MODEL

The model has three variables: two fast variables x and y, one slow variable z.

$$\begin{cases} \dot{x} = -y \\ \dot{y} = x^3 - \mu_2(z)x - \mu_1(z) - y(\nu(z) + x + x^2) \\ \dot{z} = -c(\sqrt{(x - x_s(z))^2 + y^2} - d^*) \end{cases}$$

The parameters that determine which bifurcation occurs in the system are $(\mu_2, -\mu_1, \nu)$

In reality, when going to spherical coordinates (R, theta, phi), only theta and phi matter and we can keep R fixed. Here the transform to spherical coordinates and the new equations of the system:

$$\begin{cases} \mu_2 = R\sin\theta\cos\phi \\ \mu_1 = -R\sin\theta\sin\phi \\ \nu = R\cos\theta \end{cases}$$

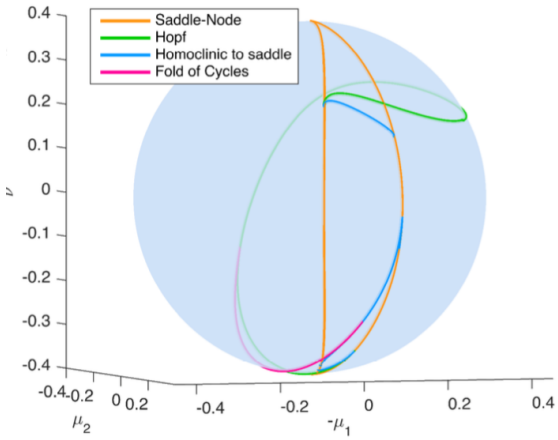
$$\begin{cases} \dot{x} = -y \\ \dot{y} = x^3 - R\sin\theta(z)\cos\phi(z)x + R\sin\theta(z)\sin\phi(z) - y(R\cos\theta(z) + x + x^2) \\ \dot{z} = -c(\sqrt{(x - x_s(z))^2 + y^2} - d^*) \end{cases}$$

We can thus plot a map of the bifurcations possible in the system, for each combination of theta and phi, on a sphere (top figure). We have Saddle-Node, Hopf, Homoclinic and Fold of Cycles.

We also have SNIC bifurcation, it is better seen in the flat cartoon representation of the map (middle figure).

Now we want the system to move back and forth between two given bifurcations. We take a point A laying on the offset bifurcation curve and a point B laying on the onset bifurcation curve. We write the equations of the arc on the sphere linking these two points and let z move the system along this path.

By changing the location of A and B on the map we can have different onset/offset pair. So A and B are the only things to be changed to change the bursting class.



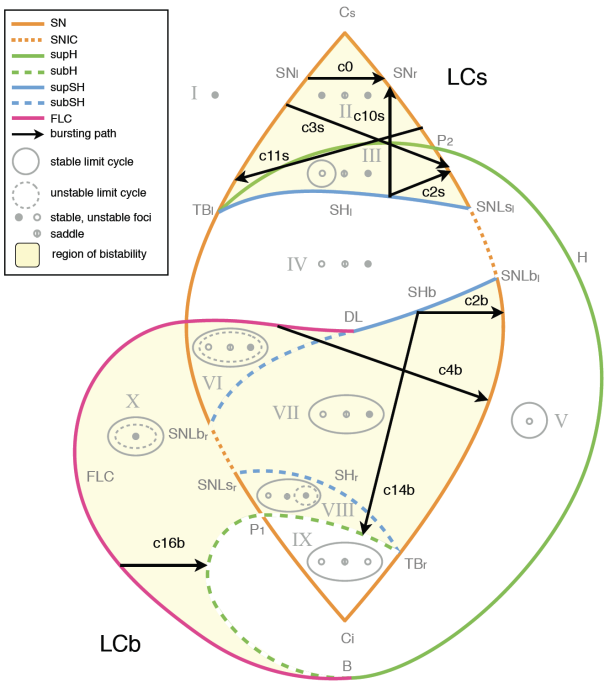


Figure from Saggio et al., 2017

offset / *onset*	**SNIC**	**SH**	**supH**	**FLC**
SN	c1 *triangular*	c2 *square-wave Type I a,b*	c3 *tapered Type V*	c4 *Type IV*
SNIC	c5 *parabolic Type II*	c6	c7	c8
supH	c9	c10	c11	c12
subH	c13	c14	c15	c16 *elliptic Type III*

In the flat cartoon map I'm showing with black arrows example for paths for several classes (the labels for the classes are in the table).

Suppose we want the saddle-node/homoclinic class. In the table you can see that the label is c2. In the cartoon map we have two types of c2: c2s and c2b. The 's' type shows baseline jump, the 'b' type doesn't. If we want c2s as in Epileptor, we have to take any point A on the blue SHl curve (homoclinic) and any point B on the orange curve (saddle-node) between P2 and SNLsl.