income deferral: these results imply that a one percent decrease in the statutory tax rate results in approximately four cents of additional deferred tax liability per dollar of revenue. The accompanying standard error on this estimate is, however, sufficiently large as to make the coefficient estimate statistically insignificant. Hence while there is some suggestion in the data that firms responded to the anticipated 1986 tax reform by shifting taxable income across time, it is impossible to reject that there was no effect that was common across firms.

Table 3.1: 1986, 1987, and 1988 Tax Rates By Fiscal Year End

1986 Tax Year	1986 Tax Rate	1987 Tax Rate	1988 Tax Rate	1985 10-K Filing Deadline
January 1, 1986 to December 31, 1986	46	40	34	April 30, 1986
February 1, 1986 to January 31, 1987	46	39	34	May 31, 1986
March 1, 1986 to February 29, 1987	46	38	34	June 30, 1986
April 1, 1986 to March 31, 1987	46	37	34	July 31, 1986
May 1, 1986 to April 30, 1987	46	36	34	August 31, 1986
June 1, 1986 to May 31, 1987	46	35	34	September 30, 1986
July 1, 1986 to June 30, 1987	46	34	34	October 31, 1986
August 1, 1986 to July 31, 1987	45	34	34	November 31, 1986
September 1, 1986 to August 31, 1987	44	34	34	December 31, 1986
October 1, 1986 to September 31, 1987	43	34	34	January 31, 1987
November 1, 1986 to October 30, 1987	42	34	34	February 29, 1987
December 1, 1986 to November 30, 1987	41	34	34	March 31, 1987

Table 3.2: Estimate of Book-Tax Depreciation Difference

	(1) DTL_DeprDiff_85	(2) DTL_DeprDiff_84
Delta_PPEN	0.0817***	0.0222**
	(0.0015)	(0.0073)
CE	-0.0247***	-0.0254*
	(0.0054)	(0.0116)
Lag_CE	0.0417***	0.0987***
	(0.0061)	(0.0124)
_cons	1.6282**	0.9169
	(0.5315)	(0.4813)
N	555	502
r2	0.9123	0.3649
F	1911.0240	95.3879

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.3: Summary Statistics For 1985 Tax Year By Fiscal Year End

Fiscal Year End	Rate Reduction	Observations	Mean DTL	1984 Mean $\frac{DTL}{Sales}$	1985 mean $\frac{DTL}{Sales}$	% Change
November	5%	12	12.46	0.0350	0.0383	9.43
October	4%	20	27.62	0.0229	0.0290	26.64
September	3%	45	33.52	0.0337	0.0339	0.59
August	2%	16	9.16	0.0209	0.0232	11.00
July	1%	18	13.54	0.0199	0.0236	18.59
June	0%	57	19.56	0.0266	0.0296	11.28
May	0%	20	23.44	0.0278	0.0294	5.75
April	0%	19	18.42	0.0245	0.0297	21.22
March	0%	22	27.53	0.0241	0.0316	31.12
February	0%	16	20.64	0.0183	0.0222	21.31
January	0%	38	18.06	0.0168	0.0187	11.31
December	0%	274	81.65	0.0430	0.0475	10.46

Table 3.4: OLS Estimation of Effect of Anticipated Tax Rate Decrease on DTL

	(1) OLS_RateDrop	(2) OLS_Treatment
Delta_PPEN	0.0551***	0.0551***
	(0.0038)	(0.0038)
CE	-0.0820***	-0.0820***
	(0.0112)	(0.0112)
Lag_CE	0.0220*	0.0220*
	(0.0111)	(0.0111)
Operating_Income	0.0617***	0.0618***
	(0.0077)	(0.0077)
Ant_Rate_Drop	0.0119	
	(0.2908)	
Jul_Nov_Dummy		0.1727
		(0.9673)
_cons	0.1625	0.1327
	(0.4050)	(0.4192)
N	555	555
r2	0.9431	0.9431
F	1960.6174	1959.4251

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.5: Quantile Regression Estimations of Effect of Anticipated Tax Rate Decrease on DTL

	(1) Quant_RateDrop	(2) Quant_Treatment
Delta_PPEN	0.0556***	0.0556***
	(0.0011)	(0.0011)
CE	-0.0734***	-0.0734***
	(0.0032)	(0.0032)
Lag_CE	0.0286***	0.0286***
	(0.0029)	(0.0029)
Operating_Income	0.0552***	0.0552***
	(0.0021)	(0.0021)
Ant_Rate_Drop	-0.0260	
	(0.1821)	
Jul_Nov_Dummy		-0.0829
		(0.5841)
_cons	-0.0052	-0.0029
	(0.2784)	(0.2828)
N	555	555
r2		
F		

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.6: OLS Estimation of Book Income Variation by Treatment

	(1) OLS_RateDrop	(2) OLS_Treatment
sales	0.2553***	0.2554***
	(0.0577)	(0.0578)
cogs	-0.1907**	-0.1908**
	(0.0633)	(0.0634)
Ant_Rate_Drop	0.9994	
	(2.2344)	
Jul_Nov_Dummy		2.9921
		(8.2631)
_cons	-10.6342	-10.6534
	(17.9824)	(18.4151)
N	555	555
r2	0.8308	0.8308
F	20.8399	24.9859

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.7: OLS Estimation Controlling for Firm Size

	(1) Anticipated Rate Drop	(2) Treatment
sales	0.0025***	0.0024***
	(0.0005)	(0.0005)
Sales_ARD	0.0369	
	(0.0361)	
Sales_Cap	-0.0000	-0.0000
	(0.0000)	(0.0000)
Sales_DeltaPPEN	0.0000	0.0000
	(0.0000)	(0.0000)
Sales_NOI	0.0000	0.0000
	(0.0000)	(0.0000)
Sales_Treat		0.0014
		(0.0012)
_cons	1.3983**	1.3735**
	(0.4430)	(0.4400)
N	555	555
r2	0.9083	0.9085
F	8042.9960	7989.2328

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3.8: Fixed Effects Estimation of Effect of Anticipated Tax Rate Decrease on DTL

	(1)
	FE_RateDrop
PPEN	0.1315***
	(0.0083)
CE	-0.1309***
	(0.0108)
Operating_Income	0.0786***
	(0.0113)
Ant_Rate_Drop	0.2056
	(0.8857)
_cons	-6.1624
	(3.5007)
N	1680
r2	0.2615
F	95.5356

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

BIBLIOGRAPHY

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[1] (????): U.S. Department of Education, Office of Planning, Evaluation and Policy Development. Evaluation of Evidence Based Practices in Online Learning: A Meta-Analysis and Review of Online Learning Studies, Washington, D.C., 2010.

[2] Ary, E. J. and C. W. Brune (2011): "A Comparison of Student Learning Outcomes in Traditional and Online Personal Finance Courses," *MERLOT Journal of Online Learning and Teaching*, 7(4), 465–474.

[3] Ben-Shahar, O. and O. Bar-Gill (2003): "The Uneasy Case for Comparative Negligence," *American Law and Economics Review*, 5, 433–469.

[4] Bernard, R., P. Abrami, Y. Lou, E. Borokhovski, C. Wade, L. Wozney, P. Wallet, M. Fiset, and B. Huang (2009): "How Does Distance Learning Compare to Classroom Instruction?" *Review of Educational Research*, 74, 379–439.

[5] Calfee, J. and R. Craswell (1986): "Deterrence and Uncertain Legal Standards," *Journal of Law, Economics and Organization*, 2, 279–303.

[6] Coates, D. and B. Humphreys (2003): "An Inventory of Learning at a Distance in Economics," *Social Science Computer Review*, 21(2), 196–207.

[7] ——— (2003): "An Inventory of Learning at a Distance in Economics," *Social Science Computer Review*, 21(2), 196–207.

[8] ——— (2004): ""No Significant" Distance between Face-To-Face and Online Instruction: Evidence from Principles of Economics," *Economics of Education Review*, 23, 533–546.

[9] Cooter, R. and T. Ulen (1986): "An Economics Case for Comparative Negligence," *New York University Law Review*, 67, 1067–1110.

[10] Cutler, D. (1998): "Tax Reform and the Stock Market: An Asset Price Approach," *American Economic Review*, 78(5), 1107–1117.

[11] Edlin, A. (1994): "Efficient Standards of Due Care: Should Courts Find More Parties Negligent under Comparative Negligence?" *International Review of Law and Economics*, 14, 21–34.

[12] Endres, A. and I. Querner (1995): "On the Existence of Care Equilibrium under Tort Law," *Journal of Institutional and Theoretical Economics*, 151, 348–357.

[13] Gratton-Lavoie, C. and D. Stanley (2009): "Teaching and Learning Principles of Microeconomics Online: An Empirical Assessment," *Journal of Economic Education*, Winter, 3–25.

[14] Guenther, D. (1994): "Earnings Management in Response to Corporate Tax Rate Changes: Evidence from the 1986 Tax Reform Act," *The Accounting Review*, January, 230–243.

[15] Haddock, D. and C. Curran (1985): "An Economic Theory of Comparative Negligence," *Journal of Legal Studies*, 14, 49–72.

[16] Lokshin, M. and Z. Sajaia (2008): "MOVESTAY: Stata module for maximum likelihood estimation of endogenous regression switching models," .

[17] Maddala, G. (1983): *Limited-Dependent and Qualitative Variables in Econometrics*, New York: Cambridge University Press.

[18] Maydew, E. (1997): "Tax-Induced Earnings Management by Firms with Net Operating Losses," *Journal of Accounting Research*, Spring, 83–96.

[19] Phillips, J., M. Pincus, and S. Rego (2003): "Earnings Management: New Evidence Based on Deferred Tax Liability," *The Accounting Review*, 78, 491–521.

[20] Scholes, M. and M. Wolfson (1992): "Firms Responses to Anticipated Reductions in Tax Rates: The Tax Reform Act of 1986," *Journal of Accounting Research*, 30, 161–185.

[21] Shachar, M. and Y. Neumann (2010): "Twenty Years of Research on the Academic Performance Differences between Traditional and Distance Learning: Summative Meta-Analysis and Trend Examination," *MERLOT Journal of Online Learning and Teaching*, 6, 318–334.

[22] Shavell, S. (1987): *Economic Analysis of Accident Law*, Cambridge, MA: Harvard University Press.

[23] Trawick, M., S. Lile, and R. Howsen (2010): "Predicting Performance for Online Students: Is It Better to be Home Alone?" *Journal of Applied Economics and Policy*, Spring, 34–46.

[24] Varian, H. (1992): *Microeconomic Analysis*, New York City, NY: W.W. Norton & Company, Inc.

[25] Wagner, S. C., S. J. Garippo, and P. Lovaas (2011): "A Longitudinal Comparison of Online Versus Traditional Instruction," *MERLOT Journal of Online Learning and Teaching*, 7(1), 68–73.

[26] Wooldrige, J. M. (2002): *Econometric Analysis of Cross Section and Panel Data*, MIT Press.